Edinburgh School of Economics

Discussion Paper Series

Number 99

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Santiago Sanchez-Pages (University of Edinburgh)

Date
2003

Published by
School of Economics
University of Edinburgh
30 -31 Buccleuch Place
Edinburgh EH8 9JT
+44 (0)131 650 8361
http://www.ed.ac.uk/schools-departments/economics

THE UNIVERSITY *of* EDINBURGH

The use of conflict as a bargaining tool against unsophisticated opponents*

Santiago Sánchez-Pagés†

Abstract

In this paper we explore the role of conflict as an informational device by means of a simple bargaining model with one-sided incomplete information: Limited conflicts reveal information about the outcome of the all-out conflict (that ends the game) because the outcomes of both types of confrontations are driven by the relative strength of the parties. We limit the analysis to the case where the uninformed party can learn the information transmitted in the battlefield but not the one conveyed by offers. The game becomes then an optimal stopping problem where the informed party has to decide at each period whether to stop, by reaching an agreement or by invoking total conflict, or to keep fighting. We show that conflict is a double-edge sword: It may paradoxically open the door to agreement when the uniformed party is too optimistic. But confrontation also occurs when agreement is possible but the informed agent has incentives to improve her bargaining position by fighting.

Keywords: Relative strength, absolute conflict, battles, unsophisticated opponent, optimal stopping.

JEL codes: C78, D74, J52, N4.

*This paper is the third chapter of my Ph.D. dissertation at Universitat Autònoma de Barcelona. I am very grateful to the kind guidance and intellectual stimula of my advisor, Clara Ponsati, who suggested me a model this paper borrows. I also want to thank the help and comments of Steven J. Brams, Aviad Heifetz, Xavier Jarque, Alastair Smith and the participants of the Meeting on Polarization and Conflict held in Barcelona on December 2002; I am especially grateful to Debraj Ray and Stergios Skaperdas for their thorough discussion there. All errors are of my entire resonsability.

†Edinburgh School of Economics, 50 George Square, EH8 9JY Edinburgh, United Kingdom.

1. Introduction

The starting point of this paper is the following idea: In order to understand *how* parties reach an agreement one has to understand first how they disagree.

The economic approach to disagreement is strongly tailored by Nash's seminal contribution. In his description of the *bargaining problem* Nash (1950) embeds disagreement in the threat point; it means to be the outcome of a hypothetical non-cooperative game played after parties fail to agree how they share the surplus of cooperation. However, no information about that game or the forces that determine the location of such point is incorporated into the description of the problem.

This paper turns around two considerations challenging this position. First, disagreement is often followed by a conflict or confrontation whose outcome is driven by the *power relationship* or *relative strength* of the parties. Examples are the renegotiation of the terms of a contract between a soccer player and his club; the negotiations between two countries on the division of some piece of territory; between workers and management on wages; among social groups on the share of political power; or simply how a just married couple will share the chores. All these bargaining situations have the common feature of occurring *in the shadow of conflict:* If parties fail to agree they can resort to coercive methods. They can go to court, they can go to war or strike; they can divorce. In any case, these are probabilistic conflicts. Their outcomes depend on military strengths, the extent of the union membership, the quality of the lawyers... That is, they depend on *power*. Consequently, any sensible agreement will be conditioned by how the conflict triggered by disagreement is resolved.

But why should disagreement be only an outside option? The second observation we want to point out is that parties do choose the way in which they disagree; they actually choose the *scope* of the conflict they are going to fight[1]: India and Pakistan have not used nuclear weapons, they only engage in skirmishes; Pepsi and Coca-cola do not engage in worldwide price wars, but only national; family arguments do not necessarily imply divorce... To assume that conflicts are just "fights to the finish" neglects the fact that the main cause for the end of a confrontation is the parties' agreement on stopping hostilities rather than the total collapse of one side. In short, it prevents us to see that conflict is *part of the bargaining process.*

Suppose that parties can only engage in conflicts aiming to the complete defeat of the opponent and consider the following situation: Two agents have incomplete information on the strength of their opponent in case of conflict; and both parties are strong but believe that they are facing a weak rival. Then, the perceived threat-point will be out of the bargaining set and the result of the negotiation will be a total confrontation[2].

[1] As Wagner (2000) pointed out, few strikes or wars would occur if their consequences were always the total defeat of one of the parties.

[2] Then our answer to the Hicks' paradox would be very extreme: Not only parties do not reach an

But what if parties can engage in a conflict of limited scope that does not entail the end of negotiations? Given that its outcome would also be determined power, it would convey information about the true relative strengths and thus it might open a range of possible agreements.

The aim of this paper is to explore the role of confrontation in negotiations by integrating these two considerations. With that purpose we explore a very simple bargaining model with one-sided incomplete information and one-sided offers. Inspired by the pioneer analysis of Clausewitz (1832) we consider two types of conflicts: *Absolute conflicts* (AC), equivalent to an outside option and that end the game when taken[3], and *Real conflicts,* that can be thought as battles and that allow the game to continue. Given that we assume that parties' winning probabilities in both types of conflicts are a function of their relative strength, Real conflicts become experiments that transmit information about it and about the expected outcome of the AC by extension. The latter is fought when one party losses completely the hope of reaching an agreement through ordinary methods; battles are fought before or during normal bargaining because they can help to improve positions by changing opponent's beliefs; therefore they do not impede the game to end in a settlement.

However, information is not only transmitted in the battlefield. Offers may also signal the type of the proponent, because the toughest players will never make certain offers. Unfortunately, the analysis when the uninformed party learns from these two sources is quite difficult and it is addressed in a companion paper (Sánchez-Pagés, 2004). We thus assume that the uninformed player is *unsophisticated*: She learns the information conveyed through confrontation but not that transmitted through ordinary bargaining.

Once battles have changed enough the expectations of the uninformed party to create a range of possible agreements, the informed agent has incentives to trigger further battles in order to gain even more advantage. Then the proponent's strategy collapses into an *optimal stopping* problem: He must decide at each period either to stop the game (by making an acceptable offer or triggering AC), or to keep gambling[4]. From this point of view our model introduces a novelty with respect to standard models of bargaining under incomplete information: The outcome of a confrontation is not subject to manipulation because it depends on the true relative strengths of the parties. The only strategic variable is the decision of invoking such conflict or not; bluffing the other party becomes a very difficult task.

We obtain the following results: Even though the game has an infinite horizon it ends

agreement when a mutually beneficial one exists; moreover, one of them will bite the dust in the ensuing (wasteful) conflict.

[3]These are a very extreme conflicts because parties aim to render the opponent defenseless and impose their most preferred outcome without opposition

[4]"Of all the branches of human activity, war is the most like a gambling game" (Clausewitz (1832)).

in a finite number of periods. The reason being that the improvement in the expectations produced by an additional victory of the informed party is decreasing both in time and in the number of partial victories already obtained. Given that players are impatient, incentives to gamble decrease with time as well. Agreement is thus immediate if costs from confrontation -either from delay or from AC- are high enough.

A second result is that the more powerful the informed agent the more likely is the use of Real conflicts during the bargaining process: In our model, the informed party owns a *persuasion device;* and the ease to "sell" information to the other party is increasing in his strength. Hence, if he is not powerful enough, he prefers to make an acceptable offer immediately. Beyond this case, we show that the existence of a bargaining range is necessary but not sufficient for a settlement to be reached. Hence, the role of conflict as a source of information would help to explain why we observe confrontation in situations where agreement is possible.

We distinguish between two scenarios: *Advantage conflicts,* where mutually beneficial agreements exist at *any* state of the game, and *Unavoidable confrontations,* where excessive optimism precludes agreement at some states. In the former case we show that the informed party's equilibrium strategy can be characterized by a sequence of integers, one for each period before the (endogenous) finite horizon. If the number of victories obtained at some period t is greater than the corresponding integer in the sequence, the informed party makes an offer that is accepted by his opponent. Otherwise, he prefers to trigger another battle. This sequence is first non-decreasing and non-increasing afterwards. In Unavoidable confrontations, a non-decreasing sequence is added, such that AC occurs if the number of victories steps below it. We are able thus to generate an *ex-post* interpretation of conflict as an outside option: If Real conflicts cannot create a bargaining range early enough or if bad luck in the battlefield makes persuasion attempts fail, the informed party loses any hope of extracting surplus and conflict is total.

The rest of the paper is structured as follows: In Section 2 we present the basic elements of the model. In Section 3 we show how the bargaining game collapses into an optimal stopping problem and in Section 4 we solve it. In Section 5 we review the related literature.

2. The model

Consider a game, denoted by $G[\delta, \theta]$, where two risk neutral players bargain over the division of a cake one euro worth. We will assign to P1 the male gender and the female gender to P2. They are impatient an discount future at a common factor $\delta \in (0, 1)$. Let us denote by $p \in [0, 1]$ the *relative strength* of player P1 in case of conflict. The p

is perfectly observed by P1 but not by P2, who believes at $t = 0$ that p is uniformly distributed[5].

Players act in discrete time under an infinite horizon. In each period $t = 0, 1, 2, ...$ player P1 chooses an action in $\{AC, B, x\}$, where $x \in [0, 1]$ denotes an *offer* to P2; AC is the outside option of *Absolute conflict* (AC henceforth) that ends the game; and B means that a *battle* between the players is fought, making the game proceed to period $t + 1$.

P2 only moves if P1 makes an offer. In that case, her available actions are $\{$*Accept*, *Reject*$\}$. If P2 accepts, agreement is reached at that period and payoffs are $(\delta^t(1-x), \delta^t x)$. Rejection triggers AC.

AC is a "fight to the finish", a confrontation where parties perfectly commit to try to defeat their opponent. Therefore, it necessarily ends the game. We model AC as a costly lottery, yielding payoffs that depend on the realization of p : With such probability P1 wins the conflict and P2 is defeated. This confrontation entails a fixed loss; the value of the cake reduces to $0 \leq \theta \leq 1$. Therefore, the payoffs from AC, conditional on p, are

$$d = (d_1, d_2) = (\theta p, \theta(1-p)).$$

Finally, a battle is a conflict of limited scope that does not entail the end of the game: Nature simply announces a winner and the next period is reached. We will assume that the outcome of these battles is a function of the relative strength p too. But since p is not known by P2, this implies that *battles convey information* about p. For simplicity, we will assume that the battle winning probabilities are precisely p and $1 - p$ respectively.

Battles are thus Bernoulli trials. The belief updating due to battles can be easily computed: Posterior beliefs on p follow a Beta distribution with parameters $(k+1, t-k+1)$ (see De Groot (1970)). The probability density function of p at period t after k successes of P1 is thus

$$f(z, k, t) = \frac{z^k(1-z)^{t-k}}{\int_0^1 u^k(1-u)^{t-k}du} \tag{1}$$

and its expectation, $E(p \mid k, t)$, is equal to $\frac{k+1}{t+2}$. This allows us to characterize completely P2's beliefs at any information set in which no offer has been made yet through a vector $h_t = (k, t)$ where k is the number of battles won by P1. Thus the set $\Theta = \{(k, t) \in \mathbb{N}^2 / k \leq t\}$ describes the possible states in the battlefield. And P2's expected payoff from AC at any state $h_t \in \Theta$ is

$$E(d_2 \mid h_t) = \theta(1 - \frac{k+1}{t+2}).$$

[5] As we will see below, this assumption can be relaxed to any priors following a Beta distribution.

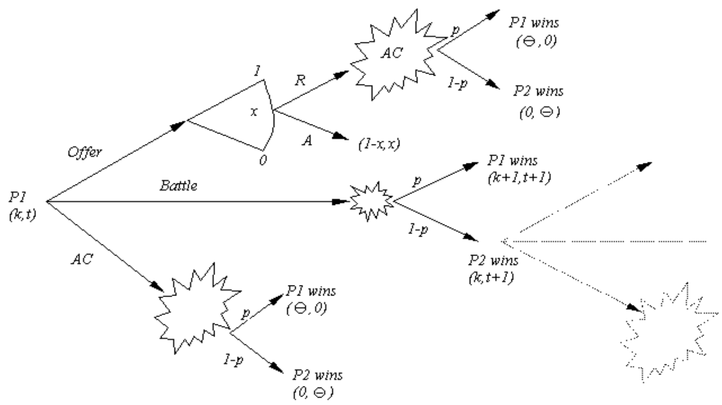


Figure 1: The game

P2 will not accept offers such that $x < E(d_2 \mid h_t)$ because she believes that she can do better by rejecting and triggering AC.

Hence, a strategy for P1 is simply a function σ_1 mapping the set of states into the set of actions $\{AC, B, x\}$; similarly, a strategy for P2 is a function σ_2 mapping states into $\{Accept,\ Reject\}$[6].

We will limit the analysis to the case when P2 only learns the information transmitted in the battlefield. In that case, we will say that P2 is *unsophisticated.*

Definition 1. *A **Unsophisticated Equilibrium** (UE) of the game $G[\delta, \theta]$ is a pair of strategies (σ_1^*, σ_2^*) such that σ_1^* maximizes P1's continuation value of the game at each h_t and P2 accepts x if and only if $x \geq E(d_2 \mid h_t)$.*

3. P1's strategy as an optimal stopping problem

In this Section we show that the P1's strategy against an unsophisticated opponent can be characterized as an optimal stopping problem.

We know that an equilibrium offer must leave P1 at least as well as under AC and give P2 at least her expected payoff in that situation. Hence, at each state (k, t) the set of relevant proposals for P1 is

$$\mathsf{X}_{k,t} = [\theta(1 - \frac{k+1}{t+2}), 1 - \theta p].$$

[6]Note that we will restrict our analysis to strategies that do not depend on the concrete order of victories and defeats.

Under complete information, agreement would be achieved immediately: P1 would offer $\theta(1-p)$ and P2 would accept. However, under incomplete information, the interval $\mathsf{X}_{k,t}$ may be empty: If P2 is too optimistic about her probability of winning AC, the sum of the perceived disagreement payoffs may be greater than one and agreement is impossible.

Remark 2. *The sum of the "perceived" disagreement payoffs exceeds the surplus at history* (k,t) *if and only if*

$$\theta(1-\frac{k+1}{t+2})+\theta p>1, \tag{2}$$

The key ingredient of our model is that the transition from period t to $t+1$ through a battle conveys information because its outcome is driven by the true relative strength of the players. Therefore conflict, in the form of battles, may actually *open the door to agreement* by making P2 less optimistic about her prospects in case of AC.

Let us denote by

$$S=\{h_t\in\Theta\ /\ \theta(1-\frac{k+1}{t+2})+\theta p\leq 1\},$$

the set of states where agreement is possible. Observe that a non-empty bargaining range at state h_t (i.e., $h_t\in S$) is not sufficient for agreement: P1 knows that he can make P2 even more pessimistic about the outcome of AC by winning one additional battle; he is endowed with a "persuasion device".

Thus P1's problem can be treated as an *optimal stopping* problem because he has to decide at each stage whether to stop or not and how (through agreement or AC). One can express P1's objective problem by means of the following value function

$$v(k,t)=\max\{r(k,t),\ \delta E(v\mid k,t)\}, \tag{3}$$

where

$$E(v\mid k,t)=p\cdot v(k+1,t+1)+(1-p)\cdot v(k,t+1),$$

is the expected continuation value of the game, and

$$r(k,t)=\max\{1-\theta(1-\frac{k+1}{t+2}),\ \theta p\},$$

is the *immediate reward function*; this is the best payoff P1 can get by stopping at (k,t): If $h_t\in S$, then $r(k,t)=1-E(d_2\mid h_t)$[7]; if not P1 would trigger AC in case he decides to stop, so $r(k,t)=\theta p$.

[7] Given that P2 knows that at each period she will never receive an offer greater than that, the continuation value of rejecting x is at most $\delta\frac{k+1}{t+2}\theta(1-\frac{k+2}{t+3})+\delta(1-\frac{k+1}{t+2})\theta(1-\frac{k+1}{t+3})=\delta\theta(1-\frac{k+1}{t+2})$; that is always dominated by the value of accepting x now.

The solution to P1's optimal stopping problem is typically a mapping from the space of states Θ to a decision of continuing or stopping.

We will say that P1 *prefers to stop the game* whenever

$$r(k,t) \geq \delta E(v \mid k,t). \tag{4}$$

Our first result is that in equilibrium P1 stops the game in finite time.

Proposition 3 (Game ends in finite time). *For each pair (δ, θ) there exists a period $\bar{t} < \infty$ such that in any UE the game ends no later than $\bar{t}$. Furthermore, $\bar{t} > 1$ only if $p < \frac{\delta}{\theta}$.*

A couple of remarks are in order: First that the battles become a relevant instrument for P1 only when the cost of delay they introduce is low relative to the loss from AC. Therefore the *endogenous finite horizon* comes *sooner the more impatient* the players are and the *higher is the conflict loss,* because the outside option becomes less attractive.

Second, the returns of conflict as an informational device are decreasing in time[8]. This, together with impatience, sets an upper bound to the number of persuasion attempts. In addition, if the game is in the set of states where the bargaining range is empty, P1 may (at some point) lose the hope of extracting some surplus, and then trigger AC.

Thus, the grid $H = \{h_t \in \Theta / \ t \leq \bar{t}\}$ covers all states of the game that are possible along any equilibrium path. Let us denote by

$$\Gamma = \{h_t \in H / \ v(k,t) = r(k,t)\},$$

the *Stopping region,* i.e. the set of states where P1 prefers to stop the game. It can be shown (see Dynkin and Yushkevich, (1969)) that it is optimal for P1 to stop at the first time the state of the game visits Γ. Therefore, to characterize the UE outcome it is sufficient to characterize Γ. The following condition will be useful for that characterization.

Weak concavity: *The immediate reward function satisfies weak concavity at state (k,t) if and only if*

$$r(k,t) \geq \delta[p \cdot r(k+1,t+1) + (1-p) \cdot r(k,t+1)],$$

[8] The increment in P2's expectation of p after a P1's victory is

$$\Delta E(p \mid k,t) = \frac{t+1-k}{(t+2)\,(t+3)},$$

It is easy to check that

$$\frac{\Delta E(p \mid k+1,t+1)}{\Delta E(p \mid k,t)} = \frac{t+2}{t+4}.$$

Weak concavity is a state-dependent property that the immediate reward function satisfies whenever the current reward is greater than the discounted expected reward in the next period. It is immediate to see that this condition is *necessary* for P1 to prefer to stop. Hence, in order to characterize Γ we must investigate when it is also *sufficient*. This is the main goal of the next Section.

4. Conflicts against unsophisticated opponents

In this Section we will characterize the *stopping region* for the case where P2 is unsophisticated. As we will see, it consists on a sequence of integers separating those states of the game where it is optimal for P1 to stop from those where it is not. By construction, this sequence will characterize the UE of our game.

Two scenarios arise here: *Advantage conflicts*, where agreements are possible at any state of the game; and *Unavoidable confrontations*, where optimism may preclude agreement and confrontation becomes the unique mean to bargain.

4.1. Advantage conflicts

In Advantage conflicts, the conflict loss $1-\theta$ is so high that AC never occurs in equilibrium. But P1 may still find incentives to delay agreement by fighting battles in order to change P2's beliefs.

In order to characterize the Stopping region Γ we will make use of the property of weak concavity. Since

$$r(k,t) = 1-\theta(1-\frac{k+1}{t+2}) \qquad \forall h_t \in H,$$

weak concavity at state (k,t) reduces to

$$1-\theta(1-\frac{k+1}{t+2}) \geq \delta(1-\theta(1-\frac{k+1+p}{t+3})). \tag{5}$$

Expression (5) implicitly defines a boundary on the set H denoted by

$$k^p(t) = \left\{ k \in \mathbb{R} \quad / \quad 1-\theta(1-\frac{k+1}{t+2}) = \delta(1-\theta(1-\frac{k+1+p}{t+3}) \right\}. \tag{6}$$

If, at a given state, $k < k^p(t)$ then weak concavity fails and consequently P1 will surely fight an additional battle.

Note that if weak concavity holds for all $h_t \in H$, i.e. $k^p(t) < 0$ $\forall t$, backwards induction from for $\bar{t}$ yields that P1 prefers to stop at any state of the game. That is, *agreement must be immediate.*

Proposition 4 (UE with Immediate agreement). *There exist some threshold* $\widetilde{p}(\delta,\theta)$ *such that in any UE agreement is immediate if the realization of* p *satisfies* $p \leq \widetilde{p}(\delta,\theta)$. *Moreover, there exists a threshold* $\widetilde{\theta}(\delta)$, *decreasing in* δ *such that, if* $\theta < \widetilde{\theta}(\delta)$, *in any UE agreement is immediate for any realization of* p.

When P1 is strong enough his "persuasion device" is very effective and the option of acquiring additional advantage is thus attractive. The threshold

$$\widetilde{p}(\delta,\theta) = \frac{1-\delta+2\sqrt{\theta\,(1-\delta)\,(1-\theta)}}{\theta\delta},$$

is decreasing in both δ and θ. Hence, as the delay cost and the loss due to AC increase, only stronger types of P1 find battles worthy as a way to improve their bargaining position. If these costs are high enough, i.e. $\widetilde{p}(\delta,\theta) > 1$, agreement is immediate *for any type* of P1.

Note that Propositions 3 and 4 together show that if p is too low or too high, battles become useless for P1 in Advantage conflicts; either because he prefers to agree immediately or to trigger AC.

For the remainder of the Section, we will focus on environments where agreement is not immediate for some P1's types.

Assumption A1 (AC not too costly): $\theta \geq \widetilde{\theta}(\delta) = (1-\delta)\frac{\left(\sqrt{2}-\sqrt{\delta}\right)^2}{(2-\delta)^2}$.

Note that this threshold is decreasing in δ. Now we are ready to characterize the stopping region and therefore the UE, with the help of the boundary (6).

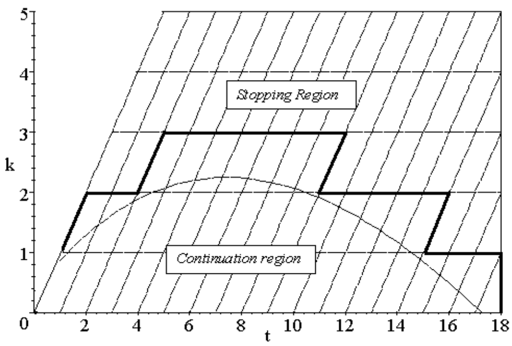
Proposition 5 (UE of Advantage conflicts). *Suppose that A1 holds. If* $p > \widetilde{p}(\delta,\theta)$ *there exists a sequence of integers* $\{k_o^p(t)\}_{t=1}^{\bar{t}}$, *that is first non decreasing and non increasing afterwards, such that the profile*

$$\sigma_1^* = \begin{cases} x^* = E(d_2 \mid h_t) & \text{if } k \geq k_o^p(t) \\ B & \text{otherwise} \end{cases}$$

$$\sigma_2^* = \begin{cases} \text{Accept} & \text{if } x \geq E(d_2 \mid h_t) \\ \text{Reject} & \text{otherwise} \end{cases}$$

constitutes the unique UE of $G^A[\delta,\theta]$.

Example 1: Let $(\theta,\delta,p) = (0.45, 0.97, 0.85)$. In this case, the type p falls above the threshold $\widetilde{p}(\delta,\theta) = 0.463$.



Stopping and Continuation regions for Example 1

The boundary $k_o^p(t)$ (the dashed curve in Figure 2) shows that the game ends in at most 18 periods since for $t > 18$, $k_o^p(t) < 0$. The thick black line characterizes the stopping region: P1 fights battles as long as the states remains below it.

Note that at $t = 0$, P1 is eager to fight. But, if the game lasts more than two periods (an event that occurs with probability 0.04), he ends up worse off than under immediate settlement even before correcting for the discounting.

4.2. Unavoidable confrontations

If P1 is sufficiently strong and the loss from AC is relatively low, P2 may be optimistic enough to make conflict unavoidable. Excessive optimism may be present from the very beginning (if $p > \frac{1-2\theta}{\theta}$ agreement is impossible at $t = 0$); or it may arise after a series of defeats on a conflict that P1 started as a simple Advantage conflict. In such cases, battles will be necessary for agreement. But if bad luck in the battlefield is pervasive, P1 resorts to AC.

In this Section we will thus make the following assumption.

Assumption A2 (Unavoidable confrontations): θ is such that at some states of the game agreement is impossible, i.e. $H \nsubseteq S$.

Denote by $G^U[\delta, \theta]$ the game satisfying A1-A2.

The main difference with the case of Advantage conflicts is thus the fact that for states $h_t \notin S$, the immediate reward function dictates P1 to stop by triggering AC, i.e. $r(k,t) = \theta p$. Therefore, when checking weak concavity one has to consider several cases depending on the values that $r(k,t)$ and $r(k,t+1)$ take[9]:

[9] If $r(k,t) = r(k+1,t+1) = \theta p$. Then the optimal decision is always to stop at (k,t).

(*i*) State $(k,t) \in S$, but $(k,t+1) \notin S$. In that case, $r(k,t) = 1 - \theta(1 - \frac{k+1}{t+2})$ and $r(k,t+1) = \theta p$. Weak concavity induces a boundary in H, denoted by

$$k_1^p(t) = \left\{ k \in \mathbb{R} \quad / \quad 1 - \theta(1 - \frac{k+1}{t+2}) = \delta p[1 - \theta(1 - \frac{k+2}{t+3})] + \delta(1-p)\theta p \right\} \quad (7)$$

If $k \geq k_1^p(t)$, then $r(k,t)$ is weakly concave.

(*ii*) State $(k,t) \notin S$, but $(k+1,t+1) \in S$. In this case, $r(k,t) = \theta p$ and $r(k+1,t+1) = 1 - \theta(1 - \frac{k+2}{t+3})$. By the same token denote as

$$k_2^p(t) = \left\{ k \in \mathbb{R} \quad / \quad \theta p = \delta p[1 - \theta(1 - \frac{k+2}{t+3})] + \delta(1-p)\theta p \right\} \quad (8)$$

If $k \leq k_2^p(t)$, then $r(k,t)$ is weakly concave. Note that weak concavity implies here that P1 will stop the game by triggering AC.

Recall that the sequence $k_o^p(t)$ applies to the case when agreement is possible at (k,t) and $(k,t+1)$. H is partitioned in three regions; in order to characterize the Stopping region weak concavity must be checked through the corresponding boundary at each state .

The boundary $k_o^p(t)$ applies only to states satisfying

$$k \geq (1 + p - \frac{1}{\theta})(t+3) - 1,$$

because $(k,t+1) \in S$. For k's below this threshold the boundary $k_1^p(t)$ starts being relevant. It is so until

$$k \leq (1 + p - \frac{1}{\theta})(t+2) - 1, \quad (9)$$

because at states satisfying this inequality the bargaining range is empty at (k,t) and P1 will trigger AC in case he prefers to stop (note that this is just the reformulation of expression (2)). For k's below this second threshold, $k_2^p(t)$ becomes the relevant boundary and if k fails to step above it P1 triggers AC.

The preceding analysis essentially describes P1's equilibrium behavior. The game is essentially an Advantage conflict as long as P1 obtains sufficiently many victories to keep the bargaining range non-empty. But if P1 is systematically defeated, and P2 becomes too optimistic in consequence, he may lose hope of extracting surplus before $\bar{t}$. So P1 will prefer to stop gambling and terminate the game in AC. This is formally stated in the following Proposition.

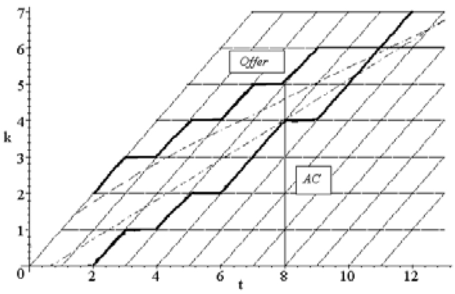
Proposition 6 (UE in Unavoidable confrontations). *Suppose that A1-A2 hold and consider an UE. Then there exists two sequences of integers* $\{\overline{k}^p(t)\}_{t=1}^{\bar{t}}$ *and* $\{\underline{k}^p(t)\}_{t=1}^{\bar{t}}$, *where the latter is non-decreasing, satisfying that* $\overline{k}^p(t) \geq \underline{k}^p(t)$ *for any* t, *such that the strategy profile*

$$\sigma_1^* = \begin{cases} x^* = E(d_2 \mid h_t) & \text{if} \quad k \geq \overline{k}^p(t) \\ AC & \text{if} \quad k < \underline{k}^p(t) \\ B & \quad \text{otherwise} \end{cases} \tag{10}$$

and $\sigma_2^* = \{Accept\}$ *if and only if* $x \geq E(d_2 \mid h_t)$ *constitutes the unique UE of* $G^U[\delta, \theta]$.

A battle is a risky gamble that P1 may use to attain advantageous positions in bargaining. However, if these limited confrontations do not help to create a bargaining range soon enough, the delay they introduce provokes the total breakdown of negotiations and precipitates AC.

Example 2: Let $(\theta, \delta, p) = (0.75, 0.97, 0.85)$. The black thick lines in Figure 2 are the boundaries $\overline{k}^p(t)$ and $\underline{k}^p(t)$. In between, P1 finds worthy to trigger additional battles. Note that the game ends in at most eight periods.



Stopping and continuation regions for Example 2.

At $t = 0$ the bargaining range is empty and P1 tries to generate a non-empty range of possible agreements by fighting one battle. At state $(1, 1)$ agreement is no longer impossible but there P1 finds incentives to improve his position, so he prefers to trigger an additional battle. On the contrary, if he is defeated in the first two battles, he knows that he will not do better for the rest of the game, so he takes the outside option. In fact, AC is the outcome of the game with probability 0.07.

5. Related literature

The present paper is related with two lines of research: The economic literature on bargaining under incomplete information and the literature on conflict, both in Economics and Political Science.

The term *bargaining in the shadow of power* was introduced by Powell (1996) as a mean to refer to those situations where agreements in negotiations must mirror the structure of disagreement. This idea has been explored in the last years from a wide variety of perspectives: In the spirit of Nash's bargaining problem Anbarci et al. (2002) analyze a model where the threat point is endogenously determined by agents' investments in arms; they modify favorably the location of the threat point but are wasteful. The authors compare several bargaining solutions and show that Equal Sacrifice is the one that induces the lower loss of efficiency. On their side, and following an axiomatic approach, Esteban and Sakovics (2002) introduce the concept of *disagreement function.* This function determines the threat point for each possible set of feasible outcomes, embedding thus the power relationship between the parties. They characterize a solution, the Agreement in the Shadow of Conflict (ASC) as the result of a sequence of partial agreements.

These papers recognize that agreements will be conditioned by the structure of disagreement. But their common pitfall is that they fail to explain the actual occurrence of conflict. Hence, incomplete information arises as an appealing explanation for the systematic difficulty of parties to reach peaceful agreements[10].

Brito and Intriligator (1985) develop a model of conflict and war that is strongly related with the pioneer work of Sobel and Takahashi (1983): There is private information about the costs of going to war. When receiving offers, the informed party has incentives to misrepresent his type by trying to look tough. War is thus the result of a *separating strategy* taken by the uninformed party. On his side, Powell (1996) presents an alternating offer game where players can impose a settlement at a cost players have private information about. In this game, like in ours, when parties become too pessimistic they take the outside option of conflict.

All the models above depict conflict simply as a costly lottery: Either one party or the other wins and captures the surplus. Thus, invoking conflict is a game-ending move, an alternative to the bargaining process.

Clausewitz (1832) coined the terms of Absolute and Real war we heavily borrow. He was the first one in noticing the distinction between those wars that seem uniquely intended to the destruction of the enemy from those that are "simply a continuation of politics by other means."[11]

Georg Simmel reconsidered this provocative idea and his work "The Sociology of Conflict" (1904, p. 501) he pointed out the following paradox:

"the most effective prerequisite for preventing struggle, the exact knowledge of the comparative strength of the two parties, is very often attainable only by the actual

[10]For a very exahustive survey of this issue see Ausubel et al. (2001). However, this does not need to be the unique explanation: Garfinkel and Skaperdas (2000) show that conflict can also occur under complete information.

[11]However, he failed to articulate these two apparently opposite motivations. See Gallie (1978).

fighting out of the conflict".

Geoffrey Blainey (1973) pursued this idea and asserted that "war itself provides the stinging ice of reality", because it helps to solve the optimism arising from conflicting expectations about the outcome of war. Furthering this reasoning, Wittman (1979) noted that if conflict is a source of information, war would occur also if there is no optimism because parties can use confrontation to extract better terms from the opponent. However, all these contributions did not provide any foundation to explain *why* parties should go to conflict when they disagree on the perception of their relative strength[12].

Wagner (2000) developed the incomplete information approach underlying Simmel and Blainey's analysis: Absolute war is indeed an outside option. However, it is not the only way to solve a situation where a contradiction in the perceived relative strengths locates the threat point outside the bargaining set. Real war arises as a solution by providing Blainey's "stinging ice of reality". Wagner also presented a model (although he did not solve it) trying to formalize Wittman (1979) ideas[13]. But, in his analysis, the author is too biased towards the study of war. This makes him assume for instance that the option of AC is only available after Real conflicts occur. Our contention is that the fact that confrontation is part of the bargaining process is common to many other contexts under the shadow of conflict; by allowing parties to *choose* the type of confrontation they want to fight, we can gain insights on the rational foundations of conflict.

Among the several attempts to formalize Wagner's ideas the two most relevant and closest contributions are the papers by Filson and Werner (2002) and Slantchev (2002). Like ours, these models introduce one-sided incomplete information but model conflict as a succession of battles only. In the former the uninformed party is the one who makes offers. They are not final, so rejection transmits information along with battles, which are resource consuming in case of losing. When her resources run out, a player can no longer fight. However, and in spite of considering only two types of informed players, the game become so complex that the authors only present the results for three-period games. In Slantchev (2002) parties use the alternating-offers protocol so the informed party screens her opponent and the uninformed signals her type through non-serious offers. Battles help to advance in a finite scale of positions. Conflict is over when a player controls all of them.

Apart from their complexity, fighting makes no sense in both models once beliefs have sufficiently converged to avoid optimism. Moreover, the *physical* meaning given

[12] Very interestengly, this idea is also hidden in the economic literature: Schnell and Gramm (1982) study empirically how unions learn by striking: Lagged strike experience is shown to reduce the propensity to strike.

[13] This lack of formalization allows Wagner to make risky statements like "bargaining does not occur until states no longer have an incentive to reveal information by fighting" (p. 472). Although our model supports it, it is very far from being an obvious statement.

to battles limit their role during the bargaining process as long as the total defeat they may induce is near. In our model and by abstracting from a particular interpretation, we are in addition able to explain the use of confrontation in situations where there is no optimism, but just uncertainty.

Finally, the informed party's optimal stopping problem is related to the literature on dynamic programming and optimal stopping problems. In particular, the game that we propose is similar to the discrete time version of the model of job matching proposed by Jovanovic (1979) and presented in Stokey and Lucas (1989): In that case an employer and a worker get matched but they do not know the quality of the match. They can learn about it by observing the output produced so far. When the expectation of the quality of the match falls below a given target, the match is terminated. One important difference with our paper is that our game can end in agreement.

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Proof of Proposition 3. First, note that when $p > \frac{\delta}{\theta}$, the optimal stopping problem is trivial: Since $E(v \mid k,t) \leq 1$, P1 will decide to stop for sure when $r(k,t) > \delta$. Then $r(k,t) \geq \theta p > \delta$ and P1 would stop for sure at any state.

So take the case when $p \leq \frac{\delta}{\theta}$. We will find a period t such that if P1 prefers to stop at $(k+1, t+1)$ then he also does at (k,t). That is, t such that

$$v(k+1,t+1) = r(k+1,t+1) \Rightarrow v(k,t) = r(k,t) \tag{11}$$

We must consider two cases depending on the value that $r(k,t)$ takes:

- Case 1: Let us consider the case when $(k,t) \in S$. The necessary condition for (11) is

$$1 - \theta + \theta\frac{k+1}{t+2} \geq \delta(1 - \theta + \theta\frac{k+2}{t+3}). \tag{12}$$

  And one can be sure that this hold for any $k \leq t$ when

$$\frac{(1-\theta)(1-\delta)}{\delta\theta} \geq \frac{t-k+1}{(t+3)(t+2)}.$$

  Given that $(k,t) \in S$ this implies[14]

$$\frac{t-k+1}{(t+3)(t+2)} \leq \frac{1-\theta p}{\theta(t+3)}$$

  Then, the condition (12) holds for sure if

$$t \geq \frac{1-\theta p}{1-p}\frac{\delta}{1-\delta} - 3.$$

  Now define the period

$$\bar{t}_1 = \left\lceil \frac{1-\theta p}{1-p}\frac{\delta}{1-\delta} - 3 \right\rceil$$

  where $\lceil \cdot \rceil$ denotes the upper integer operator. It only remains to find a period where P1 wants to stop in order to show that the game will end at t_1 since (11) holds for any state $h_t \in S$ from t_1 and on.

  Given that $E(v \mid k,t) \leq 1$, P1 will stop for sure when $r(k,t) \geq \delta$. And observe that with a sufficient number of victories any possible state $(k, \bar{t}_1) \in S$ can be traced to a state $(k+m, \bar{t}_1 + m)$ where $r(k+m, \bar{t}_1+m) \geq \delta$. Hence, for states $h_t \in S$ the game will end at most at $\bar{t}_1$.

[14] If all possible histories belong to S, one should replace this step and take $k \geq 0$.

- Case 2: For $h_t \notin S$ we follow the same procedure. If $(k+1, t+1) \notin S$ then P1 prefers to stop at (k, t), so only needs to consider the case when $(k+1, t+1) \in S$. Then the necessary condition for (11) is

$$\theta p \geq \delta(1 - \theta + \theta \frac{k+2}{t+3}).$$

It holds for any k when

$$\frac{1-\delta}{\delta} p \geq \frac{t-k+1}{(t+3)(t+2)}.$$

By using the fact that at $(k+1, t+1) \in S$ this is equivalent to condition

$$t \geq \frac{1-\theta p}{\theta p} \frac{\delta}{1-\delta} - 2.$$

Analogously, let

$$\bar{t}_2 = \left\lceil \frac{1-\theta p}{\theta p} \frac{\delta}{1-\delta} - 2 \right\rceil$$

denote the period such at states $h_t \notin S$ the game ends at most.

We can conclude that the game will end for sure no later than $\bar{t} = \max\{\bar{t}_1, \bar{t}_2\}$ when $p \leq \frac{\delta}{\theta}$. ■

Proof of Proposition 4. Let us treat t as a continuous variable. Then

$$k^p(t) = \frac{\delta\theta p(t+2) - (1-\theta)(1-\delta)(t+2)(t+3)}{\theta(t+2)(1-\delta) + \theta} - 1.$$

Then, the maximum is attained at

$$t^* = \frac{\sqrt{\delta(1-\theta+\theta p)}}{(1-\delta)\sqrt{1-\theta}} - \frac{3-2\delta}{1-\delta}.$$

One can check that this function is strictly concave:

$$\frac{\partial^2 k^p(t)}{\partial^2 t} = \frac{-2\delta(1-\delta)}{((t+2)(1-\delta)+1)^3}(1 + p - \frac{1}{\theta}) < 0.$$

A sufficient condition for agreement to be immediate is thus $k^p(t^*) < 0$. Tedious algebra shows that this holds true if and only if

$$p < \frac{1-\delta+2\sqrt{\theta(1-\delta)(1-\theta)}}{\theta\delta} = \widetilde{p}(\delta, \theta) \tag{13}$$

Finally, simple calculations yield that if

$$\theta < (1-\delta)\frac{\left(\sqrt{2}-\sqrt{\delta}\right)^2}{(2-\delta)^2} = \widetilde{\theta}(\delta) \Rightarrow \widetilde{p}(\delta,\theta) > 1. \tag{14}$$

■

Proof of Proposition 5. Let

$$j_t = \lceil k^p(t) \rceil .$$

Note that $k \geq j_t$ for all $t > t^*$ and $k \geq \lceil k^p(t^*) \rceil$ for any t are sufficient conditions for P1 to prefer to stop because at any posterior state weak concavity always holds. Then, from t^* and on weak concavity is sufficient and we can set $k_o^p(t) = j_t$.

Due to the lack of monotonicity of $k^p(t)$ it is possible that for some states h_t with $t < t^*$ and $k \geq j_t$, $r(k, t+1) < v(k, t+1)$. Weak concavity is thus no longer sufficient. However, this does not invalidate the existence of our separating sequence: Note that at $\lceil t^* \rceil$ continuation values are non-decreasing in k. By the same token fixed $t < t^*$, $E(v \mid k, t)$ is non-decreasing in k.

We know that given a $t < t^*$ there exists a $\widetilde{k} \leq \lceil k^p(t^*) \rceil$ such that P1 prefers to stop for any $k \geq \widetilde{k}$. At that states it holds that $r(\widetilde{k}, t) > \delta E_v(\widetilde{k}, t)$ but $r(\widetilde{k}-1, t) < \delta E_v(\widetilde{k}-1, t)$, implying that

$$r(\widetilde{k}, t) - r(\widetilde{k}-1, t) = \frac{\theta}{t+2} > \delta(E_v(\widetilde{k}, t) - E_v(\widetilde{k}-1, t)).$$

Hence, it cannot be the case that at t P1 does not prefer to stop if he obtains $k < \widetilde{k}$ victories but he does so if he obtains $k-1$. This implies existence of the separating sequence, that we keep denoting by $k_o^p(t)$. Its elements are most equal to $\lceil k^p(t^*) \rceil$ for $t < t^*$. It only remains to check that $k_o^p(t)$ is non decreasing before t^*. Suppose on the contrary that $k_o^p(t) > k_o^p(t+1)$. Then there exist at least one state (k, t) such that $k_o^p(t+1) \leq k < k_o^p(t)$. At this state, P1 prefers to fight one more battle, but he will stop for sure in the next period, implying that $k < k^p(t)$ necessarily, because weak concavity cannot hold. However, given that $k^p(t)$ is increasing in this range, $k < k^p(t+1)$ contradicting the fact that $k_o^p(t+1) \leq k$. ■

Proof of Proposition 6. Simple calculations show that $k_1^p(t)$ is also concave and attains a maximum at.

$$t^{**} = \frac{1}{1-\delta p}\left(\sqrt{\delta p(1+\theta\frac{1-\delta p}{1-\theta-\delta p(1-\theta p)})} - 1\right) - 2.$$

It crosses with $k_o^p(t)$ at the date

$$t_1 = \frac{1-\theta+\sqrt{(1-\theta)^2 - 4\delta\theta^2 p^2(1-\delta)}}{2(1-\delta)\theta p} - 3, \tag{15}$$

such that $k_o^p(t) \gtrless k_1^p(t)$ for $t \lessgtr t_1$. P1 becomes indifferent between triggering AC and making a definitive offer in case of losing the next battle when

$$k_o^p(t_1) = k_1^p(t_1) = (1+p-\frac{1}{\theta})(t_1+3) - 1.$$

The same happens with $k_1^p(t)$ and $k_2^p(t)$. P1 is indifferent between triggering AC and making an offer at the current state when

$$k_1^p(t_2) = k_2^p(t_2) = (1+p-\frac{1}{\theta})(t_2+2) - 1,$$

where

$$t_2 = \frac{\delta(1-\theta p)}{\theta\,(1-\delta)} - 3.$$

Again $k_1^p(t) \gtrless k_2^p(t)$ for $t \lessgtr t_2$. Note that the game ends at most at $\lceil t_2 + 1 \rceil$.

Hence, weak concavity offers a candidate to characterize the stopping-by-offer Stopping region.

$$\overline{j}(t) = \begin{cases} \lceil k_o^p(t) \rceil & \text{if} \quad t \le t_1 \\ \lceil k_1^p(t) \rceil & \text{if} \quad t \in (t_1, t_2) \\ \lceil k_2^p(t) \rceil & \quad \text{otherwise} \end{cases} \tag{16}$$

If k steps below $\overline{j}(t)$, we can be sure that P1 will not make an offer. Again the problem is that weak concavity is not sufficient to characterize Γ by the same reasons as above. Analogously, we can be sure that P1 will stop if $k > \overline{j}(t)$ for $t > t^{**}$ or if $k \ge \lceil \max\{k_o^p(t^*), k_1^p(t^{**})\} \rceil$; and that a sequence separating the stopping states exists, by following the same procedure as in Proposition 5.

In addition, we must also keep track of the possibility of AC. It generates a boundary, that is only valid when states $h_t \notin S$ do exist. This happens when

$$t \ge t_1' = \frac{1}{1+p-\frac{1}{\theta}} - 2 \tag{17}$$

It is easy to check that for those states between t_1' and t_2 the boundary defined by taking equality in expression (9), separates those states where $r(k,t) = \theta p$. Hence, at any state lying in between, the bargaining range is empty but P1 still finds worthy fighting one more battle. Weak concavity yields again a candidate for the boundary that characterizes the stopping-by-AC region

$$\underline{j}(t) = \begin{cases} \lceil (1+p-\frac{1}{\theta})(t+2) - 1 \rceil & \text{if} \quad t \in [t_1', t_2]; \\ \lceil k_2^p(t) \rceil & \text{if} \quad t > t_2. \end{cases} \tag{18}$$

After t_2, all states in which agreement is not feasible lie below $k_2^p(t)$ and P1 will opt out because there is no possibility of continuation (P1 has stopped either by making an offer or by triggering AC) so Weak concavity characterizes completely Γ. However, it is not sufficient between t_1' and t_2: At state (k,t) a victory may lead the game to the continuation region and make $v(k+1,t+1) \neq r(k+1,t+1)$.

It is easy to check that such inferior sequence exists. Suppose on the contrary that for some state $h_t \notin S$, $v(k,t) = \theta p$ but that $v(k-1,t) > \theta p$. This cannot be the case because it would imply that

$$\delta E_v(k-1,t)) > r(k-1,t) = \theta p = r(k,t) > \delta E(v \mid k,t),$$

and we know, through backwards induction from $\bar{t}$, that the expected continuation value is non-decreasing in k for a fixed t. By the same token, one can show that this inferior sequence is non-decreasing as well: On the contrary, suppose that $\underline{k}^p(t) > \underline{k}^p(t+1)$ so at state $(\underline{k}^p(t),t)$, $\theta p > \delta E(v \mid k,t)$. This implies that at state $(\underline{k}^p(t),t+1)$, P1 prefers to fight one more battle. Hence $\theta p < \delta E_V(k,t+1)$ holds true . But it is impossible because $E(v \mid k,t) \geq E_V(k,t+1)$. ■