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The Profitability of Block Trades in Auction and Dealer Markets

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Abstract

The paper compares the trading costs for institutional investors who are subject to liquidity shocks, of trading in auction and dealer markets. The batch auction restricts the institutions' ability to exploit informational advantages because of competition between institutions when they simultaneously submit their orders. This competition lowers aggregate trading costs. In the dealership market, competition between traders is absent but trades occur in sequence so that private information is revealed by observing the ‡ow of successive orders. This information revelation reduces trading costs in aggregate. We analyse the relative e¤ects on pro…ts of competition in one system and information revelation in the other and identify the circumstances under which dealership markets have lower trading costs than auction markets and vice versa.

Keywords: Market microstructure, Auction market, Dealer markets

JEL Classi…cation: G10, G15, G19

1 Introduction

Market microstructure is concerned with the organisation of trading systems in stock markets. In this paper we examine the importance of two di¤erent types of market microstructures: an auction and a dealer market, for the pro…ts of institutional investors who are trading large blocks of securities. A number of papers [Madhavan (1992), Biais (1992), Pagano and Roell (1992, 1996), Shin (1996) and Vogler (1997)] have compared the properties of auction and dealer markets, and a further string of papers [Holmstrom and Tirole (1993), Admati, P‡eiderer and Zechner (1994), Bolton and von Thadden (1998), Pagano and Roell (1998), Maug (1998)] have examined the role of large shareholders in monitoring management, given that there will be costs to this process. The contribution of this paper is to make the comparison between auction and dealer markets from the point of view of the pro…ts of large institutional traders who are subject to liquidity shocks and who trade blocks of securities at a time.

In comparing an order-driven auction market with a quote-driven dealer market, Madhavan (1992) argues that the di¤erences in the two systems lie in the sequence of trading, which leads to di¤erences in the information provided to the players and therefore in the strategic nature of the game. In the quote-driven system competition between market makers in setting quotes ensures that price quotes are competitive, and market makers make zero profits, whereas in the order-driven system competition between dealers takes the form of competition in demand schedules. Vogler (1997) extends this model to the case of a dealer market in which dealers can trade with each other in a separate inter-dealer market. Shin (1996) points out that a distinctive feature of these two systems is the move order and consequent information available to the traders when they take there respective actions. The auction market requires that all traders take their actions simultaneously, whereas in the dealership market the price setters move …rst and the buyers(sellers) take their actions after observing the price quotes of the sellers(buyers). Biais (1993) compares price formation in fragmented and centralised markets. The di¤erence between these two regimes is that a fragmented market is by de…nition less transparent than a centralised one. Pagano and Roell (1992, 1996) compare the price formation process in four alternative market trading systems, where the transparency of the current order ‡ow de…nes the di¤erences in the trading systems.

The growth in equity ownership by institutions investors has been documented for US corporations by Demsetz and Lehn (1985), and for UK companies by Nyman and Silbertson (1978) and Leech and Leahy (1991). According to Economic Trends 1993, over seventy percent of UK equity is held by institutional investors, and more than eighty …ve per cent of equity holdings are held in blocks greater than one hundred thousand pounds in value. Interest in corporate governance has prompted a number of papers to examine the trade-o¤s for institutional investors who are able to monitor company performance on account of their large stakes. Bhide (1993) in a provocative paper, suggests that the deep liquidity of equity secondary equity markets in the US are to the detriment of the monitoring responsibilities of shareholders. He argues that because the secondary markets are so liquid, shareholders have no incentive to monitor and are able "bail out" when there are any problems in the …rm. Admati, P‡eiderer and Zechner (1994) examine the trade-o¤ between the monitoring advantages of a large shareholder and the risk sharing disadvantages of large blocks for portfolio allocation decisions. Holmstrom and Tirole (1993), Bolton and von Thadden (1998), Pagano and Roell (1998), Maug (1998) have focused on the advantages of a large shareholder in terms of the incentives that they have to monitor management, but the disadvantages of large blocks because of reduced liquidity, though Burkart et al argue that too much monitoring could impose too many constraints on managers to manage e¤ectively. .

The central problem addressed in the current paper is that large institutional shareholders face liquidity shocks, but they also possess private knowledge about the …rm's value as a direct consequence of the size of their holdings. So that when liquidity shocks force the institutions to trade with dealers, they face unfavourable prices and high trading costs. The …nancial institution would like to trade in the trading system that minimises the cost of trading, and we compare the trading costs for large institutions in batch auction and dealership markets, in terms of expected pro…ts.

The batch auction restricts the institutions' ability to exploit informational advantages because of competition between institutions when they simultaneously submit their orders. This competition lowers aggregate trading costs. In the dealership market, competition between traders is absent but trades occur in sequence so that private information is revealed by observing the ‡ow of successive orders. This information revelation reduces trading costs in aggregate. We analyse the relative e¤ects on pro…ts of competition in one system and information revelation in the other and identify the circumstances under which dealership markets have lower trading costs than auction markets and vice versa.

The layout of the paper is as follows. In Section 2 we outline a model of financial institutions (traders) and dealers. Sections 3 and 4 evaluates the institutions' aggregate profits and section 5 examines the circumstances (parameter values) under which one system dominates another in this regard. Section 6 examines the effects of changing the correlation structure of liquidity shocks and of allowing institutions to collude in the auction market and section 7 provides a summary and conclusion.

2 The Model

Our model follows the approach taken in Madhavan (1992), who compares a quote driven mechanism with competing dealers, with an order driven mechanism organised as a batch auction. In the dealer market traders trade sequentially and therefore trade independently of subsequent trades. Whereas in the auction market all trades occur at the same time, so that traders act strategically when submitting their demands. In Madhavan (1992) traders may be acting on private information or because of a realisation of asset endowments which generates portfolio hedging trades which are not information motivated. In our model there are n traders in the market trading in a security, and they also trade for two reasons. Trader i observes the true value of the security v and is able to trade on the basis of this information in the secondary market. Each trader also faces a liquidity shock u_i, which is the second motive for trading. These traders are taken to be large risk-neutral institutional investors who discover the true value of the security v which is distributed $v \sim N(\bar{v}; \sigma_v^2)$ after monitoring the company on account of their large stake. In Madhavan (1992) traders in the market maximise an exponential utility function, whereas in this paper traders utilise a different objective function which emphasises the liquidity shocks that are faced by financial institutions The institutional investors trade x_i in the secondary market, following Seppi (1992) to maximise the objective function in (1)

$$\pi_i|v; u = [v - p]x_i - \frac{\gamma}{2}(x_i - u_i)^2 \qquad i = 1; 2 ::: n \tag{1}$$

The objective function (1) shows that traders generate income for each unit of stock that they hold, by trading at price p when the true value of the security is v. However these traders face a liquidity shock u_i resulting in losses which are quadratic in the difference between their holdings of the asset x_i and the liquidity shock. The relative importance of the trading profits and the liquidity shock in the investors' objective function is controlled by the parameter φ: Clearly the higher is φ the greater is the weight placed on the liquidity shock. The advantage of the specific objective function is that we are able to obtain expressions for the expected profits to an institution from trading under the two alternative microstructure systems

The institutional investors may be thought of as insurance companies who are generating premium income outside the model. A negative liquidity shock is interpreted as an unexpected insurance cash claim which must be met by the company by either selling the security or by borrowing. Under this interpretation the quadratic term $(x_i - u_i)^2$ represents increasing marginal borrowing costs. A positive liquidity shock may be interpreted as unexpected premium income and in this case costs are incurred by failing to invest this income in equities whose return exceeds that on liquid assets. In fact these costs are more likely to be linear in $(x_i - u_i)$. However, allowing for asymmetric costs would make our model analytically intractable.. The quadratic term in (1) therefore, must be viewed as approximating actual costs.

Market makers who are the only other market participants, and set prices p are not able to infer exactly the value of the security from the trading behaviour of the institutions since these institutions also trade because of liquidity shocks, which are distributed $u_i \sim N(0; \sigma_u^2)$: Note that if market makers also observed the value v, then they would set prices equal to the true value of the security, and traders could then set their demands equal to their liquidity shock to ensure no worse than zero profits. However because market makers do not observe v directly, but infer it from the trading volumes, they set prices to reduce the adverse selection problem from informed institutions trading against them, and we shall see that this reduces the profits of the institutions.

This model is an extension of the insider trading model developed by Kyle (1985), in which market makers set prices allowing for the likelihood that the aggregate demand will reflect informed trading by insiders. An institutional difference though is that the original Kyle model is a batch auction in which a single informed trader places his order in with a batch of liquidity orders. The model considered here allows for a different market microstructure in which traders deal directly with the market maker, but the market maker is unable to identify which components of trades are liquidity motivated and which are information motivated.

2.1 Oligopoly batch auction

A number of stock markets, such as the NYSE and the Paris Bourse open their daily markets with a batch auction. In the oligopoly batch auction considered here, each institutional investor submits his order to the market maker at the same time, and the market maker announces a price that will clear the market. Aggregate trading volume is $X = \sum_{i=1}^{n} x_i$. In this oligopoly batch auction we recognise that each institutional trader knows that both their own trades and their rival's will have an impact on prices.

In order to find the equilibrium solution to this model we make the conjecture that the aggregate trading volume is a linear function of the information and the liquidity shocks, and a competitive market makers set prices as a linear function of the aggregate trading volume

$$X = \beta(v - \mu) + \sum_{i=1} \gamma_i u_i \tag{2}$$

and

$$p = \mu + \lambda X \tag{3}$$

To find the optimal trading volume of each strategic institutional trader i substitute the conjectured price function (3) into the objective function (1). The reaction function for the ith investor under the Cournot assumption that each investor's demands do not affect the demands of the rival, is given by

$$x_i = \frac{v - \mu}{2\lambda + \varphi} + \frac{\varphi u_i}{2\lambda + \varphi} - \frac{\lambda(X - x_i)}{2\lambda + \varphi} \tag{4}$$

All institutions face the same problem and since aggregate trading volume is simply the sum of the n institutions' trades, summing over i=1 to n in (4) and rearranging gives the aggregate trading volume as

$$X = \frac{n(v - \mu)}{(n+1)\lambda + \varphi} + \frac{\varphi}{(n+1)\lambda + \varphi} \sum_{i=1} u_i \tag{5}$$

which is indeed a linear function of the information and the liquidity shocks. Comparing coefficients in (5) and (2) yields

$$\beta = \frac{n}{(n+1)\lambda + \varphi}; \gamma_i = \frac{\varphi}{(n+1)\lambda + \varphi} \; [= \gamma] \tag{6}$$

Turning to the problem faced by the market maker, we assume that the market maker acts competitively and sets prices as the expectation of the terminal value of the asset v conditional on the aggregate trading volume X so that prices are

$$p = E[v \mid X = \beta(v - \mu_v) + \gamma \sum_{i=1} u_i] \tag{7}$$

To compute this expectation we need to make assumptions about the correlations between the liquidity shocks. In what follows we assume the liquidity shocks are independent. This could arise for example if the insurance market was divided into several niches each niche being identified with an independent source of risk and with a firm insuring against that risk. An assumption at the other extreme would be that the liquidity shocks are perfectly correlated i.e. identical for all institutions. This would arise if all insurance companies fully diversified their risks in a secondary market so that they were only exposed to economy-wide systematic risk. Because our institutions are assumed to be risk neutral and therefore have no incentive to diversify risks the uncorrelated shocks assumption seems more appropriate and we take this as our main case. However, we examine the effect that the assumption of identical shocks has on our results later.

Joint normality of the models' variates guarantees that $E[v|X]$ and hence p is linear in X which confirms the conjecture for prices in equation (3). Taking the liquidity shocks to be iid and using the standard formula for the conditional expectation of normal variates gives λ in (3) as

$$\lambda = \{cov(v;X)/var(X)\} = \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2 + n\gamma^2\sigma_u^2} \tag{8}$$

We now have three equations in (6) and (8) and three unknowns β, λ and γ. Solving for the unknowns we may write the conjectured coefficients as

$$\lambda = \frac{\varphi\sigma_v^2}{\varphi^2\sigma_u^2 - \sigma_v^2}; \qquad \beta = \frac{n[\varphi^2\sigma_u^2 - \sigma_v^2]}{\varphi[\varphi^2\sigma_u^2 + n\sigma_v^2]}; \qquad \gamma = \frac{[\varphi^2\sigma_u^2 - \sigma_v^2]}{\varphi^2\sigma_u^2 + n\sigma_v^2} \tag{9}$$

Note that the second order condition for maximisation of the traders' profits is that $\varphi^2\sigma_u^2 > \sigma_v^2$. This condition indicates that a minimum amount of noise trade variability is required to ensure that equilibrium exists and that β and γ are strictly positive.

Throughout this paper we assume that collusion between market traders is illegal and/or infeasible. However it is interesting to compare the solution to the model in (9) and the influence of the non-cooperative game played between the n institutional investors with the problem of the "multi-plant" monopoly investor which we discuss in the Appendix. In the non-cooperative equilibrium in (9) the n institutional investors are trading too intensively relative to the collusive outcome in A9. We discuss the effects of collusion in more detail later.

We now wish to compute the expected profits to each trader before they have observed either the value of the asset or their liquidity shock. We need to substitute their optimal trades back in to the objective (1) and then integrate over the joint distribution of v and u_i.

The optimal demands for each trader are obtained by substituting (2) into (4) and rearranging. For each trader i we have

$$x_i = \frac{\sigma_u^2\gamma^2 - \sigma_v^2}{\gamma(n\sigma_v^2 + \sigma_u^2\gamma^2)}(v - \bar{v}) + \frac{\sigma_u^2\gamma^2 - \sigma_v^2}{\sigma_u^2\gamma^2}u_i + \frac{\sigma_v^2(\sigma_v^2 - \sigma_u^2\gamma^2)}{\sigma_u^2\gamma^3(n\sigma_v^2 + \sigma_u^2\gamma^2)}\sum_{j=1} u_j \quad (10)$$

Substituting (3) and (10) into the profit function (1), taking expected values over the value of the asset and the liquidity shocks and multiplying by n (expected profits are identical for each trader) gives expected profits for the n institutional traders as

$$nE\pi_i^{auction} = -\frac{n\sigma_v^2}{2\gamma^3\sigma_u^2}\left[\frac{\gamma^4\sigma_u^4 + \gamma^2\sigma_u^2\sigma_v^2 + \sigma_v^4}{\gamma^2\sigma_u^2 + n\sigma_v^2}\right] \quad (11)$$

where by abuse of notation we have used σ_u^4 and σ_v^4 to denote $(\sigma_u^2)^2$ and $(\sigma_v^2)^2$ respectively.

Equation (11) shows that expected profits are always negative. To see why this is so note that expected profits in equation (1) have two components. The first $E(v - p)x_i$ which we call trading profits represents pure expected gains or losses to the institution from trading. The second $(\gamma/2)E(x_i - u_i)^2$ which we call liquidity cost (cost because it enters institutional profits with a minus sign) is always positive.

Lemma 1 Institutional trading profits are always zero

Proof. Trading profits may be summed over all institutional firms and expanded as follows

$$\sum_{i=1} E[(v - p)x_i] = E[(v - p)X] = \int_{-\infty}^{\infty} E[(v - p)X \mid X]\, f(X)dX$$

$$= \int_{-\infty}^{\infty} E\{(v - E[v|X])X \mid X\}\, f(X)dX \qquad (12)$$

$$= \int_{-\infty}^{\infty} \{E[v|X] - E[v|X]\}X\, f(X)dX = 0 \qquad (13)$$

Aggregate institutional trading profits are always zero and it is easy to see that this implies that each institution's trading profit is also zero. k ■

Because liquidity costs are always positive, profits of each institution are always negative. The intuition as to why trading profits are zero is because faced with the adverse selection problem of trading with informed institutions, the market maker sets "fair" prices given knowledge of the current order flow i.e. he sets prices such that expected trading profits conditional on trading volume are zero. Therefore the institutions can never offset liquidity costs with trading profits. Note that if there was no adverse selection problem [$\sigma_V^2 \to 0$], then expected profits to the institutional traders in equation (11) rise to zero. In this case the market maker knows that he does not face an informed trader, and the institutions can then trade to just offset their liquidity shocks (i.e. they can trade an amount $x_i = u_i$ at a "fair" price). In the more general case [$\sigma_V^2 > 0$], the institutions are forced to trade at a loss because they are unable to credibly commit to the market maker that they are not trading on information. Of course if they make negative expected profits in the long run the institutions would cease to exist. However in reality equities pay dividends (our security does not) and insurance companies charge premiums higher than the expected insurance claims. Neither of these two sources of income are modelled here because we wished to focus on trading costs and liquidity factors alone, but they would both presumably ensure that the profits of insurance companies were positive in the long run.

Note also the market maker in the "multi-plant" monopoly case knows that the colluding institutions are acting strategically and sets a higher mark-up which actually reduces the monopoly profits. From the Result 3 in the Appendix, it can be seen that expected profits in the collusive case are actually lower than the sum of the joint profits in the non-collusive case. The anomalous effect of competition in increasing institutional profits is an important feature of oligopoly batch auctions. It is important to bear this effect in mind when we compare this case with that of the sequential dealership which where serial monopoly exists and such competitive effects on profits are absent.

2.2 Sequential dealer market

In the sequential dealer market each investor trades separately with the market maker, and therefore the market maker may offer different prices to the two investors. The investors approach the market maker sequentially, and the market maker completes a trade with the first investor before dealing with the second. Dealer markets are to be found in less-liquid stocks on the London Stock Exchange, on the foreign exchange markets and NASDAQ. As before, the first investor maximises (1), but this time we conjecture that the trading volume of the individual investor is a linear function of the information, and the market maker sets price as a linear function of the individual investor's trading volume

$$x_1 = \beta_1(v - \bar{v}) + \gamma_1 u_1 \tag{14}$$

and

$$p_1 = \bar{v} + \lambda_1 x_1 \tag{15}$$

The first investor now acts as a monopolist and therefore does not have to worry about the effect of his rival's trading volume on prices. The optimal trading volume for the first investor is

$$x_1 = \frac{v - \bar{v}}{2\lambda_1 + \varphi} + \frac{\varphi u_1}{2\lambda_1 + \varphi} \tag{16}$$

Market makers act competitively and set prices to the first investor as the expectation of the terminal value of the asset v conditional on the first investor's trading volume x_1. Under this assumption λ_1 is analogous to the λ of the previous section and is given as $\lambda_1 = cov(x_1; v)/var(x_1)$

Equating coefficients as before yields the three coefficients

$$\lambda_1 = \frac{\varphi\sigma_v^2}{\varphi^2\sigma_u^2 - \sigma_v^2}; \qquad \beta_1 = \frac{[\varphi^2\sigma_u^2 - \sigma_v^2]}{\varphi[\varphi^2\sigma_u^2 + \sigma_v^2]}; \qquad \gamma_1 = \frac{[\varphi^2\sigma_u^2 - \sigma_v^2]}{\varphi^2\sigma_u^2 + \sigma_v^2} \tag{17}$$

We again want to obtain an expression for expected profits for the trader. Using (16) and the coefficients in (17), we may write the optimal trades of the first investor as

$$x_1 = \frac{\varphi^2\sigma_u^2 - \sigma_v^2}{\varphi(\varphi^2\sigma_u^2 + \sigma_v^2)}[v - \mu + \varphi u_1] \tag{18}$$

The equilibrium price faced by the first investor is obtained by substituting (18) into the conjectured pricing rule (15)

$$p_1 = \mu + \frac{\sigma_v^2}{(\varphi^2\sigma_u^2 + \sigma_v^2)}[v - \mu + \varphi u_1] \tag{19}$$

Substituting (18) and (19) into 1) and taking expected values, we obtain the expected profits of the first institutional trader in the dealer market

$$E\pi_1^{dealer} = -\frac{\sigma_v^2}{2\varphi} \tag{20}$$

Now consider the second investor's trading strategy. This investor also trades as a monopolist and does not have to worry about the strategic implications of his rival's trading: His objective function is given by (1) which does not directly depend on previous trades. Once more we assume that the market maker sets "fair" prices ie. sets prices equal to the expectation of v conditioned on knowledge of x_1 and x_2. This would imply the market maker setting p_2 as the linear (least squares) projection of v on x_1 and x_2. However to expose the recursive structure of the problem and to simplify the solution, we taking an indirect route to the setting of prices by the market maker.

First, we conjecture that in equilibrium, optimal trades in the second period are uncorrelated with those of the first. Then, following the first trade, the market maker computes an updated distribution for v given by $v \sim N(\mu_1, \sigma_{v|1}^2)$ where

$$\mu_1 = \mu + \frac{\varphi\sigma_v^2}{\varphi^2\sigma_u^2 - \sigma_v^2}x_1 \ \{= E[v|x_1] = p_1\} \tag{21}$$

and

$$\sigma_{v|1}^2 = \frac{\varphi^2\sigma_u^2\sigma_v^2}{\varphi^2\sigma_u^2 + \sigma_v^2}\{= var[v|x_1]\} \tag{22}$$

He then sets prices to the second trader in an analogous way to the first trader, $p_2 = \mu_1 + \lambda_2 x_2$, where, λ_2 is analogous to λ_1 in (15) above and is given by $\lambda_2 = cov[x_2, (v - \mu_1)]/var[x_2|x_1]$. Using conjecture for prices in the profits function gives optimal demands for the second monopolist as

$$x_2 = \beta_2(v - \vartheta_1) + \gamma_2 u_2 \tag{23}$$

which clearly shows that optimal demands in the second period x_2, are indeed independent of those in the first x_1, (and are also normally distributed). The independence of equilibrium trades now implies that, prices in the second period satisfy

$$p_2 = \vartheta_1 + \lambda_2 x_2 = E(v|x_1) + \lambda_2 x_2 = p_1 + E(v|x_2) = E[v|x_1; x_2] \tag{24}$$

where the last equality confirms that the conjectured prices are indeed fair. Solutions for $\beta_2; \lambda_2$ and γ_2 may be computed as in (17).

It is easily seen that the recursive solution to the problem given above for the first two trades may be generalised to trade j. Solutions for β_j, λ_j, and $E\pi_j^{dealer}$, ϑ_j and $\sigma^2_{v|j}\{= var(v|x_1; x_2 ... x_j)\}$ may be obtained from equations (16) - (20) respectively by replacing the right hand side terms $\sigma^2_{v|j-1}$ and ϑ_{j-1} with σ^2_v and ϑ respectively. Adapting equation (17) in this way to give solutions for β_j and λ_j

$$\lambda_j = \frac{\phi\sigma^2_{v|j-1}}{\phi^2\sigma^2_u - \sigma^2_{v|j-1}}; \beta_j = \frac{[\phi^2\sigma^2_u - \sigma^2_{v|j-1}]}{\phi[\phi^2\sigma^2_u + \sigma^2_{v|j-1}]} \tag{25}$$

The solution shows clearly that λ_j is increasing in $\sigma^2_{v|j-1}$ and that β_j is decreasing in $\sigma^2_{v|j-1}$. Similarly we may adapt equations (20) and (22) to give

$$E\pi_j^{dealer} = -\frac{\sigma^2_{v|j-1}}{2\phi} \qquad j \geq 2 \tag{26}$$

$$\sigma^2_{v|j} = \frac{\phi^2\sigma^2_u\sigma^2_{v|j-1}}{\phi^2\sigma^2_u + \sigma^2_{v|j-1}} \tag{27}$$

respectively. Given the initial condition $\sigma^2_{v|0} = \sigma^2_v$, Equations (26) and (27) may be solved recursively to give an explicit form for the jth trader's profits for j=2,3...n. To get a closed form for expected profits for the jth trader, rearrange (26) to give

$$(\sigma^2_{v|j})^{-1} = (\sigma^2_{v|j-1})^{-1} + (\phi^2\sigma^2_u)^{-1} = (\sigma^2_v)^{-1} + j.(\phi^2\sigma^2_u)^{-1} \tag{28}$$

Using (28) on the right of (26) gives aggregate expected profits for the n institutional traders as

$$\sum_{i=1} E\pi_i^{dealer} = -\sum_{j=1} \frac{\phi\sigma_u^2\sigma_v^2}{2(\phi^2\sigma_u^2 + (j-1)\sigma_v^2)} \qquad (29)$$

Proposition 1 $\beta_j > \beta_{j-1}$ and $\lambda_j < \lambda_{j-1}$

Proof.

From (25) β_j is decreasing in $\sigma^2_{v|j-1}$ and λ_j is increasing in $\sigma^2_{v|j-1}$. Further, (28) shows that $\sigma^2_{v|j-1}$ is decreasing with j k ■

- The proposition shows that the jth trader trades more aggressively than the $j-1$th, as a consequence of the updated variance of v having fallen. Further as the covariance of the underlying value of the asset and the order flow will have fallen, the market maker set a lower mark-up to the jth trader. The information revelation that occurs as successive institutions trade increases the profits of successive traders as is clear from (29). This is an important effect in a sequential dealership market that is absent from a "one-off" batch auction where each trader's expected profits are the same. As noted in the previous section however, the fact that each institution acts as a monopolist works to reduce the institutions' profits. We examine the net impact of the two effects of competition and sequential information revelation in the next section.

3 Comparison of the two alternative market mechanisms

To compare the expected profits under the auction and dealer markets given in (11) and (29) we first take the case of two firms (i.e. the cases of duopoly and two sequential monopolists respectively) and then generalise to n firms. The following theorem which is the main result of the paper, states that whether one market mechanism is preferred to the other depends on the relative values of the uncertainty about the fundamental, and the importance of the liquidity shocks.

Theorem 1 Define the quantity

$$c = \frac{\sigma_v^2}{\varphi^2\sigma_u^2}$$

In comparing the expected profits to the institutional investors from trading in an auction or dealer market

(i) For $n = 2$ a necessary and sufficient condition for auction markets to yield highest expected profits is

$$0 < c < 0{:}36$$

(ii) For $n = 2$ a necessary and sufficient condition for dealer markets to yield highest expected profits is

$$0{:}36 < c < 1$$

(iii) For $n > 2$ condition (ii) applies with sufficiency only

(iv) A sufficient condition for auction markets to yield highest expected profits is

$$0 < c < \frac{1 + \sqrt{1 + n}}{n}$$

Proof.

For $n = 2$ comparing (11) with (29)shows that profits will be greater in the dealer market as

$$-\left[\varphi\sigma_u^2 - \frac{(\varphi^2\sigma_u^2 - \sigma_v^2)(\varphi^2\sigma_u^2 + \sigma_v^2)^2}{\varphi^3\sigma_u^2(\varphi^2\sigma_u^2 + 2\sigma_v^2)}\right] > -\frac{\sigma_v^2}{2\varphi} - \frac{\varphi\sigma_u^2\sigma_v^2}{2(\varphi^2\sigma_u^2 + \sigma_v^2)}$$

This condition simplifies to

$$\varphi^4\sigma_u^4 - 2\varphi^2\sigma_u^2\sigma_v^2 - 2\sigma_v^4 > 0$$

Rearranging and solving this inequality for c, dealer markets are preferred as $c > 0{:}36$. This establishes the one side of (i). Recall from the parameter solutions in (9) and (25) that for equilibrium we require that $\sigma_v^2 < \varphi^2\sigma_u^2$ i.e. that $\frac{\sigma_v^2}{\varphi^2\sigma_u^2} = c < 1$, which establishes the other side of (i).$\|$

In the range $0{:}36 < c < 1$ the dealer market will be preferred by these institutional traders. Therefore in the range $0 < c < 0{:}36$ the auction market will yield the highest expected profits to the traders. This establishes (ii)$\|$