Edinburgh School of Economics

Discussion Paper Series

Number 95

Inter-league competition for talent vs. competitive balance

Jonathan P Thomas (University of Edinburgh)
Tim Worral (Keel University)

Date
October 2002

Published by
School of Economics
University of Edinburgh
30 -31 Buccleuch Place
Edinburgh EH8 9JT
+44 (0)131 650 8361
http://www.ed.ac.uk/schools-departments/economics

THE UNIVERSITY *of* EDINBURGH

Unemployment Insurance under Moral Hazard and Limited Commitment: Public vs Private Provision

by

Jonathan P Thomas (University of Edinburgh)
Tim Worrall (Keele University)

Date First version: August 2000. Revised: March 2002, October 2002

Abstract This paper analyses a model of private unemployment insurance under limited commitment and a model of public unemployment insurance subject to moral hazard in an economy with a continuum of agents and an infinite time horizon. The dynamic and steady-state properties of the private unemployment insurance scheme are established.The interaction between the public and private unemployment insurance schemes is examined. Examples are constructed to show that for some parameter values increased public insurance can reduce welfare by crowding out private insurance more than one-to-one and that for other parameter values a mix of both public and private insurance can be welfare maximising.

Keywords Social Insurance, Moral Hazard, Limited Commitment, Unemployment Insurance, Crowding Out

J.E.L. Class D61, H31, H55, J65

Notes Originally titled ‘‘Social Insurance and Crowding Out’’

1 Introduction

Diamond and Mirrlees (1978) study a model of public insurance with a continuum of individuals when the individual's ability to supply labour is affected by a random variable, health, which is unobservable by government. Thus the government faces a moral hazard constraint that if unemployment insurance is too generous workers will be tempted to claim ill health when they are able to work. One assumption that Diamond-Mirrlees made was that there were no private insurance markets. The objective of this paper is to allow for optimal private insurance and to examine the interactions between public and private insurance and examine whether public insurance will crowd out private insurance and whether a mixture of public and private insurance is ever optimal.

We consider an infinite horizon version of the Diamond-Mirrlees model. The advantage of the private insurance scheme is that individuals can observe the health status of their fellow workers. Thus the private insurance scheme faces no moral hazard problem. The private insurance scheme however, cannot enforce payments in the way that the government can. The private insurance scheme is voluntary and individuals will only participate if they expect long-term benefits from the scheme.

The paper proceeds as follows. Section 2 outlines the Diamond and Mirrlees (1978) model. Section 3 develops the dynamic model of private insurance with a continnum of individuals. The steady-state solution is fully characterized and the issue of stability of the steady-state examined. Section 4 outlines the moral hazard problem faced by public insurance. Section 5 brings the previous two sections together and considers whether public insurance will crowd out private insurance and whether there is an optimum mix of public and private insurance. Section 6 concludes.

1.1 The Literature

A number of authors have considered government insurance. Diamond and Mirrlees (1978) consider moral hazard, where the government is unable to observe whether unemployed is due to inability to work or is voluntary. Whinston (1983) extends the Diamond-Mirrlees model to allow for adverse selection caused by multiple unobservable types who have different probabilities of illness. Anderberg and Andersson (2000) consider the case where workers can influence their probability of disability by choice of occupation.

Our model of private insurance builds upon the informal or implicit insurance arrangements between employers and workers considered in Thomas and Worrall (1988). This has been extended by a number of authors and a general model of mutual insurance with n-persons and storage is given by Ligon, Thomas, and Worrall (2000). The extension to a continuum of individuals is considered by Kreuger and Perri (1999) and in a finance context by Lustig (2001). In contrast to these papers, the current paper proves results on optimal private insurance using only straightforward arbitrage arguments.

The crowding out issue is considered in a static context by Arnott and Stiglitz (1991). The trade-off they examine is between internal household insurance and public insurance. The government has better opportunities to pool risk but faces a moral hazard problem not faced within the household. The contrast in this paper is not that the government has better pooling opportunities but that it has a better enforcement technology. A similar dynamic model of private insurance and crowding out is Attanasio and Ríos-Rull (2000) who examine a large number of small-sized private insurance schemes which do not interact with each other but only with the aggregate insurance provided by the government. Although Kreuger and Perri (1999) allow for a government in their model of private insurance with a continuum of individuals, they cannot consider optimal crowding out as they assume that the government faces no moral hazard constraint. This is precisely the issue we address. A dynamic model that does consider the interaction between private and public insurance is Di-Tella and MacCulloch (2002). They analyse a stationary model of private insurance with a finite set of family members and show how public insurance can crowd out private insurance but that the social optimum involves either private or public insurance and no mix of the two is optimal. Our model is a considerable advance on theirs in studying the optimal dynamic private insurance and we use this optimum to construct an example where a mix of public and private insurance indeed dominates either public or private insurance alone.

2 Static Model

This sections briefly outlines the single period social insurance model introduced by Diamond and Mirrlees (1978). There is a continuum of ex ante identical individuals. An individual is either capable of labour supply or not. There is a probability $p \in (0, 1)$ known to all that an individual is incapable of work (ill). We assume that w is the marginal product of work which is equal to the wage because of perfect competition in the labour market. We let b denote unearned income which is independent of labour supply capability. Unearned income is assumed to be at subsistence level so that consumption cannot fall below b. The utility of consumption $c \in C \subseteq \Re_+$ if working is $u(c)$. The utility of consumption if not working is $v(c)$ and the utility when not working due to illness is $v(c) - d$. Both $u(c)$ and $v(c)$ are real-valued functions.

Assumption 1 *Positive but diminishing marginal utility:* $u'(c) > 0$, $v'(c) > 0$, $u''(c) < 0$, $v''(c) < 0$.

Assumption 2 *Work is unpleasant:* $v(c) > u(c)\ \forall c$.

Assumption 3 *Employment is preferable to unemployment:* $u(w + b) > v(b)$.

We will assume that it is desirable to share risk and transfer some income from the employed to the unemployed.

Assumption 4 *Risk-sharing is desirable:* $u'(w+b) < v'(b)$.

We will also make an assumption on the disutility of labour.

Assumption 5 $v'(c^u) = u'(c^e)$ *implies* $c^e = c^u + k$ *for some constant* $w > k \geq 0$.

Remark 1 *One special case that satisfies these assumptions is when there is a fixed disutility of employment, so that* $u(c) = v(c) - x$*; in this case* $k = 0$*. Another special case is when the leisure is a perfect substitute for consumption and utility of not working is* $v(c)$ *and the utility of working is* $u(c) = v(c-x)$*, so that* $k = x$ *represents the disutility of labour supply independent of the level of consumption.*

Remark 2 *Assumption 5 implies Assumption 4 that risk-sharing is desirable.*

3 Dynamic Informal Insurance

In this section we outline the model of informal insurance in a two-person and in a continuum economy.

The time horizon is infinite and time is divided into discrete periods $t = 0, 1, 2, \ldots$. We assume that each household is ex ante identical, infinitely lived and discounts per-period utility at a constant factor of $\delta \in (0, 1)$. Per-period utility is determined by a state-dependent von Neumann-Morgernstern utility index as in Section 2. Each individual has a constant probability of illness, p, which is independent of time and other individuals. Thus by the law of large numbers, there is a constant fraction of the population, p, unable to work at any time period. There is complete information: all members of the private insurance arrangement can observe the health status of everyone else so everyone knows who is able to work and who is not. However, there is no enforcement mechanism, so any transfers must be designed to be self-enforcing. For this section there is no government, so no taxes or government transfers. We shall introduce the government public insurance in Section 4 and examine the interaction between public and private insurance in Section 5.

Let h_t denote the employment history of an individual up to and including date t. This history is simply a list of employment status at each date. Let u^t denote unemployment at date t and e^t denote employment at date t. Then h_t is a list of e's and u's. To proceed we make the following assumption of horizontal equity.

Assumption 6 *Horizontal equity: Any two agents with the same history* h_t *receive the same consumption allocation.*

Remark 3 *Thus we are ruling out random contracts or contracts in which agents with the same history alternate their consumptions. Thus we rule out the posibility that the contract allows some healthy agents to be unemployed while agents with the same history are employed.*

We imagine an informal insurance scheme in the continuum where those able to work at date t and have a history h_{t-1}, transfer an amount $\tau(h_{t-1})$ and those unable to work at date t receive $\xi(h_{t-1})$.[1] We will assume that any individual who reneges on the transfer will be excluded from future receipts and therefore will not make any further transfers.

Note too that since there is complete information, feigning ill-health will be observed and regarded as a deviation from the agreed on insurance scheme. Thus anyone deviating in this way is assumed to be punished with autarky in the future. Since, by Assumption 3, shirking yields a lower utility than working, and as a deviation need only be considered when an agent is called upon to make a positive payment, no agent would choose to deviate by shirking since this is dominated by failing to make the payment and working. Hence we can ignore shirking in this section. Thus agents are either well and employed or ill and not; the option of feigning sickness does not arise. The moral hazard problem will become important again in the Sections 4 and 5 when we examine public unemployment insurance.

The short-term loss to an employed individual of making the transfer at time t of $\tau(h_{t-1})$ relative to not making the transfer is

$$u(b + w - \tau(h_{t-1})) - u(b).$$

Let $c^e(h_{t-1}) = b + w - \tau(h_{t-1})$ be the consumption of an employed worker at date t given the history h_{t-1}. The short-term gain at date t for those unable to work is (note the d's cancel out)

$$v(b + \xi(h_{t-1})) - v(b).$$

Again let $c^u(h_{t-1}) = b + \xi(h_{t-1})$ be the consumption of an unemployed worker at date t given the history h_{t-1}. The discounted long-term gain from adhering to the agreed payments from the next period is (discounted back to period $t + 1$)

$$E\left[\sum_{j=0}^{\infty} \delta^j \left((1-p)(u(b + w - \tau(h_{t+j})) - u(b)) + p(v(b + \xi(h_{t+j})) - v(b))\right)\right].$$

where the expectation E is taken over all future histories from date t onward, $\tau(h_{t+j})$ is the payment made by an employed worker at date $t + j + 1$ given that the history up to time t was h_t and $\xi(h_{t+j})$ is the payment received by an unemployed worker at date $t + j + 1$. Letting $U(h_t)$ denote the net discounted surplus utility from date $t + 1$ in an employment state, *i.e.* where the history is $h_{t+1} = (h_t, e)$, and $V(h_t)$ be the net surplus in an unemployment state, *i.e.* where the history is $h_{t+1} = (h_t, u)$, we have the recursive equations

$$U(h_t) = u(b + w - \tau(h_t)) - u(b + w) + \delta\left((1-p)U(h_t, e) + pV(h_t, e)\right),$$

[1] We assume for now that τ and ξ are non-negative and show subsequently that this is in fact the case.

$$V(h_t) = v(b + \xi(h_t)) - v(b) + \delta\left((1-p)U(h_t, u) + pV(h_t, u)\right).$$

We view the sequence of transfers as an implicit or social contract. If an individual reneges on this social contract, then since individuals are identifiable, they will be ostracized and excluded from the contract and not receive any transfers in the future. Since there is no enforcement mechanism, an individual will only be prepared to make a transfer if the long-term benefits from doing so outweigh the short term costs. Since reneging leads to exclusion, the discounted surpluses must be non-negative at every history

$$U(h_t) \geq 0 \qquad \text{and} \qquad V(h_t) \geq 0 \qquad \forall \;\; h_t.$$

An employed worker is said to be constrained if after a sufficient relaxation of the constraint $U(h_t) \geq 0$ it would be possible to find a Pareto-improvement from date $t+1$ onward. An employed worker who is constrained has a zero surplus $U(h_t) = 0$ but an employed worker with a zero surplus is not necessarily constrained. Similar definitions apply to the unemployed worker.

With no enforcement mechanism, although risk-sharing may be desirable, it may not be feasible if δ is small or p is large as the long-term gains cannot outweigh the short-term costs of making a transfer. Thus we make a further assumption to ensure that risk-sharing in the dynamic economy is feasible absent any government transfers.

Assumption 7 *Risk-sharing is feasible:*

$$\delta > \frac{u'(w+b)}{(1-p)v'(b) + pu'(w+b)}.$$

This condition is derived by considering whether any small tax and subsidy that is constant over time can improve on autarky and satisfy the non-negative net surplus conditions. Alternatively the inequality may be expressed as a low enough probability of illness, *i.e.*:

$$p < \frac{\delta v'(b) - u'(b+w)}{\delta(v'(b) - u'(b+w))}.$$

Given Assumption 4 that $v'(b) > u'(b+w)$, Assumption 7 is satisfied for δ close enough to one. Indeed we know from the folk theorem of repeated games that for δ close enough to one, the first-best level of risk sharing with $v'(c^u) = u'(c^e) = u'(c^u - k)$ is sustainable. We will mainly be concerned with situations where the first best is not sustainable. That is for discount factors such that

$$\delta < \frac{u(b+w-p(w-k)) - u(b+w)}{(1-p)(u(b+w-p(w-k)) - u(b+w)) + p(v(b+(1-p)(w-k)) - v(b))}.$$

3.1 The two-person economy

For a two-person economy, we can use the results of Thomas and Worrall (1988). To make the comparison with the continuum economy, assume that employment status is perfectly negatively correlated. Then the transfer τ made by the employed worker is equal to the transfer received ξ by the unemployed worker. The per-capita resources of this two person economy are $b + \frac{w}{2}$ in each period and there is no aggregate uncertainty. Per capita resources in the continuum economy are $b + \frac{w}{2}$ when the probability $p = \frac{1}{2}$ which we will assume for the purposes of comparison. Let the first-best transfer be τ^{**}. By Assumption 5 the first best transfer satisfies $\tau^{**} \leq \frac{w}{2}$. From Thomas and Worrall (1988) it follows that associated with each state, employment or unemployment, there is an time-independent interval for consumption and a simple updating rule which is to keep the marginal utility of consumption constant across states or moves consumption by the smallest amount to keep it within the interval. The consumption intervals are illustrated in Figure 3.1. The top interval determines consumption in the employment state and the lower interval consumption in the unemployment state. The consumption levels at the first-best are $b+w-\tau^{**}$ and $b+\tau^{**}$. As discussed above for a large enough discount factor, the first-best transfer will be sustainable and the two intervals will extent to these first-best levels. For smaller discount factors the first-best will not be sustainable. The lower of the interval endpoints correspond to the individual getting zero net surplus whereas at the top of the endpoint the surplus is at the maximum level. By the updating rule once the employment status has switched, the employed individual will have consumption of $b + w - \tau$ at the bottom of the upper interval and the unemployed individual will have consumption of $b+\tau$ at the top of the lower interval. At the bottom of the upper interval the employed individual is not prepared to make any larger transfer as the short-term costs are too high. Thus the employed individual is constrained at this point and has a net surplus of $U = 0$. The unemployed individual has a short-run gain and thus has a positive net surplus, $V > 0$ (future surpluses are non-negative by construction).

The surplus V is determined recursively by:

$$V = v(b + \xi) - v(b) + \frac{\delta}{2}V$$

and likewise the surplus U must satisfy:

$$U = u(b + w - \tau) - u(b + w) + \frac{\delta}{2}V = 0.$$

Given that $\tau = \xi$ these two equations can be rearranged to give

$$u(b + w) - u(b + w - \tau) = \frac{(\frac{\delta}{2})(v(b + \tau) - v(b))}{(1 - \frac{\delta}{2})}$$

which can be solve for τ as a function of the parameters b, w, and δ. For $\tau = 0$ the LHS and RHS of this equation are zero. For $\tau > 0$ the LHS is positive, increasing and

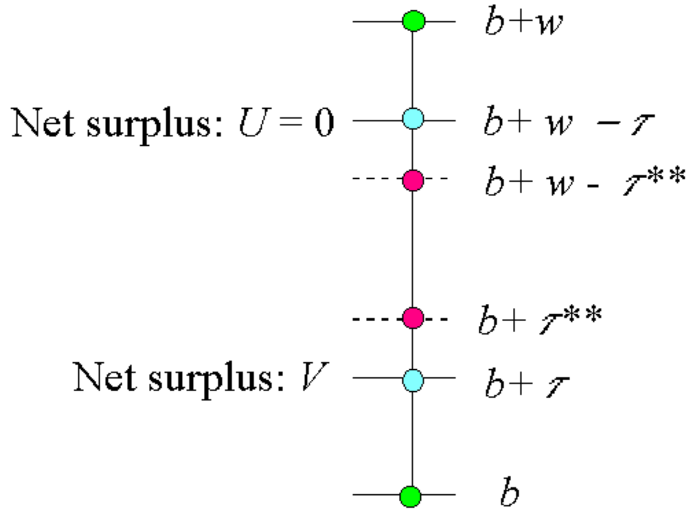

Figure 1: The Consumption Intervals

convex, the RHS is positive, increasing and concave. This is illustrated in Figure 3.1. By Assumption 4 $u'(w+b) < v'(b)$, so for δ large enough the slope of the RHS at $\tau = 0$, $(1-\frac{\delta}{2})v'(b)/(1-\frac{\delta}{2})$ is steeper than the slope of the LHS, $u'(b+w)$ but for small enough $\delta < \delta^*$ the opposite is true and the only solution is $\tau = 0$. For large discount factors $\delta > \delta^{**} > \delta^*$, the optimum transfer τ^{**} can be sustained and the two curves intersect at a $\tau > \tau^{**}$. For intermediate values of δ, the optimum transfer is determined by the intersection of the two curves. As δ increases, the optimum value τ^* increases.

3.2 Results in continuum case

The results of Ligon, Thomas, and Worrall (2002) show that in a model of informal insurance with n households, unconstrained households have the same growth rate in marginal utility and constrained households which have zero net surplus have a lower marginal utility growth rate. Lemmas 2 and 3 show that the same is true in the continuum economy. This follows from simple arbitrage arguments which consider transfers between two individuals so as to equalise the marginal rate of substitution between two dates. It is to be remembered that an individual is distinguished by their employment history. So we will denote the measure of agents with history h_{t-1} by $\mu(h_{t-1})$. Since the probability of illness is independent of history, $\mu(h_{t-1}, e^t) = (1-p)\mu(h_{t-1})$ and $\mu(h_{t-1}, u^t) = p\mu(h_{t-1})$.[2]

[2] If α is the number of periods of unemployment, then $\mu(h_t) = p^{\alpha}(1-p)^{(t-\alpha)}$.

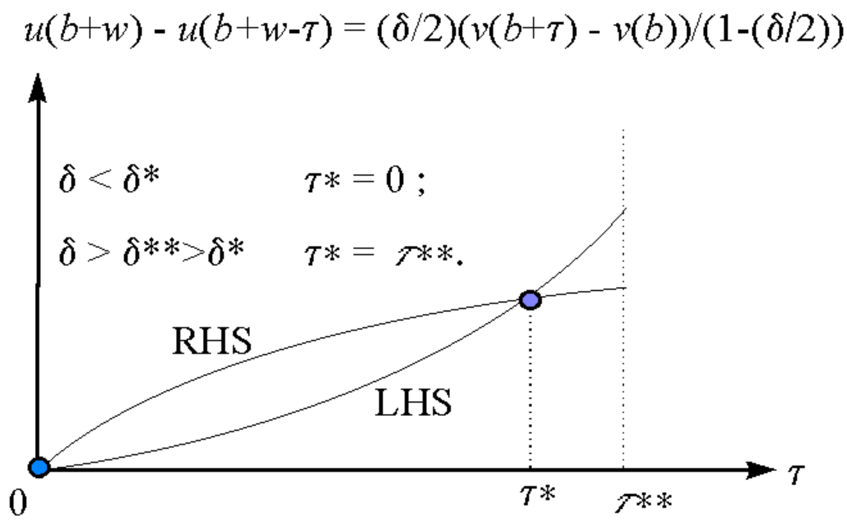

Figure 2: The Solution in the Two-Person Case

The next lemma shows that given Assumption 7 there will be some households with strictly positive net surplus.

Lemma 1 *Given Assumption 7, then there will be some individuals with strictly positive surplus at each date.*

Proof: If all individuals have zero surplus at time $t+1$, *i.e.* $U(h_t) = V(h_t) = 0$, then the only transfers that are feasible at date t are zero, *i.e.* $\tau = \xi = 0$. We now show that a small transfer of $\Delta > 0$ from the employed to the unemployed at every date forward will be beneficial. The transfer received by the unemployed is $\Gamma = \frac{(1-p)}{p}\Delta$. The change in surplus for the employed worker is

$$-u'(b+w)\Delta + \frac{\delta}{(1-\delta)}\left(pv'(b)\Gamma + (1-p)u'(b+w)\Delta\right).$$

Substituting for Γ gives the change in surplus as

$$\frac{\Delta}{(1-\delta)}\left(pu'(b+w) + (1-p)v'(b)\right)\left(\delta - \frac{u'(b+w)}{(pu'(b+w) + (1-p)v'(b))}\right).$$

This change is positive given Assumption 7 and the change in surplus for the unemployed worker is even greater. Hence if all agents have a zero surplus at any date it would be possible to find an improvement that meets all self-enforcing constraints. Thus at each date there will be some subset of agents with a strictly positive surplus. $\|$

Lemma 2 *All workers with a strictly positive surplus at date t (i.e. unconstrained workers),whether employed or unemployed, have the same growth rate in marginal utility between $t-1$ and t.*

Proof: Consider two types of individual with employment histories h_{t-1} and h'_{t-1}. Let the measure of each type be $\mu(h_{t-1})$ and $\mu(h'_{t-1})$. Suppose w.l.o.g. that the employment status for these two types over the next two time periods is (e^t, u^{t+1}) and (e^t, e^{t+1}). Suppose that neither of these types are constrained at time $t+1$ so that $V(h_{t-1}, e) > 0$ and $U(h'_{t-1}, e) > 0$. Now consider a small transfer of Δ from each of the employed at time t with history h_{t-1} with this transfer equally distributed to each of the employed at time t with history h'_{t-1}. The probability of moving to employment at date t is $(1-p)$, so the transfer received at time t is

$$\frac{\Delta\mu(h_{t-1})(1-p)}{\mu(h'_{t-1})(1-p)} = \frac{\Delta\mu(h_{t-1})}{\mu(h'_{t-1})}.$$

Suppose at time $t+1$ there is a transfer Γ in the opposite direction. Thus those with employment history (h_{t-1}, e^t, u^{t+1}) get

$$\frac{\Gamma\mu(h'_{t-1})(1-p)(1-p)}{\mu(h_{t-1})(1-p)p} = \frac{\Gamma\mu(h'_{t-1})(1-p)}{\mu(h_{t-1})p}.$$

Picking Γ and Δ small, the approximate change in utility of an employed worker at date t with history h_{t-1} is:

$$-u'(c^e(h_{t-1}))\Delta + \delta p v'(c^u(h_{t-1}, e)) \left(\frac{\Gamma\mu(h'_{t-1})(1-p)}{\mu(h_{t-1})p}\right)$$

where $c^e(h_{t-1})$ is the consumption of an employed worker at date t given the history h_{t-1} and $c^u(h_{t-1}, e)$ is the consumption of the unemployed worker at date $t+1$ given the employment history (h_{t-1}, e). We choose Γ and Δ to make the change in utility neutral and so:

$$\Gamma \approx \left(\frac{u'(c^e(h_{t-1}))\Delta}{\delta p v'(c^u(h_{t-1}, e))}\right) \left(\frac{\mu(h_{t-1})p}{\mu(h'_{t-1})(1-p)}\right)$$

Equally the change in utility for the employed worker at time t with history h'_{t-1} is approximately:

$$u'(c^e(h'_{t-1})) \left(\frac{\Delta\mu(h_{t-1})}{\mu(h'_{t-1})}\right) - \delta(1-p)u'(c^e(h'_{t-1}, e^t))\Gamma.$$

Then substituting for Γ gives the approximate change in utility for the employed worker at time t with history h'_{t-1} as

$$\Delta u'(c^e(h'_{t-1})) \left(\frac{u'(c^e(h_{t-1}))}{v'(c^u(h_{t-1}, e))}\right) \left(\frac{\mu(h_{t-1})}{\mu(h'_{t-1})}\right) \left(\frac{v'(c^u(h_{t-1}, e))}{u'(c^e(h_{t-1}))} - \frac{u'(c^e(h'_{t-1}, e))}{u'(c^e(h'_{t-1}))}\right) \quad (1)$$

Now choose the sign of Δ to be the same as the sign of the last bracket in this equation. If the bracketed term is non-zero, then this will lead to an improvement in utility for the employed at time t with history h'_{t-1}. Also by construction Δ and Γ have the same sign. Thus if $\Delta < 0$, $\Gamma < 0$ and this involves a transfer from the unemployed at time $t+1$ with history (h_{t-1}, e). But this is feasible since by assumption they are unconstrained, $V(h_{t-1}, e) > 0$. Likewise if $\Delta > 0$, this will involve a transfer from the employed at time $t+1$ with history (h'_{t-1}, e), but again this is feasible as $U(h'_{t-1}, e) > 0$. Such a change raises the discounted utility of the employed with history h'_{t-1} and by construction does not lower the discounted utility of the employed with history (h_{t-1}, e). Equally no constraint at any previous date is violated as all constraints are forward looking. Thus if the initial contact is efficient the bracketed term in the last equation must be zero and the growth rate in marginal utility for both types must be the same. It is clear that by repeating the above argument the same applies for any pair of employment histories we choose. $\|$

Lemma 3 *Any worker that is constrained and therefore has a zero surplus, has a growth rate in marginal utility that is no greater than any unconstrained worker.*

<u>Proof</u>: This follows from the previous lemma. Suppose again that there are two types of individuals with histories h_{t-1} and h'_{t-1} and assume that the employment histories at times t and $t+1$ are (e^t, u^{t+1}) and (e^t, e^{t+1}). Suppose that the type with history (h'_{t-1}, e, e) is constrained at date $t+1$ and suppose that this type has a higher growth rate in marginal utility:

$$\left(\frac{v'(c^u(h_{t-1}, e))}{u'(c^e(h_{t-1}))} < \frac{u'(c^e(h'_{t-1}, e))}{u'(c^e(h'_{t-1}))} \right).$$

Then if the type with history (h_{t-1}, e, u) is unconstrained at date $t+1$ it follows from equation (1) that the surplus of the individual with history (h', e, e) can be improved by choosing $\Delta < 0$. However, since $U(h'_{t-1}, e) = 0$, it is not possible to choose $\Delta > 0$ and hence the bracketed term in equation 1 is non-positive. $\|$

Lemma 4 *If $c^e(h_{t-1}) \neq c^u(h_{t-1})$, then the individual with the higher consumption is constrained.*

<u>Proof</u>: The agent with the higher consumption has the lower growth rate in marginal utility and thus is constrained and has a zero surplus. $\|$

Let $g(t)$ denote the growth rate in marginal utility for an unconstrained individual from date t to date $t+1$. For example if the individual is employed at both dates then

$$1 + g(t) = \frac{u'(c^e(h_{t-1}, e)}{u'(c^e(h_{t-1})}.$$

A direct implication of the fact that the constrained agents have a lower growth rate in marginal utility and thus higher consumption at date $t+1$ is that the unconstrained growth rate in marginal utility is non-negative.

Lemma 5 *Given Assumption 5, the growth rate of the marginal utility of unconstrained agents satisfies $g(t) \geq 0$.*

Proof: Assume to the contrary that $g(t) < 0$. This implies that for every agent marginal utility at time t is greater than the marginal utility at time $t+1$ no matter what the states at each date or whether the worker is constrained or not. Consider a worker with history h_{t-1}. If the history is $h_{t+1} = (h_{t-1}, e, e)$ then $c^e(h_{t-1}, e) > c^e(h_{t-1})$. Likewise for the history $h_{t+1} = (h_{t-1}, u, u)$, $c^u(h_{t-1}, u) > c^u(h_{t-1})$. For the history $h_{t+1} = (h_{t-1}, u, e)$, it follows from Assumption 5 that $c^e(h_{t-1}, u) > c^u(h_{t-1}) + k$ and similarly that for the history $h_{t+1} = (h_{t-1}, e, u)$, $c^u(h_{t-1}, e) > c^e(h_{t-1}) - k$. Since the probability of illness is independent, the histories $h_{t+1} = (h_{t-1}, e, u)$, and $h_{t+1} = (h_{t-1}, u, e)$, are equally likely. Thus summing over all possible histories h_{t-1}, it follows that aggregate consumption rises from time t to $t+1$, but this is impossible as aggregate resources are unchanged. Thus we conclude that $g(t) \geq 0$. $\|$

Lemma 6 *For a given employment status at date t, higher surplus at date t means more consumption at date t.*

Proof: We want to show that $U(h_{t-1}) > U(h'_{t-1})$ if and only if $c^e(h_{t-1}) > c^e(h'_{t-1})$ and $V(h_{t-1}) > V(h'_{t-1})$ if and only if $c^u(h_{t-1}) > c^u(h'_{t-1})$. From the recursive equations

$$U(h_{t-1}) = u(c^e(h_{t-1})) - u(b+w) + \delta\left((1-p)U(h_{t-1}, e) + pV(h_{t-1}, e)\right)$$

$$U(h'_{t-1}) = u(c^e(h'_{t-1})) - u(b+w) + \delta\left((1-p)U(h'_{t-1}, e) + pV(h'_{t-1}, e)\right)$$

First we prove sufficiency. Suppose by contradiction that we have $U(h_{t-1}) > U(h'_{t-1})$ but $c^e(h_{t-1}) \leq c^e(h'_{t-1})$. Then

$$(1-p)U(h_{t-1}, e) + pV(h_{t-1}, e) > (1-p)U(h'_{t-1}, e) + pV(h'_{t-1}, e)$$

Thus either $U(h'_{t-1}, e) < U(h_{t-1}, e)$ or $V(h'_{t-1}, e) < V(h_{t-1}, e)$ or both. It is only possible to have higher surplus if at some future point consumption is higher so w.l.o.g. take this to be period $t+1$. Suppose by way of example that $V(h_{t-1}, e) > V(h'_{t-1}, e) \geq 0$ and $c^u(h'_{t-1}, e) < c^u(h_{t-1}, e)$. But then by risk aversion

$$\frac{v'(c^u(h'_{t-1}, e))}{u'(c^e(h'_{t-1}))} > \frac{v'(c^u(h_{t-1}, e))}{u'(c^e(h_{t-1}))}.$$

This implies that (h_{t-1}, e^t) has a smaller marginal utility growth rate and thus that $V(h_{t-1}, e) = 0$ by Lemma 3 which contradicts $V(h_{t-1}, e) > 0$. A similar argument applies if $U(h_{t-1}, e) > U(h'_{t-1}, e) \geq 0$. To prove necessity again suppose by contradiction that $c^e(h_{t-1}) > c^e(h'_{t-1})$ but $U(h_{t-1}) \leq U(h'_{t-1})$. By the first part of the proof if $U(h_{t-1}) < U(h'_{t-1})$ then we have $c^e(h'_{t-1}) > c^e(h_{t-1})$; a contradiction. Thus suppose that $U(h_{t-1}) = U(h'_{t-1})$. Now choose a convex combination of the contracts so that both types get the same consumption at all dates from t onwards. By concavity this leads to a Pareto-improvement, with all the self-enforcing constraints satisfied. Equally all self-enforcing constraints at past periods are met or relaxed. Thus the original contract could not have been efficient. $\|$

Lemma 7 *For a given employment status at date t, there is a unique consumption level which delivers zero surplus at date t.*

Proof: Follows from previous lemma. $\|$

Lemma 8 (Non-crossing lemma) *If consumption is no lower at time t, then consumption in the same state at $t+1$ will also mean that consumption is no lower.*

Proof: We want to show that e.g. $c^e(h_{t-1}) \geq c^e(h'_{t-1})$ implies $c^e(h_{t-1}, e^t) \geq c^e(h'_{t-1}, e^t)$ and $c^u(h_{t-1}, e^t) \geq c^u(h'_{t-1}, e^t)$. Assume to the contrary that say $c^u(h_{t-1}, e^t) < c^u(h'_{t-1}, e^t)$. Then

$$\frac{v'(c^u(h'_{t-1}, e^t))}{u'(c^e(h'_{t-1}))} < \frac{v'(c^u(h_{t-1}, e^t))}{u'(c^e(h_{t-1}))}.$$

which implies that (h'_{t-1}, e^t, u^{t+1}) is constrained from Lemma 3. But by Lemma 6, $V(h'_{t-1}, e^t) > V(h_{t-1}, e^t) \geq 0$, a contradiction. $\|$

Let $\underline{c}^e(t)$ denote the consumption which delivers zero surplus in employment at date t and let $\underline{c}^u(t)$ denote the consumption which delivers zero surplus in unemployment at date t (this is defined so long as there is a positive measure of agents receiving zero surplus in each employment state).

Lemma 9 *At any time t, $b < \underline{c}^e(t) \leq b + w$ and $\underline{c}^u(t) = b$.*

Proof: It is obvious that $\underline{c}^e(t) \leq b + w$. If $\underline{c}^e(t) > b + w$, then there is a short-run gain for the individual but a net surplus of zero. This would imply a negative net surplus at some future date which is impossible. Equally by the same argument $\underline{c}^u(t) \leq b$. But if $\underline{c}^u(t) < b$ then there is a negative net gain at t which must be offset by some positive net gain in the future. Since the growth rate in marginal utility is non-negative this would only be possible if $\underline{c}^u(t)$ falls continuously. But this is impossible as consumption is bounded below. $\|$

We now show that provided the first-best cannot be achieved, any agent employed at date t is constrained and hence consumes $\underline{c}^e(t)$.

Lemma 10 *Assuming that the first-best is not attainable, at each t an employed agent is constrained.*

Proof: Suppose that the employed at $t-1$ are all constrained: $U(h_{t-2}) = 0$ for all h_{t-2}. To show that the employed at t are constrained, first assume that some who are employed both at $t-1$ and t have marginal utility growth equal to $g(t)$ that is $u'(c^e(\hat{h}_{t-2}, e))/u'(c^e(\hat{h}_{t-2})) = 1+g(t)$ for some $\hat{h}_{t-2}$. We show this is impossible, and hence all such agents must be constrained at t. First, the growth rate must be the same in the unemployment state at t: $v'(c^u(\hat{h}_{t-2}, e))/u'(c^e(\hat{h}_{t-2})) = 1+g(t)$ since otherwise $c^u(\hat{h}_{t-2}, e) = b$, by Lemma 9, and thus $u'(c^e(\hat{h}_{t-2}, e)) > v'(b)$, which is impossible. Suppose w.l.o.g. that thereafter, as soon as the employed state occurs, say at any time $t' \geq t+1$, total surplus discounted to t' will be zero, i.e., $U(\hat{h}_{t-2}, e_{t-1}, s_t, u_{t+1}, \ldots u_{t'-1}) = 0$ (where $s_t = u_t, e_t$). This is w.l.o.g. as we can consider the path where u is repeated from date t, until the last time that $v'(c^e(\hat{h}_{t'-2}, e_{t-1}, s_t, u_{t+1}, \ldots, u_{t''-1}))/u'(c^u(\hat{h}_{t'-2}, e_{t-1}, s_t, u_{t+1}, \ldots, u_{t''-2})) = 1+g(t'')$. Thereafter, by definition, as soon as the employed state occurs, total surplus will be zero. We can use t'' to replace t. To simplify notation, define per-period surpluses $S^e_{t-1} = u(c^e(\hat{h}_{t-2})) - u(w+b)$, $S^e_t = u(c^e(\hat{h}_{t-2}, e)) - u(w+b)$ and $S^u_{t'} = v(c^u(\hat{h}_{t-2}, e, u, \ldots, u)) - v(b)$ for all $t' \geq t$ ($t'-t-1$ periods of unemployment after $t-1$). Note that the surplus from $t+1$ on is the same after both histories $(\hat{h}_{t-1}, e)$ and $(\hat{h}_{t-1}, u)$, since marginal utilities are the same at t by assumption, and the transition from t to $t+1$ depends only on marginal utility at t. Define Z to be the total surplus from $t+1$ onwards:

$$Z = pS^u_{t+1} + \delta p^2 S^u_{t+2} + \delta^2 p^3 S^u_{t+3} \ldots \tag{2}$$

(using the fact that after date t an employment state implies total surplus from that point is zero). We have by virtue of $U(\hat{h}_{t-2}) = 0$,

$$\begin{aligned} -S^e_{t-1} &= (1-p)\delta U(\hat{h}_{t-2}, e) + p\left(\delta S^u_t + \delta^2 Z\right) && (3) \\ &\geq p\left(\delta S^u_t + \delta^2 Z\right) && (4) \end{aligned}$$

where the inequality follows from $U(\hat{h}_{t-2}, e) \geq 0$. By $U(\hat{h}_{t-2}, e) \geq 0$,

$$-S^e_t \leq \delta Z. \tag{5}$$

In view of $S^u_{t'+1} \leq S^u_{t'}$ for all $t' \geq t$ due to $g(t'+1) > 0$ (no first-best), we have $S^u_t \geq S^u_{t'}$ for all $t' > t$. From (2), this implies $pS^u_t/(1-\delta p) > Z$. Hence

$$p\left(\delta S^u_t + \delta^2 Z\right) > \delta Z \tag{6}$$

We also have $S^e_{t-1} > S^e_t$ due to $g(t'+1) > 0$. Using this and combining (4), (5) and (6), we have $-S^e_{t-1} \geq p\left(\delta S^u_t + \delta^2 Z\right) > \delta Z \geq -S^e_t > -S^e_{t-1}$, a contradiction. Since the initial time period has all employed agents making the same transfer, this transfer must be such that the employed initially have a zero net surplus and this completes the proof. ||

Theorem 1 *At any time t the transition rule from $t-1$ is determined by two numbers $b+(1-p)w \leq \underline{c}_e(t) \leq b+w$ and $g(t) \geq 0$ such that the transition between states satisfies*

1. *A transition to an employment state*

$$c^e(h_{t-1}, u) = c^e(h_{t-1}, e) = \underline{c}^e(t).$$

2. *A transition to an unemployment state*

   (a) *From an unemployment state*

$$c^u(h_{t-1}, u) = \begin{cases} v'^{-1}((1+g(t))v'(c^u(h_{t-1}))), & \text{if } c^u(h_{t-1}, u) \geq b \\ b, & \text{otherwise.} \end{cases}$$

   (b) *From an employment state*

$$c^u(h_{t-1}, e) = \begin{cases} v'^{-1}((1+g(t))u'(\underline{c}^e(t))), & \text{if } c^u(h_{t-1}, e) \geq b \\ b, & \text{otherwise.} \end{cases}$$

Proof: It has already been shown that the workers either have a growth rate in marginal utility of $g(t) \geq 0$ or are constrained. Equally it has been shown that employed workers are constrained, get a zero surplus and consume $\underline{c}^e(t)$ and unemployed workers who are constrained consume b. ||

Note that $\underline{c}_e(t)$ and $g(t)$ are jointly determined via the aggregate resource constraint.

Remark 4 *Results are easily generalized to more than two states. For example w may be state dependent. The same key features apply: unconstrained agents have the same growth rate in marginal utility and there is some unique consumption level associated with giving a zero surplus in each state.*

Remark 5 *It is possible to introduce an aggregate shock, say with w varying over time but common to all workers.*

3.3 The steady-state

In the steady-state $\underline{c}_e(t) = \underline{c}_e$ and $g(t) = g$ independent of t. All employed households have the same consumption $\underline{c}_e$ and make the same transfer $\tau = b+w-\underline{c}_e$. They are always constrained and have a zero surplus, $U(h_t) = 0$ for any past history. The unemployed are either constrained with consumption of b or are unconstrained and have a marginal utility growth rate of g. The implications are that if full insurance is not sustainable $\underline{c}_e > b+w-\tau^{**}$, then there are a finite set of consumption states. If there are $S+1$ such states indexed $s = 0, 1, \ldots, S$, with $s = 0$ indexing the employed state, then the proportion of the population in state s in the steady-state is $(1-p)p^s$ for $s = 0, 1, \ldots, S-1$ and p^S for state S.

There are two important things to note here. First, the steady-state determines the constant distribution of wealth. But although the distribution of wealth is constant over time, there is mobility of individuals within the distribution as their length of unemployment or employment status changes. Secondly the unemployed will receive a transfer from the insurance scheme, but the transfer falls with each consecutive unemployment state and eventually falls to zero after S periods of unemployment. In the optimum informal insurance scheme benefits are declining over time and are time limited.

It is easy to compute the net surplus that each agent receives in the steady-state. Let c_s denote consumption after s successive periods of unemployment. We have $c_s = v'^{-1}((1+g)v'(c_{s-1}))$ for $s = 2, 3, \ldots, S-1$, $c_s = b$ for $s \geq S$ and $c_1 = v'^{-1}((1+g)u'(\underline{c}_e))$. For notational consistency let $c_0 = \underline{c}_e$ be the consumption in the employment state in the steady-state. Then let V_s denote the net surplus of an unemployed worker who has had s successive periods of unemployment. Since the employed worker receives a zero net surplus, the surplus equations are:

$$\begin{aligned}
0 &= u(c_0) - u(b+w) + \delta p V_1 \\
V_1 &= v(c_1) - v(b) + \delta p V_2 \\
\vdots &= \vdots \\
V_s &= v(c_s) - v(b) + \delta p V_{s+1} \\
\vdots &= \vdots \\
V_{S-1} &= v(c_{S-1}) - v(b) + \delta p V_S \\
V_S &= 0
\end{aligned}$$

Since an employed worker receives no surplus, the equation for V_s consists only of the short term utility benefit $v(c_s) - v(b)$ plus the discounted value of the surplus from the subsequent unemployment period, V_{s+1}. This surplus is discounted by the adjusted discount factor $p\delta$ where the discount factor δ is adjusted by multiplying by the probability of unemployment p. Solving these equations recursively gives

$$(v(c_0) - u(c_0)) + (u(b+w) - v(b)) = \sum_{s=0}^{S-1} p^s \delta^s \left(v(c_s) - v(b)\right). \tag{7}$$

Given the distribution of consumption, there is also an aggregate constraint that aggregate consumption equals aggregate resources:

$$(1-p)\sum_{s=0}^{S-1} p^s c_s + p^S b = b + (1-p)w. \tag{8}$$

Since each c_s depends only on c_0 and g, these two equations together with the condition that $c_S = b$ determine c_0, g and S. The aggregate social welfare in the steady-state relative to autarky is

$$(1-p)(u(c_0) - u(b+w)) + \sum_{s=1}^{S-1} (1-p)p^s \left(v(c_s) - v(b)\right).$$

To see how the steady-state can be computed, consider an example where $u(c) = v(c) - x = \log_e(c) - x$ where x is the disutility of labour. In this case $c_s = \frac{c_0}{(1+g)^s}$ for $s = 0, 1, \ldots, S-1$. Since $g > 0$, a constant growth rate in marginal utility translates to a proportionate fall in consumption with successive periods of unemployment. With consumptions so determined, the surplus equation (7) can be rewritten as

$$\begin{aligned}
\log_e(b) - \log_e(b+w) &= \sum_{s=0}^{S-1} \beta^s \left(\log_e(c_0) - \log_e(b) - s\log_e(1+g)\right) \\
&= \frac{(1-\beta^S)}{(1-\beta)} \left(\log_e(c_0) - \log_e(b)\right) \\
&\quad - \left(\frac{\beta(1-\beta^S)}{(1-\beta)^2} - \frac{S\beta^S}{(1-\beta)}\right) \log_e(1+g)
\end{aligned} \tag{9}$$

where $\beta = p\delta$ is the adjusted discount factor. Let T solve the equation $c_S = b = \frac{c_0}{(1+g)^T}$, *i.e.* $T = \frac{\log_e(c_0) - \log_e(b)}{\log_e(1+g)}$. Since $c_S = b$, $S = \lceil T \rceil$ where $\lceil T \rceil$ is the smallest integer greater than or equal to T. Substituting these conditions into equation (9) gives

$$\log_e(b+w) - \log_e(b) = \log_e(1+g) \left(\frac{T}{(1-\beta)} - \frac{(T - \lceil T \rceil)\beta^{\lceil T \rceil}}{(1-\beta)} - \frac{\beta(1-\beta^{\lceil T \rceil})}{(1-\beta)^2} \right)$$

This provides a continuous mapping from T into the growth rate of marginal utility g. Equally in this case of log utility the aggregate constraint (8) becomes

$$c_0 = \left(\frac{(1-\gamma)}{(1-\gamma^{\lceil T \rceil})}\right) \left(w + \left(\frac{(1-p^{\lceil T \rceil})}{(1-p)}\right) b\right)$$

where $\gamma = \frac{p}{(1+g)}$ adjusts the probability of unemployment by the proportionate fall in consumption, so that the consumption of the employed worker, c_0 is a function of the growth rate g and T. Write $g = f(T)$ and $c_0 = h(f(T), T)$. Then the function

$$\zeta(T) = \frac{(\log_e(h(f(T), T) - \log_e(b))}{\log_e(1 + f(T))}$$

maps T back into itself. Finding a fixed point of this continuous mapping gives the steady-state solution. Note that $T = 1$ is always a fixed point of the mapping and since T is defined on $[1, \infty)$ there may be no fixed point greater than one. The two extreme cases

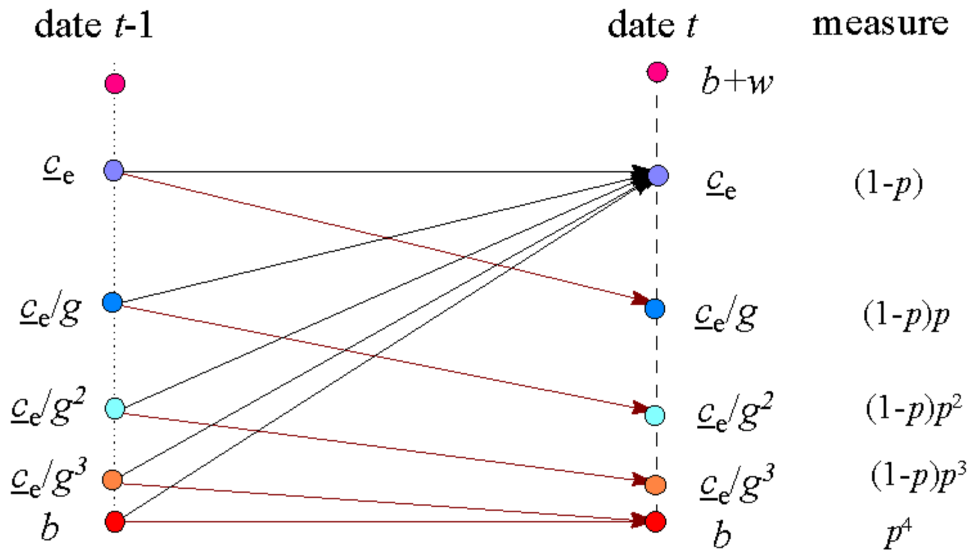

Figure 3: The Steady-State Solution

are where $T = 1$ is the only fixed point and where there is no other fixed point but the mapping diverges to infinity. The first case corresponds to the situation where no private insurance is sustainable and will occur for a low discount factor or a high probability of unemployment. The second case applies where full insurance can be sustained by the private insurance arrangement. These two cases are easily checked in the numerical analysis so that finding a relevant fixed point for T is an easy computational exercise.

A steady-state distribution is drawn in Figure 3.3. It is drawn for $u(c) = v(c) = \log_e(c)$, *i.e.* where $x = 0$ and for $S = 4$.

Example 1 *The solution when* $u(c) = v(c) = log_e(c)$, $b = 1$, $w = 3$, $p = \frac{1}{2}$, $\delta = \frac{1}{2}$, *is* $c_0 = 3.11796$, $g = 0.34132$ *and* $S = 4$. *The unemployed are excluded from benefits after four periods of unemployment and the probability of an unemployed individual receiving no benefits is* $2^{-4} = \frac{1}{16}$.

3.4 Dynamics

This section considers the dynamics of the optimal private insurance and whether there may be convergence to the steady-state solution. We consider only the simple example where $S \leq 2$. At date $t = 1$ the initial distribution has a proportion p with unemployed and receiving $c^u(1)$ and a proportion $(1 - p)$ who are employed and receiving $\underline{c}^e(1)$. In the example it is shown that for all subsequent periods $S = 2$, $(1 - p)$ are employed with consumption $\underline{c}^e(t)$, $p(1 - p)$ are in their first period of disability receiving $c^u(t)$ and the remaining proportion p^2 have a longer term unemployment and have a consumption of b. The employed are constrained in each period.

The aggregate constraint at date $t = 1$ is

$$(1-p)\underline{c}^e(1) + pc^u(1) = b + (1-p)w$$

and the aggregate constraint for $t > 1$ is

$$(1-p)\underline{c}^e(t) + p(1-p)c^u(t) + p^2 b = b + (1-p)w.$$

Given these aggregate constraints, we have a dynamic equation for $c^u(t)$ given by

$$u(w + \frac{b - pc^u(1)}{(1-p)}) - u(b+w) + \delta p(v(c^u(2)) - v(b)) = 0$$

and for $t > 1$

$$u(w + (1+p)b - pc^u(t)) - u(b+w) + \delta p(v(c^u(t+1)) - v(b)) = 0.$$

The dynamic equation is

$$c^u(t+1) = v^{-1}\left(v(b) + \frac{u(b+w) - u(w+(1+p)b - pc^u(t))}{\delta p}\right).$$

Differentiation shows that $\frac{dc^u(t+1)}{dc^u(t)} > 0$ and $\frac{d^2c^u(t+1)}{d(c^u(t))^2} > 0$. There is a unique (non-zero) stationary point which by the convexity is unstable. Clearly any unstable path is inefficient or violates a self-enforcing constraint, therefore consumption will be chosen at the stable point from $t > 1$. The next example shows that it is possible to construct the exact dynamic solution in a simple case.

Example 2 $b = 1$,$w = 3$,$p = \frac{1}{2}$, $\delta = \frac{2}{5}$, $u(c) = v(c) = log_e(c)$. *Then*

$$c^u(t+1) = e^{5\left(log_e(4) - log_e(\frac{9}{2} - \frac{1}{2}c^u(t))\right)}.$$

$\underline{c}_e(t) = 3.45$ *for all* $t \geq 1$, $c^u(1) = 1.55$ *and* $c^u(t) = 2.1$ *for all* $t > 1$.

4 Public Unemployment Insurance

In this section the static model of public unemployment insurance introduced by Diamond and Mirrlees (1978) is outlined. Unlike the private insurance scheme, the public insurance scheme must respect a *moral hazard constraint* since the government is unable to observe whether the worker is ill. Also unlike private insurance arrangements, taxes, *i.e.,* payments into the scheme, can be *enforced* by the government.

The government chooses the tax θ for the employed and level of subsidy σ for the unemployed that determine the consumption levels $c^e = b + w - \theta$ and $c^u = b + \sigma$ for the

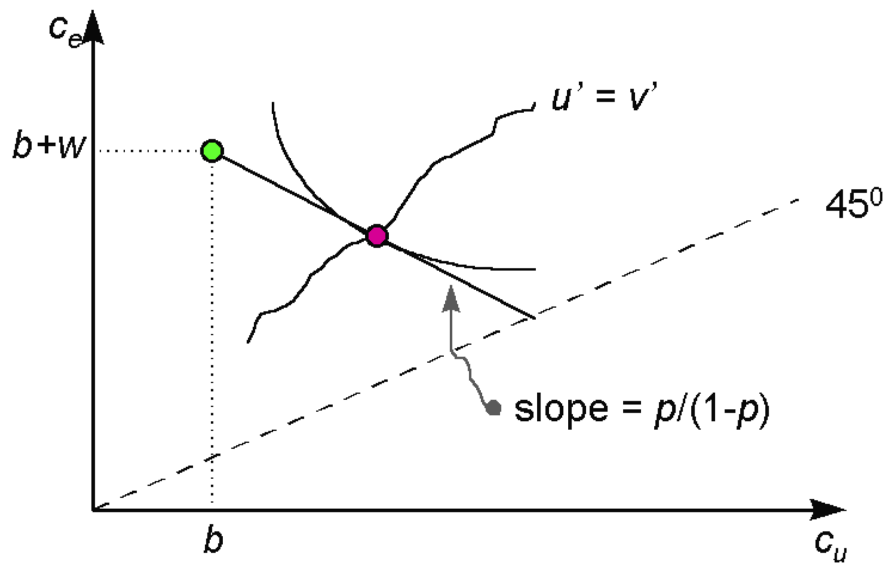

Figure 4: The First-Best Outcome

employed and unemployed respectively. Because there is a continuum of individuals and the probability of illness is independent, the aggregate resource constraint is:

$$(1-p)c^e + pc^u = b + (1-p)w$$

or $(1-p)\theta = p\sigma$. Thus the tax/subsidy scheme is assumed revenue neutral. It is assumed that $\theta \in [0, w]$ as consumption cannot fall below b. Since the government is unable to determine why an individual does not work—are they feigning ill health?—its policy must also respect the moral hazard constraint that a healthy individual has no incentive to claim to be ill rather than working,

$$u(b + w - \theta) \geq v(b + \sigma). \tag{10}$$

The additional aggregate social welfare created by the scheme over autarky is

$$(1-p)(u(b + w - \theta) - u(b + w)) + p(v(b + \sigma) - v(b)).$$

We assume that the government wants to choose $\theta \in [0, w]$ to maximize this aggregate social welfare subject to the budget balance and moral hazard constraints.

Theorem 2 (Diamond-Mirrlees) *If $u(c^e) = v(c^u)$ implies $u'(c^e) \leq v'(c^u)$ for all c^e and c^u, then the optimum is determined by the solution to $u(c^e) = v(c^u)$ and the budget balance condition $(1-p)c^e + pc^u = b + (1-p)w$.*

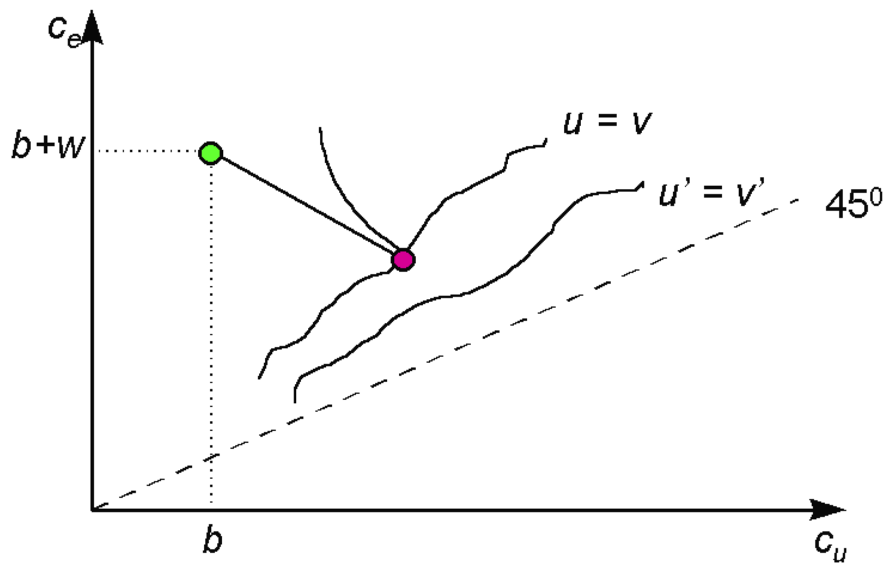

Figure 5: The Moral Hazard Constraint

Henceforth we will maintain this assumption.

Assumption 8 *The moral hazard constraint binds for the government:* $u(c^e) = v(c^u)$ *implies* $u'(c^e) \leq v'(c^u)$ *for all* c^e *and* c^u*.*

To illustrate this theorem first consider the first-best allocation of Figure 4. At the first-best only the ill will not work and the marginal utility between working and not working will be equalised. The autarkic allocation is at the point $(b, b+w)$ and a transfer of one unit from each of the $(1-p)$ proportion of the population employed will give each unemployed $p/(1-p)$ units, so the slope of the aggregate budget constraint is $p/(1-p)$. The social welfare function is $(1-p)(u(c^e)-u(b+w))+p(v(c^u)-v(b))$ and the indifference curve tangent with the budget constraint is drawn. The slope of the indifference curve is $pv'(c^u)/(1-p)u'(c^e)$, so tangency occurs at a point where $v'(c^u) = u'(c^e)$ and the locus of such points is also drawn. By diminishing marginal utility, this locus is upward sloping and the indifference curve is convex. By the assumption that risk-sharing is desirable, this locus passes below the endowment point. By Assumption 5 at the first-best optimum the employed get no less than the unemployed, so that the $v'(c^u) = u'(c^e)$ locus lies above the 45^0 line.

Figure 4 illustrates the Diamond-Mirrlees solution. It must involve equal utility for the employed and unemployed as by the convexity of the indifference curve this gets closer to the first-best insurance. The solution lies at the intersection between the budget constraint and the $u(c^e) = v(c^u)$ locus. The $u(c^e) = v(c^u)$ locus lies above the $v'(c^u) = u'(c^e)$ locus by Assumption 8 that the first best is not feasible.

5 Crowding Out

In this section we turn to the main issue in this paper. How do the public and private insurance schemes interact? Does public insurance crowd out private insurance? And can it be optimal to have a mix of public and private insurance?

To do this we bring together the analysis of Section 3 and Section 4. We consider the effect on the steady-state of private insurance from a (static) government insurance scheme of the Diamond-Mirrlees type. The public insurance scheme will affect the private insurance provision by changing the fall-back utility of both the employed and unemployed. The public insurance will provide some risk-sharing gains by reducing the variability of marginal utility for the employed and unemployed. However, in achieving these risk-sharing gains, the public insurance will make the punishment of removal of future private insurance from anyone who reneges on their private insurance payments less severe, and therefore may reduce the risk-sharing achieved by the private insurance arrangement itself. Thus it is unclear *a priori* which effect may dominate or whether a combination of public and private insurance may be optimal.

In addition to the assumption that the government cannot observe illness, it is assumed that the government cannot observe the workings of the private insurance arrangement (*i.e.*, it cannot observe the consumption of individuals), but can only observe an individual's employment status. Thus the tax or subsidy can only be based on employment status and not consumption. Further we analyse a static public insurance where tax or subsidy depends *only* on current employment status. Thus the public insurance scheme we consider here is of the same form as that examined in Section 4. It is simply a tax on the employed of $\theta \in [0, w]$ and a subsidy to the unemployed of σ. It is assumed that the public insurance is revenue neutral. Given that moral hazard problem is solved, so that the fraction of the population unemployed is indeed p, the condition for revenue neutral insurance is as before $(1-p)\theta = p\sigma$.

For given (σ, θ), the private insurance scheme will solve exactly the same problem as given in Section 3, except that income in the employment state is now $b + w - \theta$, and in the unemployment state it is $b + \sigma$.

The relevant moral hazard constraint of the government is that of a worker in the private insurance scheme but contemplating shirking and collecting government unemployment insurance even though not ill. Such an individual will be observed as shirking by his fellows and therefore will receive no future benefits from the private insurance arrangement. We now show that if we consider the steady-state of the private insurance scheme, the moral hazard constraint for the government is indeed equation (10) of Section 4.

Lemma 11 *The moral hazard constraint in the model with both public and private insurance is given by (10), i.e., it is the same as in the model with only public insurance:*

$$u(b + w - \theta) \geq v(b + \sigma).$$

Suppose that this equation holds and consider an employed agent in the private insurance arrangement who is contemplating feigning ill health to take advantage of the government subsidy to the unemployed. As demonstrated in Subsection 3.3 a healthy and employed worker at t receives $U(h_{t-1}) = 0$, where the surplus is now measured relative to what they would have outside the private insurance scheme, *i.e.* $b+w-\theta$ for the employed and $b+\sigma$ for the unemployed. If illness is feigned, the current utility gain over autarky would be $v(b+\sigma) - u(b+w-\theta)$. By assumption, the feigning of ill health would be recognized by fellow members of the private insurance scheme so that in the future the individual would not participate in private insurance and would be reliant only on government insurance. Since the public insurance scheme respects the moral hazard constraint of equation (10), no-one participating in the government insurance scheme will feign ill health and hence the future net surplus of any individual feigning ill health in the current period is zero. Thus the overall gain to an individual in the private insurance scheme of feigning ill health is $v(b+\sigma)-u(b+w-\theta)$ which is non-positive by assumption. Thus given that $U(h_{t-1}) = 0$, equation (10) is necessary and sufficient for no individual to have an incentive to feign illness at t.

Remark 6 *Since the moral hazard constraint is the same whether an agent is receiving private insurance or not our analysis also applies to the situation where there is some fraction of agents outside the private insurance scheme. Thus it is possible to undertake a welfare analysis of the effect of public insurance even when only a fraction of the agents participate in private insurance. Clearly the smaller the fraction of the population that are members of a private insurance scheme, the greater the weight that will be given to public insurance.*

Remark 7 *In line with Section 4, we do not consider negative taxes, i.e. taxation of the unemployed. Although there is a lower bound on consumption of b, negative taxes may be feasible if it could be guaranteed that the private insurance arrangements stepped in to offset any tax on the unemployed. This would be impossible if there were some fraction of agents outside the private insurance scheme. Since the government cannot by assumption observe whether an individual receives private insurance, we rule out negative taxes as infeasible.*

It is easy to construct examples such that for certain parameter values, public insurance alone will be optimal and for other parameter values private insurance will be optimal. From Assumption 7 it is known that there will be no private insurance if the risk-sharing gains are sufficiently small, *i.e.* if the discount factor is sufficiently small or if the probability of unemployment is sufficiently high. Thus if the parameter values are such that no private insurance is feasible, and the government's moral hazard constraint does not bind, then it is possible to raise welfare through public insurance alone. If the moral hazard constraint binds before any private insurance becomes feasible, then public insurance alone will be optimal. Equally if the discount factor is high enough that the first-best can be sustained by private insurance, then private insurance alone will achieve the optimum as by Assumption 8 the government is constrained from achieving the first-best by the moral hazard constraint.

As we now show it is possible to construct examples where the government can inadvertently lower welfare by increasing public insurance. That is there may be more than 1-1 crowding out of private insurance. Equally it is possible to construct examples where a combination of public insurance and private insurance can actually raise welfare. Since it is difficult to obtain analytical results, our examples are constructed numerically.

For simplicity we will consider the case where $u(c) = v(c) - x = \log_e(c) - x$ where x is the disutility of labour. The aggregate welfare generated by the public insurance relative to autarky is:

$$(1-p)(\log_e(b+w-\theta) - \log_e(b+w)) + p(\log_e(b+\sigma) - \log_e(b))$$

which is increasing in θ given Assumptions 4 and 8. The moral hazard constraint in this case is

$$\log_e(w+b-\theta) - \log_e(b+\sigma) \geq x$$

and this will limit the tax θ that can be imposed on the employed. The aggregate social welfare in the steady-state generated by private insurance relative to the fall back of only public insurance is

$$(1-p)(\log_e(c_0) - \log_e(b+w-\theta)) + \sum_{s=1}^{S-1}(1-p)p^s(\log_e(c_0) - \log_e(b+\sigma) - s\log_e(1+g)).$$

The steady-state for the private insurance can be computed as described in Subsection 3.3 and the net welfare from public insurance and private insurance calculated for different values of public taxation θ. The total welfare can then be computed[3] and the diagrams below show the welfare from public insurance, private insurance and total welfare as a percentage of the first-best welfare relative to autarky.

The following example shows that there can be more than 1-1 crowding out:

[3] It is assumed that only a fraction of agents of measure zero are outside the private insurance scheme in the calculations below.

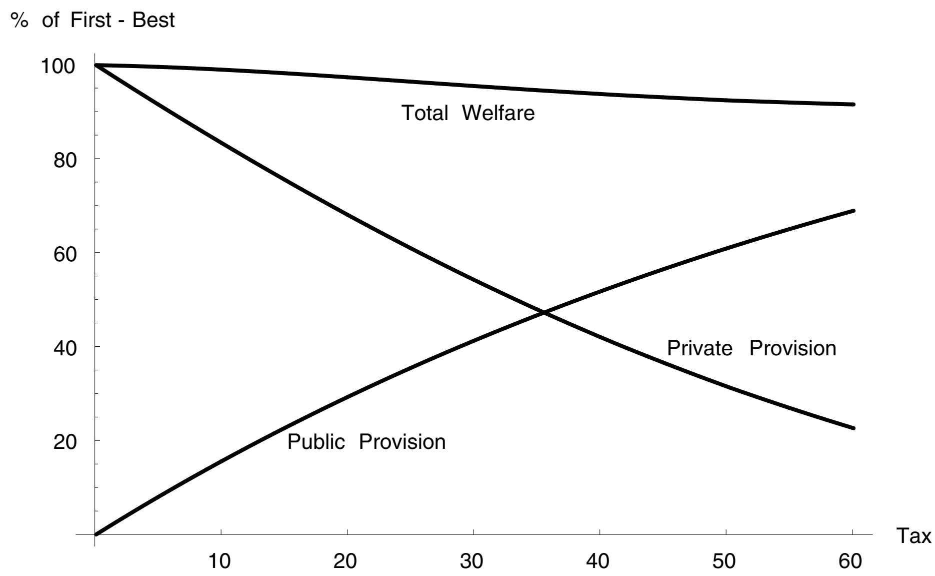

Figure 6: The Crowding-Out Effect

Example 3 *Suppose $u(c) = v(c) - x = log_e(c)$, $b = 100$, $w = 300$, $p = \frac{1}{2}$, $\delta = \frac{2}{3}$, $x = \log_e(\frac{17}{8})$. The welfare from public and private insurance and total welfare for different values of tax (consistent with the moral hazard constraint) is plotted in Figure 5. The maximum tax is $\theta = 60$. At $\theta = 60$, the public insurance scheme generates 68.90% of the first-best surplus. At this tax rate, the private insurance generates an additional 22.63% of the first-best surplus. Thus the public and private insurance schemes together generate 91.53% of the first-best surplus. At $\theta = 0$ the private insurance scheme alone generates 99.88% of the first-best surplus. So the optimum is to have no public insurance and have only private arrangements provide unemployment insurance.*

The next example demonstrates that a mixture of private and public insurance may dominate either public or private insurance alone.

Example 4 *Suppose $u(c) = v(c) - x = log_e(c)$, $b = 515$, $w = 125$, $p = \frac{4}{5}$, $\delta = \frac{19}{20}$, $x = \log_e(58/53)$. The aggregate welfare for different values of tax is plotted in Figure 5. With $x = \log_e(\frac{58}{53})$ the maximum tax is $\theta = 60$. At $\theta = 60$, the public insurance scheme generates 83.21% of the first-best surplus. At this tax rate, the private insurance generates an additional 9.66% of the first-best surplus. Thus the public and private insurance schemes together generate 92.87% of the first-best surplus. At $\theta = 0$ the private insurance scheme alone generates 89.70% of the*

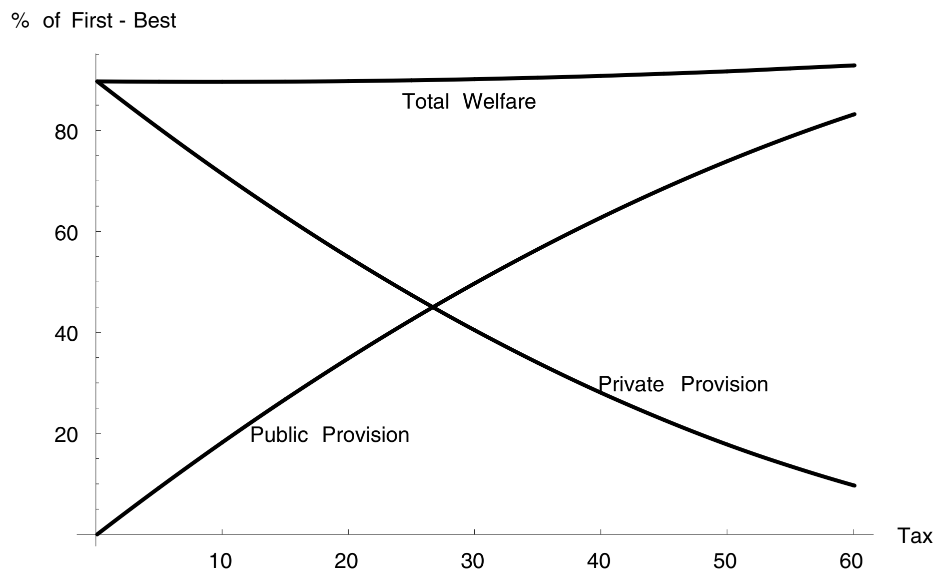

Figure 7: An Optimal Mix of Public and Private Insurance

first-best surplus but still less than that achieved with the maximum tax rate of $\theta = 60$

6 Conclusion

We have considered a model of the interaction between private and public insurance schemes. The advantage of the private insurance scheme is that it does not face the moral hazard constraint faced by government. However, the disadvantage of the private insurance scheme is that it cannot enforce payments into the scheme in the way the government scheme can.

We have developed a model of private insurance in a large economy using straightforward arbitrage arguments. The optimum private insurance scheme has a number of interesting properties. The amount received falls with the length of unemployment and is time limited. After a certain length of unemployment no insurance benefits are received. In the steady-state all the employed pay the same amount into the private insurance scheme irrespective of their past unemployment history.

We have shown that there can be more than 1-1 crowding out of private insurance by public insurance. That is a government that introduced additional public insurance may end up lowering welfare because of the private insurance it crowds out. We have also demonstrated by way of example that a mixture of public and private insurance may maximise steady-state welfare.

There remains work for further research. We have considered only the impact of the public insurance on the steady-state. It would be interesting to understand how welfare changes along any dynamic path toward a steady-state. The govenment in contrast to the private sector is assumed to adopt a static taxation system that depends only on the current employment status and not the employment history. Many government insurance schemes are responsive to employment history and exhibit some features such as declining and time limited benefits that we have found to optimal in the private insurance scheme. Again future research could address how a dynamic model of government insurance interacts with private insurance. Further in our model unemployment is exogenous and not affected by agents' decisions. Examining the welfare consequences in a model where the extent of insurance had an impact on the level of unemployment could provide an interesting extension of the model.

References

Anderberg, D., and F. Andersson (2000): "Social insurance with risk-reducing investments," *Economica*, 67, 37–56.

Arnott, R., and J. E. Stiglitz (1991): "Moral hazard and nonmarket institutions: Dysfunctional crowding out or peer monitoring?," *American Economic Review*, 81, 179–190.

Attanasio, O., and J.-V. Ríos-Rull (2000): "Consumption smoothing in island economies: Can public insurance reduce welfare?," *European Economic Review*, 44, 1225–1258.

Di-Tella, R., and R. MacCulloch (2002): "Informal Family Insurance and the Design of the Welfare State," *Economic Journal*, 112, 481–503.

Diamond, P. A., and J. A. Mirrlees (1978): "A model of social insurance with variable retirement," *Journal of Public Economics*, 10, 295–336.

Kreuger, D., and F. Perri (1999): "Risk sharing: Private insurance markets or redistributive taxes?," Mimeo.

Ligon, E., J. P. Thomas, and T. Worrall (2000): "Mutual insurance, individual savings and limited commitment," *Review of Economic Dynamics*, 3(2), 216–246.

——— (2002): "Informal insurance arrangements with limited commitment: Theory and evidence from village economies," *Review of Economic Studies*, 69(1), 209–244.

Lustig, H. (2001): "The market price of aggregate risk and the wealth distribution," Mimeo.

Thomas, J. P., and T. Worrall (1988): "Self-Enforcing Wage Contracts," *Review of Economic Studies*, 55(4), 541–554.

Whinston, M. D. (1983): "Moral hazard, adverse selection, and the optimal provision of social insurance," *Journal of Public Economics*, 22, 49–71.

KERP Keele Economics Research Papers — Recent Contributions

All papers in the KERP series are available for downloading from the Keele Economics website, via *www.keele.ac.uk/depts/ec/kerp*.

2002/21 *Damaged Durable Goods*
Jong-Hee Hahn

2002/20 *Unemployment Insurance under Moral Hazard and Limited Commitment: Public vs Private Provision* Jonathan P Thomas and Tim Worrall

2002/19 *The Water Poverty Index: an International Comparison*
Peter Lawrence , Jeremy Meigh and Caroline Sullivan

2002/18 *Price-Capping regulation as a protectionist strategy in developing countries*
Peter Lawrence and Arijit Mukherjee

2002/17 *Licensing and the Incentive for Innovation*
Arijit Mukherjee and Soma Mukherjee

2002/16 *Joint Production Games with Mixed Sharing Rules*
Richard Cornes and Roger Hartley

2002/15 *Sequential Models of Bertrand Competition for Deposits and Loans under Asymmetric Information* Frédérique Bracoud

2002/14 *Consumption Patterns over Pay Periods*
Clare Kelly and Gauthier Lanot

2002/13 *Licensing under Asymmetric Information*
Arijit Mukherjee

2002/12 *Emission Targets and Equilibrium Choice of Technique*
Martin E Diedrich

2002/11 *Dissipation In Rent-Seeking Contests With Entry Costs*
Richard Cornes and Roger Hartley

2002/10 *Advantageous or Disadvantageous Semi-collusion*
Arijit Mukherjee

2002/09 *Licensing in a Vertically Separated Industry*
Arijit Mukherjee

2002/08 *U-shaped Paths of Consumption and Phys. Capital in Lucas-type Growth Models*
Farhad Nili

2002/07 *On the Variance Covariance Matrix of the M.L. Estimator of a Discrete Mixture*
Gauthier Lanot

2002/06 *Monotonicity and the Roy Model*
Arnaud Chevalier and Gauthier Lanot

2002/05 *Capacity Commitment and Licensing*
Arijit Mukherjee

2002/04 *Household Credit and Saving: Does Policy Matter?*
Peter Lawrence

2002/03 *Innovation, Licensing and Welfare*
Arijit Mukherjee

2002/02 *Historical Schools of Economics: German and English*
Keith Tribe

KERP Keele Economics Research Papers — Recent Contributions

All papers in the KERP series are available for downloading from the Keele Economics website, via *www.keele.ac.uk/depts/ec/kerp*.

2002/01 *R&D, Licensing and Patent Protection*
Arijit Mukherjee

2001/09 *Export and Direct Investment as a Signal in Global Markets*
Arijit Mukherjee and Udo Broll

2001/08 *The Welfare Effects of Quality Degradation with Network Externalities*
Jong-Hee Hahn

2001/07 *Cost Padding in Regulated Monopolies*
Spiros Bougheas and Tim Worrall

2001/06 *Is the Unskilled Worker Problem in Developing Countries Going Away?*
Ed Anderson

2001/05 *Does Society Prefer Small Innovation?*
Arijit Mukherjee

2001/04 *Bilateral Merger in a Leadership Structure*
Tarun Kabiraj and Arijit Mukherjee

2001/03 *Imitation, Patent Protection and Welfare*
Arijit Mukherjee and Enrico Pennings

2001/02 *R&D Organization and Technology Transfer*
Arijit Mukherjee and Sugata Marjit

2001/01 *International Joint Venture and the Technology of the Future*
Sugata Marjit, Arijit Mukherjee and Tarun Kabiraj

2000/20 *Gift-Giving, Quasi-Credit and Reciprocity*
Tim Worrall and Jonathan P Thomas

2000/19 *Land Rents and Competitive Equilibrium*
Martin E. Diedrich

2000/18 *Monopoly Quality Differentiation with Top-quality Dependent Fixed Costs*
Jong-Hee Hahn

2000/17 *Time Consistency and Intergenerational Risk Sharing*
Tim Worrall

2000/16 *The Maximum Interest Rate on an Unbalanced Growth Path*
Martin E. Diedrich

2000/15 *Nonlinear Pricing of a Telecomm. Service with Call and Network Externalities*
Jong-Hee Hahn

2000/14 *Rent- seeking by Players with Constant Absolute Risk Aversion*
Richard Cornes and Roger Hartley

2000/13 *Differential Interest Rates on Unbalanced Growth Paths*
Martin E. Diedrich

2000/12 *Functional Quality Degradation of Software with Network Externalities*
Jong-Hee Hahn

2000/11 *Network Competition and Interconnection with Heterogeneous Subscribers*
Jong-Hee Hahn

ISSN 1352-8955

Department of Economics
Keele University
Keele, Staffordshire ST5 5BG
United Kingdom

tel: (44) 1782 583091
fax: (44) 1782 717577
email: economics@keele.ac.uk
web: www.keele.ac.uk/depts/ec/web/