Edinburgh School of Economics

Discussion Paper Series

Number 89

Trading Costs of Institutional Investors in Auction and Dealer Markets

Andy Snell (University of Edinburgh)
Ian Tonks (University of Bristol)

Date
January 2002

Published by
School of Economics
University of Edinburgh
30 -31 Buccleuch Place
Edinburgh EH8 9JT
+44 (0)131 650 8361
http://www.ed.ac.uk/schools-departments/economics

THE UNIVERSITY *of* EDINBURGH

Trading Costs of Institutional Investors in Auction and Dealer Markets.

Andy Snell
Dept of Economics, University of Edinburgh

Ian Tonks
Department of Economics, University of Bristol

January 2002

Abstract

This paper compares the trading costs for institutional investors who are subject to liquidity shocks, of trading in auction and dealer markets. The batch auction restricts the institutions' ability to exploit informational advantages because of competition between institutions when they simultaneously submit their orders. This competition lowers aggregate trading costs. In the dealership market, competition between traders is absent but trades occur in sequence so that private information is revealed by observing the °ow of successive orders. This information revelation reduces trading costs in aggregate. We analyse the relative e®ects on pro¯ts of competition in one system and information revelation in the other and identify the circumstances under which dealership markets have lower trading costs than auction markets and vice versa.

1 Introduction

In this paper we compare the two archetypal types of mechanism for trading financial securities, namely an auction and a dealer market, by examining the profits/costs to institutional investors of trading on these two alternative trading platforms. We are able to identify conditions under which one system is preferred to the other. Keim and Madhavan (1998) provide evidence that trading costs of institutions are different on different exchanges, and given the importance of institutional investors [Becht and Roell (1999), Myners (2001)], and their influence in determining the structure of an exchange,[1] we examine which type of trading mechanism will be preferred by investors trading blocks of shares.[2] We develop a model of trading in equities by large institutional investors that are subject to stochastic liquidity shocks and that have acquired private information through monitoring about the firm whose shares they are attempting to trade.[3] We are able to show that when asymmetric information concerns are important, the dealership system is preferred by institutional investors, but when liquidity shocks are more important, the call auction offers lower costs to the institutional investor.

These comparisons are relevant for understanding the role of competition between trading systems and exchanges. Since October 1997, when the London Stock Exchange introduced an order driven trading system to run alongside its dealership system, there have been two competing trading mechanisms for FTSE100 securities. At the opening, traders can choose whether to execute their trades on SETS (a call auction at the opening) or on SEAQ (a dealership system). Traders submit limit orders in the pre-opening period, and at the opening the limit order system closes for up to three minutes while an algorithm runs through all the FTSE100 stocks to find a market clearing price in each stock. During this three minute period while the SETS system is "closed", the SEAQ system operates and traders can deal sequentially with market makers.

Studying the liquidity of secondary equity markets is also important from the viewpoint of economic efficiency because it fundamentally affects the incentives for institutional investors to accumulate controlling stakes in companies and hence to monitor and improve their performance. Holmstrom and Tirole (1993), Bolton and von Thadden (1998), Pagano and Roell (1998), and Maug (1998) have focused on the advantages of a large shareholder in terms of the incentives that they have to monitor management, but the disadvantages of large blocks because of their reduced liquidity. In fact Bhide (1993) suggests that the deep liquidity of equity secondary equity markets in the US are to the detriment of the monitoring responsibilities of shareholders.

Our model evaluates and compares the institutional investor's trading costs on a call

[1]The recent changes on the London Stock Exchange and the introduction of an auction (SETS) trading system in the most liquid securities alongside the SEAQ dealership system in October 1997, illustrates the importance of lobbying from the users of the trading systems, through the financial regulators [Securities and Investments Board (1994, 1996)].

[2]According to Martin Dickson writing in the Financial Times on 4th May 2000, a proposed strategic alliance between London and Frankfurt in 1998 fell apart because the separate exchanges could not agree on an appropriate trading platform.

[3]Paragraph (5.81) of the Myners' Report (2001) notes that institutional investors acting on information they have received from meetings with management does not make an institution an insider

auction and dealership systems. Papers by Madhavan (1992), Biais (1993), Pagano and Roell (1992, 1996) and Shin (1996) have all examined different characteristics of alternative trading mechanisms. Madhavan (1992) argues that the differences in the two systems lie in the sequence of trading, which leads to differences in the information provided to the players and therefore in the strategic nature of the game. In the quote-driven system competition between market makers in setting quotes ensures that price quotes are competitive, and market makers make zero profits, whereas in the order-driven system competition between dealers takes the form of competition in demand schedules. Pagano and Roell (1996) emphasise the differences between alternative trading systems in terms of transparency about the history of the order flow, and compare the price formation process in four alternative market trading systems, where the transparency of the current order flow defines the trading systems. Biais (1993) compares price formation in fragmented and centralised markets, with no asymmetric information about asset payoffs. In his model, the difference between these two regimes is that a fragmented market is by definition less transparent than a centralised one, so that agents have different information about the behaviour of their competitors. Shin (1996) points out that a distinctive feature of these two systems is the move order and consequent information available to the traders when they take there respective actions. The auction market requires that all traders take their actions simultaneously, whereas in the dealership market the price setters move first and the buyers (sellers) take their actions after observing the price quotes of the sellers (buyers).

Our set up is allied to that of Madhavan (1992), but in Madhavan's model the auction market includes dealers who act as intermediaries and make positive profits, and this has implications for the properties of the two markets in terms of market efficiency. In contrast in our model both markets have intermediaries who earn zero profits, and we compare the two systems in terms of the ex ante expected trading costs to the traders of participating in these markets.[4] In our model when liquidity shocks force the institution to trade, adverse selection concerns on the part of the exchange's intermediaries, mean that the institutional investors face unfavourable prices and high trading costs. In the call auction the institution's ability to exploit informational advantages is restricted by the competition between institutions when they simultaneously submit their orders, and this lowers aggregate trading costs. On the other hand, in the dealership market, competition between traders is absent but trades occur in sequence so that (unlike the call auction) private information is revealed by observing the flow of successive orders, and this information revelation also reduces trading costs in aggregate. The net effects on institutional investor's trading costs are evaluated, and we find that the relative costs to the institutional investor of trading on the two systems hinges on a key parameter that measures the relative importance of asymmetric information to liquidity shocks. Where asymmetric information is more prevalent, the dealership system is preferred, and when liquidity shocks are more important, the call auction offers lower costs to the institutional investor.

The layout of the paper is as follows. In Section 2 we outline a model of institutional

[4] Madhavan (1992) models competition between dealers in the auction market as competition in demand schedules, and these dealers earn rents which shrink to zero as the number of dealers increase. Whereas we assume that in our two systems the market's intermediaries make zero profits at all times. Hence a comparison of our results and those of Madhavan will be valid when the number of dealers is large.

investors (traders) and dealers. Sections 3 examines the circumstances (parameter values) under which one system dominates another in terms of the institutions' aggregate profits. Section 4 examines the effects of changing the correlation structure of liquidity shocks and changing the batch auction mechanism to one where only non-price contingent trades are allowed. Section 5 provides a summary and conclusion.

2 The Model

Our model follows the approach taken in Madhavan (1992), who compares a quote driven mechanism with competing dealers, with an order driven mechanism organised as a call auction with floor traders, who are responsible for ensuring market clearing. In the current paper the dealer market is modeled as a series of sequential trades, so that traders act as monopolists, independent of subsequent trades. In contrast in the auction market all trades occur at the same time, so that traders act strategically with respect to rival traders, when submitting their demands. In our model there are n traders in the market trading in a security, and they trade for two reasons. Trader i observes the true value of the security v and is able to trade on the basis of this information in the secondary market. Each trader also faces a liquidity shock u_i, which is the second motive for trading. These traders are taken to be large risk-neutral institutional investors who discover the true value of the security v which is distributed $v \sim N(\bar{v}; \sigma_v^2)$ after monitoring the company on account of their large stake. The institutional investors trade x_i in the secondary market, following Seppi (1992) to maximise

$$\pi_i|v; u = [v - p]x_i - \frac{\phi}{2}(x_i - u_i)^2 \qquad i = 1; 2 ::: n \tag{2.1}$$

The objective function (6.30) shows that traders generate income for each unit of stock that they hold, by trading at price p when the true value of the security is v. In addition these traders face a liquidity shock u_i resulting in losses which are quadratic in the difference between their holdings of the asset x_i and the liquidity shock. The relative importance of the trading profits and the liquidity shock in the investors' objective function is controlled by the parameter ϕ: Clearly the higher is ϕ the greater is the weight placed on the liquidity shock. The advantage of the specific objective function is that we are able to obtain straightforward closed-form solutions for the expected profits to an institution from trading under the two alternative microstructure systems outlined below.[5]

The institutional investors may be thought of as insurance companies who are generating premium income outside the model. A negative liquidity shock is interpreted as an unexpected insurance cash claim which must be met by the company by either selling the security or by borrowing. Under this interpretation the quadratic term $(x_i - u_i)^2$ represents increasing marginal borrowing costs. A positive liquidity shock may be interpreted as unexpected premium income and in this case costs are incurred by failing to invest this income in equities whose return exceeds that on liquid assets. In fact these costs are more likely to be linear in (x_i- u_i), but allowing for asymmetric costs would make our model analytically

[5] Bernhardt and Hughson (1997) (Proposition 2) show that these quadratic preferences can be interpreted as the reduced-form preferences of a rational agent with exponential utility who receives a liquidity shock

intractable.. The quadratic term in (6.30) therefore, must be viewed as approximating actual costs.

Market makers who are the only other market participants, and set prices p are not able to infer exactly the value of the security from the trading behaviour of the institutions since these institutions also trade because of liquidity shocks, which are distributed $u_i \sim N(0; \sigma_u^2)$: Note that if market makers also observed the value v, then they would set prices equal to the true value of the security, and traders could then set their demands equal to their liquidity shock to ensure no worse than zero profits. However because market makers do not observe v directly, but infer it from the trading volumes, they set prices to reduce the adverse selection problem from informed institutions trading against them, and we shall see that this raises the trading costs of the institutions.

Our framework is an extension of the insider trading model developed by Kyle (1985), in which market makers set prices allowing for the likelihood that the aggregate demand will reflect informed trading by an insider. However rather then a single informed trader placing his order in with a batch of liquidity orders, the model considered here allows for a different market microstructure in which a trader deals directly with the market maker, but the market maker is unable to identify which components of trades are liquidity motivated and which are information motivated.

2.1 Periodic Call auction

A number of stock markets, such as the NYSE, London SETS and the Paris Bourse open their daily markets with a call auction. In the call auction considered here, each institutional investor simultaneously submits price contingent orders to the market, and the price is set such that market makers earn zero expected profits.[6] Aggregate trading volume is $X = \sum_{i=1}^{n} x_i$, and in this oligopoly call auction we recognise that each institutional trader knows that both their own trades and their rival's will have an impact on prices. To find the equilibrium solution to this model we make the conjecture that the aggregate trading volume is a linear function of the information and the liquidity shocks, and competitive market makers set price as a linear function of the aggregate trading volume

$$X = \beta(v - \bar{v}) + \sum_{i=1} \gamma_i u_i \tag{2.2}$$

and

$$p = \bar{v} + \lambda X \tag{2.3}$$

To find the optimal trading volume of each strategic institutional trader i substitute the conjectured price function (6.30) into the objective function (6.30). The reaction function for the ith investor under the Cournot assumption that each investor's demands do not affect the demands of the rival, is given by

$$x_i = \frac{v - \bar{v}}{2\lambda + \varphi} + \frac{\varphi u_i}{2\lambda + \varphi} - \frac{\lambda(X - x_i)}{2\lambda + \varphi} \tag{2.4}$$

[6] Following Pagano and Roell (1996) we use the term market maker to denote any speculator involved in the provision of liquidity in an auction market.

The optimal demands for trader i would appear to depend inter alia on the total order flow X which is unobserved by the traders. However, rearranging (2.4) and substituting for X using (6.30) gives demands that are linear in the price (and in observable shocks)[7]. Hence allowing institutional traders to submit price contingent demands is equivalent to allowing them to submit demands conditional on the unobserved total order flow X as in (2.4).

All institutions face the same problem and since aggregate trading volume is simply the sum of the n institutions' trades, summing over $i = 1$ to n in (2.4) and rearranging gives the aggregate trading volume as

$$X = \frac{n(v - \mu_v)}{(n+1)\lambda + \varphi} + \frac{\varphi}{(n+1)\lambda + \varphi} \sum_{i=1} u_i \tag{2.5}$$

which is indeed a linear function of the information and the liquidity shocks. Comparing coefficients in (6.30) and (6.30) yields

$$\beta = \frac{n}{(n+1)\lambda + \varphi}; \qquad \gamma_i = \gamma = \frac{\varphi}{(n+1)\lambda + \varphi} \tag{2.6}$$

We assume that the market maker acts competitively and sets prices as the expectation of the terminal value of the asset v conditional on the aggregate trading volume X so that prices are

$$p = E[v \mid X = \beta(v - \mu_v) + \gamma \sum_{i=1} u_i] \tag{2.7}$$

To compute this expectation we need to make assumptions about the correlations between the liquidity shocks. Initially we assume the liquidity shocks are independent. This could arise for example if the insurance market was divided into several niches, and each niche being identified with an independent source of risk and with a firm insuring against that risk. An assumption at the other extreme would be that the liquidity shocks are perfectly correlated i.e. identical for all institutions. This would arise if all insurance companies fully diversified their risks in a secondary market so that they were only exposed to economy-wide systematic risk. Because our institutions are assumed to be risk neutral and therefore have no incentive to diversify risks the uncorrelated shocks assumption seems more appropriate and we take this as our main case. We examine the effect that the assumption of identical shocks has on our results in Section 4 below.

Joint normality of the models' variates guarantees that $E[v|X]$ and hence p is linear in X which confirms the conjecture for prices in equation (6.30). Taking the liquidity shocks to be iid and using the standard formula for the conditional expectation of normal variates gives λ in (6.30) as

$$\lambda = \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2 + n\gamma^2\sigma_u^2} \tag{2.8}$$

[7] Explicitly, we would have $x_i = \frac{v}{\lambda+\varphi} + \frac{\varphi u_i}{\lambda+\varphi} - \frac{p}{\lambda+\varphi}$

We now have three equations in (2.6) and (2.8) and three unknowns β, λ and γ. Solving for the unknowns we may write the conjectured coefficients as

$$\lambda = \frac{\varphi\sigma_v^2}{\varphi^2\sigma_u^2 - \sigma_v^2}; \qquad \beta = \frac{n[\varphi^2\sigma_u^2 - \sigma_v^2]}{\varphi[\varphi^2\sigma_u^2 + n\sigma_v^2]}; \qquad \gamma = \frac{[\varphi^2\sigma_u^2 - \sigma_v^2]}{\varphi^2\sigma_u^2 + n\sigma_v^2} \tag{2.9}$$

Note that the second order condition for maximisation of the traders' objective is that $\varphi^2\sigma_u^2 > \sigma_v^2$. This condition indicates that a minimum amount of noise trade variability is required to ensure that equilibrium exists and that β and γ are strictly positive.[8]

We may now compute the unconditional expected profits to each trader before they have observed either the value of the asset or their liquidity shock. The optimal demands for each trader are obtained by substituting (6.30) into (2.4) and rearranging. For each trader i we have

$$x_i = \frac{\sigma_u^2\varphi^2 - \sigma_v^2}{\varphi(n\sigma_v^2 + \sigma_u^2\varphi^2)}(v - \bar{v}) + \frac{\sigma_u^2\varphi^2 - \sigma_v^2}{\sigma_u^2\varphi^2}u_i + \frac{\sigma_v^2(\sigma_v^2 - \sigma_u^2\varphi^2)}{\sigma_u^2\varphi^2(n\sigma_v^2 + \sigma_u^2\varphi^2)}\sum_{j=1} u_j \tag{2.10}$$

Substituting (6.30) and (2.10) into the objective function (6.30), taking expected values over the value of the asset and the liquidity shocks and multiplying by n (since before observing the liquidity shocks traders are identical) gives expected profits for the n institutional traders participating in the auction as

$$nE\pi_i^{auction} = -\frac{n\sigma_v^2}{2\varphi^3\sigma_u^2}\left[\frac{\varphi^4\sigma_u^4 + \varphi^2\sigma_u^2\sigma_v^2 + (n-1)\sigma_v^4}{\varphi^2\sigma_u^2 + n\sigma_v^2}\right] \tag{2.11}$$

where by abuse of notation we have used σ_u^4 and σ_v^4 to denote $(\sigma_u^2)^2$ and $(\sigma_v^2)^2$ respectively.

Equation (2.11) shows that expected profits are always negative. To see why this is so note the objective function in equation (6.30) has two components. The first $E(v - p)x_i$ which we call trading profits represents pure expected gains or losses to the institution from trading. The second $(\varphi/2)E(x_i - u_i)^2$ which we call trading cost (cost because it enters institutional profits with a minus sign) is always positive. It is easy to show that expected trading profits are zero in aggregate by writing them as

$$\sum_{i=1} E[(v - p)x_i] = E[(v - p)X]$$

Then noting that $p = E[v|X]$ gives

$$\sum_{i=1} E[(v - p)x_i] = E[(v - E[v|X])X] = 0$$

Because liquidity costs are always positive, profits of each institution are always negative. The intuition as to why trading profits are zero is because faced with the adverse selection problem of trading with informed institutions, the market maker sets "fair" prices given

[8] As has often been pointed out, without noise trades, the market would collapse because no rational market maker would trade with an informed market participant. This condition arises from such considerations.

knowledge of the current order °ow i.e. he sets prices such that expected trading pro¯ts conditional on trading volume are zero. Therefore the institutions can never o®set trading costs with trading pro¯ts. Note that if there was no adverse selection problem [¾$_V^2$! 0], then expected trading costs to the institutional traders in equation (2.11) fall to zero. In this case the market maker knows that he does not face an informed trader, and the institutions can then trade to just o®set their liquidity shocks (i.e. they can trade an amount $x_i = u_i$ at a \fair" price). In the more general case [¾$_V^2$ > 0], the institutions are forced to trade at a loss because they are unable to credibly commit to the market maker that they are not trading on information.

In Appendix A we discuss the e®ects of collusion on investors' pro¯ts (and hence on trading costs because the two are again equivalent). When the n ¯rms act as a \multi-plant" monopolist, trading costs are actually increased compared with the non-cooperative situation. This is because the market maker knows that the colluding institutions are acting strategically, and sets a higher mark-up which actually reduces the multi-plant monopolist's pro¯ts. In contrast non-colluding investors trade too aggressively, and such trading reveals more of their private information. Non-colliding investors bene¯t from this \forced" revelation of their information, due to competition from other investors. The colluding case is interesting because it illustrates that investors who act non-collusively bene¯t from being able to commit to not using their information. The anomalous e®ect of competition in reducing trading costs is an important feature of oligopoly call auctions. It is important to bear this e®ect in mind when we compare this case with that of the sequential dealership where serial monopoly exists and such competitive e®ects on trading costs are absent.[9]

2.2 Sequential dealer market

In the sequential dealer market each institutional investor trades separately with the market maker, and therefore the market maker may o®er di®erent prices to di®erent investors. In common with the existing literature, we assume that the investors approach the market maker sequentially in random order.Unfortunately, as the model stands, traders incur smaller trading costs if they defer their trades giving each trader a strong oncentive to wait before submitting their orders. The assumption of random arrival/order-submission times is a rather unsatisfactory solution to this problem and a weakness of the current analysis However, in section 4.3 and Appendix C below, we extend the model to allow for endogenous arrival times In this extension, we add the assumptions that traders incur penalties for not executing their orders within the market period (the time period taken to e®ect the call auction) and that these penalties are su±ciently large to induce trade within the period. The solution to the extended model yields an identical form for expected trading costs as that given by the exogenous arrival/order-execution assumption currently employed.

Dealer markets are to be found in less-liquid stocks on the London Stock Exchange, on the foreign exchange markets and NASDAQ. As before, the ¯rst investor maximises (6.30), but this time we conjecture that the trading volume of the individual investor is a linear

[9] We also show in Appendix A that collusion raises trading costs to investors so much, that they will always prefer to trade in the sequential dealership market regardless of the values of the parameters that a®ect their pro¯ts/costs.

function of the information, and the market maker sets price as a linear function of the individual investor's trading volume

$$x_1 = \beta_1(v - \bar{v}) + \gamma_1 u_1 \tag{2.12}$$

and

$$p_1 = \bar{v} + \lambda_1 x_1 \tag{2.13}$$

The first investor now acts as a monopolist and therefore does not have to worry about the effect of his rival's trading volume on prices. The optimal trading volume for the first investor is

$$x_1 = \frac{v - \bar{v}}{2\lambda_1 + \phi} + \frac{\phi u_1}{2\lambda_1 + \phi} \tag{2.14}$$

Market makers act competitively and set prices to the first investor as the expectation of the terminal value of the asset v conditional on the first investor's trading volume x_1. Under this assumption λ_1 is analogous to the λ of the previous section and is given as $\lambda_1 = \mathrm{cov}(x_1; v)/\mathrm{var}(x_1)$. Equating coefficients as before yields

$$\lambda_1 = \frac{\phi\sigma_v^2}{\phi^2\sigma_u^2 - \sigma_v^2}; \qquad \beta_1 = \frac{[\phi^2\sigma_u^2 - \sigma_v^2]}{\phi[\phi^2\sigma_u^2 + \sigma_v^2]}; \qquad \gamma_1 = \frac{[\phi^2\sigma_u^2 - \sigma_v^2]}{\phi^2\sigma_u^2 + \sigma_v^2} \tag{2.15}$$

Again we want to obtain an expression for expected trading costs (trading profits are zero as before) for the trader. Using (2.14) and the coefficients in (2.15), we may write the optimal trades of the first investor as

$$x_1 = \frac{\phi^2\sigma_u^2 - \sigma_v^2}{\phi(\phi^2\sigma_u^2 + \sigma_v^2)}[v - \bar{v} + \phi u_1] \tag{2.16}$$

Substituting (2.16) and (2.13) into (6.30) and taking expected values, we obtain the expected trading costs of the first institutional trader in the dealer market[10]

$$E\pi_1^{dealer} = -\frac{\sigma_v^2}{2\phi} \tag{2.17}$$

Now consider the next investor's trading strategy. This investor also trades as a monopolist and does not have to worry about the strategic implications of his rivals' trading: his objective function is given by (6.30) which does not directly depend on previous trades. Once more we assume that the market maker sets "fair" prices ie. sets prices equal to the expectation of v conditioned on knowledge of x_1 and x_2. This would imply the market maker setting p_2 as the linear (least squares) projection of v on x_1 and x_2: However to expose the recursive structure of the problem and to simplify the solution, we taking an indirect route to the setting of prices by the market maker.

[10] Note that equation 2.17 can be obtained by setting $n = 1$ in equation 2.11, which illustrates that the first phase of the sequential dealer market coincides with the single-trade case of the auction model.

First, we conjecture that in equilibrium, optimal trades in the second period are uncorrelated with those of the first. Then, following the first trade, the market maker computes an updated distribution for v given by $v \sim N(\bar{v}_1; \sigma^2_{v|1})$ where

$$\bar{v}_1 = E[v|x_1] = \bar{v} + \frac{\varphi\sigma_v^2}{\varphi^2\sigma_u^2 - \sigma_v^2} x_1 = p_1 \qquad \text{and} \qquad \sigma^2_{v|1} = var[v|x_1] = \frac{\varphi^2\sigma_u^2\sigma_v^2}{\varphi^2\sigma_u^2 + \sigma_v^2} \tag{2.18}$$

He then sets prices to the second trader in an analogous way to the first trader, $p_2 = \bar{v}_1 + \lambda_2 x_2$; where, λ_2 is analogous to λ_1 in (2.13) above and is given by $\lambda_2 = cov[x_2; (v - \bar{v}_1)]/var[x_2|x_1]$. Using the conjecture for prices in the objective function gives optimal demands for the second monopolist as $x_2 = \beta_2(v - \bar{v}_1) + \gamma_2 u_2$ which clearly shows that optimal demands in the second period x_2, are indeed independent of those in the first x_1, (and are also normally distributed). The independence of equilibrium trades now implies that, prices in the second period satisfy

$$p_2 = \bar{v}_1 + \lambda_2 x_2 = E(v|x_1) + \lambda_2 x_2 = p_1 + E(v|x_2) = E[v|x_1; x_2] \tag{2.19}$$

where the last equality confirms that the conjectured prices are indeed fair. Solutions for $\beta_2; \lambda_2$ and γ_2 may be computed as in (2.15).

It is easily seen that the recursive solution to the problem given above for the first two trades may be generalised to trade j. Solutions for β_j, λ_j, and $E\pi_j^{dealer}$, $\bar{v}_j$ and $\sigma^2_{v|j}\{= var(v|x_1; x_2 \ldots x_j)\}$ may be obtained from equations (2.15) to (2.18) respectively by replacing the right hand side terms $\sigma^2_{v|j-1}$ and $\bar{v}_{j-1}$ with σ_v^2 and $\bar{v}$ respectively. Adapting equation (2.15) in this way to give solutions for β_j and λ_j

$$\lambda_j = \frac{\varphi\sigma^2_{v|j-1}}{\varphi^2\sigma_u^2 - \sigma^2_{v|j-1}}; \beta_j = \frac{[\varphi^2\sigma_u^2 - \sigma^2_{v|j-1}]}{\varphi[\varphi^2\sigma_u^2 + \sigma^2_{v|j-1}]} \tag{2.20}$$

The solution shows clearly that λ_j is increasing in $\sigma^2_{v|j-1}$ and that β_j is decreasing in $\sigma^2_{v|j-1}$. Similarly we may adapt equations (2.17) and (2.18) to give

$$E\pi_j^{dealer} = -\frac{\sigma^2_{v|j-1}}{2\varphi} \qquad j \geq 2 \tag{2.21}$$

$$\sigma^2_{v|j} = \frac{\varphi^2\sigma_u^2\sigma^2_{v|j-1}}{\varphi^2\sigma_u^2 + \sigma^2_{v|j-1}} \tag{2.22}$$

respectively. Given the initial condition $\sigma^2_{v|0} = \sigma_v^2$, Equations (2.21) and (2.22) may be solved recursively to give an explicit form for the jth trader's profits for $j = 2; 3 \ldots n$. To get a closed form for expected profits for the jth trader, rearrange (2.22) to give

$$(\sigma^2_{v|j})^{-1} = (\sigma^2_{v|j-1})^{-1} + (\varphi^2\sigma_u^2)^{-1} = (\sigma_v^2)^{-1} + j.(\varphi^2\sigma_u^2)^{-1} \tag{2.23}$$

Using (2.23) on the right of (2.21) gives aggregate expected profits and hence trading costs for the n institutional traders as

$$\sum_{i=1} E\pi_i^{dealer} = -\sum_{j=1} \frac{\varphi\sigma_u^2\sigma_v^2}{2(\varphi^2\sigma_u^2 + (j-1)\sigma_v^2)} \tag{2.24}$$

Note that because β_j is decreasing in $\sigma^2_{v|j-1}$ and λ_j is increasing in $\sigma^2_{v|j-1}$ [see (2.20)] and because $\sigma^2_{v|j-1}$ is decreasing with j (see (2.22))then β_j increases and λ_j declines with j. The jth trader trades more aggressively than the $j-1$th because the updated variance of v has fallen and because the covariance of the underlying value of the asset and the order flow has also fallen the market maker set a lower mark-up to the jth trader. The information revelation that occurs as successive institutions trade, reduces the trading costs of successive traders as is clear from (2.24),. [11] This is an important effect in a sequential dealership market that is absent from a "one-off" call auction where each trader's expected trading costs are the same. As noted in the previous section however, the fact that each institution acts as a monopolist works to increase the institutions' costs. We examine the net impact of the two effects of competition and sequential information revelation in the next section.

3 Comparison of the two alternative market mechanisms

To compare the expected trading costs under the auction and dealer markets given in (2.11) and (2.24) we first take the case of two firms (i.e. the cases of duopoly and two sequential monopolists respectively) and then generalise to n firms. The following theorem which is the main result of the paper, states that whether one market mechanism is preferred to the other depends on the relative values of the uncertainty about the fundamental, and the importance of the liquidity shocks.

Theorem 3.1 Define the quantity

$$c = \frac{\sigma_v^2}{\varphi^2\sigma_u^2}$$

In comparing the expected trading costs to the institutional investors from trading in an auction or dealer market

(i) For $n > 2$ a sufficient condition for auction markets to yield lowest expected trading costs than the dealer market is

$$0 < c < \frac{\sqrt{1+n}-1}{n}$$

(ii) For $n = 2$ this condition is both necessary and sufficient for auction markets to yield lowest expected trading costs than the dealer market

Proof.

For $n = 2$ comparing (2.11) with (2.24) shows that trading costs will be smaller in the dealer market as

$$\left[\varphi\sigma_u^2 - \frac{(\varphi^2\sigma_u^2-\sigma_v^2)(\varphi^2\sigma_u^2+\sigma_v^2)^2}{\varphi^3\sigma_u^2(\varphi^2\sigma_u^2+2\sigma_v^2)}\right] > \frac{\sigma_v^2}{2\varphi} + \frac{\varphi\sigma_u^2\sigma_v^2}{2(\varphi^2\sigma_u^2+\sigma_v^2)}$$

[11] So that without the assumption of random arrival times, which is common in models of sequential dealership markets, arrival times would be endogenous and the solution to the model would change.

This condition simplifies to

$$\varphi^4\sigma_u^4 - 2\varphi^2\sigma_u^2\sigma_v^2 - 2\sigma_v^4 > 0$$

Rearranging and solving this inequality for c, dealer markets are preferred as $c > \frac{\sqrt{3}-1}{2}$. Recall from the parameter solutions in (2.9) and (2.20) that for equilibrium we require that $\sigma_v^2 < \varphi^2\sigma_u^2$ i.e. that $\frac{\sigma_v^2}{\varphi^2\sigma_u^2} = c < 1$. In the range $\frac{\sqrt{3}-1}{2} < c < 1$ the dealer market will be preferred by these institutional traders. Therefore in the range $0 < c < \frac{\sqrt{3}-1}{2}$ the auction market will yield the lowest expected trading costs to the traders. This establishes (ii)||

To prove (i), define the difference in trading costs between the two trading systems as a function of n

$$\pi^d(n) = nE\pi_i^{auction} - \sum_{i=1} E[\pi_i^{dealer}] \qquad n \geq 2$$

Define also the increment to $\pi^d(n)$ as n increases by one as a function of n

$$\Delta\pi^d(n) = \pi^d(n) - \pi^d(n-1) \qquad n \geq 2$$

"Differencing" (2.11) and (2.24) and subtracting the latter "difference" from the former gives incremental profit differences as

$$\Delta\pi^d(n) = -(n-1)\sigma_v^4 \frac{n\sigma_v^4 + 2\varphi^2\sigma_v^2\sigma_u^2 - \varphi^4\sigma_u^4}{2\varphi^3\sigma_u^2[(n-1)\sigma_v^2 + \varphi^2\sigma_u^2][n\sigma_v^2 + \varphi^2\sigma_u^2]} \qquad n \geq 2$$

Clearly $\Delta\pi^d(n) < 0$ if the numerator in the fraction is positive. Dividing this term by the positive quantity $\varphi^4\sigma_u^4$ gives the expression

$$nc^2 + 2c - 1$$

which can be factorised as

$$nc^2 + 2c - 1 = n\left(c + \frac{\sqrt{1+n}+1}{n}\right)\left(c - \frac{\sqrt{1+n}-1}{n}\right)$$

Hence $\Delta\pi^d(n) < 0$ iff $c > \frac{\sqrt{1+n}-1}{n}$, and for $n \geq 2$ this condition is sufficient to ensure $\Delta\pi^d(n) < 0$. Noting that

$$\pi^d(n) \equiv \pi^d(2) + \sum_{j=3} \Delta\pi^d(j) \tag{3.25}$$

we see that if $c > \frac{\sqrt{1+n}-1}{n}$ is satisfied, all terms on the right hand side of (3.25) are negative. Also note that for $n \geq 2$; $\Delta\pi^d(n) > 0$ iff $c < \frac{\sqrt{1+n}-1}{n}$, and all terms on the right of (3.25) are positive which proves (i). || ■

It was established above that in the call auction, expected trading costs to the institutions are reduced by the presence of competition between traders (since traders benefit from being able to commit to revealing their private information) but impaired by the inability of trades to reveal information sequentially, whereas the opposite is true in the dealer market where expected trading costs are reduced by the sequential revelation of information, but harmed by the lack of competition. In the Theorem, c measures the ratio of information volatility to that of (normalised) noise trade volatility. Hence when c is high, asymmetric information is prevalent, whereas if c is low, liquidity effects are more important. The fact that when c is high dealer markets yield lower trading costs shows that the size of the information revelation effect is more sensitive to the degree of inside information than is the competition effect. In other words, in markets where inside information has a relatively large influence on stock price movements the value of sequential trading in revealing information to the market and so reducing financial institutions' trading costs is high. In markets where liquidity trading is the predominant source of stock price volatility then the value of competitive bidding in reducing financial institutions' trading costs is high.

4 Sensitivity analyses

In this section we briefly discuss a) what happens when liquidity shocks are perfectly correlated (instead of independent), b) the effect of allowing traders to submit market orders rather than price-contingent trades, and c) an extension to the model that allows the arrival time of institutional investors to the dealer market to be endogenous. Derivations for a) are obvious and suppressed, while those for b) can be found in Appendix B, and for c) in Appendix C.

4.1 Perfectly correlated liquidity shocks

In the analysis above, our institutions were perceived as insurance companies in a segmented market where firm i offers insurance against idiosyncratic risk u_i. It may be however, that these insurance companies pool their risk in a secondary insurance market so that the only risk encountered is systematic risk. This assumption is the polar opposite of the one used in above because instead of liquidity shocks being uncorrelated, they would become perfectly correlated i.e. identical. The analysis is much more simple under this assumption and we suppress derivations here to give just the main results. The institutions' trading costs (again these are the same as profits) under the auction and dealership markets are now respectively

$$E(\pi)^{\text{Auction}} = -\frac{\sigma_v^2(\sigma_v^2 + n^2\sigma_u^2\varphi^2)}{2n\varphi(\sigma_v^2 + \varphi^2\sigma_u^2)} \tag{4.26}$$

and

$$E(\pi)^{\text{Dealer}} = -\frac{\sigma_v^2}{2\varphi} \tag{4.27}$$

The difference between the trading costs from the two market systems is

$$E(\pi)^{\text{Auction}} - E(\pi)^{\text{Dealer}} = \frac{(n-1)\sigma_v^2(c-n)}{2n\varphi(c+1)} < 0 \tag{4.28}$$

Equation (4.28) shows that in the case of identical liquidity shocks, the dealership always yields lower trading costs. The intuition is that the sequential revelation of information has now become very powerful because the market maker will be able to infer the value of v exactly after only two trades. Hence, trading costs to all except the ¯rst institutional trader in the dealership are zero. No amount of competition between traders in the call auction, where only the single observation of aggregate trading volume is available to the market maker, can generate such low trading costs.

4.2 Non-price contingent batch auction

In the call auction traders submit price contingent demands or limit orders. We now examine the e®ect of disallowing limit orders and con¯ning traders to submit market orders to the auction. The problem in this case is the analytical complexity arising from the asymmetric information that investors have about each other's liquidity shocks and the resulting Bayes-Nash solution to the problem. For the n = 2 case, however, we show in Appendix B that trading costs are uniformly lower in the market order auction than in either the limit order or in the sequential dealership. We also show that the market depth parameter ¸ is uniformly lower in market order versus limit order auction.

The results in Appendix B show that in contrast to the price contingent auction, the pure call auction with market orders is unambiguously preferred by institutional investors. The intuition behind the reduced trading costs for the pure call auction lies in the fact that allowing investors to condition bids on total order °ow is equivalent to knowing your rivals' liquidity shocks. In turn, knowing your rivals' liquidity shocks generates a response that amounts to increased \collusion" in aggregate. This situation is similar to the results on \sharing of information" in Shapiro's (1986) model of oligopoly.[12] Analogous to Shapiro where oligopolists share information about marginal costs of production, in the current paper, the institutional investors are able to share information about their liquidity shocks, although this takes place only implicitly through the ability to submit price contingent demands. Unlike the standard oligopoly set up however, increased collusion in the form of information sharing in our batch auction is recognised by the market maker and leads to higher mark ups (¸) and increased trading costs (see the multi plant monopolist case in Appendix A). Hence our result on information sharing is the complete reverse of that of Shapiro. The crux of this reversal lies in the fact that in Shapiro's goods market, the consumer demand curve is ¯xed so that increased \agressiveness" of reaction functions resulting from information sharing suceeds in raising pro¯ts. By contrast, in our batch auction, the market maker's \demand curve" is not ¯xed but shifts adversely against the institutional investors in response to the more \aggressive" trading that information sharing generates. Once again, the extra information implicitly granted by the ability to submit price contingent trades is a curse in the same way that the information about v is a curse in the sense that investors would bene¯t if they could credibly commit to not using it [see the discussion above after Equation (2.11)]. In the pure call auction with market orders, investors do not know anything about their

[12] In Shapiro (1986) oligopolists have private information about their own costs, and Shapiro considers the two equilibria where the oligopolists choose to share and not to share their cost information. Shapiro shows that oligopolists will prefer to share information, because low cost ¯rms can induce their rivals to reduce their output, by making their own relatively agressive reaction functions known, and hence raise aggregate oligopoly pro¯ts.

rivals' liquidity shocks and there is no implicit or explicit mechanism for credibly sharing information. Disallowing limit orders therefore reduces trading costs and investors would prefer to trade on the pure call auction market than limit order auction market.

Finally we note that appendix B additionally establishes that for $n = 2$ trading costs in the pure call auction are also uniformly lower than in the sequential dealership. This is in contrast to the comparison between the limit order auction and the sequential dealership where the preferred system hinged on parameter values and serves to reinforce how powerful is the effect on trading costs of disallowing information sharing during an auction.

4.3 Endogenising arrival times in the dealer market

In the analysis of the dealer market we made the standard assumption that traders arrive in random order. [See, for example Glosten and Milgrom (1985), Glosten (1989), Madhavan (1992), and Shin (1996)]. Without the assumption of random arrival times, all institutions will wait until the last round of trading, since from (2.24) the trading costs of successive traders is reduced. Though if everyone waits until the last round then either there is an auction, or there is some probability of no trade.[13] We now relax the assumption of exogenous arrival times and allow institutional investors to choose when they trade. We assume that if two or more traders submit at the same time then the market maker processes their trades in a random order.[14] Building on the idea that if agents wait too long there is some probability of no trade and a resultant large financial penalty we can show that an equilibrium exists where all traders choose to submit orders simultaneously. The trades are processed in a random order by the market maker and hence the traders receive identical expected profits In this equilibrium, no trader will incur the waiting costs and total expected profits of the institutional investors has an identical form to that given in the analysis above.

Suppose that during the trading period the execution of orders submitted up to some time T is guaranteed. However traders submitting after T face a probability of not having their order processed. If an order fails to be processed, the trader incurs a penalty, the size of which increases with the trader's optimal trade. In this case the objective function (6.30) becomes

$$\pi_i | v; u = [v - p]x_i - \frac{\varphi}{2}(x_i - u_i)^2 + \delta w x_i^{*2} \qquad i = 1, 2, \ldots, n \tag{4.29}$$

where $\delta w x_i^{*2}$ is a penalty from failing to trade. δ is a binary variable, which is zero if trader i trades within the period in the ith sequential dealership market but is unity otherwise, x_i^* denotes optimal demands for a trader attempting to trade within the period in market i, and w is the penalty cost The quadratic form of the penalty function is driven by considerations of analytical tractability. However it does have the property of scaling the fine to the seriousness of the "transgression". Note that the weight φ used in the "penalty" term $\frac{\varphi}{2}(x_i - u_i)^2$ is different from w because the implications of not trading at all are assumed to be far more serious than trading at least some amount. A failure to trade may mean

[13] Alternatively, institutions with large liquidity shocks will trade early, signalling that they have large liquidity shocks: but this effect would require a different modelling strategy.

[14] This could be taken as a metaphor for the physical system in existence where traders communicate their orders by phone and the phones are answered sequentially.

the wholesale breaking of a legally binding contractual obligation with the trader's client. and such transgressions are likely to be of a di®erent order of seriousness to those involving partial satisfaction of contractual obligations (as is the case when at least some amount of trade is made to o®set the liquidity shock)

In Appendix C we demonstrate that if w is large enough then all traders in the sequential dealer market will want to trade at the same time T. All institutions trading at the same time forces there to be some random order to trading. In the equilibrium of our model, all traders submit at time T and are processed by the market maker in random order. The physical process that this could correspond to is one where traders phone the market maker and are held in a "queue" at time T. Once the ith trader ¯nally gets through to the market maker, he is o®ered a price schedule appropriate to the ith sequential market, and submits the corresponding optimal demand.

5 Conclusions

In this paper we have examined two alternative secondary market microstructures: a sequential dealer market and a call auction, where institutional investors trade non-collusively in the underlying security. The main result of this paper has been to identify the conditions under which one market microstructure is preferred to another in terms of the expected pro¯ts to the institutional investors, or conversely where trading costs are lowest. We established that in the call auction, trading costs to the institutions are reduced by the presence of competition between traders but increased by the inability of trades to reveal information sequentially whereas the opposite is true in the dealer market. The paper is not without its limitations since the formal modelling of the two alternative trading systems is restrictive, and important aspects such as transparency, time and reputation are absent. Models that account for one or more of these features is the subject of future research.

The main insight of this paper has been to demonstrate that the extent of information revelation, and the inferences of market intermediaries di®ers across trading systems: so that there is not a single optimal trading structure. The structure which yields the lowest trading costs to the traders depends on parameter values which govern the relative importance of liquidity shocks to information. Speci¯cally when asymmetric information is prevalent the dealer markets yield higher pro¯ts because the size of the information revelation e®ect is more sensitive to the degree of inside information than is the competition e®ect. In other words, in markets where inside information has a relatively large in°uence on stock price movements the value of sequential trading in revealing information to the market and so reducing ¯nancial institutions' trading costs is high. In markets where liquidity trading is the predominant source of stock price volatility then the value of competitive bidding in reducing ¯nancial institutions' trading costs is high, in which case the auction market is the preferred market microstructure.

We also compared a limit order auction market with a market order auction market and demonstrated that an auction market with market orders yield lower trading costs, because the ability to submit price contingent demands, is equivalent to allowing some degree of collusion between traders, which in aggregate increases trading costs.

.

References

[1] Becht, M. and A. Roell (1999) \Blockholdings in Europe: an international comparison", European Economic Review, vol. 43, 1049-1056.

[2] Bernhardt, D. and E. Hughson (1997) \Splitting orders" Review of Financial Studies, Vol. 10, No. 1, 69-101.

[3] Bhide, A. (1993) \The hidden costs of stock market liquidity", Journal of Financial Economics, vol. 34, 31-51.

[4] Biais, B. (1993) \Price formation and equilibrium liquidity in fragmented and centralized markets", Journal of Finance, vol. 48, no. 1, March, 157-186.

[5] Bolton, P. and E-L von Thadden (1998) \Blocks, liquidity and corporate control", Journal of Finance, vol. 53, no. 1, 1-26.

[6] Glosten, L. (1989), \Insider trading, liquidity, and the role of the monopoly specialist, Journal of Business, vol. 62, 211-236.

[7] Glosten, L. and P. Milgrom (1985), \Bid, ask, and transaction prices in a specialist market with heterogeneously informed traders", Journal of Financial Economics, vol. 13, 71-100

[8] Holmstrom, B. and J. Tirole (1993) \Market liquidity and performance monitoring", Journal of Political Economy, vol. 101, 678-709.

[9] Keim, D.B. and A. Madhavan (1998) \The cost of institutional equity trades", Financial Analysts Journal, 50-69.

[10] Kyle, A.S. (1985) \Continuous auctions and insider trading", Econometrica, vol. 53, 1315-1335.

[11] Madhavan, A. (1992) \Trading mechanisms in securities markets", Journal of Finance, vol. 47, no. 2, 607-641.

[12] Maug, E. (1998) \Large shareholders as monitors: Is there a trade-o® between liquidity and control?", Journal of Finance, vol. 53, no. 1, 65-98.

[13] Myners, P. (2001) \Institutional Investment in the United Kingdom: A Review", (Report to HM Treasury).

[14] Pagano, M. and A. Roell (1992) \Auction and dealership markets: what is the di®erence?", European Economic Review, vol. 36, nos. 2/3, 613-23.

[15] Pagano, M. and A. Roell (1996) \Transparency and liquidity: a comparison of auction and dealer markets with informed trading" Journal of Finance, vol. 51, no. 2, 579-612.

[16] Pagano, M. and A. Roell (1998) \The choice of stock ownership structure: agency costs, monitoring and the decision to go public", Quarterly Journal of Economics, vol 113.

[17] Securities and Investments Board (1994) \Regulation of the United Kingdom Equity Markets", SIB Discussion Paper, February

[18] Securities and Investments Board (1996) \Maintaining Enhanced Market Liquidity", SIB Consultative Paper 97, May

[19] Seppi, D. (1992) \Block trading and information revelation around quarterly earnings announcements", Review of Financial Studies, vol. 5, 281-305

[20] Shapiro, C. (1986) \Exchange of cost information in oligopoly", Review of Economic Studies, vol. 53, 433-446.

[21] Shin, H.S. (1996) \Comparing the robustness of trading systems to higher-order uncertainty", Review of Economic Studies, vol. 63, 39-59.Appendix C

6 Appendix A

6.0.1 The effects of collusion.

Consider equilibrium for a "multi-plant" monopoly institutional trader, with n liquidity shocks who must choose optimal x_i. We might think of this financial institution as an investment bank with insurance company or pension fund subsidiaries, with each subsidiary suffering a liquidity shock. The trader maximises

$$\pi | v, u = \sum_{i=1} \{[v - p(X)]x_i - \frac{\varphi}{2}(x_i - u_i)^2\} \tag{A1}$$

To find the optimal trading volume of the ith institutional trader substitute the conjectured price function (3) into the objective function (A1). The reaction function for the ith subsidiary is

$$x_i = \frac{v - \bar{v}}{2\lambda_m + \varphi} + \frac{\varphi u_i}{2\lambda_m + \varphi} - \frac{2\lambda_m(\sum_{j=1}^{n} x_j - x_i)}{2\lambda_m + \varphi} \tag{A4}$$

and similarly for the second subsidiary, so that the aggregate trading volume is

$$X_m = \frac{n(v - \mu_m)}{2n\lambda_m + \varphi} + \frac{\varphi}{2n\lambda_m + \varphi} \sum_{i=1} u_i \tag{A5}$$

so that

$$\beta_m = \frac{n}{2n\lambda_m + \varphi}; \ \gamma_m = \frac{\varphi}{2n\lambda_m + \varphi} \tag{A6}$$

As before market makers act competitively and set prices as the conditional expectation of the terminal value of the asset v conditional on the aggregate trading volume X. The implicit value of λ_m is given by

$$\lambda_m = \{cov(v, X)/var(X)\} = \frac{\beta_m \sigma_v^2}{\beta_m^2 \sigma_v^2 + n\gamma_m^2 \sigma_u^2} \tag{A8}$$

Solving for the three unknown coefficients using (A6) and (A8), gives

$$\lambda_m = \frac{\varphi \sigma_v^2}{\varphi^2 \sigma_u^2 - n\sigma_v^2}; \quad \beta_m = n\frac{[\varphi^2 \sigma_u^2 - n\sigma_v^2]}{\varphi^3 \sigma_u^2 + n\varphi\sigma_v^2}; \quad \gamma_m = \frac{[\varphi^2 \sigma_u^2 - n\sigma_v^2]}{\varphi^2 \sigma_u^2 + n\sigma_v^2} \tag{A9}$$

We can now compare the values of β and λ from (9) with their counterparts β_m and λ_m from (A9).

Proposition 6.1

$$\lambda < \lambda_m \text{ and } \beta > \beta_m$$

The proposition shows that competition between rival institutional investors means that trading intensity is greater under the non-cooperative oligopoly than under monopoly, and hence the mark-up of prices by the competitive market maker is less under non-cooperative duopoly than under the collusive outcome. Combining these two effects we can examine the overall impact on prices.

Proposition 6.2 For a given sequence of realisations of u_i and v

$$p_m = p$$

The form of the price function in both the collusive and competitive cases depends on the product of λ and β. Proposition A1 shows that the former is higher and the latter lower under collusion and Proposition A2 shows that these two effects cancel to leave price exactly the same under non-cooperative oligopoly as in the multi-plant monopoly.

Note that the comparative static properties from (9) and (A9) are that

$$\frac{d\lambda}{d\sigma_v} > 0 \text{ and } \frac{d\lambda}{d\sigma_u} < 0$$

and

$$\frac{d\beta}{d\sigma_v} < 0 \text{ and } \frac{d\beta}{d\sigma_u} > 0$$

which are exactly the same as in the Kyle model. When σ_u is high there is so much noise in the system that the market maker cannot distinguish whether the insider is trading or not and λ is insensitive to order flow, likewise the informed trader can easily disguise his trades in the liquidity trades and trading intensity is relatively high. When σ_v is high, because the mm knows that the informed trader has strong reasons to trade, there is a large covariance between the value of the asset and the order flow, so that the order flow is relatively informative about the underlying value of the asset and market makers set a high mark-up.

The value of expected profits for the multiplant monopolist $E\pi_m$ is given by substituting the parameter values from (A9) into (A4), the result and (A5) into the objective function (A1) and then taking expectations over v and u_i to give

$$E\pi_m = -\frac{n\sigma_v^2}{2\gamma} \tag{A11}$$

We may compare (A11) with the sum of the expected profits of the oligopolistic traders in the call auction given in equation (11). Surprisingly the expected profits of the multiplant monopolist are lower. This is because the market maker knows that in the multiplant monopolist case the trader is deliberately hiding his information through a less intensive trading strategy, and the market maker's response (from Remark 1) is to increase the price mark-up. This results in lower expected profits for the multiplant monopolist. We may also compare the profits here with those of the sequential dealership monopolist. We may write profits under the latter as

$$E(\pi)^{\text{Dealer}} = -\left[\frac{\sigma_v^2}{2\gamma}\right]\left[\frac{\gamma^2\sigma_u^2}{\gamma^2\sigma_u^2+\sigma_v^2} + \frac{\gamma^2\sigma_u^2}{\gamma^2\sigma_u^2+2\sigma_v^2} + \ldots\frac{\gamma^2\sigma_u^2}{\gamma^2\sigma_u^2+n\sigma_v^2}\right] \tag{A12}$$

The second term in square braces is clearly less than n so that dealership profits are less than $-n[\frac{\sigma_v^2}{2\gamma}]$ which is as we showed above are the profits under collusion in the call auction.

6.1 Appendix B

6.1.1 Results for the non-price contingent auction case

Here we drop the assumption that institutional investors may submit price contingent orders and consider a batch auction in its simplest and purest form where participants simply submit quantities. Although this market mechanism does not exist in its purest form in the real world, it has been examined in the theoretical literature and is therefore worth some attention here also. In the batch auction mechanism considered in the paper, it did not matter that investor i did not observe investor j's liquidity shock because investor i could submit a continuum of orders representing an optimal response to each and every possible X value. Here however, investors will maximise expected profits given their information because they may only submit a single quantity order. Hence we seek a Bayes-Nash solution equilibrium for the game. As in the main model in the paper, price contingent trades are an irrelevance in the sequential dealership market because there is only one trader submitting orders to the market maker and each investor can work out in advance what equilibrium price he/she will face.

Unfortunately, tractible analytical results are hard to obtain for the general n investor batch auction case but we can make some headway for $n = 2$. With regards to the batch auction, the first order conditions for firm i now become

$$x_i = \frac{v - \bar{v}}{2\lambda + \varphi} + \frac{\varphi u_i}{2\lambda + \varphi} - \frac{\lambda E(x_j|u_i)}{2\lambda + \varphi} \qquad i; j = 1; 2 \qquad i \neq j \tag{B1}$$

Following for example Shapiro(1986) we conjecture that in a symmetric Bayes-Nash equilibrium, optimal equilibrium strategies for all institutional investors may be written as

$$x_i = \rho(v - \bar{v}) + \varpi u_i \tag{B2}$$

Using (B2) to compute $E(x_j|u_i)$ and substituting in to (B1) gives

$$x_i = \frac{v - \bar{v}}{3\lambda + \varphi} + \frac{\varphi u_i}{2\lambda + \varphi} \qquad i = 1; 2 \tag{B3}$$

Following now familiar procedures, we sum (B3) over $i = 1; 2$ and compute λ as $\mathrm{cov}(X; v)$ divided by $\mathrm{var}(X)$. This gives an implicit (cubic) form for λ as

$$\lambda(3\lambda + \varphi)^2 - c(\lambda + \varphi)(2\lambda + \varphi)^2 = 0 \tag{B4}$$

Now noting that as in the paper trading profits are once again zero[15], firm i's expected profits are just $-\frac{\varphi}{2}E(x_i - u_i)^2$ which using (B3) may be written as

$$E(\pi_i) = -\frac{\varphi}{2}\left(\frac{\sigma_v^2}{(3\lambda + \varphi)^2} + \frac{4\lambda^2\sigma_u^2}{(2\lambda + \varphi)^2}\right) \tag{B5}$$

[15] The unconditional expectation of total trading profits is $E\{(v - p)X\}$. Substituting for p as $E(v|X)$ gives $E\{(v - E(v|X))X\}$ which is zero by definition.

Substituting for $(3\lambda + \varphi)^2$ in (B5) using (B4) and manipulating the result gives a simpler form for expected profits as

$$E(\pi_i^{auction}) = -\frac{\varphi}{2}E(x_i - u_i)^2 = -\frac{\varphi\lambda\sigma_u^2}{2(\lambda + \varphi)} \tag{B6}$$

The term $E(x_i - u_i)^2[= -\frac{2}{\varphi}E(\pi_i^{auction})]$ is proportional to expected trading costs per trader in the auction market, henceforth referred to as just "costs". For simplicity and without losing generality we switch from comparing profits to comparing these "costs". We compare costs in the current market order auction market with the sequential dealership and with our earlier limit order auction market. Costs in the market order auction market (C^{moa}) are given by (B6) which may be written as

$$C^{moa} = \frac{\lambda^*\sigma_u^2}{(\lambda^* + 1)} \tag{B7}$$

where $\lambda^* = \frac{\lambda}{\varphi}$. A form for costs in the sequential dealership market (C^{sd}) is obtained by setting $n = 2$ in equation (2.24) of the main text to get

$$C^{sd} = \frac{2\varphi^2\sigma_u^2\sigma_v^2 + \sigma_v^4}{2\varphi^2(\varphi^2\sigma_u^2 + \sigma_v^2)} = \frac{c(2 + c)}{2(1 + c)}\sigma_u^2 \tag{B8}$$

where we have used and using $c = \frac{\sigma_v^2}{\varphi^2\sigma_u^2}$ in deriving the second equality. The equivalent form for the auction market with limit orders (C^{loa}) is given by setting $n = 2$ in (2.11) to get

$$C^{loa} = \frac{\sigma_v^2}{\varphi^4\sigma_u^2} \cdot \frac{\varphi^4\sigma_u^4 + \varphi^2\sigma_u^2\sigma_v^2 + \sigma_v^4}{\varphi^2\sigma_u^2 + 2\sigma_v^2} = c\frac{1 + c + c^2}{1 + 2c}\sigma_u^2 \tag{B9}$$

We now show that C^{moa} lies below C^{sd} and below C^{loa} for all admissible values of c (i.e. $\forall c \in [0; 1]$).

(B7) and (B9) imply that

$$C^{moa} \le C^{loa} \text{ iff } \frac{\lambda^*}{1 + \lambda^*} \le \frac{c + c^2 + c^3}{1 + 2c}; \text{ and} \tag{B10}$$

$$C^{moa} \le C^{sd} \text{ iff } \frac{\lambda^*}{1 + \lambda^*} \le \frac{c(2 + c)}{2(1 + c)} \tag{B11}$$

We deal with (B10) first.

We may rearrange the far right inequality in (B10) to give

$$C^{moa} \le C^{loa} \text{ iff } \lambda^* \le \frac{c + c^2 + c^3}{1 + c - c^2 - c^3} \tag{B12}$$

We may rewrite the cubic that determines λ (B4) in terms of λ^* as follows

$$g(\lambda^*) = \lambda^*(3\lambda^* + 1)^2 - c(\lambda^* + 1)(2\lambda^* + 1)^2 = 0 \tag{B13}$$

We now show that all solutions to $g(\lambda^*) = 0$ lie to the left of $\lambda^* = \frac{c+c^2+c^3}{1+c-c^2-c^3}$.

If we evaluate $g(\lambda^*)$ in (B13) at $\lambda^* = \frac{c+c^2+c^3+\delta}{1+c-c^2-c^3}$ for some $\delta \geq 0$ and simplify we get

$$\begin{aligned} g\left(\frac{c+c^2+c^3+\delta}{1+c-c^2-c^3}\right) &= (1-c)^{-3}(1+c)^{-6}\left(A+B\delta+C\delta^2+D\delta^3\right) \\ \text{where } A &= c^2+6c^3+18c^4+37c^5+32c^6+27c^7+10c^8+4c^9; \\ B &= 1+9c+24c^2+23c^3+48c^4+17c^5+14c^6-c^7; \\ C &= 6+25c+c^2+17c^3-4c^4); \quad \text{and} \\ D &= 9-4c \end{aligned}$$

which is always positive for nonzero c and for any $\delta \geq 0$. Hence, all real positive solutions for λ^* must lie to the left of $\frac{c+c^2+c^3+d}{1+c-c^2-c^3}$. This establishes that $C^{moa} \leq C^{loa}$.

To show $C^{moa} \leq C^{sd}$ we follow an identical procedure.

We may rearrange the far right inequality in (B11) to give

$$C^{moa} \leq C^{sd} \text{ iff } \lambda^* \leq \frac{2c+c^2}{2-c^2}$$

Following familiar arguments we evaluate $g(\lambda^*)$ above at $\lambda^* = \frac{2c+c^2}{2-c^2}$ and show that it is weakly positive for all admissible c and for all $\delta \geq 0$. Substituting $\lambda^* = \frac{2c+c^2}{2-c^2}$ into $g(\lambda^*)$ and simplifying gives

$$\begin{aligned} g\left(\frac{2c+c^2+\delta}{2-c^2}\right) &= \left(2-c^2\right)^{-3}(E+F\delta+G\delta^2+H\delta^3) \\ \text{where } E &= 12c^2+40c^3+36c^4+14c^5+2c^6; \\ F &= 4+28c+64c^2+24c^3-c^5; \\ G &= 12+38c-3c^2-4c^3; \quad \text{and} \\ H &= 9-4c \end{aligned}$$

which is always positive for nonzero c and for any $\delta \geq 0$. Hence, all real positive solutions for λ^* must lie to the left of $\frac{2c+c^2}{2-c^2}$. This establishes that $C^{moa} \leq C^{sd}$.

It is easily established that the mark up parameter λ in the market order auction is always less than its counterpart in the limit order auction. It is easy to show that in the limit order auction $\lambda^{loa} = \frac{c}{1-c}$: Hence all we need to establish is that

$$\lambda^*[=\frac{\theta}{\gamma}] \leq \lambda^{*loa}[=\frac{\lambda^{*loa}}{\gamma}] = \frac{c}{1-c} \tag{B14}$$

We showed above that $\lambda^* \leq \frac{c+c^2+c^3}{1+c-c^2-c^3}$ and it is easily verified that $\frac{c+c^2+c^3}{1+c-c^2-c^3} \leq \frac{c}{1-c} = \lambda^{*loa}$. Hence, $\lambda \leq \lambda^{*loa}$ so that the markup in the limit order auction exceeds its market order counterpart.

6.2 Appendix C

6.2.1 Justification for exogenous arrival time in the dealer market

We now consider an extension to the sequential dealer market that allows for a penalty function $\delta w x_i^{*2}$ if traders fail to trade before the market closes at time T. The objective function is now given by

$$\pi_i|v;u = [v - p]x_i - \frac{\varphi}{2}(x_i - u_i)^2 + \delta w x_i^{*2} \qquad i = 1, 2 \ldots n \tag{C1}$$

We conjecture that provided w is sufficiently large, an equilibrium exists where all traders submit (telephone) orders at time T. In the equilibrium of our model, all traders submit at time T and are processed by the market maker in random order. To establish this we compare the expected profits conditional on knowing u and v of a single trader who submits at time T with those obtained from submitting after T. If the first exceeds the second for any agent then there will be no incentive for any agent to deviate from the strategy of submitting at time T and such a strategy will be an equilibrium one.

If all traders submit at time T they each have a $\frac{1}{n}$ chance of being processed first, a $\frac{1}{n}$ chance of being processed second and so on. All such traders would be guaranteed order execution so δ is zero in the profits functions. Using (C1) we may write expected profits for a particular trader who has suffered a liquidity shock u and who has observed v and who, like all other traders, follows the equilibrium strategy of submitting at time T as

$$= \frac{1}{n}\sum_{i=1}^{n}\left(\beta_i(1 - \lambda_i\beta_i) - \frac{\varphi}{2}\right)(v + \varphi u_i)^2 - \frac{\varphi}{2}u^2 \tag{C2}$$

Now we compute the expected profits to this trader of deviating from this strategy and waiting until after T to submit his order. This trader will have his order executed in market n (the $n - 1$ other traders will already had their trades executed) with probability $(1 - q)$ and will fail to trade with probability q. Expected profits from this strategy of deviating from equilibrium are therefore

$$E(\pi_d|v;u) = (1 - q)\left(\left(\beta_n(1 - \lambda_n\beta_n) - \frac{\varphi}{2}\right)(v + \varphi u)^2 - \frac{\varphi}{2}u^2\right) + q\left(-wx_n^2 - \frac{\varphi}{2}u^2\right) \tag{C3}$$

Subtracting (C2) from (C3) gives the condition for no deviation as

$$E(\pi_e|v;u) - E(\pi_d|v;u) = \frac{1}{n}\sum_{i=1}^{n}\left(\beta_i(1 - \lambda_i\beta_i) - \frac{\varphi}{2}\right)(v + \varphi u)^2 - \tag{C4}$$

$$(1 - q)\left(\beta_1(1 - \lambda_1\beta_1) - \frac{\varphi}{2}\right)(v + \varphi u)^2 + q\left(-wx_n^2\right) > 0$$

Now, using the fact that $x_n = \beta_n(v + \varphi u)$ and simplifying gives the condition in terms of w as

$$w > \frac{1}{q\beta_n^2}\left(\frac{1}{n}\sum_{i=1}^{n}\left(\beta_i(1 - \lambda_i\beta_i) - \frac{\varphi}{2}\right) - (1 - q)\left(\beta_1(1 - \lambda_1\beta_1) - \frac{\varphi}{2}\right)\right) \tag{C5}$$

In sum, provided that (C5) is satisfied the equilibrium described above exists. Note that it is obvious that deviating from the equilibrium strategy by submitting before T is dominated by submitting at T, so there is no need to analyse this possibility.

Finally w above was a common minimum waiting cost. It is trivially true that the analysis applies to the case where each trader has distinct waiting costs w_i provided that $w_i \geq w$.