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A Panel Data Approach to Testing Anomaly Effects in Factor Pricing Models*

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Abstract

There has been a large anomaly literature where firm specific characteristics such as earnings-to-price ratio and book-to-market ratio as well as size help explain cross sectional returns. These anomalies that have been attributed to market inefficiency could be the result of a misspecification of the underlying factor pricing model. The most popular approach to detecting these anomaly effects has been the two pass (TP) cross-sectional regression models. However, it is well-established that the TP method suffers from the errors in variables problem, because estimated betas are used in the second stage cross sectional regression. In this paper we address the issue of testing for factor price misspecification via the panel data approach. Perhaps one of the main reasons for the neglect of benefits of using panel data technique is that in factor pricing models, all betas are heterogeneous in the first pass time series regression. However, if our interest lies solely in testing the significance of the firm's characteristics in factor pricing models, we can show how to construct a theoretically coherent example to which panel data techniques dealing with both homogeneous and heterogeneous parameters can be applied. Panel-based anomaly tests have one clear advantage over TP-based tests; they are based on full information maximum likelihood estimates so that they do not suffer from the errors in variable problem and have all the usual asymptotic properties associated with likelihood tests. The empirical illustration shows the importance of book to market equity and market value in helping explain asset returns in the UK over 1968-2002 even in the three factor models.

JEL Classification: C12, C13, G12.
Key Words: Excess returns, factor pricing models, anomaly effects, partially heterogeneous panels, pooled ML estimation.

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1 Introduction

The central prediction of the asset pricing models of Sharpe (1964), Lintner (1965), and Black (1972) is that the market portfolio of invested wealth is mean-variance efficient. This implies that expected returns on securities are an exact positive linear function of their market betas. But, there have been several empirical findings which contradict the prediction of these models. The most prominent is the size effect of Banz (1981), who finds that the market value of equity adds to the explanation of the cross-section of average returns provided by market betas. More recently, there has been a large anomaly literature where firm specific characteristics such as leverage, past returns, dividend-yield, earnings-to-price ratios and book-to-market ratios as well as size help explain cross sectional returns. See for example Keim (1983), Fama and French (1992, 1996), Berk (1995) and Gauer (1999).

To accommodate these anomaly effects, a general procedure pursued in the literature is as follows. First, find characteristics that may prospectively be associated with average returns. Then sort portfolios based on those characteristics, compute betas for the portfolios and check whether differences in average return are accounted for only by the differences in the betas. Fama and French's (1993, 1996) model successfully explains the average returns of the 25 size and book-to-market sorted portfolios with three factors, namely, returns on the market, returns on a small minus big (SMB) portfolio and returns on a high minus low (HML) portfolio. Even though the choice of factors is motivated mostly by empirical experience and thus somewhat arbitrary, their three factor model has been widely used in evaluating various expected return puzzles.

One practically important issue is to check whether the factor pricing models need to be augmented by asset specific characteristics. For example, momentum effects, where a portfolio consists of short-term winners, have been found to be important [see Jagadeesh and Titman (1993)], violating the Fama and French three factor model, (see Fama and French ,1996). In a similar vein, Daniel and Titman (1997), and Daniel, Titman and Wei (2001) have advanced ways to distinguish between factor models and characteristic models.

These anomalies that have been attributed to market inefficiency could be the result of a mis-specification of the underlying factor pricing model. The most popular approach to detect these anomaly effects has been the two pass (TP) cross-sectional regression methods, advanced by Black, Jensen and Scholes (1972) and Fama and MacBeth (1973), which have been widely used to evaluate linear factor pricing models, including the capital asset pricing model (CAPM), the arbitrage pricing theory (APT) and their variants [see Cochrane (2001) for an excellent survey]. In the first stage, the asset betas are estimated by time series linear regression of the asset's return on a set of common factors. Then, the cross sectional regression of mean returns on betas and characteristics is estimated, and the significance of asset specific regressors are evaluated along with factor risk premia estimation. The same approach could be applied to evaluating momentum anomaly effects using asset specific proxy variables for the past short term performance of portfolios (such as lagged portfolio returns).

However, it is well-established that the TP method suffers from the errors in variables (EIV) problem, because estimated betas are used in place of true betas in the second stage cross sectional regression. In this regard, many econometricians have suggested several ways to derive the EIV corrected standard errors of the TP estimators under different set of assumptions. A detailed treatment of TP estimation and associated asymptotic theories can be found in Shanken (1985, 1992), Jagannathan and Wang (1998), and Ahn and Gadarowski (2001).

In this paper we address the issue of testing for factor price mis-specification via the panel data approach. It is a salient fact that conventional approaches have completely ignored the benefits of using panel data techniques. Perhaps one of the main reasons for this neglect is that in factor pricing models, all betas are heterogeneous in the first pass time series regression. As a result there is no room for exploiting the panel dimension since there are no homogeneous coefficients to estimate. Instead, the validity of the null hypothesis that the time series factor pricing model is correctly specified is in fact tested in the second pass cross sectional regression, for example, of pricing errors on characteristics.

If our interest lies solely in testing the significance of these characteristics, we can show how to construct a panel data regression model with one set of variables varying over time such as common factors and another set of variables varying both over time and over asset portfolios. A statistical model where the parameters on factors are heterogenous and the parameters on characteristics are homogeneous is required to analyse the existence of anomalies in factor pricing models such as the CAPM or APT.

The current paper provides a theoretically coherent example to which panel data techniques dealing with both homogeneous and heterogeneous parameters can be applied. This partially heterogeneous panel data model shares common features with the econometric framework recently proposed by Pesaran, Shin and Smith (1999), who develop dynamic heterogeneous panel estimation techniques that allow the simultaneous investigation of both homogenous long-run relationships and heterogenous short-run dynamic adjustment towards that long run relationship. Though similar, in spirit the exact econometric methodology developed and used in this paper is different from that of Pesaran, Shin and Smith (1999), and is therefore developed separately here.

Our suggested panel-based anomaly tests have one clear advantage over TP-based tests; they are based on full information maximum likelihood estimates so that they do not suffer from the EIV problem and have all the usual asymptotic properties associated with likelihood tests. In addition the panel technique adopted here yields parameter estimates of firm specific effects that (under the alternative) are fully efficient.

The empirical illustration shows the importance of book to market equity and market value in helping explain asset returns. When such terms are added to the simple CAPM version of the model their significance is enormous. This confirms results from similar studies done on both US and UK data. Interestingly enough, however, we find, contrary to the results of Fama and French (1996), that (i) adding size and book-to-market variables do not drive out the significance of a standard CAPM market factor, and (ii) book-to-market variable remains mostly significant even after the basic CAPM factor is augmented by Fama French SMB and HML factors.

The next section outlines a heterogenous panel model within which factor pricing anomalies can be analysed and Section 3 derives the econometric theory required for this analysis. Section 4 gives an empirical illustration of the techniques applied to the UK excess stock returns. Section 5 concludes.

2 Overview on Modelling Issues

It is nowadays standard to assume that returns on the individual portfolio (or the individual stock returns) are linearly generated by multiple common factors,

$$r_{it} = a_i + \boldsymbol{\beta}_i' \mathbf{f}_t + \varepsilon_{it},\ i = 1, ..., N,\ t = 1, ..., T, \tag{2.1}$$

where r_{it} is the excess return of assets in the portfolio i at time t, $\mathbf{f}_t$ is the k vector of factors, a_i is the portfolio-specific intercept term, $\boldsymbol{\beta}_i$ is the k vector of betas (factor loadings) of portfolio i corresponding to $\mathbf{f}_t$, and ε_{it} is assumed to be the zero mean idiosyncratic error for portfolio i at time t. This model includes the standard CAPM as a special case. The (linear) beta-pricing restrictions imposed on (2.1) is given by

$$H_0 : E(r_{it}) = \gamma_0 + \boldsymbol{\beta}_i' \boldsymbol{\gamma}_1,\ i = 1, ..., N, \tag{2.2}$$

where $E(r_{it})$ is the expected return on assets in the portfolio i, γ_0 is an unknown constant (*e.g.* zero-beta expected return), $\boldsymbol{\gamma}_1$ is the k vector of associated factor risk premia. If (2.2) holds then asset markets are efficient in the sense that there are no (asymptotic) gains to arbitrage. However, there is mounting empirical evidence that asset specific factors are also priced. To the extent that these asset specific factors have idiosyncratic components (*i.e.* sources of risk that are diversifiable), then their pricing is incompatible with zero (asymptotic) arbitrage.[1] Specifically, previous studies, most famously that by Banz (1981) have added asset specific regressors to (2.2) and have estimated alternative models of the form

$$H_A : E(r_{it}) = \gamma_0 + \boldsymbol{\beta}_i' \boldsymbol{\gamma}_1 + \mathbf{s}_{it}' \boldsymbol{\gamma}_2,\ i = 1, ..., N, \tag{2.3}$$

where $\mathbf{s}_{it}$ is a q vector of asset specific variables such as size or book-to-market value for assets in the portfolio i at time t, $\boldsymbol{\gamma}_2$ is a $q \times 1$ vector of unknown parameters of return premiums associated with $\mathbf{s}_{it}$.

To test the null model against the alternative model H_A, a traditional two pass (TP) regression method has been applied to (2.3). To estimate $\boldsymbol{\gamma} = (\gamma_0, \boldsymbol{\gamma}_1', \boldsymbol{\gamma}_2')'$, we run the second pass cross sectional regression (CSR),

$$\bar{r}_i = \gamma_0 + \hat{\boldsymbol{\beta}}_i' \boldsymbol{\gamma}_1 + \bar{\mathbf{s}}_i' \boldsymbol{\gamma}_2 + \eta_i,\ i = 1, ..., N, \tag{2.4}$$

where $\bar{r}_i = T^{-1} \sum_{t=1}^{T} r_{it}$, $\bar{\mathbf{s}}_i = T^{-1} \sum_{t=1}^{T} s_{it}$, and $\hat{\boldsymbol{\beta}}_i$ are the OLS estimates of $\boldsymbol{\beta}_i$ obtained from the first pass time series regression (2.1). Alternatively, Fama and MacBeth (1973) considered a rolling CSR in each time period t,

$$r_{it} = \gamma_{0t} + \hat{\boldsymbol{\beta}}_i' \boldsymbol{\gamma}_{1t} + \mathbf{s}_{it}' \boldsymbol{\gamma}_{2t} + \eta_{it},\ i = 1, ..., N. \tag{2.5}$$

[1] Fama and French (1996) argue that most of the asset specific variables (particularly size and book to market) that generate anomalies in this way can be accounted for by additional pricing factors. We do not enter this debate here but focus on tests of pre-specified factor models against asset specific alternatives. Interestingly, Fama and French also admit that their factors cannot drive out the significance of own lagged returns in the cross section. The inclusion of variables such as own lagged returns makes the model a heterogenous dynamic panel but does not raise any problems for our approach as wiil be shown below.

where $\hat{\boldsymbol{\beta}}_i$ is estimated using time series observations 1 through $t-1$. Once the consistent TP estimator of $\boldsymbol{\gamma}$, denoted $\hat{\boldsymbol{\gamma}}'_{TP} = \left(\hat{\gamma}_{0,TP}, \hat{\boldsymbol{\gamma}}'_{1,TP}, \hat{\boldsymbol{\gamma}}'_{2,TP}\right)'$, is obtained, the validity of the asset price restriction (2.2) can be evaluated by testing $H_0 : \boldsymbol{\gamma}_2 = 0$, using for example a Wald test statistic given by

$$Wald = \hat{\boldsymbol{\gamma}}'_{2,TP} \left[Var\left(\hat{\boldsymbol{\gamma}}_{2,TP}\right)\right]^{-1} \hat{\boldsymbol{\gamma}}_{2,TP}, \tag{2.6}$$

which is distributed as χ^2_q under the null.

A well-known problem with this TP-based estimation is that the use of estimated betas in the second pass regression generates an errors in variables (EIV) problem. There has been a large literature attempting to derive the EIV corrected standard errors of the TP estimators under different sets of assumptions. In particular, with arbitrary positive definite weighting matrix, the TP estimator can be obtained by OLS, GLS, or GMM estimation. [For a treatment of TP estimation and associated asymptotic theories, see Shanken (1985, 1992) and Jagannathan and Wang (1998).]

An alternative method used to avoid the EIV problem is the ML estimation of Gibbons (1982). These authors express the null model in (2.2) as

$$H_0^* : a_i = \lambda_0 + \boldsymbol{\beta}'_i \boldsymbol{\lambda}_1, \ i = 1, ..., N, \tag{2.7}$$

where a_i is the individual intercept in the first-pass regression (2.1), $\boldsymbol{\lambda}_1$ is an unknown $k \times 1$ vector. Notice that the following relationships hold between the γ's and λ's (see Ahn and Gadarowski, 2001, p.6):[2]

$$\lambda_0 = \gamma_0, \ \ \boldsymbol{\lambda}_1 = \boldsymbol{\gamma}_1 - E\left(\mathbf{f}_t\right)$$

Similarly, the alternative model in (2.3) can be equivalently written as

$$H_A^* : a_i = \lambda_0 + \boldsymbol{\beta}'_i \boldsymbol{\lambda}_1 + \mathbf{s}'_{it} \boldsymbol{\lambda}_2, \ i = 1, ..., N, \tag{2.8}$$

where

$$\lambda_0 = \gamma_0, \ \ \boldsymbol{\lambda}_1 = \boldsymbol{\gamma}_1 - E\left(\mathbf{f}_t\right), \ \ \boldsymbol{\lambda}_2 = \boldsymbol{\gamma}_2.$$

Thus, the validity of the null H_0^* can be checked now by testing the restriction $\boldsymbol{\lambda}_2 \left(= \boldsymbol{\gamma}_2\right) = 0$. Applying the minimum distance approach to (2.7) and (2.8) in terms of TP estimation, Ahn and Gadarowski (2001) have developed several robust methods to estimate $\lambda = (\lambda_0, \lambda'_1, \lambda'_2)'$, but also provide EIV corrected standard errors of the TP estimators, such that the validity of asset pricing models can be evaluated under a general set of assumptions.

Suppose now that we are interested solely in testing the significance of the asset characteristics, as envisaged either by (2.3) or (2.8). More specifically, under (2.8), the time series linear factor pricing regression can be extended to the following panel data regression:

$$r_{it} = \alpha_i + \boldsymbol{\delta}' \mathbf{s}_{it} + \boldsymbol{\beta}'_i \mathbf{f}_t + \varepsilon_{it}, \ i = 1, ..., N, \ t = 1, ..., T, \tag{2.9}$$

[2]Notice here that the factor risk premia $\boldsymbol{\gamma}_1$ are now decomposed into the population mean vector of the factors $E\left(\mathbf{f}_t\right)$, and the so-called lambda component $\lambda_1 = \gamma_1 - E\left(\mathbf{f}_t\right)$. This lambda component can be interpreted as the vector of factor mean adjusted risk premia, see Zhou (1998).

where $\alpha_i = \lambda_0 + \boldsymbol{\beta}_i'\boldsymbol{\lambda}_1$ and $\boldsymbol{\delta} = \boldsymbol{\lambda}_2$. If certain asset characteristics are statistically significant for explaining excess returns, then these anomaly effects can be regarded as evidence against the underlying multi-factor models. The aim of this paper is to provide a panel data-based test for the null model (2.7) against the alternative model (2.8) in the context of multi-beta pricing models. We propose this be done via a simple Wald test of $\boldsymbol{\delta} = \mathbf{0}$ in (2.9). Because this does not require second pass cross sectional estimation, the panel-based test will not suffer from the EIV problem discussed above. Further, the fact that we use a Wald test gives the procedure all of the desirable (asymptotic) inferential properties associated with likelihood based tests. Finally, a by product of the method is that it generates full information ML estimates of all the model's parameters under the alternative and these estimates will be fully efficient.

As we noted above, one possible reason why previous authors have completely ignored the potential efficiency gains associated with ML panel data estimation is that under the null, all betas are heterogeneous so that there are no homogeneous coefficients to estimate and no efficiency gains (apart from those arising from imposing the null restrictions on the intercepts) to be made from system wide ML estimation. However under the alternative as (2.9) clearly shows, a panel-based analysis becomes not only natural but desirable from the point of view of efficient estimation and inference.

We close this section with some brief comments on the panel model. First, there are two different types of regressors: the asset pricing factors, which vary over time but are constant across assets/portfolios and the asset specific characteristic variables, which vary over both time and assets/portfolios. By contrast, factor loadings $\boldsymbol{\beta}_i$, are heterogenous across portfolios whilst the parameters on characteristics, $\boldsymbol{\delta}$, are homogeneous across portfolios. Hence, the panel data model (2.9) shares common features with the econometric framework recently proposed by Pesaran, Shin and Smith (1999), who develop dynamic heterogeneous panel estimation techniques that allow the simultaneous investigation of both homogenous long-run relationships and heterogenous short-run dynamic adjustment towards that long run relationship. Though similar, in spirit the exact econometric methodology developed and used in this paper is different from that of Pesaran, Shin and Smith (1999). Hence we must develop the underlying econometric theory for estimation and inference using (2.9) anew. This is achieved in the next section.

3 Heterogeneous Panel Data Methodology

In this section we formally develop the underlying econometric theory. To this end it will be convenient to generalize notation. Explicitly, we consider the heterogeneous panel regression model,

$$y_{it} = \boldsymbol{\delta}'\mathbf{x}_{it} + \boldsymbol{\beta}_i'\mathbf{f}_t + u_{it},\ i = 1, ..., N,\ t = 1, ..., T, \tag{3.10}$$

with error components,

$$u_{it} = \alpha_i + \varepsilon_{it}, \tag{3.11}$$

where y_{it} is a scalar dependent variable, $\mathbf{x}_{it}$ is a q vector of explanatory variables, $\mathbf{f}_t$ is the k vector of common factors, α_i contains individual effects, and ε_{it}'s are independently distributed (over time and cross-section) with mean zero and heterogeneous variance, σ_i^2. We assume that α_i are identically and independently distributed with zero mean and variance σ_α^2, and that α_i are uncorrelated with ε_{jt} for all i, j and t.

In this panel data model, some parameters ($\boldsymbol{\beta}_i$) are allowed to be heterogenous, but others ($\boldsymbol{\delta}$) are homogenous. Under the assumption that ε_{it} are normally distributed with heterogenous variances, σ_i^2, we obtain the following (concentrated) log-likelihood function:[3]

$$\ell_T(\boldsymbol{\varphi}) = -\frac{T}{2}\sum_{i=1}^{N} \ln 2\pi\sigma_i^2 - \frac{1}{2}\sum_{i=1}^{N}\frac{1}{\sigma_i^2}\left(\mathbf{y}_i - \mathbf{x}_i\boldsymbol{\delta}\right)' \mathbf{H}_i \left(\mathbf{y}_i - \mathbf{x}_i\boldsymbol{\delta}\right), \tag{3.12}$$

where $\mathbf{y}_i = (y_{i1}, ..., y_{iT})'$, $\mathbf{x}_i = (\mathbf{x}_{i1}, ..., \mathbf{x}_{iT})'$, $\mathbf{H}_i = \mathbf{I}_T - \mathbf{W}_i(\mathbf{W}_i'\mathbf{W}_i)^{-1}\mathbf{W}_i'$, $\mathbf{I}_T$ is an identity matrix of order T, $\mathbf{W}_i = (i_T, \mathbf{f})$ with $i_T = (1, ..., 1)'$ and $\mathbf{f} = (\mathbf{f}_1, ..., \mathbf{f}_T)'$, and $\boldsymbol{\varphi} = (\boldsymbol{\delta}', \sigma_i^2, ..., \sigma_i^2)'$.

The maximum likelihood estimator of the homogeneous parameters $\boldsymbol{\delta}$ can be obtained by maximizing (3.12) with respect to $(\boldsymbol{\delta}, \sigma_1^2, ..., \sigma_N^2)$, respectively. It is then straightforward to obtain the following formula for $\hat{\boldsymbol{\gamma}}$, and $\hat{\sigma}_i^2$:

$$\hat{\boldsymbol{\delta}} = \left(\sum_{i=1}^{N}\frac{1}{\hat{\sigma}_i^2}\mathbf{x}_i'\mathbf{H}_i\mathbf{x}_i\right)^{-1}\left(\sum_{i=1}^{N}\frac{1}{\hat{\sigma}_i^2}\mathbf{x}_i'\mathbf{H}_i\mathbf{y}_i\right), \tag{3.13}$$

$$\hat{\sigma}_i^2 = T^{-1}\left(\mathbf{y}_i - \mathbf{x}_i\hat{\boldsymbol{\delta}}\right)'\mathbf{H}_i\left(\mathbf{y}_i - \mathbf{x}_i\hat{\boldsymbol{\delta}}\right),\ i = 1, ..., N. \tag{3.14}$$

These need to be solved iteratively. Starting with an initial estimate of $\boldsymbol{\delta}$, say $\hat{\boldsymbol{\delta}}^{(0)}$, estimates of σ_i^2 can be computed using (3.14), which can then be substituted in (3.13) to obtain new estimate of $\boldsymbol{\delta}$, say $\hat{\boldsymbol{\delta}}^{(1)}$, and so on until convergence is achieved. Alternatively, these estimators can be computed by the familiar Newton-Raphson algorithm which makes use of both first and the second derivatives.

In order to derive the asymptotic distribution of the pooled ML estimators of $\boldsymbol{\varphi}$, we assume that all the underlying variables are stationary, in which case under fairly standard conditions the consistency and the asymptotic normality of the pooled ML and mean group estimators (see below) of the parameters in (3.12) can be easily established. In particular, as both $T \to \infty$ and $N \to \infty$, the pooled ML estimator of $\boldsymbol{\delta}$ has the following asymptotic distribution:

$$\sqrt{NT}\left(\hat{\boldsymbol{\delta}} - \boldsymbol{\delta}\right) \overset{a}{\sim} N\left\{\mathbf{0}, \left[\frac{1}{N}\sum_{i=1}^{N}\frac{1}{\sigma_{i0}^2}\mathbf{Q}_{x_i x_i}\right]^{-1}\right\}, \tag{3.15}$$

where $\mathbf{Q}_{x_i x_i}$ are the probability limits of $T^{-1}\mathbf{x}_i'\mathbf{H}_i\mathbf{x}_i$.[4] The proof can be easily established using the results in Pesaran and Smith (1995) and Pesaran, Shin and Smith (1999).

[3] Normality can be relaxed in which case a quasi-ML approach would be invoked.

[4] For this result to hold it is necessary that the limit of $N^{-1}\sum_{i=1}^{N}\frac{1}{\sigma_{i0}^2}\mathbf{Q}_{x_i x_i}$ as $N \to \infty$ is a positive definite matrix.

Using these results, the joint null hypothesis $\boldsymbol{\delta} = 0$ can be tested simply by a Wald statistic given by

$$Wald = \hat{\boldsymbol{\delta}}' \left[Var\left(\hat{\delta}\right)\right]^{-1} \hat{\boldsymbol{\delta}} = \left(\sum_{i=1}^{N} \frac{1}{\hat{\sigma}_i^2} \mathbf{x}_i' \mathbf{H}_i \mathbf{y}_i\right)' \left(\sum_{i=1}^{N} \frac{1}{\hat{\sigma}_i^2} \mathbf{x}_i' \mathbf{H}_i \mathbf{x}_i\right)^{-1} \left(\sum_{i=1}^{N} \frac{1}{\hat{\sigma}_i^2} \mathbf{x}_i' \mathbf{H}_i \mathbf{y}_i\right), \tag{3.16}$$

where $\hat{\sigma}_i^2$ is the final consistent estimate of σ_i^2 . Then, under the null, we have

$$Wald \overset{a}{\sim} \chi_q^2,$$

where q is the dimension of $\mathbf{x}_{it}$. As a special case the single null of $\delta_i = 0$, $i = 1, 2, ..., q$, can be tested using the t-test given by

$$t = \frac{\hat{\delta}_i}{\sqrt{Var(\hat{\delta}_i)}}, \tag{3.17}$$

where $\hat{\delta}_i$ is an ith element of $\hat{\boldsymbol{\delta}}$, which converges to the standard normal distribution under the null.

4 Empirical Illustration

In this section we apply the panel data test to a sample of UK firms. We focus on the significance and importance of size and book-to-market effects in explaining returns within the context of both a single factor or CAPM model and of a three factor model. Several other authors have already discovered "anomalous" size and book-to-market effects in UK data. See for example Levis (1985, 1989), Strong and Xu (1997) and Hussain, Diacon and Toms (2000).

4.1 Data Description

The data consist of 408 monthly observations from July 1968 to June 2002 on 5603 UK firms quoted in the London Stock Exchange. Stock market returns for the Financial Times All Share Index and for the individual companies are obtained by transforming the associated (monthly) return indices from *Datastream* into the monthly percentage returns (for example, the return index, RI, is the growth in value of a share holding over a month, assuming that dividends are re-invested to purchase additional units of an equity at the closing price applicable on the ex-dividend date). The excess return on the market portfolio, denoted r^m is obtained as the difference between the monthly market return and the monthly return on the risk-free asset which we take to be a 3-Month UK Treasury Bill. To test the model 25 new portfolios are formed on size and Book-to-Market equity. At the end of each month from July 1968 to June 2002 all the LSE stocks are ranked on market value and on Book-to-Market independently and then split into five size and five book-to-market groups. We construct 25 portfolios from the intersections of the size and Book-to-Market quintiles and calculate the value-weighted monthly excess returns on the portfolios, denoted r, as the difference between the respective weighted returns and the return on the risk-free asset. We also calculate Book-to-Market value and Market value for the 25 portfolios as the simple average of Book-to-Market value and Market value within each portfolio. Book-to-Market value (BTM) and Market value (MV) are as given by *Datastream,* and proxy the firm characteristics of financial distress and size, respectively. BTM is defined as the ratio between book value and market value, where book values, measured in millions of pounds, are defined as net tangible assets, excluding intangible assets, less total liabilities, minority interest and preference stock. MV, measured also in millions of pounds, is the share price multiplied by the number of ordinary shares in issue.[5] The 25 portfolios formed on size and book-to-market equity produce a wide range of average excess returns from 0.51 % to 2.73 % (see Table 1). The portfolios confirm the Fama and French (1992) and Fama and French (1993) evidence that there is a negative relation between size and average return, and a positive relation between average return and Book-to-Market equity. In all the BTM quintiles average returns tend to decrease with portfolio firm size. With the exception of a slight dip in the second BTM quintile, average returns increase with BTM for any given size quintile. Finally, taken together, the five portfolios in the largest size quintile account for on average about 56 % of the total value. The portfolio in both the largest size and the second BTM quintile alone accounts for more than 26 % of the combined value of the 25 portfolios. This finding slightly differs from Fama and French (1993) where the portfolio in both the largest size and the lowest BTM quintile contains the highest concentration of market values.

Following Fama and French (1996), we construct the two additional factors meant to mimic the underlying risk factors in returns related to size and Book-to-Market. We use six portfolios that are formed much like the 25 portfolios discussed earlier. At the end of each month from July 1968 to June 2002 all the LSE stocks are ranked on MV and BTM, independently. The median of MV value is used to split the sample into two groups, small and big. LSE stocks are also split into three BTM equity groups based on the break points for the bottom 30%, middle 40%, and top 30% of the ranked values of BTM for LSE stocks. The use of three groups for BTM but only two for MV is consistent with Fama and French (1993). Our portfolio SMB (small minus big), meant to mimic the risk factor in returns related to size, is the each month difference between the simple average of returns on the small-stock portfolios (S) and the simple average of returns on the big-stock portfolios (B). This difference should be largely free of the influence of BTM equity, focusing instead on the different return behaviour of small and big stocks. The portfolio HML (high minus big) meant to mimic the risk factor in returns related to Book-to-Market equity is similarly constructed: HML is the each month difference between the simple average of returns on the high BTM portfolios (H) and the average of returns on the low BTM portfolios (L). The difference between the two returns should be largely free of the size factor return, focusing instead on the different return behaviour of high and low Book-to-Market equity. The evidence of the success of this procedure is that the correlation from 1968 to 2002 monthly returns for the size and Book-to-Market factors is only 0.06. True mimicking portfolios for the common risk factors in returns minimize the variance of firm-specific factors. The six

[5]Berk (1995) points out that even if size and Book-to-Market effects were not priced, by their definition and construction, these variates will always covary with market price and hence returns. This spurious source of correlation will generate significant anomaly effects even when none are present.

size-Book-to-Market portfolios in SMB and HML are value-weighted. Using value-weighted components is in the spirit of minimizing the variance, since return variances are negatively related to size. Notice also that on average only 59 firms (out of 5603) per year have negative book equity. There are no firms with negative book equity till 1972 and they are very rare before 1985. The negative book equity firms are mostly concentrated in the last 12 years of the sample 1990-2002, and we do not include them in the test.

Table 1 about here

Table 2 presents annual returns for r^m, r, SMB and HML and firm characteristics (MV and BTM) for the thirty-five years but also provides the mean, the standard error and the number of negative values over the full sample. To accommodate the possible time varying nature of the underlying factor pricing models (including structural breaks), we also examine the three different decades, separately. The first subsample goes from 1968 to 1980, the second from 1981 to 1990 and the third from 1991 to 2002. Excess return and excess market returns are clearly correlated and their cyclical pattern follows the main events that hit the UK economy during the last thirty-five years. The mean of r is 4.9% and might represent a proxy for the average equity risk premium over a relatively long time period. It is higher than the 3.90% figure derived from a shorter period using UK data (Hussain, Diacon and Toms, 2000). Excess return and excess market returns reflect the recession between 1973 and 1974. Generally the '80s witnessed a rapid expansion of equity markets worldwide, which was accompanied by a particularly strong increase in international equity trading . As reported in the Bank of England Quarterly Bulletin (1988), the exceptionally rapid growth in international equity trading over the period can be attributed to an attempt to reduce risk through international portfolio diversification and the pursuit of international arbitrage opportunities emerging in the context of changing macroeconomic, tax and regulatory environments. The expansion of international equities has been associated with a greater interdependence of national stock markets and greater international interdependence of stock market ought to result in greater market efficiency. To the extent that funds can flow freely into markets in which assets are undervalued and out of markets where they are overvalued, prices tend to be based on more uniform risk-return criteria, enabling funds to be channelled into their most productive uses. On the other hand, the events of October 1987 interrupted the trend towards expansion. The negative values of $r^m$and r show the effect of the recession that affected the UK economy in the second half of 1990 and in 1991. Following the recession in the early '90s three other major events negatively influence $r^m$and r: namely, the Mexican crisis and political uncertainty in Europe between 1994 to 1995, the Asian crisis between 1997 and 1998 and the global slowdown starting from early 2001.

Table 2 about here

We now turn to SMB and HML. The SMB appears to be positive mainly for the first half of the study and negative for the second half of the study. In total, there are sixteen negative years out of thirty-five for SMB implying that small firms only outperformed large firms in nineteen out of thirty-five years. Clearly, there is evidence of a cyclical pattern in this factor. On the other hand, the HML variable appears to be mainly positive with only six negative years out of thirty-five and with these six years falling in the last two decades. This implies that high BTM firms outperformed low Book-to-Market firms for 83% of the sample's time span.

4.2 Empirical Results

Table 3 shows our suggested panel-based ML estimation and test results for anomaly effects for the single factor model and for the three factor model. Under the null hypothesis the Wald test examines the joint significance of the homogeneous coefficients on MV and BTM in explaining excess returns. The rejection of the null suggests that the underlying factor pricing model is mis-specified. We also re-estimate excluding each of the variables in turn and perform a t-test for the significance of the included variable's homogenous coefficient. We present the results for the sample as a whole and present the analogous results for the three selected subsamples.

Looking at the full sample estimates in the single factor (r^m) model, we see that when the model includes both MV and BTM, the value of the Wald test indicates massive significance of these terms. When only one characteristic is included in the regression, the coefficient on BTM is positive and significant whereas the coefficient on MV is negative and significant. These findings are consistent with Fama and French's (1996) argument that MV and BTM proxy for a macro"distress" factor with low BTM/MV firms being more exposed to bankruptcy risk and therefore, paying a higher return. But, when BTM and MV are jointly included in the regression, the accounting variable Book-to-Market equity has consistent explanatory power for average returns whereas the coefficient on MV becomes insignificantly positive. Strong and Xu (1997) have also obtained similar results that when BTM is included in the regression, the coefficient on MV turns out to be insignificant.

Turning to the analysis of the subperiods, we also find that the value of the Wald test for the joint significance of MV and BTM still indicates high significance of these terms for all subperiods. Next, when only one characteristic is included in the regression, the coefficients on BTM are always positive and significant whereas the coefficients on MV are always negative and significant. When BTM and MV are included in the regression, the coefficient on MV remains negative but becomes significant only for the second subsample of '80s. On the other hand, the accounting variable Book-to-Market equity has consistent explanatory power for average returns with t-statistics in the range 2.7 to 7.5.

Overall these findings are consistent with other estimation and test results in the literature. Our results in particular confirm for the UK the importance of Book-to-Market and market equity found previously for the US and for the UK, and justify the use of the SMB and HML variables for the UK.

Fama and French (1996) suggest that firm specific characteristics such as size and distress proxies are really picking up the effects of missing factors. They propose two additional factors SMB and HML that when used in TP regressions, destroy the significance of all of the usual characteristic variables. Table 3 also summarizes the results for the tests of anomaly effects for the three-factor model. In the full sample, the coefficient on MV is insignificant and small, but it becomes significant and negative when the model excludes BTM although its value is very close to zero. In the first two subperiods the coefficients on MV turn out to be significant but for the third, its value is close to zero and insignificant. The coefficients on MTB are significant except for the first period.

More importantly, comparing the three and one factor models, on average, the magnitude of the MTB coefficients are slightly lower in the former but the signs of the coefficients are the same in both. Although the values of the Wald test are generally lower than in the single factor model, they still indicate that the variables that proxy for characteristics are on the whole highly significant. Strictly speaking, this implies that the three factor model is still mis-specified, although, in terms of fit, it is an improvement over a single factor model.

Tables 3 about here

Finally, we carry out a mean group test advanced by Pesaran and Smith (1995) and assess the "average" significance of market betas and intercepts in the panel as a whole[6]. In particular, we test the joint null, $H_0 : \beta_i = 0$, $i = 1, ..., N$ against the (one-sided) alternative hypotheses $H_1 : \beta_i > 0$ for $i = 1, ..., N$, and thus construct the mean group t statistic as

$$\bar{t}_{NT}(\boldsymbol{\beta}) = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} t_T(\beta_i), \tag{4.18}$$

where $t_T(\beta_i)$ is an individual t-test for $\beta_i = 0$. Under the null as $N, T \to \infty$ and $\frac{N}{T} \to 0$, it would be possible to show under certain additional assumptions that [see Shin and Snell (2001)]

$$\bar{t}_{NT}(\boldsymbol{\beta}) \to_d N(0, 1).$$

The test results summarized in Table 4 indicate that market betas remain significant on average despite the introduction of our two asset specific size and book-to-market distress effects, moreover their values change only slightly when the characteristics proxies are added. Table 4 also shows the results for intercepts. The intercepts are always found to be significant. As in Fama and French (1993) the two-factor regressions intercepts are on average larger (in absolute value) than the intercepts in both the single and the three factor model. Adding the excess market return to the regressions lowers the intercepts although their value never turns out to be significantly close to zero.

Tables 4 about here

In sum, our results contradict most of the empirical findings to date which tend to show that the inclusion of firm specific factors, particularly size proxies, causes the coefficient on asset betas to become insignificant. We tentatively suggest that the efficient estimation and corresponding high power that we would expect from our test procedure may be the reason for this.

[6] In some ways, the current application is an ideal environment for mean group testing because the error terms are cross sectionally independent (*i.e.* idiosyncratic) under the null, an assumption which is required by this analysis but which for many other applications may be considered too strong.

5 Concluding Remarks

In this paper as an alternative of the conventional two path regression we have presented a logically natural and theoretically coherent panel data framework within which to analyse asset return anomalies. We have derived the appropriate estimation and inference techniques within this framework together with their relevant asymptotic properties. In an empirical application, we find, contrary to the results of Fama and French (1996) that (i) adding size and Book-to-Market variables do not drive out the significance of a standard CAPM market factor, and (ii) firm size and Book-to-Market variables remain mostly significant even after the basic CAPM factor is augmented by Fama French size and book to market factors.

Table 1. Descriptive statistics for 25 stock portfolios formed on size and Book-to-Market equity

		Book-to-Market				
Average MV^1		Low	2	3	4	High
	Small					
	2	187.6	211.3	142.9	71.49	34.11
	2	190.4	214.1	145.7	74.26	36.88
Size	3	196.6	220.4	151.9	80.54	43.16
	4	218.2	241.9	173.5	102.1	64.74
	Big	780.1	803.8	735.4	663.9	626.6
Average BTM^2						
	Small	0.96	1.12	1.25	1.43	2.01
	2	0.71	0.86	0.99	1.17	1.75
Size	3	0.61	0.76	0.89	1.07	1.65
	4	0.55	0.71	0.84	1.02	1.59
	Big	0.52	0.67	0.82	0.99	1.5
Average r^3						
	Small	1.3	1.22	1.73	2.05	2.73
	2	0.81	0.72	1.23	1.55	2.23
Size	3	0.67	0.59	1.09	1.42	2.11
	4	0.66	0.58	1.09	1.42	2.11
	Big	0.58	0.51	1.01	1.34	2.03

Notes: [1]MV is measured in millions of pounds. [2]BTM is measured in millions of pounds. [3]r is measured in percentage per month .

Table 2. Annual returns and firm's characteristics

Date	HML	SMB	r^m	r	BTM	MV
1968	13.81	-3.69	15.41	36.05	0.74	39.79
1969	2.99	3.11	-16.31	-32.24	0.84	34.66
1970	5.12	3.22	-4.55	16.33	0.97	30.27
1971	7.86	21.39	26.24	71.91	1.04	29.25
1972	13.26	23.96	14.91	39.25	0.79	29.23
1973	13.78	13.02	-34.32	-71.04	0.83	26.01
1974	31.75	1.43	-68.19	-109.71	1.58	15.91
1975	27.91	-18.73	84.43	144.72	1.99	17.89
1976	11.78	4.45	-16.22	9.22	1.76	23.27
1977	22.32	26.11	39.38	88.42	1.71	29.05
1978	7.17	19.77	4.88	24.44	1.42	38.05
1979	1.41	-0.79	-3.02	-1.43	1.38	44.66
1980	-8.42	-21.22	14.63	7.25	1.65	49.43
1981	18.04	10.04	-0.74	12.19	1.72	57.57
1982	-8.09	-7.57	13.82	19.96	1.67	63.48
1983	18.93	7.72	10.82	37.85	1.45	80.89
1984	8.77	-1.88	14.26	28.77	1.18	94.9
1985	11.76	2.59	8.73	15.38	1.03	128.1
1986	16.75	11.61	10.84	34.24	0.86	163.4
1987	12.97	23.04	1.13	28.11	0.62	235.4
1988	11.11	4.85	8.23	6.26	0.66	220.5
1989	-4.21	-22.71	13.82	11.23	0.65	262.5
1990	5.12	-21.51	-16.34	-49.03	0.82	272.5
1991	-17.35	-2.62	5.13	6.21	0.95	315.1
1992	7.59	-21.85	8.65	7.17	1.01	348.5
1993	34.7	17.12	15.62	56.93	0.88	423.5
1994	5.41	9.75	-4.51	-8.74	0.67	463.7
1995	-0.003	-6.42	12.9	19.07	0.71	461.1
1996	11.02	-18.4	7.69	17.11	0.69	492.8
1997	8.37	-14.17	14.11	21.32	0.65	545.7
1998	-11.23	-19.03	3.53	-9.7	0.71	658.8
1999	24.74	24.76	16.28	53.82	0.79	829.3
2000	37.35	7.26	-6.14	8.27	0.73	947.4
2001	28.7	-4.62	-14.04	-24.58	0.77	856.3
2002	23.21	-3.03	-9.9	-15.25	0.89	821.1
Negative	6	16	12	10	0	0
Mean	11.14	2.08	4.95	13.17	1.06	256.5
Std Dev	3.21	3.57	6.24	10.63	0.41	275.1

Table 3. Pooled Estimation and Test Results for Anomaly Effects

	Single Factor Model[1]				Three Factor Model[2]			
	68-02	68-80	81-90	91-02	68-02	68-80	81-90	91-02
Joint[3]								
δ_{BTM}	1.86 (10.70)	2.12 (6.87)	.92 (2.77)	5.27 (7.48)	1.50 (8.97)	.11 (0.37)	.65 (1.99)	4.78 (7.25)
δ_{MV}	.0004 (1.88)	−.034 (−0.32)	−.005 (−3.99)	−.002 (−0.79)	.0001 (0.77)	−.02 (−2.89)	−.007 (−5.42)	.0004 (1.48)
$Wald$	118.2	67.4	49.8	62.2	87.5	9.12	63.8	52.4
Single[4]								
δ_{BTM}	1.74 (10.71)	2.29 (7.55)	1.63 (5.81)	5.39 (7.85)	1.45 (9.33)	0.25 (0.88)	1.61 (5.82)	4.57 (7.09)
δ_{MV}	−.0003 (−1.97)	−.048 (−4.46)	−.008 (−6.67)	−.0008 (−2.55)	−.0004 (−2.64)	−.03 (−2.99)	−.008 (7.73)	−.0001 (−0.13)

Notes: The values inside () indicate the t-ratio for the significance of an individual coefficient. [1]The results are obtained from the panel data regression of r on r^m, BTM, and\or MV. [2]The results are obtained from the panel data regression of r on r^m, HML, SML, BTM, and\or MV. [3]Both BTM and MV are jointly used as regressors. [4]BTM and MV are used, separately.

Table 4. Mean Group Estimation and Test Results for Alphas and Betas

		68-02	68-80	81-90	90-02
Model 1	$\bar{\beta}_{r_m}$	1.29	1.29	1.34	1.23
	$\bar{t}_{NT}\,(\beta_{r_m}=0)$	111.7	74.2	56.5	52.0
	$\bar{\alpha}$	0.68	0.80	0.58	0.63
	$\bar{t}_{NT}\,(\bar{\alpha}=0)$	9.42	5.86	4.60	6.05
Model 2	$\bar{\beta}_{r_m}$	1.28	1.27	1.33	1.22
	$\bar{t}_{NT}\,(\beta_{r_m}=0)$	110.7	73.5	56.4	51.7
	$\bar{\alpha}$	-1.38	-0.88	0.53	-3.35
	$\bar{t}_{NT}\,(\bar{\alpha}=0)$	-19.2	-6.57	3.91	-33.4
Model 3	$\bar{\beta}_{r_m}$	1.29	1.23	1.38	1.24
	$\bar{t}_{NT}\,(\beta_{r_m}=0)$	117.5	76.4	58.8	54.9
	$\bar{\alpha}$	0.59	0.64	0.67	0.57
	$\bar{t}_{NT}\,(\bar{\alpha}=0)$	8.20	6.94	5.04	5.75
Model 4	$\bar{\beta}_{r_m}$	1.29	1.23	1.36	1.23
	$\bar{t}_{NT}\,(\beta_{r_m})$	116.9	76.6	58.8	54.7
	$\bar{\alpha}$	-1.02	1.75	1.14	-3.42
	$\bar{t}_{NT}\,(\bar{\alpha}=0)$	-14.1	12.5	8.65	-34.6

Notes: The models correspond to the following panel data regressions: Model 1: r on r^m, Model 2: r on r^m, BTM and MV, Model 3: r on r^m, SMB, HML and Model 4: r on r^m, SMB, HML, BTM, and MV. In Models 2 and 4, all estimates are computed conditional on the pooled ML estimates of the coefficients on BTM, and MV.

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