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Running to Keep in the Same Place: Consumer Choice as a Game of Status

Abstract

We investigate consumer choice where individuals care not only about the absolute values of consumption, but also about their status. This is defined as their ordinal rank in the distribution of consumption of one "positional" good. In such a situation, the consumer's problem becomes strategic as her utility will depend on the consumption choices of others. In the symmetric Nash equilibrium of the resulting game, each individual spends more on visible consumption than in the absence of a concern for status and has lower utility. Treating status endogenously allows us to analyze how exogenous changes in the distribution of income can affect individual choices. In a more affluent society, individuals spend a higher proportion of their income on the positional good, which leads to a reduction in utility at each income level. In a more equal society those with lower incomes spend more on conspicuous consumption and are worse off. We go on to analyze externality-correcting consumption taxes and subsidies.

1   Introduction

> "Now *here*, you see, it takes all the running you can do to keep in the
> same place."
>
> Lewis Carroll (1871), "Through the Looking-Glass"

Neoclassical economic theory assumes that an agent's utility depends solely on the absolute level of personal consumption. An alternative assumption, that utility or happiness depends at least in part in the comparison of one's own consumption to that of others, dates back at least to Veblen's seminal work of 1899. More recently, compelling evidence has accumulated that people tend to evaluate what they consume in the light of the consumption of others. For example, a number of studies have found that self-reported happiness is sensitive to relative as well as to absolute income.[1] There is also now a developing theoretical literature that examines the implications of the presence of relative concerns in agents' preferences.

In this paper, we take a new approach that emphasizes the strategic nature of concerns for relative position and analyzes their interaction with the distribution of income. Assume that an agent's status depends on her consumption relative to that of others. Assume further that her utility depends at least in part on her status. Then the choice of levels of consumption is necessarily strategic, because each agent must anticipate the consumption decisions of others in making her optimal consumption decision. Here we model concern for status, as indicated by ordinal rank in the distribution of consumption, as a simultaneous move game. In the symmetric Nash equilibrium, an individual's position in the distribution of consumption coincides with his position in the distribution of income. That is, everyone increases conspicuous consumption in order to improve status, but any gain in status is cancelled out by the similarly increased expenditure of others. Such an economy can be described as a Lewis Carroll "Red Queen" economy, in which "it takes all the running you can do to keep in the same place".[2]

Furthermore, treating status endogenously allows us to analyze how exogenous changes in income distribution can affect individual choices. For the form of preferences that we analyze, the problem of the consumer, on a formal level, is very similar to that of a bidder participating in a first price auction. We are able to employ techniques from auction theory to show that as income in society increases (in the sense of a refinement of first order stochastic dominance), the more significant is the "Red Queen" effect: the proportion of income spent on conspicuous consumption increases and equilibrium utility falls at each level of income. Partly this is because, as a society becomes richer, those whose incomes do not grow spend more on conspicuous consumption in an attempt to keep up. Second, we show that if the income becomes more equally distributed (in a sense of a new refinement of second order stochastic dominance, or,

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[1]For a survey of the evidence on happiness see Oswald (1997). Further empirical evidence on the importance of relative concerns can be found in Frank (1985a), van de Stadt, Kapteyn, van de Geer (1985), Clark and Oswald (1996), Solnick and Hemenway (1998), Neuwark and Postlewaite (1998).

[2]The idea of using Lewis Carroll's Red Queen as a metaphor for competition has already been used in evolutionary biology and in evolutionary game theory.

equivalently, generalized Lorenz dominance), those with lower incomes spend more on conspicuous consumption and their utility falls. Finally, we consider some policy implications of the model and find that a suitable consumption tax can be welfare improving. Perhaps surprisingly, however, we find that if the income distribution changes in the direction of greater equality, the marginal rate of taxation for those with low incomes should rise, and for those with high incomes the marginal rate should fall.

The idea that the poor might lose from greater equality is somewhat surprising, but it reflects the old phrase "misery loves company". Consider those with low income who are left behind when others' incomes are raised as consequence of an increase in overall equality. These people now see fewer people with similar or lower incomes. Furthermore, they observe the increased consumption expenditure of those who have benefitted from the change in income distribution. There is therefore social pressure to raise their own consumption levels. In contrast, at the top of the income distribution an increase in equality will reduce the competition for status as it thins the ranks of the rich. Consequently, the rich may gain from an increase in equality.

These results appears to be particularly timely in that the relationship between happiness, income and inequality has been subject to much recent empirical work. Indeed, there is some empirical support for our finding that inequality and happiness can be positively linked. Alesina, di Tella and MacCulloch (2001) find that there is greater satisfaction with inequality amongst the poor than the rich in the US. Clark (2000), using British data, finds a positive relationship between inequality and self-reported happiness. Equally, however, Alesina, di Tella and MacCulloch find that in Europe inequality and happiness are negatively related, a result echoed in work on German data by Schwarze and Härpfer (2002). The difference in results perhaps reflects different social norms and dynamic issues such as social mobility, not captured by our static model. However, one result that is consistent across these studies is that relative income seems to matter for happiness, an observation first made by Easterlin (1974).

Assuming that people care about their relative position leaves unanswered how exactly such preferences should be modelled. Indeed, we can divide the existing literature on this topic into two strands. The first approach, stemming from Duesenberry (1949) and Pollack (1976), employs interdependent preferences represented by utility functions that depends not only on the absolute value of consumption, but also on the average level of consumption (referred to as "the Joneses") within a population.[3] In this paper, however, we concentrate on the other formulation of interdependent preferences that involves concern with one's status, as indicated by the ordinal rank in the distribution of consumption but also potentially income or wealth. It was pioneered by Frank (1985b) in a study of the demand for positional and non-positional goods. Robson (1992) considered preferences over absolute wealth as well as ordinal rank in wealth. Direr (1999) considers preferences over absolute and relative consumption in each of two periods of the lifetime of an individual. However, the existing models of status as rank approach the problem largely non-strategically. That is, they assume that each individual makes

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[3]Further works in this growing literature include Abel (1990), Gali (1994), Harbaugh (1996), Corneo and Jeanne (1997) and Clark and Oswald (1998).

a decision with the actions of others fixed. A notable exception is Cooper, García-Peñalosa and Funk (2001) who noticed the strategic nature of the problem but analyse a model where agents only interact with other agents of the same wealth. What is of a greater interest to us is the interaction between the distribution of income, conspicuous consumption and welfare. This, to our knowledge, has not been explored before.

The possible reasons why people may possess a concern for status are also diverse. First, this type of preference may be intrinsic or "hard-wired", a fundamental human characteristic. Many economists would be happier with the alternative possibility that agents can have an "instrumental" concern for status, that is, they do not value status itself but seek it because high status allows better consumption opportunities. This second approach has been advocated by Postlewaite (1998). Consumption and saving decisions when status is instrumental were studied extensively by Cole, Mailath and Postlewaite (1992, 1995, 1998), Corneo and Jeanne (1998). In their models, status as indicated by ordinal rank in the distribution is instrumental to a (marriage) matching. That is, those with higher rank have higher consumption because they marry better. Our model is consistent with either underlying motivation.

The paper is organized as follows. Section 2 introduces the game of status and shows the existence of a unique symmetric equilibrium. Section 3 shows how comparative statics predictions can be obtained on equilibrium consumption behavior, and outlines the implications of these results for welfare. Section 4 explores the possibilities of a corrective consumption tax. Section 5 explores the robustness of the comparative statics results. Section 6 concludes.

2   A Game of Status

Consider the problem of a consumer who must decide how to divide her income between the purchase of two different goods. The neoclassical solution to this problem can be found in any textbook on microeconomics. However, now imagine that expenditure on one of the two goods provides some form of status which gives an agent utility distinct from the direct utility from its consumption. For example, if the Joneses buy a large car, this may arouse the envy and admiration of their neighbors, pleasing the Joneses. Suppose in particular that this status arises from relative and not absolute levels of consumption. That is, it is not just that the car is big but that it is bigger than those owned by the neighbors that also matters. Then, first, the good is *positional* in sense of Hirsch (1976). Second, the standard problem of choosing consumption levels becomes a game between consumers. Individuals will engage in competition in terms of conspicuous consumption of the positional good, that is, in a game of status.

There may be various reasons of why people care about status. As Veblen (1899) and later Duesenberry (1949) argued, people may aspire for higher status as an end in itself. Individuals gain psychological satisfaction from being better off than others and feel uneasy when they see others doing better. To be more specific, let $F(\cdot)$ be the distribution of the consumption of the positional good in society. For an individual

whose own level of conspicuous consumption is x, $F(x)$ gives the expected frequency with which she will be able to make these pleasurable favorable comparisons in terms of visible prosperity between herself and another individual. Her utility will then be increasing in $F(x)$. However, it is also possible that an agent's position in the distribution of conspicuous consumption can enter his utility function for conventional economic reasons, as Postlewaite (1998) suggested. That is, individuals may care about their status mostly "instrumentally", as societies frequently allocate goods according to one's status rather than through markets. An example is when marriage arrangements are such that one's status determines possible marriage partners. Conspicuous consumption bestows status and thus allows better marriage opportunities as it signals wealth that is otherwise unobservable.[4]

We do not try to adjudicate between these two possible explanations as to why people may have relative concerns.[5] Instead, we analyze the behavior of agents possessing such preferences. This is done in the context of a simultaneous move game of incomplete information. We assume an economy consisting of a continuum of agents, identical except in terms of income. Each agent is endowed with a level of income z which is private information and is an independent draw from a common distribution. This is described by a distribution function $G(z)$ which is strictly increasing and continuous on some interval $[\underline{z}, \bar{z}]$ with $\underline{z} \geq 0$. We follow Frank (1985b) and assume that each agent must choose how to allocate his income between a visible (positional) good which carries status and another (non-positional) good, the consumption of which is not directly observable by other agents. Let x be the amount consumed of the positional good, and y the amount of the non-positional good. We will refer to consumption of x as conspicuous consumption. Agents' choices of conspicuous consumption are aggregated in a distribution of conspicuous consumption $F(\cdot)$. Finally, for an agent who chooses a level of consumption x, let her status be defined as $F(x)$, her rank within that distribution. We assume that each agent faces the following problem,

$$\max_{x,y} U(x, y, F(\cdot)) = V(x, y)F(x), \text{ subject to } px + y \leq z, \ x \geq 0, \ y \geq 0 \qquad (1)$$

where p is the price of the positional good. The price of the non-positional good is normalized to one. We assume that $V(\cdot)$ is strictly increasing in both its arguments, quasiconcave and twice differentiable. We further assume that $V_{ii} \leq 0$ for $i = 1, 2$ and that $V_{ij} \geq 0$ for $i \neq j$.[6]

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[4]Marriage matching for a continuum population has been extensively explored in Cole, Mailath and Postlewaite (1992, 1995, 1998). The link between matching, status and conspicuous consumption under incomplete information is studied by Corneo and Jeanne (1998). For a finite population, Eeckhout (2000) demonstrates that when each sex can rank each member of the other according to a common criterion, which here we would take to be status, the only stable voluntary matching mechanism is the one in which each woman is matched to a man whose rank in the distribution of males is equal to her rank in the distribution of females.

[5]Indeed, they are not mutually exclusive. Perhaps it is because such marriage matching problems were important for our ancestors that we have preferences for status now. Rege (2001), for example, finds that a concern for social status can be evolutionarily stable in a matching market.

[6]Maskin and Riley (2000b) adopt a similar assumption which they term "weak supermodularity". They use it, as do we, to ensure that optimal strategies are strictly increasing. See Lemma 1 in the Appendix.

The fact that the status term $F(x)$ enters multiplicatively into the preferences (1) brings out the formal resemblance of the problem to a first-price sealed-bid auction, where a bidder gains a utility $V(x, y)$ if she wins with a bid x and $F(x)$ is the probability of winning. Increasing one's expenditure on the positional good leads to a trade-off between the increase in status and the lowering of direct utility from decreased consumption of the non-positional good, just as a bidder in an auction must trade off increasing his probability of winning against lower realized profits in the event of winning. It is this formal resemblance to an auction that permits clear comparative statics results.

This specification also has the implication that the lowest ranked agent will always gain zero utility. This requires a few words of justification. As indicated above, we have in mind that social rank may have a significant impact on marriage prospects with, at least in our evolutionary past, low rank often leading to failure to reproduce. In an evolutionary context, this is the lowest payoff possible. Even today, one could argue that very low social status is associated with unemployment, poor marriage prospects and social exclusion. It is in this context that it is possible that even people with very low incomes may try hard to increase their status. We examine the robustness of our results by analyzing an alternative specification that allows for the lowest ranked individual to gain positive utility in Section 5.

In this context, a *symmetric equilibrium* will be a Nash equilibrium in which all agents use the same strategy, that is, the same mapping $x(z)$ from income to expenditure. Suppose for the moment that the equilibrium strategy $x(z)$ is strictly increasing (we will go on to show that such an equilibrium exists). Note, first, that this would imply that in equilibrium an agent's status $F(x)$ is equal to his rank $G(z)$ in the distribution of income. Second, note that the marginal increase in status from an increase in an agent's expenditure on x can be calculated as $dF(x)/dx = g(z)x'(z)^{-1}$. If we continue with the maximization problem (1), substituting using the budget constraint and with a bit of manipulation, the resultant first order conditions can be written as the single equation,

$$V_1(x, z - px) - pV_2(x, z - px) + V(x, z - px)\frac{f(x)}{F(x)} = V_1 - pV_2 + \frac{V}{x'(z)}\frac{g(z)}{G(z)} = 0. \quad (2)$$

Note that the first two terms in (2) are the first order conditions for the standard consumer problem. Now there is an additional term that represents the additional marginal return to expenditure on x due to enhanced status. The equation (2) implies the following first-order differential equation:

$$x'(z) = \frac{g(z)}{G(z)}\frac{V}{pV_2 - V_1} = \frac{g(z)}{G(z)}\psi(x, z) \quad (3)$$

We can show that the solution to this differential equation is a symmetric equilibrium of the game of status.

Proposition 1 *The game of status has a unique symmetric Nash equilibrium given by the unique solution to the differential equation (3). Equilibrium conspicuous consumption $x(z)$ is strictly increasing in income z so that rank in the positional good is equal to rank in income, that is, $F(x) = G(z)$.*

While it is only possible to obtain an explicit characterization of equilibrium for specific utility functions and income distributions, we are able obtain quite general results in terms of comparative statics. But first, we make one observation about the equilibrium of the game of status. We define the equivalent of what Frank (1985b) calls the *cooperative case*, which will prove useful as a point of comparison, particulary when we consider corrective taxes in Section 4. It simply assumes that each agent makes a consumption choice (x_c, y_c) according to the standard tangency condition. That is,

$$\frac{V_1(x_c, y_c)}{V_2(x_c, y_c)} = p. \tag{4}$$

Note that, as it has been assumed that both goods are normal, $x_c(z)$ is strictly increasing. This would imply that the richest agent will consume the most and therefore have the highest status. More generally the distribution of conspicuous consumption and hence status will be identical to the distribution of income, just as they are in the Nash equilibrium.

If one then compares (4) with the noncooperative first order conditions (2), it is clear that spending on conspicuous consumption in the noncooperative equilibrium is higher than in the cooperative case. Yet in both cases, status exactly reflects the original income distribution. In the equilibrium, the additional expenditure on conspicuous consumption has no net effect on individual's position in the societal hierarchy, and thus it is "wasteful" in the sense it leads to a Pareto inferior outcome.[7] Frank (1985b) obtained a similar result taking status as exogenous. If all agents could agree to stick to the cooperative solution, everyone would be better off. But this is not a Nash equilibrium.

The advantage of determining status endogenously is that it allows individual consumption behavior to be analyzed in terms of the original income distribution. In fact, it is possible to obtain quite general comparative static results on the effect of changes in the entire income distribution on equilibrium behavior, and consequently the implications for welfare. We will explore this in the next section.

3   The Distribution of Income and Conspicuous Consumption

Taking status endogenously not only seems to be a more reasonable approach but it also allows us to consider the effect of a change in the entire income distribution on consumer choice. From our analysis in the previous section, we know that equilibrium demand is given by a solution to the differential equation (3). To obtain an explicit solution to this differential equation, one has to place strong restrictions on the form of

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[7]This may be true from the point of view of the participants in the game of status, but maybe not for society as a whole. Some forms of conspicuous expenditure, extravagant presents, lavish dinners for example, may represent transfers to those one is trying to impress.

the utility function and the distribution function. Luckily, however, some comparative static analysis of the equilibrium consumptions decisions is possible even without an explicit solution.

Consider two societies, A and B, which differ only in terms of income distributions given by cumulative distributions G_A and G_B respectively, both having support on $[\underline{z}, \bar{z}]$. We first consider how changes in the distribution of income which make society better off affect conspicuous consumption. To do this, we need a way to order two income distributions, so that we can say one is "higher" than the other. The most common ordering of this kind is (first order) stochastic dominance. However, stochastic dominance, though a strong condition in itself, is not sufficient to obtain clean comparative statics in the games of status we consider. Thus we employ the following refinement of first order stochastic dominance.[8]

Definition (MPR): The two distributions G_A, G_B satisfy the Monotone Probability Ratio (MPR) order and we write $G_A \succ_{MPR} G_B$ if the probability ratio $G_A(z)/G_B(z)$ is strictly increasing on $(\underline{z}, \bar{z}]$.

The monotone probability ratio order implies that the ratio $\frac{G_A(z)}{G_B(z)}$ is strictly increasing over $(\underline{z}, \bar{z}]$ and as $G_A(\bar{z}) = G_B(\bar{z}) = 1$ that $G_A(z) < G_B(z)$ for all $z \in (\underline{z}, \bar{z})$. This shows that the MPR order implies that G_A (first order) stochastically dominates G_B.[9] Moreover, average income in society A is greater than that in society B, or $\mu_A = \int z dG_A > \int z dG_B = \mu_B$. The relationship also implies that

$$\frac{g_A(z)}{G_A(z)} > \frac{g_B(z)}{G_B(z)} \tag{5}$$

for all $z \in (\underline{z}, \bar{z})$.

We will also wish to consider changes in the level of inequality in society. Second order stochastic dominance (or, equivalently, generalized Lorenz dominance), since the work of Atkinson (1970), has become a standard way in which to rank income distributions in terms of inequality. However, like first order stochastic dominance it is not sufficient to obtain comparative statics results. We now introduce a strengthening of second order stochastic dominance analogous to the monotone probability ratio order.[10]

Definition (UPR): Two distributions G_A, G_B satisfy the Unimodal Probability Ratio (UPR) order and we write $G_A \succ_{UPR} G_B$ if the ratio of their distribution functions $G_A(z)/G_B(z)$ is unimodal and $\mu_A \geq \mu_B$. That is, it is strictly increasing for $z < \hat{z}$ and

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[8]Researchers have resorted to similar refinements to obtain monotone comparative statics in games of incomplete information, such as auctions. The use of this particular refinement can also be found in Eeckhoudt and Gollier (1995), Lebrun (1998) and Maskin and Riley (2000a). This ordering has also been referred to as the "reverse hazard rate order" by Shaked and Shanthikumar (1994). Maskin and Riley's "conditional stochastic dominance" is more general.

[9]The fact that G_A first order stochastic dominates G_B can be interpreted as implying that an individual with a given income z occupies a lower hierarchical position in the society A than in the society B.

[10]The properties and implications of this concept are explored further in Hopkins and Kornienko (2001).
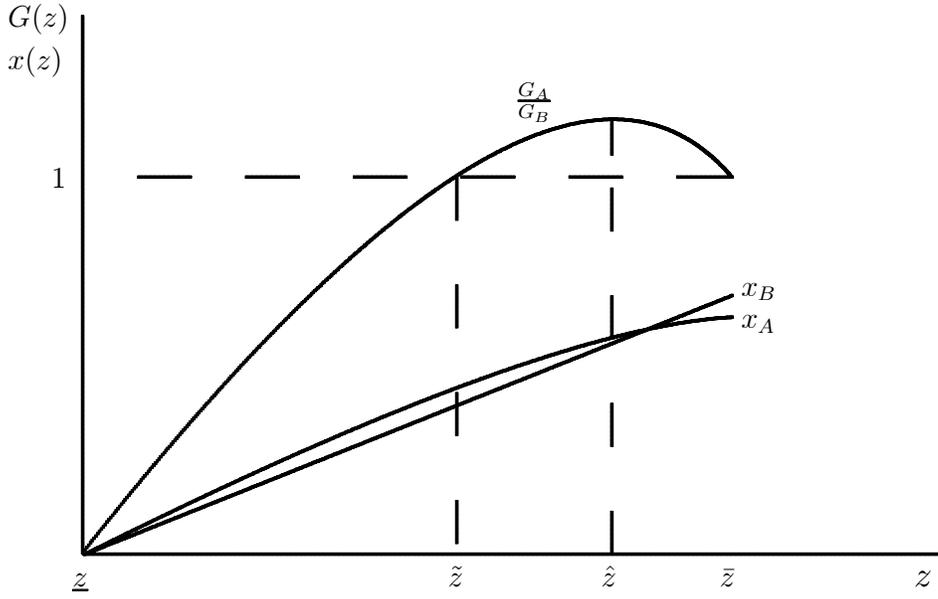
Figure 1: Comparative statics of solutions under unimodality of the probability ratio.

it is strictly decreasing for $z > \hat{z}$ for some $\hat{z} \in (\underline{z}, \bar{z}]$.

In simple terms, if an income distribution $G_A \succ_{UPR} G_B$, then G_A is more equal and less dispersed than G_B. More precisely, it can be shown that if $G_A \succ_{UPR} G_B$, then G_A also second order stochastically dominates G_B (Hopkins and Kornienko, 2001, Proposition 2). This in turn implies that if $G_A \succ_{UPR} G_B$ and the means are in fact equal, then G_B is a mean preserving spread of G_A. If $G_A \succ_{UPR} G_B$ then the ratio G_A/G_B will have a unique maximum on $(\underline{z}, \bar{z}]$.[11] Let $\hat{z}$ be that value of z that maximizes the ratio. If $\hat{z} = \bar{z}$, then the above condition reduces to the monotone probability ratio order, or in other words, the monotone probability ratio order implies the unimodal probability ratio order.

We are now ready to state our main comparative static result.

Proposition 2 *Suppose $x_A(z)$ and $x_B(z)$ are the equilibrium choices of the positional good for distributions G_A and G_B, respectively. If $G_A(z) \succ_{UPR} G_B(z)$, then either $x_A(z) > x_B(z)$ almost everywhere, or there exists a point $z^* > \hat{z} = \arg\max G_A(z)/G_B(z)$ such that $x_A(z^*) = x_B(z^*)$, $x_A(z) > x_B(z)$ for all $z \in (\underline{z}, z^*)$ and $x_A(z) < x_B(z)$ for all $z \in (z^*, \bar{z})$.*

Stated less formally, the conspicuous consumption of the poor, that is, those with income less than $\hat{z}$, will rise as the income distribution becomes more equal in the sense of a second order dominance relationship. The conspicuous consumption of the rich,

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[11]The condition that $\mu_A \geq \mu_B$ rules out the possibility that $G_A(z) > G_B(z)$ for all interior z.

that is, agents with incomes more than $\hat{z}$, may or may not rise. This is illustrated in Figure 1, which shows sample solutions for two income distributions where G_B is a mean preserving spread of G_A. Because in addition, $G_A \succ_{UPR} G_B$, x_A, conspicuous consumption in society A, is necessarily greater than x_B at income levels up to $\hat{z}$. At income levels above $\hat{z}$, the two solutions may cross once as depicted in Figure 1. But equally for some pairs of distributions conspicuous consumption will be unambiguously higher in the more equal society.

We are also interested in what happens to the demand for positional goods as a society's income increases. As stated above the MPR order implies the UPR order. Hence, we can derive the following corollary as the special case of the above proposition when $\hat{z} = \bar{z}$.

Corollary 1 *Suppose $x_A(z)$ and $x_B(z)$ are the equilibrium choices of the positional good for distributions G_A and G_B, respectively. If $G_A(x) \succ_{MPR} G_B(z)$, then $x_A(z) > x_B(z)$ for all $z \in (\underline{z}, \bar{z})$.*

The above corollary says that equilibrium demands for the positional good are higher (and, thus, equilibrium demands for the non-positional good are lower) in the more affluent society A. That is the proportion of income spent at each income level on the positional good is higher (thus proportion spent on the non-positional good is lower) in the society A, i.e. $x_A(z)/z > x_B(z)/z$ and $y_A(z)/z < y_B(z)/z$ for all z.

Example 1 *Suppose utility is given by $U = xyF(x)$ and that $G_B(z) = z$ on [0,1], that is a uniform distribution, and $G_A = 3z^2 - 2z^3$, a Beta distribution which is unimodal and hence less dispersed than G_B (both distributions have mean 1/2). It is easily verified that $G_A \succ_{UPR} G_B$ and the graph of the ratio $G_A/G_B = z(3 - 2z)$ is similar to that in Figure 1, with a crossing point of the distributions at $\tilde{z} = 1/2$ and a maximum of their ratio at $\hat{z} = 3/4$. An explicit solution exists for the uniform case given by $x_B = 2z/3$. By numerical solution, we find that the solution x_A crosses x_B from above with a value of z^* of approximately 0.86.*

We now investigate the implications of these comparative statics results for welfare. Our results concern the utility of someone whose income remains unchanged as the distribution of income in society changes. Under classical assumptions, if an individual's income remained unchanged while the distribution of income changes, her utility would be unchanged. Here, this is not the case and it happens for two reasons. First, a change in the distribution of income will alter an individual's equilibrium status $G(z)$. Second, equilibrium expenditure on conspicuous consumption will change. As we will see, an individual who suffers a fall in status as others' incomes rise will be forced to increase conspicuous consumption in an attempt to keep up, leading to yet lower utility. We first look at the effect of an increase of equality in the sense of the UPR order. Let $U^*(z) = V(x^*(z), y^*(z))G(z)$ be the individual utility gained in the symmetric equilibrium.

Proposition 3 *If $G_A \succ_{UPR} G_B$ then there is at most one interior point $\tilde{z}$ such that $G_A(\tilde{z}) = G_B(\tilde{z})$. Furthermore, $U_A^*(z) < U_B^*(z)$ for all $z \in (\underline{z}, \tilde{z})$.*

That is, the "poor" are better off under the more unequal distribution G_B. As equality rises, people will typically spend more on conspicuous consumption, which will tend to worsen welfare. The intuition is clear: the closer are people together, the greater the incentive to differentiate oneself, that is, in a more equal society the marginal return to conspicuous consumption is higher. However, the effect will be asymmetric with different implications for the rich and the poor.

To see this, take an agent with income z at the lower end of the distribution. Given a fixed income, in a more equal society A she will occupy a lower social position, as for low incomes G_A is less than G_B. Furthermore, by Proposition 2 she will spend more on conspicuous consumption. Thus, the poor would be worse off in a more equal society. As the poor have higher utility when there are more poor people around with whom they can make favorable comparisons, we can describe this result as an example of "misery loves company". For the relatively rich the comparison is ambiguous. For a given income, she occupies a lower hierarchical position in a more unequal society, but by Proposition 2 she may spend less on the conspicuous good, too. Which effect dominates depends on the relative importance of status in her society.

Example 2 *Suppose as in the previous example that utility is given by $U = xyF(x)$ and that $G_B(z) = z$ on [0,1], that is, a uniform distribution, and $G_A = 3z^2 - 2z^3$, a unimodal Beta distribution. We have in this case $U_B^* = 2z^3/9$. Numerical calculation reveals that $U_A^* < U_B^*$ for $z < 0.61$ but U_A^* is higher than U_B^* thereafter.*

We now turn to the effect of a increase in social income in the sense of the MPR order. We note that as the MPR order implies first-order stochastic dominance, if $G_A \succ_{MPR} G_B$ there is no interior crossing point of the two distribution functions, rather they meet only at the upper bound $\bar{z}$. Under the MPR order, Proposition 3 will hold but with $\tilde{z}$ equal to $\bar{z}$, which gives us the following.

Corollary 2 *If $G_A(\cdot) \succ_{MPR} G_B(\cdot)$ then $U_A^*(z) < U_B^*(z)$ for all $z \in (\underline{z}, \bar{z})$.*

That is, as society becomes more affluent, the above proposition indicates that utility falls at each level of income. This happens for two reasons. First, there is what we could call envy: one's status decreases as the incomes of those around rises, because $G_A(z) < G_B(z)$ on all of $(\underline{z}, \bar{z})$. But this is not all. From Corollary 1, if $G_A \succ_{MPR} G_B$, expenditure on x will be higher. That is, although the individual's own wealth is unchanged, competition for status forces him to increase his expenditure on conspicuous consumption as the incomes of his rivals increase.

We have looked at the effect of changes of the distribution of income on utility at each level of income. Our results make it clear that even an increase in prosperity in

the sense of the MPR order will have an overall effect that is ambiguous. In the more affluent income distribution, G_A, utility is lower at each income level. But equally the fact that $G_A \succ_{MPR} G_B$ mean that a positive mass of the population will have higher incomes under G_A than G_B, a rise that may or may not be enough to offset the fall in utility at each income level. Similar issues arise using the UPR order, but because of the lack of monotonicity in behavior established in Propositions 2 and 3, clear welfare comparisons are even more difficult.

4 Consumption Taxes and Subsidies

Robert Frank, who as noted above was the first to model status as rank in the distribution of consumption, more recently (Frank, 1999) has likened spending on conspicuous consumption to pollution, in that it imposes a negative externality on other consumers. He has advocated a consumption tax, in effect a Pigouvian tax, as a potential correction (Frank, 1985a, 1985b, 1999) for the externality. Many governments in the past have have labelled certain products as luxuries and levied taxes on them. However, what is and what is not a luxury is somewhat subjective and taxes imposed on this basis seem likely to produce unwanted distortions. Frank has suggested instead that the tax fall on total consumption (this gives the non-conspicuous good y an attractive interpretation as saving). We try to identify a tax policy that could implement the cooperative solution identified in Section 2, that if achieved, would represent a Pareto improvement on the Nash equilibrium.

A Pigouvian tax on an externality involves raising the price of the good or activity that causes the externality until it reaches its social cost. In the present model, the return to additional expenditure on x is typically different at different levels of income. Therefore, in order for the government to implement the cooperative solution it may have to use what amounts to perfect price discrimination, charging a different level of consumption tax and/or offering a different level of subsidy at each level of income. Denote the post tax price as $p_\tau(z) = p(1 + \tau(z))$ where p is the initial relative price. In particular, suppose there is a policy $\tau(z)$ such that in Nash equilibrium the cooperative solution (4) is chosen, then one can write the differential equation (3) that defines the equilibrium as

$$x'(z, p_\tau) = \frac{g(z)}{G(z)} \frac{V(x_c, y_c)}{p_\tau V_2(x_c, y_c) - V_1(x_c, y_c)} = \frac{g(z)}{G(z)} \frac{V(x_c, y_c)}{p\tau(z)V_2(x_c, y_c)} \tag{6}$$

And, if the solution of this equation is indeed equal to the cooperative solution, then at each z, it must hold that $x'(z, p_\tau) = x'_c(z, p)$. Substituting this into the above equation and solving for $\tau(z)$, one obtains,

$$\tau(z) = \frac{g(z)}{G(z)} \frac{V(x_c, y_c)}{pV_2(x_c, y_c)x'_c(z, p)} \tag{7}$$

Thus, there exists a continuous function $\tau(z)$ that, if $p_\tau(z) = p(1 + \tau(z))$, implements the cooperative solution. That is, $x(z, p_\tau) = x_c(z, p)$ and $y(z, p_\tau) = y_c(z, p)$ at each income level z.

Note that there are several ways in which the appropriate relative price can be obtained. One would be simply to tax x. However, since our model for the present lacks a public sector, we concentrate on revenue neutral policies. One such policy would be to tax x and redistribute the revenue raised back to the public. This, however, could potentially be counterproductive. If it were handed back as a per capita refund, this would represent an increase in income in the sense of the MPR order. We have seen in Section 3 that such shifts in the income distribution increase the proportion of income spent on conspicuous consumption. Finally, the revenue raised could be used to subsidize the price of non-conspicuous consumption. We give an example of this.

Example 3 *Suppose again that $U = xyF(x)$ and that z is distributed uniformly on $[0, 1]$, then in the equilibrium each consumer demands $2z/(3p)$ units of x and $z/3$ units of y. In this case the "cooperative" allocation is $x_c(z) = z/(2p)$ and $y_c(z) = z/2$. From the equation (7), one can calculate that in this case $\tau(z) = 1$, a constant. If initially, $p = 1$, then one policy that would implement the first best case and which would be revenue neutral would be to tax x at tax rate of $1/3$ and subsidize y at a subsidy rate of $1/3$.*

More generally, a tax policy derived in this way is unlikely to be constant. This in turn raises the question whether it will be progressive or regressive: should the marginal tax rate on conspicuous consumption be highest at low or high levels of income? What we can show is that the comparative statics techniques developed in the previous section can also be fruitfully applied in this context.

Proposition 4 *Suppose there are two income distributions G_A, G_B such that $G_A \succ_{UPR} G_B$ with the ratio G_A/G_B achieving a maximum at $\hat{z} \in (\underline{z}, \bar{z}]$. Then, under the two different distributions, the two resulting consumption taxes, $\tau_A(z), \tau_B(z)$ respectively, as defined by (7), satisfy $\tau_A(z) > \tau_B(z)$ on $[\underline{z}, \hat{z})$ and $\tau_A(z) < \tau_B(z)$ on $(\hat{z}, \bar{z}]$.*

We have already seen in Section 3 that an increase in equality in terms of the unimodal probability ratio order will lead to an increase in conspicuous consumption by the poor and possibly a decrease by the rich. This fact is reflected in the above proposition which establishes that if there is an increase in equality, the marginal tax rate should be increased for those with low income and decreased for those with high income. But does this imply that taxes should actually be regressive? Now, we have seen in Example 3 above that, for Cobb-Douglas type preferences and a uniform distribution of income, the consumption tax is constant. Thus, for these preferences and for any distribution which dominates the uniform in the sense of the UPR order, the consumption tax will be regressive, having a higher marginal rate of tax for the poor than the rich. Note that any distribution that has a mean that is no lower and a unimodal density will dominate the uniform in the sense of the UPR order. Thus, for any distributions that resemble actual empirical income distributions, and for preferences as in Example 3, this analysis suggests that a consumption tax should be regressive. This may be surprising, but note that a regressive tax, combined with appropriate subsidies will implement the

cooperative solution and make everybody better off. We now offer a simple example of this.

Example 4 *If $G_A(z) = 3z^2 - 2z^3$ (a unimodal Beta distribution) on $[0,1]$ then if $G_B(z) = z$ (a uniform distribution) then G_B is a mean preserving spread of G_A and $G_A \succ_{UPR} G_B$. For the preferences $U = xyF(x)$, one can calculate that $\tau_A(z) = 6(1 - z)/(3 - 2z)$. This can be implemented revenue-neutrally by a tax on x equal to $\tau_x(z) = 3(1 - z)/(6 - 5z)$ and an equal subsidy on y. The function τ_x is strictly decreasing with $\tau_x(0) = 0.5$ and $\tau_x(1) = 0$. That is, the poorest face a marginal tax rate of 50% and the rich a zero marginal rate.*

Ireland (2001) recently has also investigated the possibility of Pareto improving taxes in the presence of concern for status. He analyzes a game similar to ours where agents of different abilities signal their abilities through wasteful consumption. He finds that taxes on signaling does increase welfare. He also reports that this model gives little support for progressive taxation, even if it does not support regressive taxation. Earlier work on this topic was done by Seidman (1987). In a non-strategic setting where each individual's income has a negative externality on other people, he finds that the optimal income tax is progressive.

5 An Alternative Specification

Perhaps the most surprising implication of comparative statics results is that, when there are concerns for status, the welfare of the poor may be greater in a more unequal society, something we called "misery loves company". Skeptics may believe that we obtain this result only because the poor are in misery by assumption. That is, as noted in Section 2, our original specification of utility (1) ensures that those at the very bottom of the status ladder must obtain zero utility. In this section, in order to test the robustness of our previous results, we explore a different specification that avoids enforced misery. We find that indeed this changes the equilibrium behavior of the poorest in society, but that our "misery loves company" result still holds.

Suppose now that being at the bottom of distribution of status gives positive utility. For example, the marriage prospects of an individual with zero status may be worse than those of the rich but are still quite adequate. The simplest way to implement this idea is to assume that a consumer's problem is now given by, for some $\alpha > 0$,

$$\max_{x,y} U(x, y, F(\cdot), \alpha) = V(x, y)(\alpha + F(x)), \text{ subject to } px + y \le z, \ x \ge 0, \ y \ge 0 \quad (8)$$

Clearly, an agent with zero status now obtains $V(x, y)\alpha$ and not zero. The symmetric equilibrium will now arise as a solution to the modified differential equation

$$x'(z, \alpha) = \frac{g(z)}{\alpha + G(z)} \frac{V}{pV_2 - V_1} = \frac{g(z)}{\alpha + G(z)} \psi(x, z) \quad (9)$$

It is not too difficult to establish that at any income level expenditure on conspicuous consumption is lower in this new specification.

Proposition 5 *Let $x(z)$ and $x(z, \alpha)$ be the equilibrium conspicuous consumption solutions of the differential equations (3) and (9) respectively. Then, for any $\alpha > 0$, $x(z) > x(z, \alpha)$ for all $z \in (\underline{z}, \bar{z}]$.*

We were able to obtain comparative statics results on solutions to the original differential equation (3) by making use of the ratio G_A/G_B. The ratio that performs a similar role in our current analysis is $(\alpha + G_A)/(\alpha + G_B)$. It typically has different properties. As Lemma 3 in the Appendix shows, if the distribution G_A dominates G_B in the sense of the UPR order then the ratio $(\alpha + G_A)/(\alpha + G_B)$ has at most two extremal points, $\hat{z}_-$, where the ratio is at minimum, and $\hat{z}_+$, where the ratio is at maximum, with $\underline{z} < \hat{z}_- < \tilde{z} < \hat{z} < \hat{z}_+ < \bar{z}$. An example of this is illustrated in Figure 2. Together with $\tilde{z}$, these two extremal points provide a convenient interpretation. If individual's income z is in the interval $[\underline{z}, \hat{z}_-]$, we say that this individual is *poor*; if $z \in (\hat{z}_-, \tilde{z})$, she belongs to *the lower middle class*; if $z \in [\tilde{z}, \hat{z}_+]$, she belongs to *the upper middle class*; if $z \in (\hat{z}_+, \bar{z}]$, she is *rich*. We obtain somewhat different comparative statics than previously.

Proposition 6 *Suppose, for some $\alpha > 0$, $x_A(z, \alpha)$ and $x_B(z, \alpha)$ are the equilibrium choices of the positional good for distributions G_A and G_B, respectively. If $G_A(z) \succ_{UPR} G_B(z)$, then $x_A(z, \alpha)$ crosses $x_B(z, \alpha)$ at most twice. Moreover, $x_A(z, \alpha) < x_B(z, \alpha)$ for all $z \in [\underline{z}, \hat{z}_-]$ (possibly, empty), and $x_A(z, \alpha) > x_B(z, \alpha)$ for all $z \in [\tilde{z}, \hat{z}_+]$.*

The single but important difference in this result as compared to our earlier Proposition 2 is that now the conspicuous consumption of the poor may fall rather than rise in response to greater equality or an increase in income. One possible interpretation of this result is that, as the poor are no longer desperate to escape low status, an increase in the competitiveness of their environment induces them to try less hard at competing for status. A typical example is illustrated in Figure 2, which shows sample solutions for two income distributions where G_B is a mean preserving spread of G_A.

Example 5 *Suppose utility is given by $U = xy(\alpha + F(x))$ with $\alpha = 0.1$ and that $G_B(z) = z$ on [0,1], that is a uniform distribution, and $G_A = 3z^2 - 2z^3$, a Beta distribution which is unimodal and hence less dispersed than G_B (both distributions have mean 1/2). It is easily verified that $G_A \succ_{UPR} G_B$ as $G_A/G_B = z(3 - 2z)$ has a maximum at $\hat{z} = 3/4$, and cross 1 at $\tilde{z} = 1/2$. At the same time, $(\alpha + G_A)/(\alpha + G_B)$ has an internal minimum at $\hat{z}_- \approx 0.12$ and an internal maximum at $\hat{z}_+ \approx 0.755$. By numerical solution, we find that the solution x_A crosses x_B from below at approximately 0.135 and from above at approximately 0.861.*

What is quite surprising given the differences between Propositions 2 and 6 is that our original result on welfare still holds.
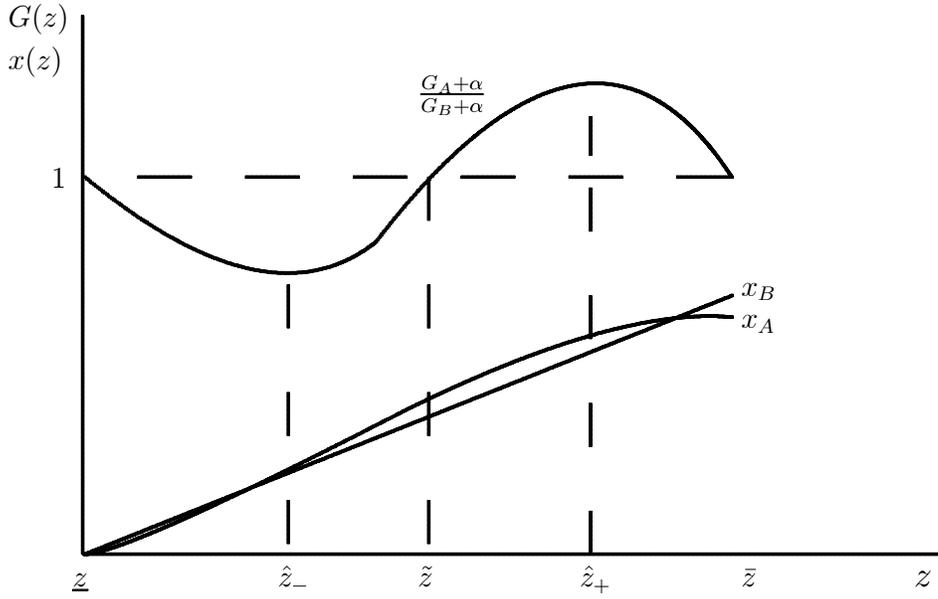
Figure 2: Comparative statics of solutions with $\alpha > 0$

Proposition 7 *For some $\alpha > 0$, if $G_A \succ_{UPR} G_B$ then $U_A^*(z, \alpha) < U_B^*(z, \alpha)$ for all $z \in (\underline{z}, \tilde{z})$.*

That is, the "lower" classes (i.e. both the poor and the lower middle class) are still better off under the more unequal distribution G_B. For the rest of the society (both the rich and the middle class) the comparison remains ambiguous.

6 Conclusion

We considered a game of status, in which individuals are concerned with the level of their consumption relative to that of others as well as its absolute level. Individual status is determined by the consumption choices of others in the equilibrium of the resulting game. In the symmetric Nash equilibrium, an individual's status, that is, her position in the distribution of consumption, coincides with her position in the income distribution. Hence, rank in the income distribution can be inferred from one's consumption behavior. But this signalling is costly and the Nash equilibrium is Pareto dominated by the state where agents take no account of status in their consumption decisions.

The advantage of a simultaneous move game approach is that it is possible to analyze how exogenous changes in the distribution of income affect individual decisions. We show that, for the class of preferences considered, an increase in average income, in the sense of a refinement of first order stochastic dominance, will lead to an increase in

conspicuous consumption. However, given that everyone now spends a greater proportion of income on conspicuous consumption, this implies a reduction in utility at each income level. Hence, an increase in average income may be consistent with a decrease in social welfare.

We also analyze the effect of changes in inequality on conspicuous consumption and welfare. We find that greater equality induces greater conspicuous waste on the part of those with low incomes (the effect is ambiguous for the richer members of society). This enables us to show that the poor are made worse off by greater equality. Consequently, when we consider corrective taxes on conspicuous consumption, we find that the more equally is income distributed, the higher should marginal tax rates be for those with low incomes, and the lower should be marginal rates for those with high incomes. Indeed, we are able to show that in some circumstances a regressive tax, with higher tax rates (but also higher subsidies) for those with low incomes, can lead to a Pareto improvement. These results may seem counterintuitive, but recent empirical work by Clark (2000) and Alesina, di Tella and MacCulloch (2001) finds that happiness in the US and the UK may have a positive relationship with inequality. Data from continental Europe gives quite different results, however. This suggests that actual social attitudes towards inequality are likely to depend on a number of different factors. For example, as well as status effects, one's belief in the possibility of upward mobility, the political system and other institutions may all be important in determining how people respond to changes in the level of inequality.

A related question is whether a different model of relative concerns could allow for greater equality to have a positive impact on welfare. It is true that the sharpness of our results do depend on the particular form of preferences that we employ. Nonetheless, in Section 5 of this paper we were able to show that some of our results are robust to changes in the specification of utility. Meanwhile, Samuelson (2001) investigates a model where agents are concerned about the consumption decisions of others not for reasons of status but because consumption patterns provide information about economic opportunities. In such a society an increase in equality, in that it increases the precision of information on the underlying environment, may be welfare improving. This difference in outcome is largely due to the fact that, in the model of Samuelson, an individual's consumptions choices do not impose negative externalities on others.

This is important because, as our analysis indicates, the exact relation between inequality and consumption behavior has important implications. It seems to imply that policy interventions to reduce inequality might have unintended consequences. Furthermore, while in the framework we consider here a consumption tax will be welfare-improving, the form it should take depends on the distribution of income in unexpected ways. These findings should not be interpreted as a call for the immediate overhaul of existing attitudes and policies toward inequality. Our current understanding of relative concerns, their place in human happiness and the interaction with income inequality is much too rudimentary for that. However, we think that our present work has illuminated some connections between status and inequality which are both complex and unexpected. We hope that we have shown that this interaction is worthy of further

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study and that the present work provides a methodology for its analysis.

Appendix: Proofs

Proof of Proposition 1. We first prove two lemmas to verify that any equilibrium strategy is increasing and to establish a boundary condition.

Lemma 1 *If the strategy $x^*(z)$ is a best response to other agents' consumption choices then it is strictly increasing.*

Proof:. This result is adapted from Proposition 1 of Maskin and Riley (2000b) for high bid auctions. If an agent i with income z_i adopts the choice $x_i = x^*(z_i)$ which is a best response to choices of other agents summarized by the distribution $F(\cdot)$, then for any other choice, $\hat{x}_i < x_i$, necessarily

$$V(x_i, z_i - px_i)F(x_i) \geq V(\hat{x}_i, z_i - p\hat{x}_i)F(\hat{x}_i).$$

The next step is to establish the following inequality.

$$\frac{\partial V}{\partial z_i}(x_i, z_i - px_i)F(x_i) > \frac{\partial V}{\partial z_i}(\hat{x}_i, z - p\hat{x}_i)F(\hat{x}_i). \tag{10}$$

One can write the left-hand side of the above as

$$\frac{\partial V}{\partial z_i}(x_i, z_i - px_i)F(\hat{x}_i) + \frac{\partial V}{\partial z_i}(x_i, z_i - px_i)(F(x_i) - F(\hat{x}_i)).$$

Now, the first term in the above must be at least as big as the right-hand side of (10) by our assumptions that $V_{ij} \geq 0$ and that $V_{ii} \leq 0$. Now, if x_i is greater than $x_c(z_i)$ the cooperative level defined in (4), $V(x_i, z_i - px_i)$ is decreasing in x_i.[12] Hence, if x_i is a best response then $F(x_i) > F(\hat{x}_i)$ or the agent i could lower x_i to $\hat{x}_i$ with no loss of status and an increase in utility. Then the second term in the above equation must be strictly positive as V_2 is strictly positive by assumption. We have established the inequality (10), so that at x_i $\partial U/\partial z$ is strictly increasing in x. But equally this implies an increase in income leads to an increase in the marginal return to x, and the optimal choice of x necessarily increases. ∎

Lemma 2 *In equilibrium $x^*(\underline{z}) = \underline{z}/p$.*

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[12]If x_i is a best response, we have necessarily $x_i \geq x_c(z_i)$, as it is strictly dominated to choose a level of consumption below the cooperative level. Suppose that equality holds, that is, the best reply for an agent with income z_i is to choose $x_c(z_i)$. But then the choice of x is strictly increasing in income simply because by assumption x is a normal good.

Proof: In a symmetric equilibrium, an individual with income $\underline{z}$ has rank 0. That is, her equilibrium utility is $U^*(\underline{z}) = V(x,y)G(\underline{z}) = 0$. The only way she can increase her utility would be to raise her rank. Thus, in equilibrium she must spend all her income on x, otherwise she could increase her expenditure on x and thus her rank, which would be a profitable deviation. ∎

From Lemma 1, in equilibrium $x = x(z)$ is a strictly increasing and continuous function of an agent's income. Then the probability that an individual i has higher status than an another individual j is therefore

$$F(x_i(z_i)) = \Pr[x_i(z_i) > x_j(z_j)] = \Pr[x_j^{-1}(x_i(z_i)) > z_j] = G(x_j^{-1}(x_i(z_i))) = G(z_i) \quad (11)$$

The final step follows from the assumption of a symmetric equilibrium, where $x_i(z_i) = x_j(z_i)$. Hence, $x_j^{-1}(x_i(z_i)) = z_i$.

We are left with establishing that the differential equation arising from the the first order conditions (2) describes a unique symmetric equilibrium. A sufficient condition to ensure that the first order conditions identify a maximum is pseudoconcavity. That is, U is increasing in x for $x < x^*(z)$ and decreasing for $x > x^*(z)$. One takes the first-order conditions and differentiates with respect to z to obtain

$$\frac{\partial^2 U}{\partial x \partial z} = V_{12} - pV_{22} + V_2 f(x_i) F(x_i)^{-1} > 0.$$

Now, take $x_0 < x^*(z)$ and let $\hat{z}$ be such that $x^*(\hat{z}) = x_0$. Then, $\hat{z} < z$. Hence, for any $x < x^*(z)$,

$$\frac{dU(x,z)}{dx} \geq \frac{dU(x_0, \hat{z})}{dx} = 0$$

This shows that U is increasing in x for $x < x^*(z)$. One can similarly show that it is decreasing for $x > x^*(z)$. Lemma 2 provides the boundary condition for the differential equation (3). However, if we evaluate $x'(z)$ at $z = \underline{z}$, we have the denominator equal to zero and there is a potential failure of Lipschitz continuity (a problem known from the analysis of auctions), and therefore there are potentially multiple solutions with the same boundary condition. We can rule this out here. Note that as (3) is continuously differentiable on $(\underline{z}, \bar{z}]$, it has a unique solution on $(\underline{z}, \bar{z}]$. Thus, any potential multiple solutions cannot cross on $(\underline{z}, \bar{z}]$. But then any two solutions, say $x_1(z), x_2(z)$ must satisfy $x_2(z) > x_1(z)$ for $z > \underline{z}$. Note, first, that for $\psi(x,z) = V/(pV_2 - V_1)$, given that V is quasiconcave $V_{ii} \leq 0$ and $V_{ij} \geq 0$, we have

$$\frac{\partial \psi}{\partial x} = \frac{-(pV_2 - V_1)^2 - V(-p^2 V_{22} + pV_{21} - V_{11} + pV_{12})}{(pV_2 - V_1)^2} < 0 \quad (12)$$

Examining the equation (3) we have $x_2'(z) < x_1'(z)$ for $z > \underline{z}$. This would imply $\int_{\underline{z}}^{\bar{z}} x_1'(z)dz = x_1(\bar{z}) - x_1(\underline{z}) > x_2(\bar{z}) - x_2(\underline{z}) = \int_{\underline{z}}^{\bar{z}} x_2'(z)dz$, which, as $x_1(\underline{z}) = x_2(\underline{z})$, is a contradiction. ∎

Proof of Proposition 2. Denote $\arg\max G_A/G_B$ as $\hat{z}$ so that the ratio $G_A(z)/G_B(z)$ is increasing on $[\underline{z}, \hat{z})$ and decreasing on $(\hat{z}, \bar{z}]$. Examining the differential equation (3),

one can see that if x_A and x_B cross at all on the interior of $[\underline{z}, \bar{z}]$, then at such crossing points $\psi(x_A, z) = \psi(x_B, z)$. Because G_A/G_B is unimodal, the inequality (5) holds on $(\underline{z}, \hat{z})$ and the reverse inequality holds on $(\hat{z}, \bar{z})$. Hence, if $x_A(z) = x_B(z)$ on $(\underline{z}, \hat{z})$, then $x'_A(z) > x'_B(z)$ and if $x_A(z) = x_B(z)$ on $(\hat{z}, \bar{z})$ then $x'_A(z) < x'_B(z)$. Therefore, it must be that x_A crosses x_B at most twice - from below to the left of $\hat{z}$ and from above to the right of $\hat{z}$. Denote these points of intersection z_+ and z_- with $z_+ < \hat{z} < z_-$. Since x_A crosses x_B from below at z_+ and from above at z_-, so that the sequence of sign of the difference $x_A - x_B$ is $-, +, -$. By the boundary condition (see Lemma 2) $x_A(\underline{z}) = x_B(\underline{z})$. Than there must exist a point $\check{z} \in (\underline{z}, z_+)$ where the difference $x_B - x_A$ is maximized. At this point, the slopes of x_A and x_B must be equal, i.e. $x'_A(\check{z}) = x'_B(\check{z})$. Since from (12) $\frac{\partial \psi(x,z)}{\partial x} < 0$ we have $\psi_A(\check{z}) > \psi_B(\check{z})$. But this implies that $\frac{g_A}{G_A} < \frac{g_B}{G_B}$ at $\check{z}$, which is a contradiction. Thus, x_A crosses x_B at most once, from above, and to the right of $\operatorname{argmax} G_A/G_B$. ∎

Proof of Proposition 3. As $G_A(\bar{z})/G_B(\bar{z}) = 1$ and as $G_A(z)/G_B(z)$ is unimodal, it must be that $G_A(\underline{z})/G_B(\underline{z}) \leq 1$ or G_A would be almost everywhere greater than G_B which would imply $E[Z_A] < E[Z_B]$. The ratio reaches a unique maximum at some point $\hat{z}$, and $G_A(\hat{z})/G_B(\hat{z}) \geq 1$, with equality only if $\hat{z} = \bar{z}$. Therefore, if the maximum is interior, that is, G_A and G_B do not satisfy the MPR ratio order, then necessarily there is a unique point $\tilde{z} < \hat{z}$ such that $G_A(\tilde{z})/G_B(\tilde{z}) = 1$.

At any income level $z \in (\underline{z}, \tilde{z})$, $G_A(z) < G_B(z)$, and so the individual with income $z < \tilde{z}$ has lower status under distribution G_A. Second, by Proposition 2, $x_A(z) > x_B(z)$ on $(\underline{z}, \hat{z}]$ which implies that for $z \leq \tilde{z} < \hat{z}$, $x_A(z) > x_B(z) > x_c(z)$, the cooperative or efficient level. Given that $V(\cdot)$ is increasing and quasiconcave by assumption, this implies that $V(x_A(z), y_A(z)) < V(x_B(z), y_B(z))$. Thus $U_A^*(z) = V(z)G_A(z) < V(z)G_B(z) = U_B^*(z)$ for all $z \in (\underline{z}, \tilde{z}]$. ∎

Proof of Proposition 4: If $G_A \succ_{UPR} G_B$ then $g_A(z)/G_A(z) > g_B(z)/G_B(z)$ for $z \in (\underline{z}, \hat{z})$ and $g_A(z)/G_A(z) < g_B(z)/G_B(z)$ for $z \in (\hat{z}, \bar{z}]$. Given the definition of $\tau(z)$ in (7), and that (x_c, y_c) are independent of the distribution of income, the result follows. ∎

Proof of Proposition 5. We first need to establish a boundary condition for the differential equation (9). Given that in equilibrium an agent with income $\underline{z}$ receives a utility of $\alpha V(x(\underline{z}), \underline{z} - px(\underline{z}))$, her equilibrium choice must maximize V. That is, it must be the cooperative choice $x_c(\underline{z})$, so that for $\underline{z} > 0$, $x(\underline{z}, \alpha) > x(\underline{z})$ for any positive α.[13] Second, if $x(z) = x(z, \alpha)$ at some point z_+ then comparing (3) with (9) it is clear that $x'(z_+) > x(z_+, \alpha)$, that is, $x(z, \alpha)$ crosses $x(z)$ from above and there can be at most one such crossing. If $x(\underline{z}, \alpha) < x(\underline{z})$ then the proof is complete. If $x(\underline{z}, \alpha) = x(\underline{z}) = 0$, then there must be a point in $(\underline{z}, z_+)$ where the difference $x(\underline{z}, \alpha) - x(\underline{z})$ is maximized and so that $x'(\underline{z}, \alpha) = x'(\underline{z})$. But given that by (12) $\psi(z, x)$ is decreasing in x, this generates a contradiction. ∎

Proof of Proposition 6. We first establish a lemma.

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[13]This implies, for $\underline{z} > 0$, $\lim_{\alpha \to 0} x(\underline{z}, \alpha) \neq x(\underline{z})$. That is, there is a discontinuous fall in the conspicuous consumption of agents with income $\underline{z}$ as α moves from zero to positive.

Lemma 3 *Let* $P(\alpha, z) = \frac{\alpha + G_A(z)}{\alpha + G_B(z)}$. *If* $G_A(z) \succ_{UPR} G_B(z)$ *then for all* $\alpha > 0$, $P(\alpha, z)$ *has two extremes, at minimum at* $\hat{z}_-$ *and a maximum at* $\hat{z}_+$, *such that* $\underline{z} < \hat{z}_- < \tilde{z} < \hat{z}_+ < \bar{z}$.

Proof: Note first that whenever $P(0, z) > 1$, $P(\alpha, z) > 1$ as well, and whenever $P(0, z) < 1$, $P(\alpha, z) < 1$. Also note that $P(\alpha, \underline{z}) = P(\alpha, \tilde{z}) = P(\alpha, \bar{z}) = P(0, \tilde{z}) = P(0, \bar{z}) = 1$. The rest is obvious. ∎

One can see that if x_A and x_B cross at all on the interior of $[\underline{z}, \bar{z}]$, then at such crossing points $\psi(x_A, z) = \psi(x_B, z)$. Whenever $\frac{g_A}{\alpha + G_A} < \frac{g_B}{\alpha + G_B}$ (or when $z \in [\underline{z}, \hat{z}_-)$ or $(\hat{z}_+, \bar{z}]$), $x'_A < x'_B$ and whenever $\frac{g_A}{\alpha + G_A} > \frac{g_B}{\alpha + G_B}$ (when $z \in (\hat{z}_-, \hat{z}_+)$) $x'_A > x'_B$ at the points of intersection. Therefore, it must be that x_A crosses x_B at most three times - from above to the left of $\hat{z}_-$ and to the right of $\hat{z}_+$, and from below in between. Denote these points of intersection z_1, z_2 and z_3 with $z_1 < \hat{z}_- < z_2 < \hat{z}_+ < z_3$. Then the sequence of sign of the difference $x_A - x_B$ is $+, -, +, -$. By the boundary condition, $x_A(\underline{z}) = x_B(\underline{z})$. Then there must exist a point $\check{z} \in (\underline{z}, z_1)$ where the difference $x_A - x_B$ is maximized. At this point, the slopes of x_A and x_B must be equal, i.e. $x'_A(\check{z}) = x'_B(\check{z})$. Since from (12) $\frac{\partial \psi(x, z)}{\partial x} < 0$ we have $\psi_A(\check{z}) < \psi_B(\check{z})$. But this implies that $\frac{g_A}{G_A} > \frac{g_B}{G_B}$ at $\check{z}$, which is a contradiction. Thus, x_A crosses x_B at most twice, first from below, then from above. ∎

Proof of Proposition 7. One can see that if U_A^* and U_B^* cross at all on the interior of $[\underline{z}, \bar{z}]$, then at such crossing points $V_A < V_B$ (and thus $x_A > x_B$) whenever $G_A > G_B$, and $V_A > V_B$ (and thus $x_A < x_B$) whenever $G_A < G_B$. By the envelope theorem, $\frac{\partial U^*(z)}{\partial z} = V_2^*(\alpha + G(z))$. As Lemma 1 established the inequality (10), we have $\frac{\partial U^*(z)}{\partial z}$ increasing in x. Thus, at the points of intersection U_A^* is steeper than U_B^* when $G_A > G_B$ and flatter when for $G_A > G_B$. Therefore, it must be that U_A^* crosses U_B^* at most twice - from above to the left of $\tilde{z}$ and from below to the right of $\tilde{z}$. Denote these points of intersection $\grave{z}$ and $\acute{z}$ with $\grave{z} < \tilde{z} < \acute{z}$. The sequence of sign of the difference $U_A^* - U_B^*$ is $+, -, +$. By the boundary condition $U_A^*(\underline{z}) = U_B^*(\underline{z})$. Than there must exist a point $\dot{z} \in (\underline{z}, \grave{z})$ where the difference $U_A^* - U_B^*$ is maximized. At this point, the slopes of U_A^* and U_B^* must be equal. Because $G_A(\dot{z}) < G_B(\dot{z})$, it must be that $V_{2A}(\dot{z}) > V_{2B}(\dot{z})$. Given that $V_{ij} \geq 0$ by assumption, the only way this would be possible is that $x_A(\dot{z}) > x_B(\dot{z})$. But this, together with $G_A(\dot{z}) < G_B(\dot{z})$ means that $U_A^*(\dot{z}) < U_B^*(\dot{z})$, which is a contradiction. Thus, U_A^* crosses U_B^* at most once and from below. We now need to establish where that might be. Let us consider point $\ddot{z}$ where the difference $U_B^* - U_A^*$ is maximized. At this point, the slopes of U_A^* and U_B^* must be equal. If $G_A(\ddot{z}) > G_B(\ddot{z})$, then $V_{2A}(\ddot{z}) < V_{2B}(\ddot{z})$, so that $x_A(\ddot{z}) < x_B(\ddot{z})$. But this, together with $G_A(\ddot{z}) > G_B(\ddot{z})$ means that $U_A^*(\ddot{z}) > U_B^*(\ddot{z})$, which is a contradiction. Thus, $\ddot{z} \in (\underline{z}, \tilde{z})$. This means that U_A^* is steeper than U_B^* at $\tilde{z}$. Since $G_A(\tilde{z}) = G_B(\tilde{z})$, $V_{2A}(\tilde{z}) > V_{2B}(\tilde{z})$, so that $x_A(\tilde{z}) > x_B(\tilde{z})$, which implies that $U_A^*(\tilde{z}) < U_B^*(\tilde{z})$. It is easy to observe that the case where $U_A^* > U_B^*$ for all z is false since $U_A^* < U_B^*$ for all $z \in (\hat{z}_-, \tilde{z})$. However, we can not rule out the possibility of $U_A^* < U_B^*$ for all z. ∎

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