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Forecast Uncertainties in Macroeconometric Modelling: An Application to the UK Economy*

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Abstract

This paper argues that probability forecasts convey information on the uncertainties that surround macro-economic forecasts in a straightforward manner which is preferable to other alternatives, including the use of confidence intervals. Point and probability forecasts obtained using a small macro-econometric model are presented and evaluated using recursive forecasts generated from the model over the period 1999q1-2000q1. Out of sample probability forecasts of inflation and output growth are also provided over the period 2001q2-2003q1, and their implications discussed in relation to the Bank of England's inflation target and the need to avoid recessions, both as separate events and jointly. It is also shown how the probability forecasts can be used to provide insights on the inter-relationship of output growth and inflation at different horizons.

Keywords: Probability Forecasting, Long Run Structural VARs, Macroeconometric Modelling, Forecast Evaluation, Probability Forecasts of Inflation and Output Growth
JEL Classifications: C32, C53, E17

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1 Introduction

With few exceptions, macroeconomic forecasts are presented in the form of point forecasts and their uncertainty is characterized (if at all) by forecast confidence intervals.[1] Focusing on point forecasts is justified when the underlying decision problems faced by agents and the government are linear in constraints and quadratic in the loss function; the so-called LQ problem. But for most decision problems, reliance on point forecasts will not be sufficient and event probability forecasts will be needed (see, for example, Granger and Pesaran, 2000a,b). It is also important that statements about economic policy are made in probabilistic terms, since the public's perception of the credibility of the policy has important implications for its success or failure, irrespective of whether the underlying decision problem is of the LQ type or not. A prominent example, discussed in Peel and Nobay (1998), is the choice of an optimal monetary policy in an economy where the government loss function is asymmetric around the inflation target. In this context, a stochastic approach to the credibility of the monetary policy will be required, and policy announcements should be made with reference to probabilistic statements, such as "the probability that inflation will fall in the range (π_L, π_U) is at least α per cent". Policy targets expressed in terms of a fixed range only partially account for the uncertainty that surrounds policy making.[2]

One of the main advantages of the use of probability forecasts as a means of conveying the uncertainties surrounding forecasts is their straightforward use in decision theoretic contexts. In a macroeconomic context, the motivation for the current monetary policy arrangements in the UK is that it provides for transparency in policy-making and an economic environment in which firms and individuals are better able to make investment and consumption decisions. The range of possible decisions that a firm can make regarding an investment plan, for example, represents the firm's action space. The 'states of nature' in this case are defined by all of the possible future out-turns for the macro-economy. For example, the investment decision might rely on output growth in the next period, or the average output growth over some longer period, remaining positive; or interest might focus on the future path of inflation and output growth considered together. In making a decision, the firm should define a loss function which evaluates the profits or losses associated with each point in the action space and given any 'state of nature'. Except for LQ decision problems, decisions rules by individual households and firms will generally require probability forecasts with respect to different threshold values reflecting their specific cost-benefit ratios. For this purpose, we need to provide estimates of the totality of the probability distribution function of the events of interest, rather than particular forecast intervals which are likely to be relevant only to the decision problem of a few.

The need for probability forecasts is acknowledged by a variety of researchers and institutions. In the statistics literature, for example, Dawid (1984) has been advocating the use of probability forecasting in a sequential approach to the statistical analysis of data; the so-called "prequential approach".[3] In the econometric modelling literature, Fair (1980, 1993)

[1]For a recent articulation of this point see Chris Giles, "Bamboozled by Statistics", *Financial Times*, December 18, 2001, London.

[2]For example, see the discussion on the design of inflation targets in Yates (1995).

[3]The name *prequential* is derived by combining *probability* forecasting with *sequential pre*diction. See

was one of the first to compute probability forecasts using a macroeconometric model of the US economy. The Bank of England routinely publishes a range of outcomes for its inflation and output growth forecasts (see Britton, Fisher and Whitley, 1998, or Wallis, 1999); the National Institute use their model to produce probability statements alongside their central forecasts (their methods are described in Blake, 1996, and Poulizac et al., 1996); and in the financial sector, J.P. Morgan presents 'Event Risk Indicators' in its analysis of foreign exchange markets.[4] However, it remains rare for forecasters to provide probability forecasts in a systematic manner. One explanation may be due to the difficulty in measuring the uncertainties associated with forecasts in the large-scale macroeconometric models typically employed. Another explanation relates to the various types of uncertainty that are involved in forecasting. For example, probability forecasts typically provided in the literature deal with future uncertainty only, assuming that the parameters of the underlying model are known with certainty. This is true of the probability forecasts published by the National Institute, for example. Further complications arises if parameter and model uncertainties are also to be allowed for in the computation of probability forecasts.

This paper considers event probability forecasting in the context of a small long-run structural vector error correcting autoregressive model (VECM) of the UK economy. Particular events of interest include inflation falling within a pre-specified target range and/or output growth remaining positive over two subsequent quarters. Other events or combinations of events can also be considered over a sequence of time periods, or different time intervals in the future. For this purpose, we provide a general simulation framework for the computation of probability forecasts and the characterizations of different forms of uncertainty that surrounds them. The probability forecasts presented in this paper are based on a revised and updated version of the model developed by Garratt *et al.* (2001). This version, specifically updated for forecasting purposes, employs the long-run relations estimated over a long sample period starting in 1965q1, but bases the estimation of the short-run coefficients on a shorter sample period starting 1985q1. We use the model both in a probability forecast evaluation exercise over the period 1999q1-2001q1, as well as for generating out-of-sample point and probability forecasts of inflation and output growth over the period 2001q2-2003q1. The forecast evaluation exercise is carried out recursively and provides significant statistical evidence of improved forecasting ability when the theory-based long-run restrictions are imposed.

In generating out-of-sample probability forecasts, amongst the many possible macroeconomic events of interest, we focus on the possibility of a "recession" and the likelihood of the inflation rate falling within the range 1.5%-3.5%, the target range currently considered by the Monetary Policy Committee (MPC) of the Bank of England. We consider these and a number of related events both singly and jointly. In particular, based on information available at the end of 2001q1, we estimate the probability of inflation falling within the Bank of England's target range to be relatively high, with only a small probability of a recession.

The probability estimates reported in this paper illustrate the clarity with which event probability statements can convey some of the uncertainties that are associated with forecasts and demonstrate their potential value in policy debates. The predictive distribution functions

Dawid (1984, pp.278-279).

[4]For an academic reference see for example, Berkowitz (1999).

clearly exhibit both the range of potential outcomes and how these will evolve at different forecast horizons. The use of event probabilities can also yield important insights into the properties of the underlying data generating process, as well as providing us with invaluable tools in decision-making.

The lay-out of the rest of the paper is as follows. Section 2 considers the alternative approaches that are available for characterizing forecast uncertainty and notes the advantages of probability forecasts particularly in decision making contexts. Section 3 considers probability forecasts in more detail, discussing the concept and estimation of probability forecasts in general terms both in the presence and absence of parameter uncertainty. The remainder of the paper is concerned with an application of the probability forecasting approach to the UK economy. Section 4 describes the model, provides estimates of the model's parameters, and discusses the results of the forecast evaluation exercise where alternative approaches to the evaluation of probability forecasts are reviewed and applied to our model's forecasts. Macroeconomic events of interest and the associated probability forecasts are discussed in Section 5. This section provides a brief account of inflation targeting in the UK, comments on the relationship between the fan charts published by the Bank of England and probability forecasting, and presents single and joint event probability forecasts involving output growth and inflation objectives at different forecast horizons. Section 6 offers some concluding remarks.

2 Alternative Approaches to Characterising Forecast Uncertainty

All model-based forecasts are subject to four types of uncertainties:

- Measurement uncertainty (data inadequacies and measurement errors);
- Model uncertainty (including policy uncertainty);
- Parameter uncertainty (for a given model);
- Future uncertainty.

This paper focuses on future and parameter uncertainties and how to allow for them in the computation of probability forecasts. Measurement and model uncertainties pose special problems of their own and will not be addressed in this paper.[5] Future uncertainty refers to the effects of unobserved future shocks on forecasts, while parameter uncertainty is concerned with the robustness of forecasts to the choice of parameter values, assuming a given forecasting model.

The standard textbook approach to taking account of future and parameter uncertainties is through the construction of forecast intervals. For the purpose of exposition, initially we abstract from parameter uncertainty and consider the following simple linear regression model:

$$y_t = \mathbf{x}'_{t-1}\boldsymbol{\beta} + u_t, \qquad t = 1, 2, ..., T,$$

[5] For a discussion on the problem of model uncertainty, see Draper (1990) and Chatfield (1995).

where $\mathbf{x}_{t-1}$ is a $k \times 1$ vector of pre-determined regressors, $\boldsymbol{\beta}$ is a $k \times 1$ vector of fixed but unknown coefficients, and $u_t \sim N(0, \sigma^2)$. The optimal forecast of y_{T+1} at time T (in the mean squared error sense) is given by $\mathbf{x}_T'\boldsymbol{\beta}$. In the absence of parameter uncertainty, the calculation of a probability forecast for a specified event is closely related to the more familiar concept of forecast confidence interval. For example, suppose that we are interested in the probability that the value of y_{T+1} lies below a specified threshold, say a, conditional on $\Omega_T = (y_T, \mathbf{x}_T, y_{T-1}, \mathbf{x}_{T-1}, ...)$, the information available at time T. For given values of $\boldsymbol{\beta}$ and σ^2, we have

$$\Pr(y_{T+1} < a \mid \Omega_T) = \Phi\left(\frac{a - \mathbf{x}_T'\boldsymbol{\beta}}{\sigma}\right),$$

where $\Phi(\circ)$ is the standard Normal cumulative distribution function while the $(1-\alpha)\%$ forecast interval for y_{T+1} (conditional on Ω_T) is given by $\mathbf{x}_T'\boldsymbol{\beta} \pm \sigma\Phi^{-1}\left(1 - \frac{\alpha}{2}\right)$.

The two approaches, although related, are motivated by different considerations. The point forecast provides the threshold value $a = \mathbf{x}_T'\boldsymbol{\beta}$ for which $\Pr(y_{T+1} < a \mid \Omega_T) = 0.5$, while forecast interval provides the threshold values $c_L = \mathbf{x}_T'\boldsymbol{\beta} - \sigma\Phi^{-1}\left(1 - \frac{\alpha}{2}\right)$, and $c_U = \mathbf{x}_T'\boldsymbol{\beta} + \sigma\Phi^{-1}\left(1 - \frac{\alpha}{2}\right)$ for which $\Pr(y_{T+1} < c_L \mid \Omega_T) = \frac{\alpha}{2}$ and $\Pr(y_{T+1} < c_U \mid \Omega_T) = 1 - \frac{\alpha}{2}$. Clearly, the thresholds values, c_L and c_U, associated with the $(1-\alpha)\%$ forecast interval may or may not be of interest.[6] Only by chance will the forecast interval calculations provide information in a way which is directly useful in specific decision making contexts.

The relationship between probability forecasts and interval forecasts becomes even more obscure when parameter uncertainty is also taken into account. In the context of the above regression model, the point estimate of the forecast is given by $\widehat{y}_{T+1} = \mathbf{x}_T'\widehat{\boldsymbol{\beta}}_T$, where

$$\widehat{\boldsymbol{\beta}}_T = \mathbf{Q}_{T-1}^{-1}\mathbf{q}_T,$$

is the Ordinary Least Squares (OLS) estimate of $\boldsymbol{\beta}$, with

$$\mathbf{Q}_{T-1} = \sum_{t=1}^{T} \mathbf{x}_{t-1}\mathbf{x}_{t-1}', \text{ and } \mathbf{q}_T = \sum_{t=1}^{T} \mathbf{x}_{t-1} y_t.$$

The relationship between the actual value of y_{T+1} and its time T predictor can be written as

$$\begin{aligned} y_{T+1} &= \mathbf{x}_T'\boldsymbol{\beta} + u_{T+1} \\ &= \mathbf{x}_T'\widehat{\boldsymbol{\beta}}_T + \mathbf{x}_T'(\boldsymbol{\beta} - \widehat{\boldsymbol{\beta}}_T) + u_{T+1}, \end{aligned} \quad (1)$$

so that the forecast error, ξ_{T+1}, is given by

$$\xi_{T+1} = y_{T+1} - \widehat{y}_{T+1} = \mathbf{x}_T'(\boldsymbol{\beta} - \widehat{\boldsymbol{\beta}}_T) + u_{T+1}.$$

This example shows that the point forecasts, $\mathbf{x}_T'\widehat{\boldsymbol{\beta}}_T$, are subject to two types of uncertainties, namely that relating to $\boldsymbol{\beta}$ and that relating to the distribution of u_{T+1}. For any given

[6] The association between probability forecasts and interval forecasts are even weaker when one considers *joint* events. For example, it would be impossible to infer the probability of the joint event of a positive output growth and an inflation rate falling within a pre-specified range from individual, variable-specific forecast intervals. Many different such intervals will be needed for this purpose.

sample of data, Ω_T, $\widehat{\boldsymbol{\beta}}_T$ is known and can be treated as fixed. On the other hand, although $\boldsymbol{\beta}$ is assumed fixed at the estimation stage, it is unknown to the forecaster and, from this perspective, it is best viewed as a random variable at the forecasting stage. Hence, in order to compute probability forecasts which account for future as well as parameter uncertainties, we need to specify the joint probability distribution of $\boldsymbol{\beta}$ and u_{T+1}, conditional on Ω_T. As far as u_{T+1} is concerned, we continue to assume that

$$u_{T+1}|\Omega_T \sim N(0, \sigma^2),$$

and to keep the exposition simple, for the time being we shall assume that σ^2 is known and that u_{T+1} is distributed independently of $\boldsymbol{\beta}$. For $\boldsymbol{\beta}$, noting that

$$\left(\widehat{\boldsymbol{\beta}}_T - \boldsymbol{\beta}\right)|\Omega_T \sim N\left(\mathbf{0}, \sigma^2 \mathbf{Q}_{T-1}^{-1}\right), \tag{2}$$

we assume that

$$\boldsymbol{\beta}\,|\Omega_T \sim N\left(\widehat{\boldsymbol{\beta}}_T, \sigma^2 \mathbf{Q}_{T-1}^{-1}\right), \tag{3}$$

which is akin to a Bayesian approach with non-informative priors for $\boldsymbol{\beta}$. Hence

$$\xi_{T+1} \mid \Omega_T \sim N\left[0, \sigma^2\left(1 + \mathbf{x}_T'\mathbf{Q}_{T-1}^{-1}\mathbf{x}_T\right)\right].$$

The $(1-\alpha)\%$ forecast interval in this case is given by

$$c_{LT} = \mathbf{x}_T'\widehat{\boldsymbol{\beta}}_T - \sigma\left\{1 + \mathbf{x}_T'\mathbf{Q}_{T-1}^{-1}\mathbf{x}_T\right\}^{1/2}\Phi^{-1}\left(1 - \frac{\alpha}{2}\right), \tag{4}$$

and

$$c_{UT} = \mathbf{x}_T'\widehat{\boldsymbol{\beta}}_T + \sigma\left\{1 + \mathbf{x}_T'\mathbf{Q}_{T-1}^{-1}\mathbf{x}_T\right\}^{1/2}\Phi^{-1}\left(1 - \frac{\alpha}{2}\right). \tag{5}$$

When σ^2 is unknown, under the standard non-informative Bayesian priors on (β,σ^2), the appropriate forecast interval can be obtained by replacing σ^2 by its unbiased estimate, $\hat{\sigma}_T^2 = (T-k)^{-1}\sum_{t=1}^{T}(y_t - \mathbf{x}_{t-1}'\widehat{\boldsymbol{\beta}}_{\mathbf{T}})'(y_t - \mathbf{x}_{t-1}'\widehat{\boldsymbol{\beta}}_{\mathbf{T}})$, and $\Phi^{-1}\left(1-\frac{\alpha}{2}\right)$ by the $(1-\frac{\alpha}{2})\%$ critical value of the standard t-distribution with $T-k$ degrees of freedom. Although such interval forecasts have been discussed in the econometrics literature, the particular assumptions that underlie them are not fully recognized.

Using this interpretation, the effect of parameter uncertainty on forecasts can also be obtained via stochastic simulations, by generating alternative forecasts of y_{T+1} for different values of $\boldsymbol{\beta}$ (and σ^2) drawn from the conditional probability distribution of $\boldsymbol{\beta}$ given by (3). Alternatively, one could estimate probability forecasts by focusing directly on the probability distribution of y_{T+1} for a given value of $\mathbf{x}_T$, simultaneously taking into account both parameter and future uncertainties. For example, in the simple case where σ^2 is known, this can be achieved by simulating $y_{T+1}^{(j)}$, $j = 1, 2, ..., J$, where

$$y_{T+1}^{(j)} = \mathbf{x}_T'\widehat{\boldsymbol{\beta}}^{(j)} + u_{T+1}^{(j)},$$

$\widehat{\boldsymbol{\beta}}^{(j)}$ is the j-th random draw from $N\left(\widehat{\boldsymbol{\beta}}_T, \sigma^2 \mathbf{Q}_{T-1}^{-1}\right)$, and $u_{T+1}^{(j)}$ is the j-th random draw from $N(0, \sigma^2)$.[7] This is an example of the parametric "bootstrap predictive density" discussed in Harris (1989). In large samples, the stochastic simulation approach will be equivalent to the analytical methods discussed above. However, as argued below, it is more generally applicable and will be used in our empirical application.

An alternative approach to allowing for the effects of future and parameter uncertainties on prediction of y_{T+1} would be to follow the literature on "predictive likelihoods", where a predictive density for y_{T+1} conditional on Ω_T is derived directly.[8] In the case of the regression example, the problem has been studied by Levy and Perng (1986) who show that the optimal prediction density for y_{T+1}, in the Kullback-Leibler information-theoretic sense, is the Student t distribution with $T-k$ degrees of freedom, having the location $\hat{y}_{T+1} = \mathbf{x}_T' \widehat{\boldsymbol{\beta}}_T$ and the dispersion $\hat{\sigma}_T^2 \left\{1 + \mathbf{x}_T' \mathbf{Q}_{T-1}^{-1} \mathbf{x}_T\right\}$. This is the same as the Bayes predictive density of $y_{T+1}|\Omega_T$ with a non-informative prior on $(\boldsymbol{\beta}, \sigma^2)$. In this way Levy and Perng provide a non-Bayesian interpretation of Bayes predictive density in the context of linear regression models. However, while this is the optimal prediction density in the original linear model, Harris (1989) demonstrates that the bootstrap prediction density performs well in a number of important cases.

3 A General Framework for Probability Forecasting

To formalize the discussion of probability forecasts, suppose we are interested in a decision making process that requires probability forecasts of an event defined in terms of the m-variable vector, $\mathbf{z}_t = (z_{1t}, z_{2t}, ..., z_{mt})'$. Assume also that the forecasts are made with reference to a parametric family of models, denoted by $M(\boldsymbol{\theta})$, and characterized by the joint density function of $\mathbf{z}_t$ over the estimation and the forecast periods $t = 1, 2, ..., T$, and $T+1, T+2, ..., T+h$, respectively. The probability model, $M(\boldsymbol{\theta})$, is a set of density functions, each describing the probability of obtaining specified values for the observed and forecasted data, and indexed by the unknown $k \times 1$ parameter vector $\boldsymbol{\theta}$ assumed to lie in the compact parameter space, Θ:

$$M(\boldsymbol{\theta}) = \left\{ f\left(\mathbf{z}_1, \mathbf{z}_2, ..., \mathbf{z}_T, \mathbf{z}_{T+1}, \mathbf{z}_{T+2}, ..., \mathbf{z}_{T+h}; \boldsymbol{\theta}\right), \quad \boldsymbol{\theta} \in \Theta \right\}. \tag{6}$$

Throughout we shall assume that the true value of $\boldsymbol{\theta}$, which we denote by $\boldsymbol{\theta}_0$, is fixed and remains constant across the estimation and the prediction periods and lies in the interior of Θ. We denote the maximum likelihood estimate of $\boldsymbol{\theta}_0$ by $\widehat{\boldsymbol{\theta}}_T$, and assume that $f(.)$ satisfies the usual regularity conditions so that

$$\sqrt{T}\left(\widehat{\boldsymbol{\theta}}_T - \boldsymbol{\theta}_0\right) \overset{a}{\backsim} N\left(\mathbf{0}, \mathbf{V}_\theta\right),$$

[7] In the realistic case where σ^2 is unknown it is replaced by $\hat{\sigma}_T^2$.

[8] A large number of different predictive likelihoods have been suggested in the statistics literature. Bjørnstad (1990) provides a review.

where $\overset{a}{\backsim}$ stands for "asymptotically distributed as", and $\mathbf{V}_\theta$ is a positive definite covariance matrix.[9] Under these assumptions, parameter uncertainty only arises when T is finite. The case where $\boldsymbol{\theta}_0$ could differ across the estimation and forecast periods poses new difficulties and can be resolved in a satisfactory manner if one is prepared to formalize how $\boldsymbol{\theta}_0$ changes over time.

The density function $f(.)$ can be decomposed in two ways. First, a sequential conditioning decomposition can be employed to write $f(.)$ as the product of the conditional distributions on successive observations on the $\mathbf{z}_t$,

$$f\left(\mathbf{z}_1, \mathbf{z}_2, ..., \mathbf{z}_t; \mathbf{z}_0, \boldsymbol{\theta}\right) = \prod_{s=1}^{t} f\left(\mathbf{z}_s \mid \mathbf{z}_1, \mathbf{z}_2, ..., \mathbf{z}_{s-1}; \mathbf{z}_0, \boldsymbol{\theta}\right),$$

for given initial values $\mathbf{z}_0$. And second, since we frequently wish to distinguish between variables which are endogenous, denoted by $\mathbf{y}_t$, and those which are exogenous, denoted by $\mathbf{x}_t$, we can write $\mathbf{z}_t = (\mathbf{y}_t', \mathbf{x}_t')'$ and use the factorization:

$$f\left(\mathbf{z}_t \mid \mathbf{z}_1, \mathbf{z}_2, ..., \mathbf{z}_{t-1}; \mathbf{z}_0, \boldsymbol{\theta}\right) = f_y\left(\mathbf{y}_t \mid \mathbf{z}_1, \mathbf{z}_2, ..., \mathbf{z}_{t-1}, \mathbf{x}_t; \mathbf{z}_0, \boldsymbol{\theta}_y\right) \times f_x\left(\mathbf{x}_t \mid \mathbf{z}_1, \mathbf{z}_2, ..., \mathbf{z}_{t-1}; \mathbf{z}_0, \boldsymbol{\theta}_x\right), \tag{7}$$

where $f_y\left(\mathbf{y}_t \mid \mathbf{z}_1, \mathbf{z}_2, ..., \mathbf{z}_{t-1}, \mathbf{x}_t; \mathbf{z}_0, \boldsymbol{\theta}_y\right)$ is the conditional distribution of $\mathbf{y}_t$ given $\mathbf{x}_t$ and the information available at time $t-1$, $\Omega_{t-1} = (\mathbf{z}_{t-1}, \mathbf{z}_{t-2}, ...)$, and $f_x\left(\mathbf{x}_t \mid \mathbf{z}_1, \mathbf{z}_2, ..., \mathbf{z}_{t-1}; \mathbf{z}_0, \boldsymbol{\theta}_x\right)$ is the marginal density of $\mathbf{x}_t$ conditional on Ω_{t-1}. Note that the unknown parameters $\boldsymbol{\theta}$ are decomposed into the parameters of interest, $\boldsymbol{\theta}_y$, and the parameters of the marginal density of the exogenous variables, $\boldsymbol{\theta}_x$. In the case where $\mathbf{x}_t$ is strictly exogenous, knowledge of the marginal distribution of $\mathbf{x}_t$ does not help with the estimation of $\boldsymbol{\theta}_y$, and estimation of these parameters can therefore be based entirely on the conditional distribution, $f_y\left(\mathbf{y}_t \mid \mathbf{x}_t, \Omega_{t-1}; \boldsymbol{\theta}_y\right)$.

Despite this, parameter uncertainty relating to $\boldsymbol{\theta}_x$ can continue to be relevant for probability forecasts of the endogenous variables, $\mathbf{y}_t$, and forecast uncertainty surrounding the endogenous variables is affected by the way the uncertainty associated with the future path of the exogenous variables is resolved. In practice, the future values of $\mathbf{x}_t$ are often treated as known and fixed at pre-specified values. The resultant forecasts for $\mathbf{y}_t$ are then referred to as *scenario (or conditional) forecasts*, with each scenario representing a different set of assumed future values of the exogenous variables. This approach under-estimates the degree of forecast uncertainties. A more plausible approach would be to treat $\mathbf{x}_t$ as strongly (strictly) exogenous at the estimation stage, but to allow for the forecast uncertainties of the endogenous and the exogenous variables jointly. The exogeneity assumption will simplify the estimation process but does not eliminate the need for a joint treatment of future uncertainties associated with the exogenous variables and the shocks to the endogenous variables.

Now, suppose the *joint event* of interest is defined by

$$\varphi_l(\mathbf{z}_{T+1}, \mathbf{z}_{T+2}, \ldots, \mathbf{z}_{T+h}) < a_l \text{ for } l = 1, 2, ..., L,$$

[9] In the case of cointegrating vector autoregressive models analysed later in this paper, a more general version of this result is needed. This is because the cointegrating coefficients converge to their asymptotic distribution at a faster rate than the other parameters in the model. However, the general results of this section are not affected by this complication.

or, equivalently,

$$\boldsymbol{\varphi}\left(\mathbf{z}_{T+1},\mathbf{z}_{T+2},\ldots,\mathbf{z}_{T+h}\right)<\mathbf{a},$$

where $\boldsymbol{\varphi}(.)$ and $\mathbf{a}$ are defined by the $L\times 1$ vectors $\boldsymbol{\varphi}(.)=(\varphi_1(.),\varphi_2(.),...,\varphi_L(.))'$, $\mathbf{a}=(a_1,a_2,...,a_L)'$, $\varphi_l(\mathbf{z}_{T+1},\mathbf{z}_{T+2},\ldots,\mathbf{z}_{T+h})$ is a scalar function of the variables over the forecast horizon $T+1$,..., $T+h$ and a_j is the "threshold" value associated with $\varphi_j(.)$. To simplify the exposition, we denote this joint event by $\mathfrak{A}_\varphi$. The (conditional) probability forecast associated with this event is given by

$$\pi\left(\mathbf{a},h;\boldsymbol{\varphi}(.),\boldsymbol{\theta}\right)=\Pr\left[\boldsymbol{\varphi}\left(\mathbf{z}_{T+1},\mathbf{z}_{T+2},\ldots,\mathbf{z}_{T+h}\right)<\mathbf{a}\mid\Omega_T;M(\boldsymbol{\theta})\right]. \tag{8}$$

In practice, we might also be interested in computing probability forecasts for a number of alternative threshold values over the range $a_j\in[a_{\min},a_{\max}]$.

If the model is known to be $M(\boldsymbol{\theta})$ defined by (6) but the value of $\boldsymbol{\theta}$ is not known, a *point estimate* of $\pi\left(\mathbf{a},h;\boldsymbol{\varphi}(.),\boldsymbol{\theta}\right)$ can be obtained by

$$\pi\left(\mathbf{a},h;\boldsymbol{\varphi}(.),\widehat{\boldsymbol{\theta}}_T\right)=\int_{\mathfrak{A}_\varphi}f(\mathbf{z}_{T+1},\mathbf{z}_{T+2},\ldots,\mathbf{z}_{T+h}\mid\Omega_T;\widehat{\boldsymbol{\theta}}_T)d\mathbf{z}_{T+1}\ldots d\mathbf{z}_{T+h}, \tag{9}$$

where $f(\mathbf{z}_{T+1},\mathbf{z}_{T+2},\ldots,\mathbf{z}_{T+h}\mid\Omega_T;\widehat{\boldsymbol{\theta}}_T)$ is the joint density of $\mathbf{z}_{T+1},\ldots,\mathbf{z}_{T+h}$ conditional on Ω_T and evaluated at $\boldsymbol{\theta}=\widehat{\boldsymbol{\theta}}_T$. This probability density function (viewed as a function of $\mathbf{a}$), also known as the "profile predictive likelihood",[10] takes account of future uncertainties arising from the model's stochastic structure and the future uncertainty with respect to the evolution of the model's exogenous variables. It does not, however, take account of model or parameter uncertainties, as it is computed for a given density function within $M(\boldsymbol{\theta})$ and for a given value of $\boldsymbol{\theta}$, namely $\widehat{\boldsymbol{\theta}}_T$. To allow for parameter uncertainty, we assume that conditional on Ω_T, the probability distribution function of $\boldsymbol{\theta}$ is given by $g\left(\boldsymbol{\theta}\mid\Omega_T\right)$. Then

$$\tilde{\pi}\left(\mathbf{a},h;\boldsymbol{\varphi}(.)\right)=\int_{\boldsymbol{\theta}\in\Theta}\pi\left(\mathbf{a},h;\boldsymbol{\varphi}(.),\boldsymbol{\theta}\right)\;g\left(\boldsymbol{\theta}\mid\Omega_T\right)\;d\boldsymbol{\theta}, \tag{10}$$

or, equivalently,

$$\tilde{\pi}\left(\mathbf{a},h;\boldsymbol{\varphi}(.)\right)=\int_{\boldsymbol{\theta}\in\Theta}\int_{\mathfrak{A}_\varphi}f(\mathbf{z}_{T+1},\ldots,\mathbf{z}_{T+h}\mid\Omega_T;\boldsymbol{\theta})\;g\left(\boldsymbol{\theta}\mid\Omega_T\right)d\mathbf{z}_{T+1}\ldots d\mathbf{z}_{T+h}d\boldsymbol{\theta}.$$

In practice, computation of $\tilde{\pi}\left(\mathbf{a},h;\boldsymbol{\varphi}(.),\widehat{\boldsymbol{\theta}}_T\right)$ is typically carried out by stochastic simulations. For further details, see Section 5 and the Appendix.

In a Bayesian context, $g\left(\boldsymbol{\theta}\mid\Omega_T\right)$ could be derived from *a prior* distributional assumptions on $\boldsymbol{\theta}$ at the start of the *estimation* period. Alternatively, in the case where the asymptotic normal theory applies to $\widehat{\boldsymbol{\theta}}_T$, it may be reasonable to compute the probability density function assuming

$$\boldsymbol{\theta}\mid\Omega_T\overset{a}{\backsim}N\left(\widehat{\theta}_T,T^{-1}\widehat{\mathbf{V}}_\theta\right).$$

[10]See, for example, Bjørnstad (1990).

In this case the point estimate of the probability forecast, $\pi\left(\mathbf{a},h;\boldsymbol{\varphi}(.),\widehat{\boldsymbol{\theta}}_T\right)$, and the alternative estimate, $\tilde{\pi}\left(\mathbf{a},h;\boldsymbol{\varphi}(.)\right)$, that allows for parameter uncertainty are asymptotically equivalent as $T \to \infty$. The latter is the "bootstrap predictive density" described in Harris (1989) and its application to a cointegrating VAR model will be discussed in Section 4. Also, both of these estimates tend to $\pi\left(\mathbf{a},h;\boldsymbol{\varphi}(.),\boldsymbol{\theta}\right)$, which is the profile predictive likelihood for a known value of $\boldsymbol{\theta}$. But for a fixed T, the two estimates could differ substantially, as the applications in Section 5 demonstrate.

4 An Application to the UK Economy

4.1 A Cointegrating VAR Model of the UK Economy

In principle, probability forecasts can be computed using any macroeconometric model, although the necessary computations would become prohibitive in the case of most large scale macroeconometric models, particularly if the objective of the exercise is to compute the probabilities of joint events at different horizons. At the other extreme, the use of small unrestricted VAR models, while computationally feasible, may not be satisfactory for the analysis of forecast probabilities over the medium term. An intermediate alternative that we shall follow here is to use a cointegrating VAR model that takes account of the long-run relationships that are likely to exist in a macro-economy. A model of this type has been developed for the UK by Garratt *et al.* (2001, 2002). This model is based on a number of long-run relations derived from arbitrage conditions in goods and capital markets, solvency and portfolio balance conditions. The model comprises six domestic variables whose developments are widely regarded as essential to a basic understanding of the U.K. economy; namely, output, inflation, the exchange rate, the domestic relative to the foreign price level, the nominal interest rate and real money balances. It also contains three foreign variables: foreign output, foreign interest rate and oil prices (see Table 1 for a detailed description of the variables used).

The five long-run equilibrium relationships of the model outlined in Garratt *et al.* (2001) are given by:

$$p_t - p_t^* - e_t = b_{10} + b_{11}t + \xi_{1,t+1}, \tag{11}$$

$$r_t - r_t^* = b_{20} + \xi_{2,t+1}, \tag{12}$$

$$y_t - y_t^* = b_{30} + \xi_{3,t+1}, \tag{13}$$

$$h_t - y_t = b_{40} + b_{41}t + \beta_{42}r_t + \beta_{43}y_t + \xi_{4,t+1}, \tag{14}$$

$$r_t - \Delta p_t = b_{50} + \xi_{5,t+1}, \tag{15}$$

where p_t is the logarithm of domestic prices, p_t^* is the logarithm of foreign prices, e_t is the logarithm of nominal exchange rate (defined as the domestic price of a unit of the foreign currency, so that a depreciation of the home currency increases e_t), y_t is the logarithm of real per capita domestic output, y_t^* is the logarithm of real per capita foreign output, r_t is the domestic nominal interest rate variable, r_t^* is the foreign nominal interest rate variable, h_t is the logarithm of the real per capita money stock, we also use the variable p_t^o which is the logarithm of oil prices and $\xi_{i,t+1}$, $i = 1, 2, .., 5$, are stationary reduced form errors.

A detailed account of the framework for long run macro-modelling, describing the economic theory that underlies the relationships in (11) - (15), is provided in Garratt *et al.* (2001). In brief, we note here that (11) is the Purchasing Power Parity (PPP) relationship, which allows for a trend in the real exchange rate, based on international goods market arbitrage; (12) is an Interest Rate Parity (IRP) relationship, and is based on arbitrage between domestic and foreign bond holdings; (13) is an "output gap" (OG) relationship, based on a stochastic version of the Solow growth model in which there is common technological progress in production at home and abroad;[11] (14) is a real money balance (RMB) relationship, based on the condition that the economy must remain solvent in the long run; and (15) is the Fisher Interest Parity (FIP) relationship.

The five long-run relations of the model, (11) - (15), can be written compactly as:

$$\boldsymbol{\xi}_t = \boldsymbol{\beta}'\mathbf{z}_{t-1} - \mathbf{b}_1(t-1) - \mathbf{b}_0, \tag{16}$$

where

$$\mathbf{z}_t = \left(p_t^o, e_t, r_t^*, r_t, \Delta p_t, y_t, p_t - p_t^*, h_t - y_t, y_t^*\right)', \tag{17}$$

$$\mathbf{b}_0 = (b_{01}, b_{02}, b_{03,}b_{04}, b_{05})',\ \ \mathbf{b}_1 = (b_{11}, 0, 0, b_{41}, 0),$$

$$\boldsymbol{\xi}_t = (\xi_{1t}, \xi_{2t}, \xi_{3t}, \xi_{4t}, \xi_{5t})',$$

and

$$\boldsymbol{\beta}' = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & -\beta_{42} & 0 & -\beta_{43} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{18}$$

Under the assumption that oil prices are long-run forcing, efficient estimation of the parameters can be based on the following *conditional* error correction model,[12]

$$\Delta\mathbf{y}_t = \mathbf{a}_y - \boldsymbol{\alpha}_y\left[\boldsymbol{\beta}'\mathbf{z}_{t-1} - \mathbf{b}_1(t-1)\right] + \sum_{i=1}^{p-1}\Gamma_{yi}\Delta\mathbf{z}_{t-i} + \boldsymbol{\psi}_{yo}\Delta p_t^o + \mathbf{u}_{yt}, \tag{19}$$

where $\mathbf{y}_t = (e_t, r_t^*, r_t, \Delta p_t, y_t, p_t - p_t^*, h_t - y_t, y_t^*)'$, $\mathbf{a}_y$ is an 8×1 vector of fixed intercepts, $\boldsymbol{\alpha}_y$ is a 8×5 matrix of error-correction coefficients, $\{\Gamma_{yi}, i = 1, 2, ..., p-1\}$ are 8×9 matrices of short-run coefficients, $\boldsymbol{\psi}_{yo}$ is an 8×1 vector representing the impact effects of changes in oil prices on $\Delta\mathbf{y}_t$, and $\mathbf{u}_{yt}$ is an 8×1 vector of disturbances assumed to be $IID(0, \Sigma_y)$, with Σ_y being a positive definite matrix. This specification embodies the economic theory's long-run predictions by construction, in contrast to the more usual approach where the starting

[11] Our use of the term 'output gap relationship' to describe (13) should not be confused with the more usual use of the term which relates more specifically to the difference between a country's actual and potential output levels (although clearly the two uses of the term are related).

[12] See Pesaran, Shin and Smith (2000) for details.

point is an unrestricted VAR model with some vague priors about the nature of the long-run relations.

For multi-step ahead forecasting, we also need to augment the conditional model, (19), with a long-run forcing oil price equation. Such a general specification is given by

$$\Delta p_t^o = a_o + \sum_{i=1}^{p-1} \Gamma_{oi} \Delta \mathbf{z}_{t-i} + u_{ot}, \tag{20}$$

where Γ_{oi} is a 1×9 vector of fixed coefficients and u_{ot} is a serially uncorrelated error term distributed independently of $\mathbf{u}_{yt}$. This specification encompasses the random walk model as a special case. It is the absence of the error correction terms, $\boldsymbol{\beta}'\mathbf{z}_{t-1} - \mathbf{b}_1(t-1)$, in (20) which renders oil price changes long-run forcing for $\mathbf{y}_t$.

Combining (19) and (20) solving for $\Delta \mathbf{z}_t$ yields the following reduced form equation

$$\Delta \mathbf{z}_t = \mathbf{a} - \boldsymbol{\alpha} \left[\boldsymbol{\beta}' \mathbf{z}_{t-1} - \mathbf{b}_1(t-1) \right] + \sum_{i=1}^{p-1} \Gamma_i \Delta \mathbf{z}_{t-i} + \mathbf{v}_t, \tag{21}$$

where

$$\mathbf{a} = \begin{pmatrix} a_o \\ \mathbf{a}_y - \boldsymbol{\psi}_{yo} a_o \end{pmatrix}, \ \boldsymbol{\alpha} = \begin{pmatrix} 0 \\ \boldsymbol{\alpha}_y \end{pmatrix}, \ \Gamma_i = \begin{pmatrix} \Gamma_{oi} \\ \Gamma_{yi} - \boldsymbol{\psi}_{yo} \Gamma_{oi} \end{pmatrix}, \tag{22}$$

and

$$\mathbf{v}_t = \begin{pmatrix} u_{ot} \\ \mathbf{u}_{yt} - \boldsymbol{\psi}_{yo} u_{ot} \end{pmatrix}, \tag{23}$$

is the vector of reduced form errors assumed to be $iid(0, \Sigma)$,where Σ is a positive definite matrix.

4.2 The Estimated Model

Estimation of the parameters of the conditional model, (19), can be carried out using the long-run structural modelling approach described in Pesaran and Shin (2001) and Pesaran, Shin and Smith (2000). With this approach, having selected the order of the underlying VAR model (using model selection criteria such as the Akaike Information Criterion (AIC) or the Schwarz Bayesian Criterion (SBC)), we test for the number of cointegrating relations using the conditional model, (19), with unrestricted intercepts and restricted trend coefficients. As shown in Pesaran, Shin and Smith (2000), these restrictions ensure that the solution of the model in levels of $\mathbf{z}_t$ will not contain quadratic trends. We then compute Maximum Likelihood (ML) estimates of the model's parameters subject to exact and over-identifying restrictions on the long-run coefficients.[13] If there is empirical support for the existence of five long-run relationships, as suggested by theory, exact identification in our model requires five restrictions on each of the five cointegrating vectors (each row of $\boldsymbol{\beta}$), or a total of

[13]The computations were carried out using Pesaran and Pesaran's (1997) Microfit 4.1.

twenty-five restrictions on $\boldsymbol{\beta}$. These represent only a subset of the restrictions suggested by economic theory as characterized in (18), however. Estimation of the model subject to all the (exact- and over-identifying) restrictions given in (18) enables a test of the validity of the over-identifying restrictions, and hence the underlying long-run economic theory, to be carried out. Of course, it also provides the means for generating point as well as probability forecasts.

Such an empirical exercise is conducted by Garratt *et al.* (2001) using UK data over the period 1965q1-1999q4. Their results showed that: (i) a VAR(2) model can adequately capture the dynamic properties of the data; (ii) there are five cointegrating relationships amongst the nine macroeconomic variables; and that (iii) the over-identifying restrictions suggested by economic theory, and described in (11) - (15) above, cannot be rejected. For the present exercise, we re-estimated the model on the more up-to-date sample, 1965q1-2001q1. The results continue to support the existence of 5 cointegrating relations, and are qualitatively very similar to those described in Garratt *et al.* (2001). For example, the interest rate coefficient in the real money balance equation, β_{42}, was estimated to be 75.68 (standard error 35.34), compared to 56.10 (22.28) in the original work, while the coefficient on the time trend, b_{41}, was estimated to be 0.0068 (0.0010), compared to 0.0073 (0.0012).

Since the modelling exercise here is primarily for the purpose of forecasting, we next re-estimated the model over the shorter period of 1985q1-2001q1, taking the long-run relations as given. The inclusion of the long-run relations estimated over the period 1965q1-2001q1 in a cointegrating VAR model estimated over the shorter sample period 1985q1-2001q1, is justified on two grounds: (i) as argued by Clement and Hendry (1999, 2002) and Barassi, Coporale and Hall (2001), the short-run coefficients are more likely to be subject to structural change as compared to the long-run coefficients; and (ii) the application of Johansen's cointegration tests are likely to be unreliable in small samples. Following this procedure, we are able to base the forecasts on a model with well-specified long-run relations, but which is also data-consistent, capturing the complex dynamic relationships that hold across the macroeconomic variables over recent years.

Table 2 gives the estimates of the individual error correcting relations of the model estimated over the 1985q1-2001q1. These estimates show that the error correction terms are important in most equations and provide for a complex and statistically significant set of interactions and feedbacks across commodity, money and foreign exchange markets. The estimated error correction equations pass most of the diagnostic tests, and compared to standard benchmarks fit the historical observations relatively well. In particular, the $\overline{R}^2$ of the domestic output and inflation equations, computed at 0.549 and 0.603 respectively, are quite high. The diagnostic statistics for tests of residual serial correlation, functional form and heteroskedasticity are well within the 90 per cent critical values, although there is evidence of non-normal errors in the case of some of the error correcting equations. Non-normal errors is not a serious problem at the estimation and inference stage, but can lead to biased probability forecasts. To deal with this problem, we shall also consider the use of non-parametric (re-sampling) techniques for the computation of one-step ahead probability forecasts.[14]

[14]But as explained in the Appendix the use of non-parametric techniques for computation of multi-step ahead forecasts can be problematic.

4.3 Evaluation of Probability Forecasts

In addition to the above in-sample diagnostics, an out-of-sample evaluation of the model's performance is also desirable. For this purpose, we considered a number of simple events and two versions of the cointegrating VAR model: a restricted version as set out in Table 2 which imposes the over-identifying restrictions on the long-run relations based on economic theory, and an alternative, purely cointegrating specification which does not. This alternative model is also a VAR model in the nine variables with the same lag order, a cointegrating rank of 5, and unrestricted short-run dynamics, but it does not incorporate any of the long-run over-identifying restrictions.[15] Our objective here is to see if the imposition of long-run restrictions yield better forecasts. Note that forecasts from an exactly identified cointegrating VAR model are invariant to the way the long-run relations are identified.

Using these alternative specifications of the conditional model, (19), combined with an estimated version of the oil price equation, (20), we generated two sets of forecasts for a number of simple events over the period 1999q1-2001q1 in a recursive manner, where we first estimated the models over the period 1985q1-1998q4 and computed one-step-ahead probability forecasts for 1999q1, and ended with forecasts for 2001q1 based on models estimated over the period 1985q1-2000q4.[16] The probability forecasts were computed for directional events of interest. In the case of $p_t - p_t^*$, e_t, r_t, r_t^* and $\Delta\widetilde{p}_t$, we computed the probability that these variables rise next period, namely $\Pr[\Delta(p_t - p_t^*) > 0 \mid \Omega_{t-1}]$, $\Pr[\Delta e_t > 0 \mid \Omega_{t-1}]$), and so on. For the remaining trended variables, ($y_t, y_t^*, h_t - y_t$ and p_t^o), we considered the event that the rate of change of these variables rise from one period to the next, namely $\Pr[\Delta^2 y_t > 0 \mid \Omega_{t-1}]$, $\Pr[\Delta^2 y_t^* > 0 \mid \Omega_{t-1}]$), and so on. The probability forecasts are computed recursively using the parametric stochastic simulation technique which allow for future uncertainty and the nonparametric bootstrap technique which allow for parameter uncertainty. Both techniques are described in the Appendix. To evaluate the probability forecasts, we adopted a statistical approach, using a threshold probability of 0.5, so that an event was forecast to be realized if its probability forecast exceeded 0.5.[17] Formal statistical comparisons of forecasts and realizations were made using Kuipers score (KS), Pesaran and Timmermann (PT) (1992) directional (market timing) statistic and the probability integral transform as proposed by Dawid (1984) and developed further in Diebold, Gunther and Tay (1998).

Table 3a reports the incidence of the four possible combinations of our directional forecasts based on the over-identified model, and Table 3b gives the results for the exactly identified specification. For each variable, nine event forecasts are generated over the period 1999q1-2001q1 (nine quarters), thus providing 81 forecasts for evaluation purposes. These

[15] For both the over and exactly identified models, the coefficients of the cointegrating relationships are estimated using data for the longer period 1965q1-1998q4.

[16] It is worth emphasising that the two-stage estimation procedure used to obtain our model, in which estimates of the long-run and the short-run relationships were based on a samples starting in 1965q1 and 1985q1 respectively, was also employed in the evaluation exercise. But the end date of the sample rolled forward in the recursion so that no out-of-sample information was used in the construction of the forecasts.

[17] As an alternative, we could conduct a decision-theoretic approach to forecast evaluation as advocated in Granger and Pesaran (2000) and reviewed in Pesaran and Skouris (2001), which bases the evaluation of the probability forecasts on their implied economic value in a specific decision-making context. However, this demands a complete specification of the decision problem and this has been rather rare in macro-economic policy evaluation.

event forecasts are compared with their realizations and grouped under the headings, 'UU', indicating forecasts and realizations are in the same upward direction, 'UD' indicating an upward forecast with a realized downward movement, and so on. High values for UU and DD indicate an ability of the model to forecast upward and downward movements correctly, while high values of UD and DU suggest poor forecasting ability.

Using the information in Tables 3a and 3b, the Kuipers score is defined by $H-F$, where H is the proportion of ups that were correctly forecast to occur, and F is the proportion of downs that were incorrectly forecast.[18] It provides a measure of the accuracy of directional forecasts of the model, with high positive numbers indicating high predictive accuracy. The probability forecasts based on the model subject to the long-run over-identifying restrictions yielded the more accurate forecasts. It also turned out that allowing for parameter uncertainty in the computation of probability forecasts resulted in marginally poorer forecasts. For example, the Kuipers score of forecasts based on the over-identified model that allow only for future uncertainty was 0.342, as compared to 0.216 for the exactly identified model.[19] The corresponding estimates when parameter uncertainty is also allowed for were 0.252 and 0.120, respectively.

The Kuipers score is a useful summary measure but does not provide a statistical test. Pesaran and Timmermann (1992) provide a formal statistical test of the directional forecasting performance which, as shown in Granger and Pesaran (2000b), turns out to be equivalent to a test based on the Kuipers score. The PT statistic is defined by

$$PT = \frac{\widehat{P} - \widehat{P}^*}{\left\{\widehat{V}(\widehat{P}) - \widehat{V}(\widehat{P}^*)\right\}^{\frac{1}{2}}},$$

where $\widehat{P}$ is the proportions of correctly predicted upward movements, $\widehat{P}^*$ is the estimate of the probability of correctly predicting the events under the null hypothesis that forecasts and realisations are independently distributed, and $\widehat{V}(\widehat{P})$ and $\widehat{V}(\widehat{P}^*)$ are the consistent estimates of the variances of $\widehat{P}$ and $\widehat{P}^*$, respectively. Under the null hypothesis, the PT statistic has a standard normal distribution. For the forecasts based on the over-identified model, we obtained $PT = 3.16$ when only future uncertainty was allowed for, and $PT = 2.35$ when both future and parameter uncertainties were taken into account. Both of these statistics are statistically highly significant. However, the probability forecast results based on the exactly identified model were much less convincing. The associated PT statistics for the case of future uncertainty only, and when both future and parameter uncertainties were taken into account were 1.95 and 1.09, respectively. These results clearly highlight the potential importance of imposing theory-based long-run restrictions for probability forecasting. It would be interesting to see if this result also holds in other similar applications.

[18]These two proportions are known as the "hit rate" and "false alarm rate" respectively. In the case where the outcome is symmetric, in the sense that we value the ability to forecast ups and downs equally, then the score statistic of zero means no accuracy, whilst high positive and negative values indicate high predictive power.

[19]These statistics are based on probability forecasts where future uncertainty is taken into account using a parametric procedure. The results are hardly affected if a non-parametric procedure used instead. For example, for the over-identified model the Kuipers score based on the probability forecasts computed using the non-parametric approach was 0.345, compared to 0.342 obtained with the parametric procedure.

An alternative approach to probability forecast evaluation would be to use the probability integral transforms

$$u(\mathbf{z}_t) = \int_{-\infty}^{\mathbf{z}_t} p_t(\mathbf{x})\, d\mathbf{x}, \qquad t = T+1, T+2, ..., T+n,$$

where $p_t(\mathbf{x})$ is the forecast probability density function, and $\mathbf{z}_t$, $t = T+1, T+2, ..., T+n$, the associated realizations. Under the null hypothesis that $p_t(\mathbf{x})$ coincides with the true density function of the underlying process, the probability integral transforms will be distributed as iid $U[0,1]$. This result is due to Rosenblatt (1952), and has been recently applied in time series econometrics by Diebold, Gunther and Tay (1998).[20] In our application, we first computed a sequence of one step ahead probability forecasts (with and without allowing for parameter uncertainty) from the over-identified and exactly identified models for the nine simple events set out above over the nine quarters $1999q1, 1999q2, ..., 2001q1$, and hence the associated probability integral transforms, $u(\mathbf{z}_t)$. To test the hypothesis that these probability integral transforms are random draws from $U[0,1]$, we calculated the Kolmogorov-Smirnov statistic, $D_n = \sup_x |F_n(x) - U(x)|$, where $F_n(x)$ is the empirical cumulative distribution function (CDF) of the probability integral transforms, and $U(x) = x$, is the CDF of iid $U[0,1]$. Large values of the Kolmogorov-Smirnov statistic, D_n, indicate that the sample CDF is not similar to the hypothesized uniform CDF.[21] For the over-identified specification, we obtained the value of 0.111 for the Kolmogorov-Smirnov statistic when only future uncertainty was allowed for, and the larger value of 0.148 when the underlying probability forecasts took account of both future and parameter uncertainties. The corresponding statistics for the exactly identified model turned out to be 0.111 and 0.136, respectively. All these statistics are well below the 5% critical value of Kolmogorov-Smirnov statistic (which for $n = 81$ is equal to 0.149), and the hypothesis that the forecast probability density functions coincide with the true ones cannot be rejected. This is in line with the results of the directional tests and provides further support for the use of the over-identified specification in forecasting. With this in mind, we now proceed to the generation of out-of-sample forecast probabilities of interest using the over-identified model.

5 Probability Forecasts of Inflation and Output Growth

In this section, we present out-of-sample probability forecasts of two events of particular interest for the analysis of macro-economic policy in the UK, namely inflation targeting and output growth. Inflation targets have been set explicitly in the UK since October 1992, following the UK's exit from the European Exchange Rate Mechanism (ERM). The Chancellor's stated objective at the time was to achieve an average annual rate of inflation of 2%, while keeping the underlying rate of inflation within the 1%-4% range. In May 1997, the policy of targeting inflation was formalized further by the setting up of the Monetary Policy Committee (MPC), whose main objective is to meet inflation targets primarily by

[20] Also see Diebold, Hahn and Tay (1999) and Berkowitz (1999).

[21] For details of the Kolmogorov-Smirnov test and its critical values see, for example, Neave and Worthington (1992, pp.89-93).

influencing the market interest rate through fixing the base rate at regular intervals. Its current remit, as set annually by the Chancellor, is to achieve an average annual inflation rate of 2.5%, with the rate falling in the target range 1.5%-3.5%.

The measure of inflation used by the MPC is the Retail Price Index, excluding mortgage interest payments, (RPI-x), and the time horizon over which the inflation objective is to be achieved is not stated. Inflation rates outside the target range act as a trigger, requiring the Governor of the Bank of England to write an open letter to the Chancellor explaining why inflation had deviated from the target, the policies being undertaken to correct the deviation, and how long it is expected before inflation is back on target. The Bank is also expected to conduct monetary policy so as to support the general economic policies of the government, in so far as this does not compromise its commitment to its inflation target.

Since October 1992, the Bank of England has produced a quarterly *Inflation Report* which describes the Bank's assessment of likely inflation outcomes over a two-year forecast horizon. In addition to reviewing the various economic indicators necessary to place the inflation assessment into context, the *Report* provides forecasts of inflation over two year horizons, with bands presented around the central forecast to illustrate the range of inflation outcomes that are considered possible (the so-called fan charts). The forecasts are based on the assumption that the base rate is left unchanged. Since November 1997, a similar forecast of output growth has also been provided in the *Report*, providing insights on the Bank's perception of the likely outcome for the government's general economic policies beyond the maintenance of price stability. For a critical assessment of the Bank's approach to allowing for model and parameter uncertainties, see Wallis (1999).

The fan charts produced by the Bank of England are an important step towards acknowledging the significance of forecast uncertainties in the decision making process and it is clearly a welcome innovation. However, the approach suffers from two major shortcomings. First, it seems unlikely that the fan charts can be replicated by independent researchers. This is largely due to the subjective manner in which uncertainty is taken into account by the Bank, which does not readily lend itself to independent analysis even though it may be justified from a real time decision-making perspective. Second, the use of fan charts is limited for the analysis of uncertainty associated with joint events. Currently, the Bank provides separate fan charts for inflation and output growth forecasts, but in reality one may also be interested in joint events involving both inflation and output growth, and it is not clear how the two separate fan charts could be used for such a purpose. Here, we address both of these issues using the long-run structural model developed in the previous section.[22]

In what follows, we present plots of estimated predictive distribution functions for inflation and output growth at a number of selected forecast horizons using the over-identified version of the cointegrating model, (19), augmented with the oil price equation, (20). These plots provide us with the necessary information with which to compute probabilities of a variety of events, and demonstrate the usefulness of probability forecasts in conveying the future and parameter uncertainties that surround the point forecasts. But our substantive discussion of the probability forecasts focuses on two central events of interest; namely, keeping the rate of inflation within the announced target range of 1.5 to 3.5 per cent, and

[22]We do not address the important issue of model uncertainty, but our approach could be adapted to deal with it following a method similar to that discussed in Draper (1990).

avoiding of a recession. Following the literature, we define a recession as the occurrence of two successive negative quarterly growth rates. Other concepts of recessions, such as a negative annualised rate of output growth, could also be considered, although we do not expect the probability estimates presented in this section would be much affected by which one of these notions is adopted.[23]

5.1 Point and Interval Forecasts

Before reporting the probability forecasts, it is worth briefly summarizing the point and interval forecasts to help place the probability forecasts in context. Tables 4a and 4b provide the point forecasts for domestic inflation rates and output growth over the period 2001q1-2003q1 together with their 95% confidence intervals. Table 4a presents the four quarterly growth rate forecasts, while Table 4b gives the forecasts of annualised quarter-on-quarter growth rates.[24]

The model predicts the average annual rate of inflation to fall from 2.5% in 2001q1 to 1.8% in 2001q2. This is followed by further falls for the rest of 2001 before returning to approximately 2% to the end of the forecast horizon, 2003q1. These point forecasts are lower than the inflation rates realized during 2000, as illustrated by the historical data on inflation presented in Figure 1a. Output growth is predicted to be positive throughout the forecast horizon, falling from an average annual rate of 2.8% in 2000 to 1.4% by the end of 2001, before rising to around 2.0% thereafter. (See Table 4a). Therefore, based on these point forecasts, we may be tempted to rule out the possibility of a recession occurring in the UK over the 2001-2003 period.

However, these point forecasts are subject to a high degree of uncertainty, particularly when longer forecast horizons are considered. For example, at the two year forecast horizon the point forecast of annual inflation in 2003q1 is predicted to be 1.9%, which is well within the announced inflation target range. But the 95% confidence interval covers the range -0.8% to +4.6%. For the quarter on quarter definition, the uncertainty is even larger, with a range of -3.5% to 7.5% around a point forecast of approximately 2.0%. Similarly, the point forecast of the annual rate of output growth in 2003q1 is 2.1%, but its 95 per cent confidence interval covers the range -2.6% to +6.7%. As we have noted, it is difficult to evaluate the significance of these forecast intervals for policy analysis and a more appropriate approach is to directly focus on probability forecasts as a method of characterising the various uncertainties that are associated with events of interest. This is the topic that we shall turn to now.

5.2 Predictive Distribution Functions

In the case of single events, probability forecasts are best represented by means of probability distribution functions. Figures 2a-2b give the estimates of these functions for the four quar-

[23] Harding and Pagan (2000) make the argument that, by focusing on turning points in output levels, this definition of recession matches the most widely-held view of the business cycle and one which decision makers find most useful. As a point of comparison, in this paper, we also consider the event 'output growth prospects are poor', defined when the four quarterly moving average of output growth falls below 1 per cent.

[24] It is worth noting that the inflation target is expressed in terms of RPI-x while our model provides forecasts of RPI.

terly moving averages of inflation and output growth for selected forecast horizons: 1, 4 and 8 quarters ahead. These estimates are computed using the simulation techniques described in detail in the Appendix and take account of both future and parameter uncertainties. We shall refer to these as the bootstrap predictive distribution (BPD) functions, and denote them by $\widetilde{\pi}$.

Figure 2 presents the BPD function estimates of inflation for the threshold values ranging from 0% to 4% per annum at the three selected forecast horizons. Perhaps not surprisingly, the estimates for the one-quarter ahead forecast horizon is quite steep, but starts to become flatter as the forecast horizon is increased. Above the threshold value of 2.0%, the estimated probability distribution functions shift to the right as longer forecast horizons are considered, showing that the probability of inflation falling below thresholds greater than 2.0% declines with the forecast horizon. For example, the forecast probability that inflation lies below 3.5% becomes smaller at longer forecast horizons, falling from close to 100% in 2001q2 to 70% in 2003q1. These forecast probabilities are in line with the recent historical experience, as set out in Figure 1. Over the period 1985q1-2001q1, the average annual rate of inflation fell below 3.5% for 53.9 per cent of the quarters, but were below this threshold value throughout the last two years of the sample, 1999q1-2001q1.

Figure 2b plots the BPD functions for output growth. These estimates also become flatter as the forecast horizon is increased, reflecting the greater uncertainty associated with growth outcomes at longer forecast horizons. These plots also suggest a weakening of the growth prospects in 2001 before recovering a little at longer horizons. For example, the probability of a negative output growth in 2001q2 is estimated to be almost zero, but rises to 14% in 2002q1 before falling back to 12% in 2003q. Therefore, a rise in the probability of a recession is predicted, but the estimate is not sufficiently high for it to be much of a policy concern (at least viewed from the end of our sample period 2001q1).

5.3 Event Probability Forecasts

Here we consider three single events of particular interest:

A : Achievement of inflation target, defined as the four-quarterly moving average rate of inflation falling within the range 1.5%-3.5%.

B : Recession, defined as the occurrence of two consecutive quarters of negative output growth.

C : Poor growth prospects, defined to mean that the four-quarterly moving average of output growth is less than 1%.

and the joint events

$A \cap \overline{B}$: Inflation target is met *and* recession is avoided.

$A \cap \overline{C}$: Inflation target is met *combined* with reasonable growth prospects.

where $\overline{B}$ and $\overline{C}$ denote the complement of events B and C.

5.3.1 Inflation and the Target Range

Two sets of estimates of $\Pr(A_{T+h} \mid \Omega_T)$ are provided in Table 5a (for $h = 1, 2, ..., 8$) and depicted in Figure 3 over the longer forecast horizons $h = 1, 2, ..., 24$; namely the profile predictive likelihoods (PPL or π) which only take account of future uncertainty, and the bootstrap predictive distribution (BPD's or $\widetilde{\pi}$) functions that allow for future as well as parameter uncertainties. As we shall see, π and $\widetilde{\pi}$ both convey a similar message in these Tables and Figures, but there are nevertheless some differences between them, at least at some forecast horizons, so that it is important that both estimates are considered in practice.[25]

Based on these estimates, and conditional on the information available at the end of 2001q1, the probability that the Bank of England will be able to achieve the government inflation target is estimated to be high in the short-run but falls in the longer run, reflecting the considerable uncertainty surrounding the inflation forecasts at longer horizons. Specifically, the probability estimate is high in 2001q2, at 0.87 (0.80) for $\widetilde{\pi}$ (π), but it falls rapidly at the end of 2001/early 2002 as the likelihood that inflation will lie below the 1.5% threshold rises. In fact, forecast levels of inflation are low by historical standards throughout the forecast period: as shown in Figure 4b, for example, the probability that inflation will fall below 3.5%, the upper threshold of the target range, is estimated to be close to 1.0 initially and settles down to around 0.70 (0.77) using $\widetilde{\pi}$ (π) in the longer term. Ultimately, the estimated probability of achieving inflation within the target range settles to 0.38 (0.35) for $\widetilde{\pi}$ (π) in 2003q1. However, at the same time, the probabilities of inflation falling below and above the target range are 0.32 and 0.30, respectively, using $\widetilde{\pi}$ (or 0.42 and 0.23 using π), so these figures reflect the relatively high degree of uncertainty associated with inflation forecasts even at moderate forecast horizons. Hence, while the likely inflation outcomes are low by historical standards and there is a reasonable probability of hitting the target range, there are also comparable likelihoods of undershooting and overshooting the inflation target range at longer horizons.

5.3.2 Recession and Growth Prospects

Figures 5 and 6 show the estimates of the recession probability, $\Pr(B_{T+h} \mid \Omega_T)$, and the low growth probability, $\Pr(C_{T+h} \mid \Omega_T)$, over the forecast horizons $h = 1, 2, ..., 24$. For these events, the probability estimates that allow for parameter uncertainty (i.e. $\widetilde{\pi}$) exceed those that do not (i.e. π) at shorter horizons, but the opposite is true at longer horizons. For event B, π and $\widetilde{\pi}$ are very similar in size across the different forecast horizons and suggest a very low probability of a recession. Based on the $\widetilde{\pi}$ estimate, the probability of a recession occurring in 2001q2 is estimated to be around zero, rising to 0.09 in 2002q1. (See Table 5b). However, the probability that UK faces poor growth prospects is much higher, in the region of 0.3-0.4 during 2001, falling to 0.3 in 2003q1 according to the $\widetilde{\pi}$ estimates.

[25]For a given model and sample size, allowing for parameter uncertainty could increase or decrease the probability of an event of interest and there can be no general rule as to the order of magnitudes of the two types of probability estimates.

5.3.3 Joint Event Probabilities

Single events are clearly of interest but very often decision makers are concerned with joint events involving, for example, both inflation and output growth outcomes. As examples here, we consider the probability estimates of the two joint events, $A_{T+h} \cap \overline{B}_{T+h}$, and $A_{T+h} \cap \overline{C}_{T+h}$ over the forecast horizons $h = 1, 2, ..., 24$. Probability estimates of these events (based on $\widetilde{\pi}$) are presented in Table 5b and Figures 7a and 7b. Both events are of policy interest as they combine the achievement of the inflation target with alternative growth objectives. Also provided in each of the Figures is a plot of the product of the single event probabilities; that is $\Pr(A_{T+h} \mid \Omega_T) \times \Pr(\overline{B}_{T+h} \mid \Omega_T)$, $h = 1, 2, ..24$ in Figure 7a and $\Pr(A_{T+h} \mid \Omega_T) \times \Pr(\overline{C}_{T+h} \mid \Omega_T)$, $h = 1, 2, ..24$ in Figure 7b. For the event $A_{T+h} \cap \overline{B}_{T+h}$, the joint probability forecasts are similar in magnitude to those that relate to $\Pr(A_{T+h} \mid \Omega_T)$ alone at every time horizon. Of course, this is not surprising since the probability of a recession is estimated to be small at most forecast horizons and therefore the probability of avoiding recession is close to one.[26] More interesting though are the plots provided in Figure 5b where the joint probabilities are clearly distinct from either of the single event probabilities. In this case, it is also interesting to compare the joint probabilities with the product of the two single event probabilities since this provides an indication of the degree of dependence/independence of the two events. As it turns out, these two sets of probabilities are very close at most forecast horizons, thus indicating little dependence between output growth prospects and inflation outcomes. This result is certainly compatible with the long-term neutrality hypothesis that postulates independence of inflation outcomes from output growth outcomes in the long-run.

6 Concluding Remarks

One of the many problems economic forecasters and policy makers face is conveying to the public the degree of uncertainty associated with point forecasts. Policy makers recognise that their announcements, in addition to providing information on policy objectives, can themselves initiate responses which effect the macroeconomic outcome. This means that Central Bank Governors are reluctant to discuss either pessimistic possibilities, as this might induce recession, or more optimistic possibilities, since this might induce inflationary pressures. There is therefore an incentive for policy makers to seek ways of making clear statements regarding the range of potential macroeconomic outcomes for a given policy, and the likelihood of the occurrence of these outcomes, in a manner which avoids these difficulties.

In this paper, we have argued for the use of probability forecasts as a method of charaterising the uncertainties that surround forecasts from a macroeconomic model believing this to be superior to the conventional way of trying to deal with this problem through the use of confidence intervals. We argue that the use of probability forecasts has an intuitive appeal, enabling the forecaster (or users of forecasts) to specify the relevant "threshold values" which define the event of interest (e.g. a threshold value corresponding to an inflation target range of 1.5% to 3.5%). This is in contrast to the use of confidence intervals which define threshold values only implicitly, through the specification of the confidence interval widths, and

[26] Of course, even relatively minor differences in probabilities can have an important impact on decisions if there are large, discontinuous differences in the net benefits of different outcomes.

these values may or may not represent thresholds of interest. A further advantage of the use of probability forecasts compared with the use of confidence intervals and over other more popular methods is the flexibility of probability forecasts, as illustrated by the ease with which the probability of joint events can be computed and analysed. Hence, for example, we can consider the likelihood of achieving a stated inflation target range whilst simultaneously achieving a given level of output growth, with the result being conveyed in a single number. In situations where utility or loss functions are non-quadratic and/or the constraints are non-linear the whole predictive probability distribution function rather than its mean is required for decision making. This paper shows how such predictive distribution functions can be obtained in the case of long-run structural models, and illustrates its feasibility in the case of a small macro-econometric model of the UK.

The empirical exercise of the paper provides a concrete example of the usefulness of event probability forecasting both as a tool for model evaluation and as a means for conveying the uncertainties surrounding the forecasts of specific events of interest. The model used represents a small but comprehensive model of the UK macro-economic which incorporates long-run relationships suggested by economic theory so that it has a transparent and theoretically-coherent foundation. The model evaluation exercise not only demonstrates the statistical adequacy of the forecasts generated by the model but also highlights the considerable improvements in forecasts obtained through the imposition of the theory-based long-run restrictions. The predictive distribution functions relating to single events and the various joint event probabilities presented in the paper illustrate the flexibility of the functions in conveying forecast uncertainties and, from the observed independence of probability forecasts of events involving inflation and growth, in conveying information on the properties of the model. The estimated probability functions also show the importance of taking into account parameter uncertainty as well as future uncertainty in deriving the probability forecasts.

The various probability forecasts presented in the paper are encouraging from the point of view of the government's inflation objectives. Taking account of future as well as parameter uncertainties, the probability of inflation falling within the target range is quite high in the short run, accompanied with only a small probability of a recession. Over a longer forecast horizon the probability of inflation falling within the target range starts to decline, primarily due to a predicted rise in the probability of inflation falling below 1.5%, the lower end of the target range. Overall, however, based on information available at the end of 2001q1, the probability that the inflation objective is achieved with moderate output growths in the medium term is estimated to be reasonably high, certainly higher than the probabilities of inflation falling above or below the target range.[27]

[27] Of course, these probability forecasts have taken into account future uncertainty and parameter uncertainty only and have not accommodated model uncertainty. It is possible that different conclusions would be drawn if we extended the analysis to consider other models in addition to that which we have estimated, and on which our probability forecasts are based.

A Appendix: Computation of Probability Forecasts by Stochastic Simulation

In this Appendix, we consider some of the computational issues that will typically be encountered in the calculation of probability forecasts and note how these can be dealt with through the use of stochastic simulation methods. These methods involve repeatedly simulating the future values of the variables under study, $\mathbf{z}_t$, (say S times), using the assumed stochastic structure of the data generating process, to obtain $\mathbf{z}_{T+h}^{(s)}(\widehat{\boldsymbol{\theta}})$, $h = 1, 2, ...,$ $s = 1, 2, ..., S$. On each simulation, the occurrence or non-occurrence of the event of interest is noted and, eventually, the probability of the event occurring can be calculated as the proportion of the S simulations in which the event was observed to occur. We illustrate the use of these methods below in the context of the general vector error correcting model used in the paper.

For forecasting purposes we first write the reduced form error correction model (21) in the following form

$$\mathbf{z}_t = \sum_{i=1}^{p} \Phi_i \mathbf{z}_{t-i} + \mathbf{a}_0 + \mathbf{a}_1 t + \mathbf{v}_t, \ t = 1, 2, ..., T, \tag{24}$$

where

$$\begin{aligned} \Phi_1 &= \mathbf{I}_m - \boldsymbol{\alpha}\boldsymbol{\beta}' + \Gamma_1, \ \Phi_i = \Gamma_i - \Gamma_{i-1}, \ i = 2, 3, ..., p-1, \ \Phi_p = -\Gamma_{p-1}, \\ \mathbf{a}_0 &= \mathbf{a}_y - \boldsymbol{\alpha}_y \mathbf{b}_1, \ \mathbf{a}_1 = \boldsymbol{\alpha}_y \mathbf{b}_1, \end{aligned}$$

and $\mathbf{v}_t$ is assumed to be a serially uncorrelated *iid* vector of shocks with zero means and a positive definite covariance matrix, Σ. In what follows, we consider the calculation of probability forecasts using (24), first assuming that the parameters are known and then taking into account parameter uncertainty.

A.1 Forecasts in the absence of parameter uncertainty

Suppose that the ML estimators of Φ_i, $i = 1, 2, \ldots, p$, $\mathbf{b}_0$, $\mathbf{b}_1$ and Σ are given and denoted by $\hat{\Phi}_i$, $i = 1, 2, \ldots, p$, $\hat{\mathbf{b}}_0$, $\hat{\mathbf{b}}_1$ and $\hat{\Sigma}$, respectively. Then the point estimates of the h-step ahead forecasts of $\mathbf{z}_{T+h}$ conditional on Ω_T, which we denote by $\hat{\mathbf{z}}_{T+h}$, can be obtained recursively by

$$\hat{\mathbf{z}}_{T+h} = \sum_{i=1}^{p} \hat{\Phi}_i \hat{\mathbf{z}}_{T+h-i} + \hat{\mathbf{a}}_0 + \hat{\mathbf{a}}_1 (t+h), \ h = 1, 2, \ldots, \tag{25}$$

where the initial values, $\mathbf{z}_T, \mathbf{z}_{T-1}, \ldots, \mathbf{z}_{T-p+1}$, are given. To obtain probability forecasts using stochastic simulation methods, we need to simulate the values of $\mathbf{z}_{T+h}$ by

$$\mathbf{z}_{T+h}^{(s)} = \sum_{i=1}^{p} \hat{\Phi}_i \mathbf{z}_{T+h-i}^{(s)} + \hat{\mathbf{a}}_0 + \hat{\mathbf{a}}_1 (t+h) + \mathbf{v}_{T+h}^{(s)}, \ h = 1, 2, \ldots, \ s = 1, 2, ..., S, \tag{26}$$

where superscript '(s)' refers to the s-th replication, and $\mathbf{z}_T^{(s)} = \mathbf{z}_T$, $\mathbf{z}_{T-1}^{(s)} = \mathbf{z}_{T-1}, \ldots, \mathbf{z}_{T-p+1}^{(s)} = \mathbf{z}_{T-p+1}$ for all s. The $\mathbf{v}_{T+h}^{(s)}$'s can be drawn by parametric or nonparametric methods as described in section B below. The probability that $\boldsymbol{\varphi}\left(\mathbf{z}_{T+1}^{(s)}(\widehat{\boldsymbol{\theta}}), \mathbf{z}_{T+2}^{(s)}(\widehat{\boldsymbol{\theta}}), \ldots, \mathbf{z}_{T+h}^{(s)}(\widehat{\boldsymbol{\theta}})\right) < \mathbf{a}$, can then be computed as

$$\pi\left(\mathbf{a}, h; \boldsymbol{\varphi}(.), \widehat{\boldsymbol{\theta}}\right) = \frac{1}{S}\sum_{s=1}^{S} I\left[\mathbf{a} - \boldsymbol{\varphi}\left(\mathbf{z}_{T+1}^{(s)}(\widehat{\boldsymbol{\theta}}), \mathbf{z}_{T+2}^{(s)}(\widehat{\boldsymbol{\theta}}), \ldots, \mathbf{z}_{T+h}^{(s)}(\widehat{\boldsymbol{\theta}})\right)\right],$$

where $I[\cdot]$ is an indicator function which takes the value of one if $\mathbf{a} - \boldsymbol{\varphi}\left(\mathbf{z}_{T+1}^{(s)}(\widehat{\boldsymbol{\theta}}), \mathbf{z}_{T+2}^{(s)}(\widehat{\boldsymbol{\theta}}), \ldots, \mathbf{z}_{T+h}^{(s)}(\widehat{\boldsymbol{\theta}})\right.$ $\mathbf{0}$, and zero otherwise.

A.2 Forecasts in the presence of parameter uncertainty

To allow for parameter uncertainty, we first use the simulation method to obtain R (*within sample)* simulated values of $\mathbf{z}_t$, $t = 1, 2, ..., T$, denoted by $\mathbf{z}_t^{(r)}$,

$$\mathbf{z}_t^{(r)} = \sum_{i=1}^{p} \hat{\Phi}_i \mathbf{z}_{t-i}^{(r)} + \hat{\mathbf{a}}_0 + \hat{\mathbf{a}}_1 t + \mathbf{v}_t^{(r)},\ t = 1, 2, ..., T,\ r = 1, 2, .., R,$$

where actual observations on the initial values, $\mathbf{z}_{-1}, \ldots, \mathbf{z}_{-p}$ are used for this purpose. Again, the $\mathbf{v}_t^{(r)}$'s can be drawn either by parametric or nonparametric methods. Having obtained the R set of the simulated in-sample values, $\left(\mathbf{z}_1^{(r)}, \mathbf{z}_2^{(r)}, \ldots, \mathbf{z}_T^{(r)}\right)$, the $VAR(p)$ model (24) is estimated R times to obtain new maximum likelihood estimates, $\hat{\Phi}_i^{(r)}$, $i = 1, 2, \ldots, p$, $\hat{\mathbf{b}}_0^{(r)}$, $\hat{\mathbf{b}}_1^{(r)}$ and $\hat{\Sigma}^{(r)}$, $r = 1, 2, ..., R$. For each of these replications, we employ the simulation technique described above to obtain the associated probability forecasts which we denote by $\pi^{(r)}$, $r = 1, 2, ..., R$. Then, the empirical mean of the probability forecast is obtained by

$$\bar{\pi}_R = \frac{1}{R}\sum_{r=1}^{R} \pi^{(r)},$$

and the associated 100α % lower and upper confidence bands computed as the R α-th smallest and largest values of $\pi^{(r)}$, $r = 1, 2, ..., R$, respectively.[28]

B Generating Simulated Errors

In this section, we briefly comment on the alternative methods that can be used to simulate errors for use in the stochastic simulations and the bootstrap exercises described above,

[28] We can also obtain the empirical mean of the point forecasts by

$$\bar{\mathbf{z}}_{T+h} = \frac{1}{R}\sum_{r=1}^{R}\frac{1}{S}\sum_{i=1}^{S}\mathbf{z}_{T+h}^{(i,r)},\ h = 1, 2, ...,$$

and construct their associated 100α % lower and upper confidence bands as the RS α-th smallest and largest values of $\mathbf{z}_{T+h}^{(i,r)}$, $i = 1, 2, ..., S$, $r = 1, 2, ..., R$, respectively.

allowing for the contemporaneous correlations that exist across the errors in the different equations of the model. The first is *parametric* method where the errors are drawn from an assumed probability distribution function. Alternatively, one could employ a *non-parametric* procedure. These are slightly more complicated and are based on re-sampling techniques in which the simulated errors are obtained by a random draw from the observed errors (see, for example, Hall (1992)).

In what follows the application of these two approaches to generate forecast probabilities at different horizons, $T+h$, $h=1,2,...,H$, will now be described.

B.1 Parametric Approach

In our application of the parametric approach we assume that the errors are drawn from a multivariate Normal distribution with zero mean and the covariance matrix, Σ. The procedure makes use of the matrix $\mathbf{P}$, where $\mathbf{P}^{-1}$ is the lower triangular Choleski decomposition of Σ such that $\Sigma = \mathbf{P}\mathbf{P}'$. In this case, $\boldsymbol{\varepsilon}_t = \mathbf{P}^{-1}\mathbf{v}_t$ is an $m \times 1$ vector of standard normal disturbances. To obtain simulated errors for m variables over h periods, say, we generate mh draws from the standard normal distribution, denoted by $\{\boldsymbol{\varepsilon}_{T+1}, \boldsymbol{\varepsilon}_{T+2}, \ldots, \boldsymbol{\varepsilon}_{T+h}\}$, and these are used to obtain $\{\mathbf{v}_{T+1}, \mathbf{v}_{T+1,} \ldots, \mathbf{v}_{T+h}\}$ via the transformation $\mathbf{v}_{T+h} = \mathbf{P}\boldsymbol{\varepsilon}_{T+h}$.

B.2 Non-Parametric Approaches

The most obvious non-parametric approach to generating simulated errors, $\mathbf{v}_{T+h}$, which we shall denote 'Method 1', is simply to take h random draws from the observed errors $\{\mathbf{v}_1, \ldots, \mathbf{v}_T\}$, replacing the chosen error vector after each draw. The simulated errors thus obtained clearly have precisely the same distribution and covariance structure as that observed in the original sample. However, this method is susceptible to the criticism, discussed below, that serial independence is introduced at longer forecast horizons since there is a set of just T observations from which we sample each time.

An alternative non-parametric method for generating simulated errors, 'Method 2', makes use of the Choleski decomposition of the estimated covariance employed in the parametric approach. Having identified the matrix $\mathbf{P}$ for which $\Sigma = \mathbf{P}\mathbf{P}'$, we can obtain a set of mT transformed error terms $\{\boldsymbol{\varepsilon}_1, \ldots, \boldsymbol{\varepsilon}_T\}$ where $\boldsymbol{\varepsilon}_t = \mathbf{P}^{-1}\mathbf{v}_t$, $t=1,...,T$. The mT individual error terms are uncorrelated with each other, but retain the distributional information (relating to extreme values, and so on) contained in the original observed errors. A set of mh simulated errors can be obtained through random draw, with replacement, from the transformed errors, and these can be arranged into a set of $m \times 1$ vectors $\{\boldsymbol{\varepsilon}_{T+1}, \ldots, \boldsymbol{\varepsilon}_{T+h}\}$ which can be used to obtain $\{\mathbf{v}_{T+1}, \ldots, \mathbf{v}_{T+h}\}$ using $\mathbf{v}_{T+h} = \mathbf{P}\boldsymbol{\varepsilon}_{T+h}$. Given that the $\mathbf{P}$ matrix is used to generate the simulated errors, it is clear that the $\mathbf{v}_{T+h}$ again have the same covariance structure as the original estimated errors. And being based on errors drawn at random from the transformed originals, these generated simulations will also display the same distributional features. Further, given that the re-sampling occurs from the mT transformed error terms, Method 2 also has the advantage over Method 1 that the serial dependence introduced through sampling with replacement will be problematic only at longer time horizons.

B.3 Choice of Approach

The non-parametric approaches described above have the advantage over the parametric approach that they make no distributional assumptions on the error terms, and are better able to capture the uncertainties arising from (possibly rare) extreme observations. However, they suffer the disadvantage that they require random sampling *with replacement*. Replacement is essential as otherwise the draws at longer forecast horizons are effectively 'truncated' and unrepresentative. On the other hand, for a given sample size, it is clear that re-sampling from the observed errors with replacement inevitably introduces serial dependence in the simulated forecast errors at longer horizons as the same observed errors are drawn repeatedly. When generating simulated errors over a forecast horizon, therefore, this provides an argument for the use of non-parametric methods over shorter forecast horizons, but suggests that a greater reliance might be placed on the parametric approach for the generation of probability forecasts at longer time horizons.

Table 1
List of Variables and their Descriptions in the Core Model

y_t : natural logarithm of the UK real per capita GDP at market prices (1995 = 100).

p_t : natural logarithm of the UK Retail Price Index, All Items (1995 = 100).

r_t : is computed as $r_t = 0.25 \ln(1 + R_t/100)$, where R_t is the 90 day Treasury Bill average discount rate per annum.

h_t : natural logarithm of UK real per capita M0 money stock (1995 = 100).

e_t : natural logarithm of the nominal Sterling effective exchange rate (1995 = 100).

y_t^* : natural logarithm of the foreign (Total OECD) real per capita GDP at market prices (1995 = 100).

p_t^* : natural logarithm of the foreign (Total OECD Consumer Price Index) (1995 = 100).

r_t^* : is computed as $r_t^* = 0.25 \ln(1 + R_t^*/100)$, where R_t^* is the weighted average of 90 day interest rates per annum in the United States, Germany, Japan and France.

p_t^o : natural logarithm of oil prices, measured as the Average Price of Crude Oil.

t : time trend, taking the values $1, 2, 3, \ldots$, in $1965q1, 1965q2, 1965q3, \ldots$, respectively.

Notes: The data set used in the probability forecasting exercise is based on the European Standard of Accounts. For more detail of the data sources and a description of the construction of the series see the Data Appendix in Garratt et al. (2002).

Table 2
Error Correction Specification for the Over-identified Model: 1985q1-2001q1

Equation	$\Delta(p_t\text{-}p_t^*)$	Δe_t	Δr_t	Δr_t^*	Δy_t	Δy_t^*	$\Delta(h_t\text{-}y_t)$	$\Delta^2\widetilde{p}_t$
$\hat{\xi}_{1t}$	$-.020^*$ (.010)	$.136^*$ (.071)	.003 (.004)	.0006 (.001)	.010 (.009)	.002 (.006)	$.031^*$ (.017)	$-.014^*$ (.008)
$\hat{\xi}_{2t}$	$-.775$ (.664)	-2.59 (4.63)	$-593^\dagger$ (.281)	.117 (.075)	.541 (.592)	.063 (.418)	-1.31 (1.09)	$-1.05^\dagger$ (.508)
$\hat{\xi}_{3t}$	.022 (.060)	.073 (.414)	.029 (.025)	$-.003$ (.007)	$-.061$ (.050)	.057 (.037)	$.271^\dagger$ (.098)	$.087^*$ (.045)
$\hat{\xi}_{4t}$	$.010^*$ (.006)	.003 (.043)	.004 (.003)	$-.001$ (.0007)	$-.012^\dagger$ (.005)	.0004 (.004)	$-.003$ (.010)	.005 (.005)
$\hat{\xi}_{5t}$	.131 (.239)	2.04 (1.67)	.007 (.101)	$-.014$ (.027)	.315 (.203)	.060 (.150)	.257 (.393)	1.26 (.183)
$\Delta(p_{t-1}\text{-}p_{t-1}^*)$	.275 (.176)	$-.588$ (1.23)	$-.030$ (.074)	.007 (.020)	.136 (.149)	.031 (.111)	$-.066$ (.289)	.163 (.134)
Δe_{t-1}	.020 (.022)	.210 (.155)	$-.0001$ (.009)	.0004 (.003)	.019 (.029)	$-.012$ (.014)	.059 (.037)	$-.025$ (.017)
Δr_{t-1}	$-.025$ (.404)	-3.90 (2.81)	.214 (.171)	.053 (.046)	.190 (.342)	.025 (.254)	$-.296$ (.665)	$.960^\dagger$ (.309)
Δr_{t-1}^*	$-.839$ (1.23)	5.74 (8.59)	$-.120$ (.522)	$.407^\dagger$ (.139)	.784 (1.05)	$-.732$ (.775)	-2.42 (2.03)	1.15 (.943)
Δy_{t-1}	$-.090$ (.177)	-1.47 (1.23)	.009 (.075)	$-.017$ (.020)	$.439^\dagger$ (.150)	$.343^\dagger$ (.111)	$-.782^\dagger$ (.291)	$.252^*$ (.135)
Δy_{t-1}^*	$-.052$ (.229)	.489 (1.51)	.131 (.097)	$.072^\dagger$ (.026)	$.351^*$ (.194)	.184 (.053)	.386 (.377)	.147 (.175)
$\Delta(h_{t-1}\text{-}y_{t-1})$	.023 (.086)	$-.081$ (.588)	$-.029$ (.036)	$-.001$ (.010)	$-.057$ (.073)	$-.007$ (.053)	$-.255^*$ (.141)	$-.023$ (.066)
$\Delta^2\widetilde{p}_{t-1}$	$-.064$ (.171)	.860 (1.19)	$-.012$ (.072)	$-.008$ (.019)	$-.019$ (.145)	$-.049$ (.107)	$-.194$ (.281)	.017 (.131)
Δp_{t-1}^o	$-.005$ (.005)	.006 (.036)	$-.0001$ (.002)	$-.0009$ (.0006)	$.012^\dagger$ (.004)	.005 (.003)	.006 (.009)	.003 (.004)
Δp_{t-1}^o	$-.010^\dagger$ (.005)	$-.019$ (.032)	.002 (.002)	$-.0007$ (.0005)	$-.010^\dagger$ (.004)	$-.001$ (.003)	$-.001$ (.007)	.004 (.003)
$\overline{R}^2$	.365	.089	.017	.476	.549	.371	.378	.603
$\hat{\sigma}$	.005	.032	.002	.001	.004	.003	.008	..003
$\chi^2_{SC}[4]$	4.31	3.16	9.40^*	1.91	5.74	7.29	7.40	5.89
$\chi^2_{FF}[1]$	3.04	0.76	3.49^*	2.26	0.86	2.31	0.02	0.98
$\chi^2_{N}[2]$	3.53	$11.2^\dagger$	$7.13^\dagger$	0.27	1.91	1.47	$33.9^\dagger$	$26.0^\dagger$
$\chi^2_{H}[1]$	0.01	0.01	1.08	0.01	0.83	0.84	0.17	.057

Notes: The five error correction terms, estimated over the period 1965q1-2001q1, are given by

$$
\begin{aligned}
\hat{\xi}_{1,t+1} &= p_t - p_t^* - e_t - 4.8566, \\
\hat{\xi}_{2,t+1} &= r_t - r_t^* - 0.0057, \qquad \hat{\xi}_{3,t+1} = y_t - y_t^* + 0.0366, \\
\hat{\xi}_{4,t+1} &= h_t - y_t + \underset{(35.34)}{75.68}\, r_t + \underset{(0.001)}{0.0068}\, t + 0.1283, \\
\hat{\xi}_{5,t+1} &= r_t - \Delta\widetilde{p}_t - 0.0037.
\end{aligned}
$$

Standard errors are given in parenthesis. "*" indicates significance at the 10% level, and "†" indicates significance at the 5% level. The diagnostics are chi-squared statistics for serial correlation (SC), functional form (FF), normality (N) and heteroscedasticity (H).

Table 3a
Forecast Evaluation of the Over-identified Model

Variable	Future Uncertainty				Future/Parameter Uncertainty			
	UD	DD	DU	UU	UD	DD	DU	UU
p_t^o	0	6	1	2	1	5	1	2
e_t	5	0	0	4	5	0	0	4
r_t^*	0	3	2	4	2	1	2	4
r_t	5	0	0	4	5	0	0	4
$\Delta\widetilde{p}_t$	1	3	0	5	2	2	0	5
y_t	2	2	1	4	2	2	1	4
p_t-p_t^*	2	5	2	0	3	4	2	0
h_t-y_t	0	4	1	4	0	4	1	4
y_t^*	2	3	3	1	2	3	2	2
Total	17	26	10	28	22	21	9	29
Hit Rate	54/81=0.67				50/81=0.62			

Table 3b
Forecast Evaluation of the Exactly-identified Model

Variable	Future Uncertainty				Future/Parameter Uncertainty			
	UD	DD	DU	UU	UD	DD	DU	UU
p_t^o	0	6	1	2	0	6	1	2
e_t	2	3	4	0	2	3	3	1
r_t^*	1	2	1	5	2	1	1	5
r_t	5	0	0	4	5	0	0	4
$\Delta\widetilde{p}_t$	3	1	0	5	4	0	0	5
y_t	2	2	1	4	3	1	1	4
p_t-p_t^*	2	5	1	1	3	4	2	0
h_t-y_t	1	3	2	3	0	4	2	3
y_t^*	3	2	3	1	3	2	4	0
Total	19	24	13	25	22	21	14	24
Hit Rate	49/81=0.61				45/81=0.56			

Notes: The forecast evaluation statistics are based on one-step-ahead forecasts obtained from models estimated recursively, starting with the forecast of events in 1999q1 based on models estimated over 1985q1-1998q4 and ending with forecasts of events in 2001q1. The events of interest are described in Section 4. In the column headings the first letter denotes the direction of the forecast (U=up, D=down) and the second letter the direction of the outcome (U=up, D=down). For example, UU indicates an upward movement was correctly forecast. Hit rate is defined as $(DD + UU) / (UD + DD + DU + UU)$.

Table 4a
Point and Interval Forecasts of Inflation and Output Growth
(Four Quarterly Moving Averages)

Forecast Horizon	**Output Growth** Per cent, Per annum	**Inflation** Per cent, Per annum
2001q2	1.84 (1.02, 2.65)	1.80 (1.11, 2.49)
2001q3	1.30 (-0.13, 2.73)	1.61 (0.34, 2.88)
2001q4	1.28 (-0.62, 3.18)	1.37 (-0.36, 3.11)
2002q1	1.27 (-1.05, 3.51)	1.69 (-0.44, 3.82)
2002q2	1.42 (-1.10, 3.94)	2.08 (-0.31, 4.47)
2002q3	1.65 (-1.08, 4.37)	2.01 (-0.51, 4.52)
2002q4	1.89 (-1.04, 4.81)	1.92 (-0.69, 4.52)
2003q1	2.02 (-1.08, 5.12)	1.93 (-0.75, 4.60)

Notes: Forecasts are based on the model reported in Table 2, combined with an estimate of the oil price equation, (20). The figures in parenthesis are the lower and upper 95% confidence intervals. The four quarterly moving average output growth is defined as $400 \times \ln(GDP_{T+h}/GDP_{T+h-4})$, where GDP_T is the real Gross Domestic Product in 2001q1, which is computed from the forecasts of per capita output, y_{T+h}, assuming a population growth of 0.22% per annum. The four quarterly moving average inflation rate is defined as $400 \times (p_{T+h} - p_{T+h-4})$ where p_T is the natural logarithm of the retail price index in 2001q1.

Table 4b
Point and Interval Forecasts of Inflation and Output Growth
(Quarter on quarter changes)

Forecast Horizon	**Output Growth** Per cent, Per annum	**Inflation** Per cent, Per annum
2001q2	1.30 (-1.96, 4.55)	0.28 (-2.49, 3.06)
2001q3	1.16 (-2.61, 4.91)	2.22 (-2.05, 6.50)
2001q4	1.12 (-2.83, 5.07)	2.31 (-2.40, 7.04)
2002q1	1.53 (-2.59, 5.64)	1.93 (-3.01, 6.87)
2002q2	1.89 (-2.37, 6.15)	1.86 (-3.28, 7.00)
2002q3	2.05 (-2.36, 6.45)	1.91 (-3.39, 7.21)
2002q4	2.08 (-2.45, 6.61)	1.95 (-3.47, 7.37)
2003q1	2.08 (-2.56, 6.71)	1.97 (-3.54, 7.49)

Notes: See Table 4a. Output growth is defined as $400 \times \ln(GDP_{T+h}/GDP_{T+h-1})$, while inflation is defined as $400 \times (p_{T+h} - p_{T+h-1})$.

Table 5a
Probability Estimates of Single Events Involving Inflation

Forecast Horizon	Pr(Inf < 1.5%)		Pr(Inf < 2.5%)		Pr(Inf < 3.5%)		Pr(1.5% < Inf < 3.5%))	
	π	$\widetilde{\pi}$	π	$\widetilde{\pi}$	π	$\widetilde{\pi}$	π	$\widetilde{\pi}$
2001q2	0.206	0.135	0.978	0.920	1.000	1.000	0.795	0.865
2001q3	0.437	0.275	0.884	0.732	0.996	0.963	0.560	0.688
2001q4	0.541	0.364	0.849	0.682	0.974	0.899	0.433	0.535
2002q1	0.451	0.292	0.721	0.533	0.893	0.761	0.442	0.469
2002q2	0.367	0.244	0.597	0.441	0.801	0.652	0.434	0.408
2002q3	0.405	0.285	0.611	0.484	0.785	0.683	0.381	0.398
2002q4	0.424	0.315	0.625	0.514	0.792	0.705	0.368	0.390
2003q1	0.422	0.321	0.607	0.515	0.772	0.702	0.351	0.381

Notes: The probability estimates for inflation relate to the four quarterly moving average of inflation defined by $400 \times (p_{T+h} - p_{T+h-4})$, where p is the natural logarithm of the retail price index. The probability estimates (π and $\widetilde{\pi}$) are computed using the model reported in Table 2, where π is the "Profile Predictive Likelihood" that only takes account of future uncertainty, whereas $\widetilde{\pi}$ is the "Bootstrap Predictive Distribution" function and accounts for both future and parameter uncertainties. The computations are carried out using 2,000 replications. See the Appendix for computational details.

Table 5b
Probability Estimates Involving Output Growth and Inflation

Forecast Horizon	Pr(Recession)	Pr(output growth <1%)	Pr(1.5% < Inf < 3.5%, No Recession)	Pr(1.5% < Inf <3.5%, output growth < 1%)
	$\widetilde{\pi}$	$\widetilde{\pi}$	$\widetilde{\pi}$	$\widetilde{\pi}$
2001q2	0.000	0.040	0.865	0.832
2001q3	0.111	0.319	0.629	0.500
2001q4	0.084	0.343	0.499	0.381
2002q1	0.092	0.371	0.426	0.300
2002q2	0.092	0.312	0.373	0.278
2002q3	0.088	0.314	0.365	0.273
2002q4	0.090	0.305	0.358	0.272
2003q1	0.092	0.295	0.350	0.270

Notes: The probability estimates for output growth are computed from the forecasts of per capita output, assuming a population growth of 0.22% per annum. Recession is said to have occured when output growth (measured, quarter on quarter, by $400 \times \ln(GDP_{T+h}/GDP_{T+h-1})$) becomes negative in two consecutive quarters. Also see the notes to Tables 5a and 4a.

Figure 1a: Inflation (four-quarter moving average)

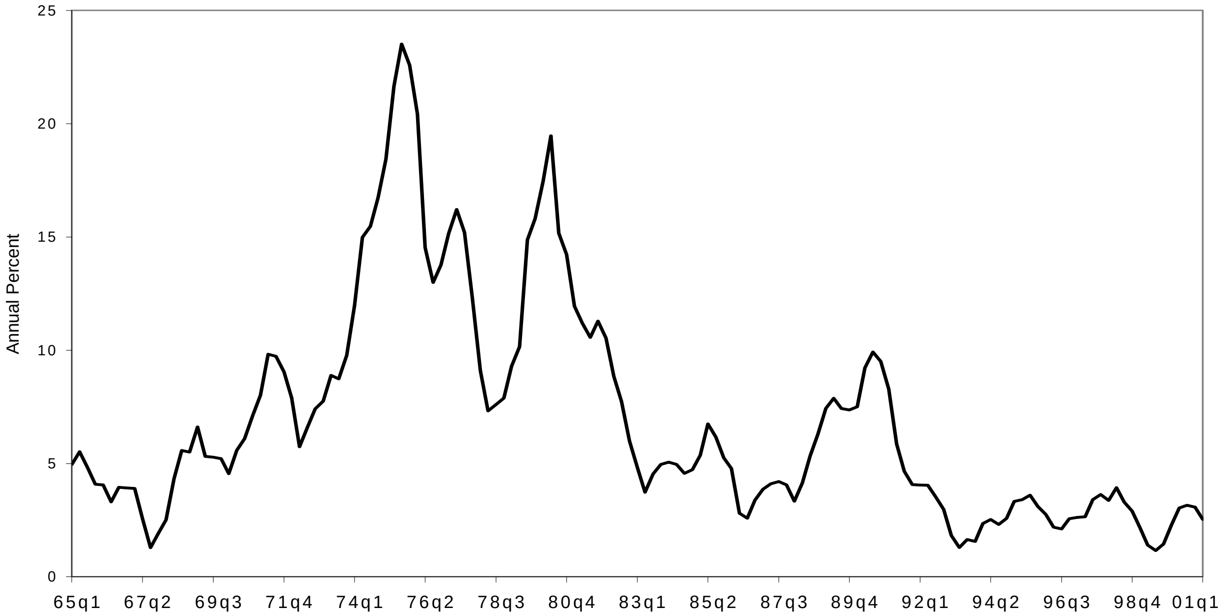


Figure 1b: Output Growth (four-quarter moving average)

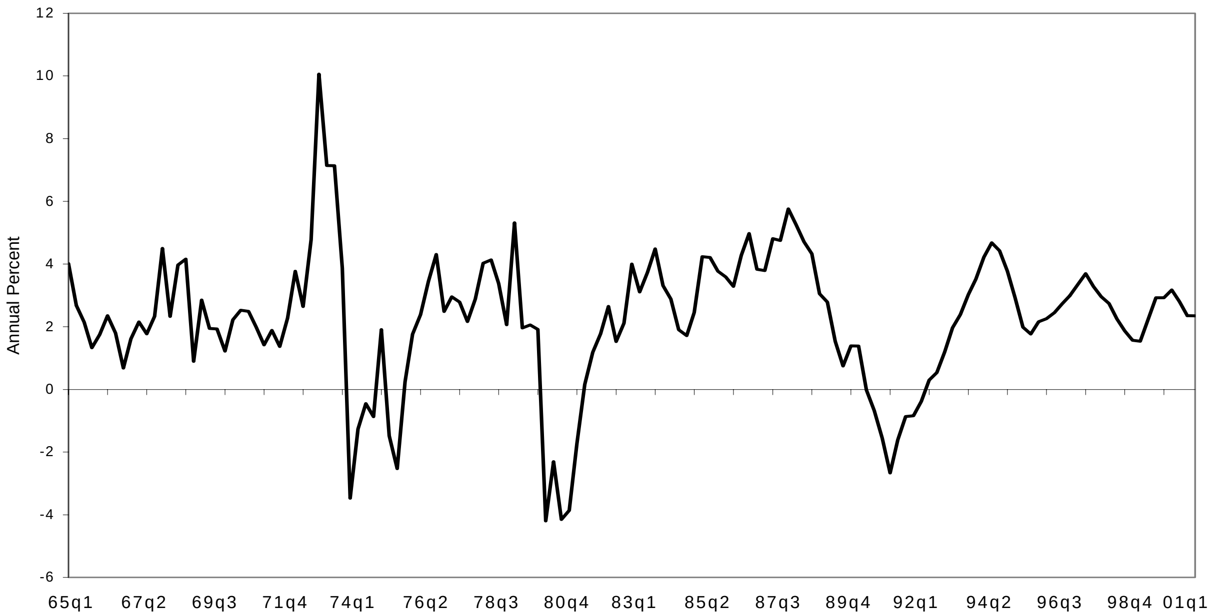
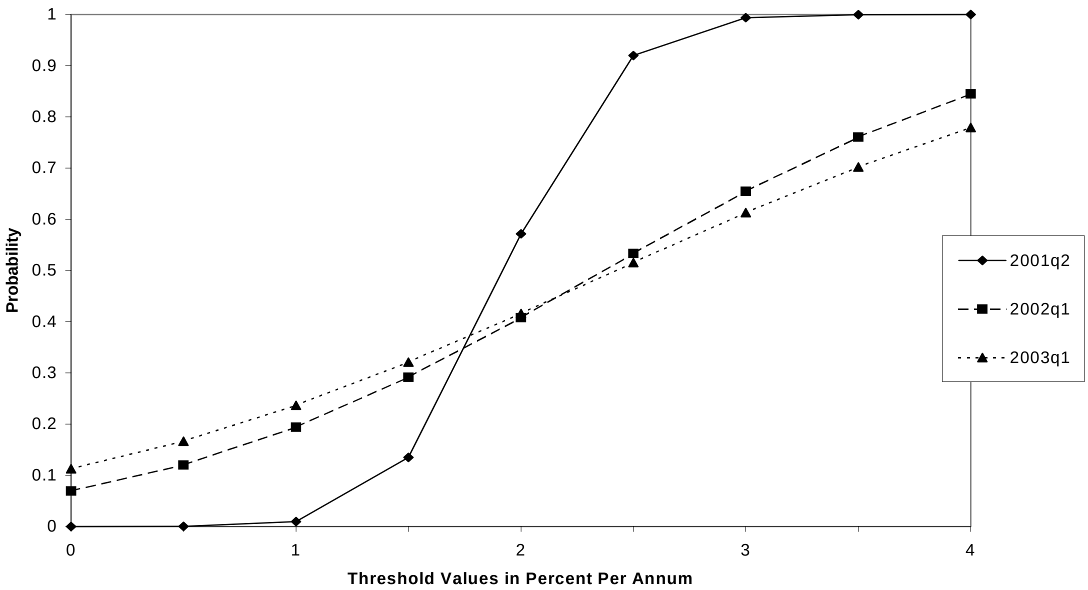
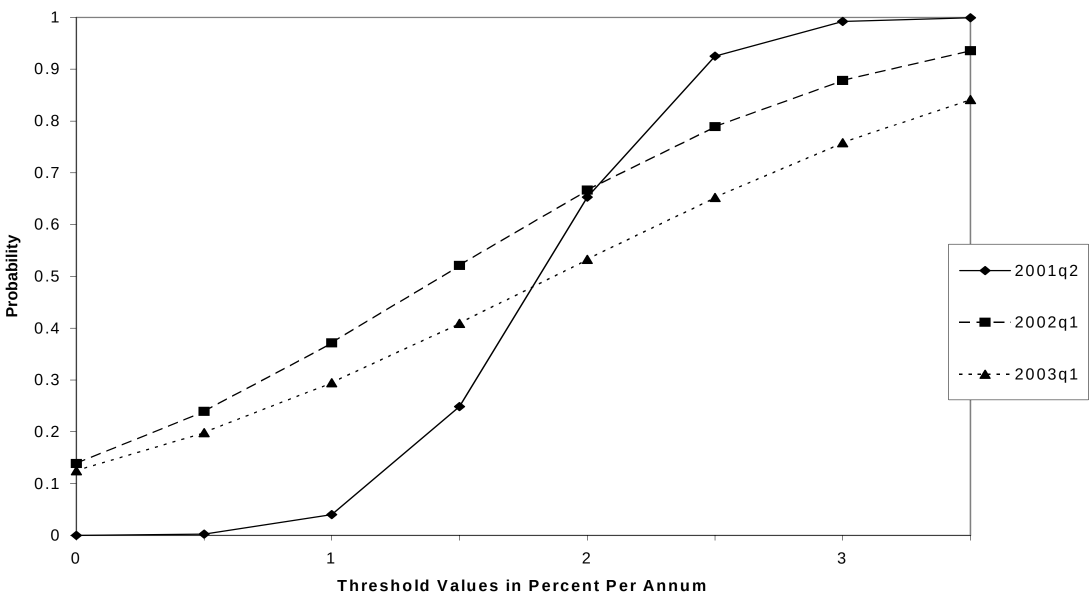


Figure 2a: Bootstrap Predictive Distribution Functions for Inflation Forecasts at Different Horizons†



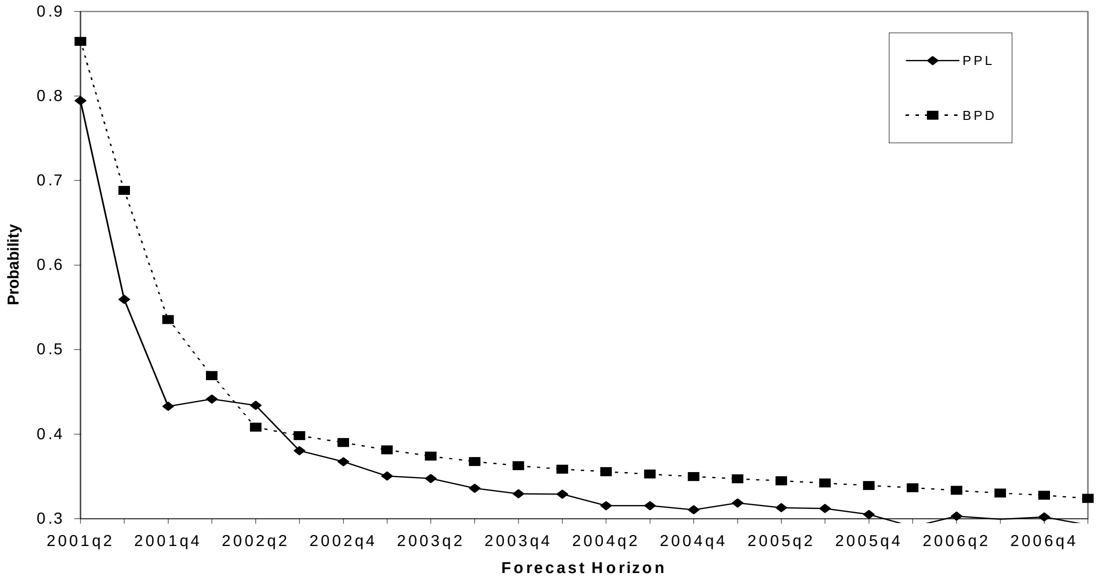
† The results take into account both future and parameter uncertainties.

Figure 2b: Bootstrap Predictive Distribution Functions for Output Growth Forecasts at Different Horizons†



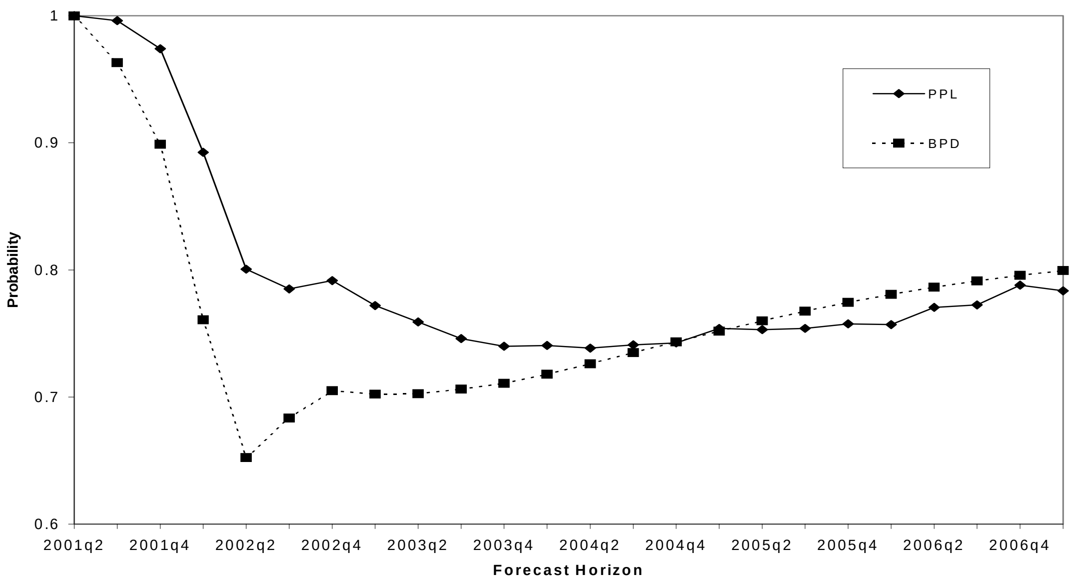
† See the note to Figure 2a.

Figure 3: Probability Estimates of Inflation Falling within the Target Range (A)†



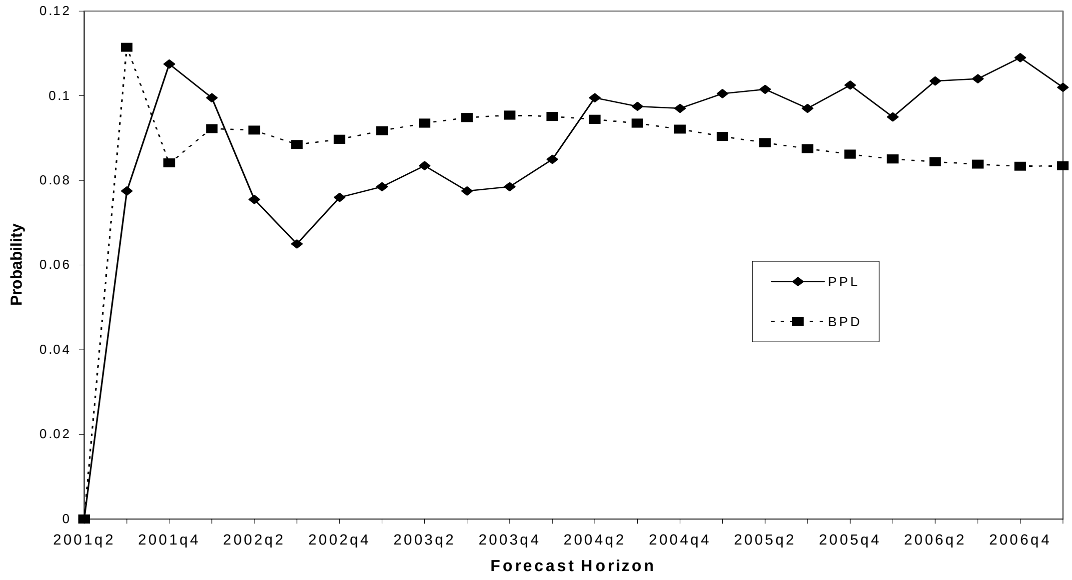
† The inflation event (A) refers to inflation falling within the range 1.5-3.5 per cent. PPL is the Profile Predictive Likelihood which takes into account future uncertainties. BPD is the Bootstrap Predictive Distribution function which takes into account both future *and* parameter uncertainties.

Figure 4: Probability Estimates of Inflation Falling Below the 3.5% Threshold†



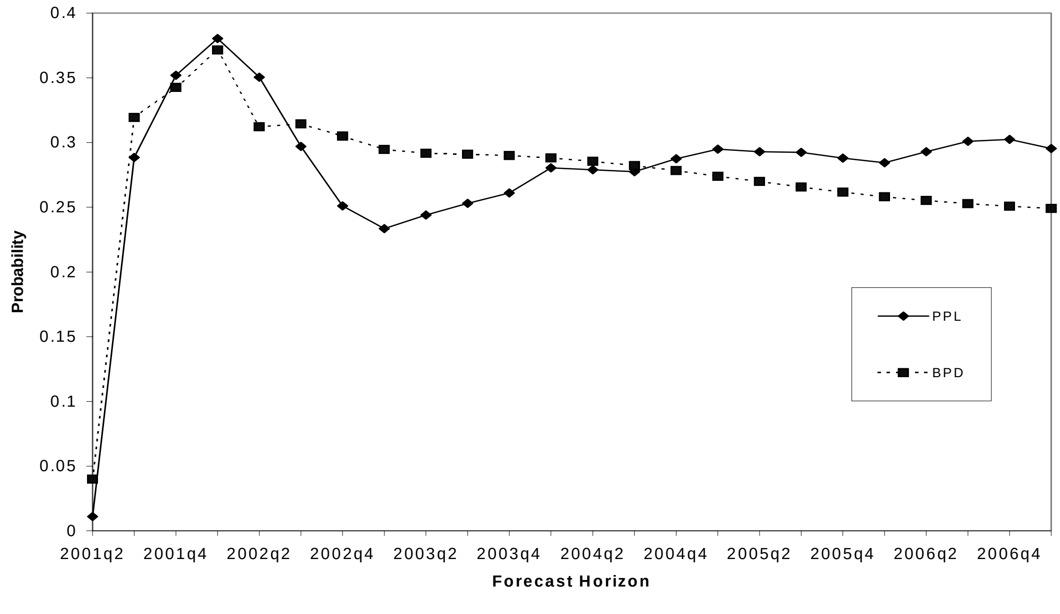
† See the notes to Figure 4a.

Figure 5: Probability Estimates of a Recession (B)†



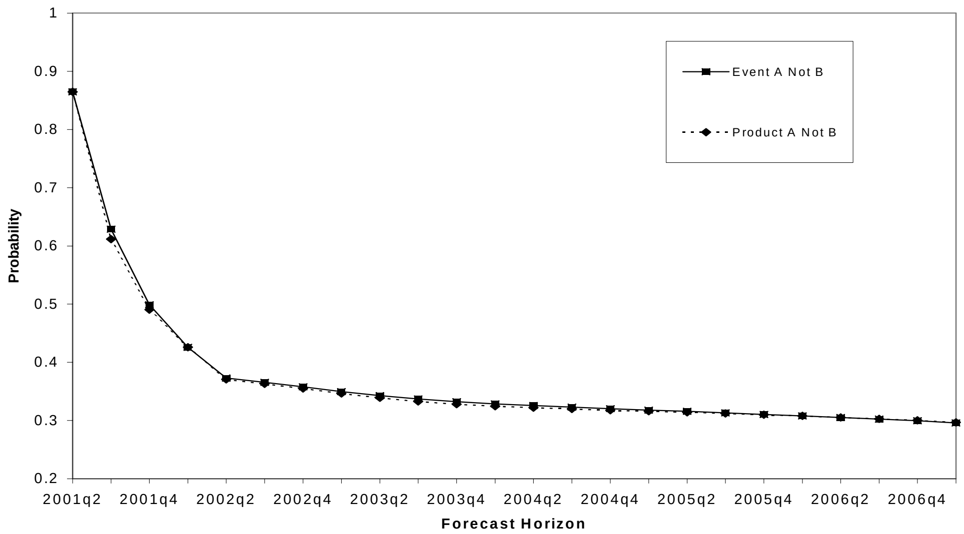
†A recession (event B) is defined as the occurrence of negative growth rates during two consecutive quarters. The PPL and BPD estimates are defined in the footnote to Figure 3.

Figure 6: Probability Estimates of a Low Growth Scenario (C)†



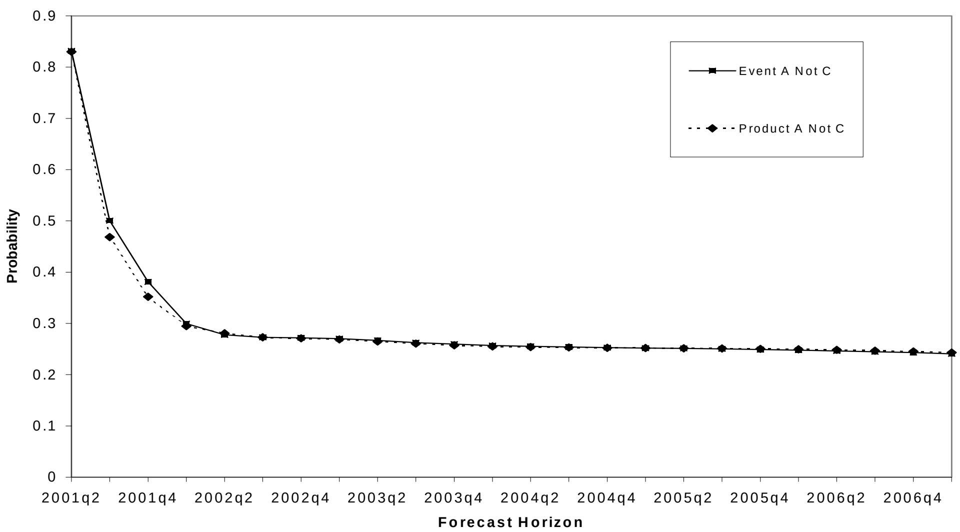
† The low growth event (C) is defined as four-quarter moving average output growth lying below 1 per cent. The PPL and BPD estimates are defined in the footnote to Figure 3.

Figure 7a: Probability Estimates of Achieving the Inflation Objective† without a Recession ($A \cap \overline{B}$)



† The difference between the product and joint event probabilities measures the degree of independence between events A and $Not\ B$. All probability estimates plotted are BPDs.

Figure 7b: Probability Estimates of Achieving the Inflation Objective Combined with Reasonable Growth Prospects ($A \cap \overline{C}$)



†The difference between the product and joint event probabilities measures the degree of independence between events A and $Not\ C$. All probability estimates plotted are BPDs. Also see the footnote to Figure 6.

References

[1] Barassi, M.R., Caporale, G.M. and S.G. Hall (2001), "Testing for Changes in the Long-Run Causal Structure of Cointegrated Vector Auto regressions," *Imperial College Discussion Paper* ICMS13.

[2] Berkowitz, J. (1999), "Evaluating the Forecasts of Risk Models," *Federal Reserve Bank Finance and Economic Discussion Paper Series No. 1999-11.*

[3] Blake, A.P. (1996), "Forecast Error Bounds By Stochastic Simulation," *National Institute Economic Review*, 72-79.

[4] Bjørnstad (1990), "Predictive Likelihood: A Review," *Statistical Science*, 5, 242-265.

[5] Britton, E., P. Fisher and J. Whitley (1998), "The Inflation Report Projections: Understanding the Fan Chart," *Bank of England Quarterly Bulletin*, 38, 30-37.

[6] Chatfield, C. (1995), "Model Uncertainty, Data Mining and Statistical Inference," *Journal of the Royal Statistical Society*, 158, 419–444.

[7] Clements, M.P. and D.F Hendry, (1999), "Forecasting Non-Stationary Economic Time Series," The MIT Press, Cambridge,Massachusettes.

[8] Clements, M.P. and D.F Hendry, (2002), "Explaining Forecast Failure in Macroeconomics," forthcoming in Clements, M.P. and D.F Hendry (eds.), *Companion to Economic Forecasting*, Blackwells, Oxford.

[9] Dawid, A. P. (1984), "Present Position and Potential Developments: Some Personal Views - Statistical Theory, The Prequential Approach," *Journal of Royal Statistical Society Series A,* 147, 278-292.

[10] Diebold, F.X., T.A. Gunther and A.S. Tay (1998), "Evaluating Density Forecasts, with Applications to Financial Risk Management," *International Economic Review*, 39, 863-884.

[11] Diebold, F.X., J. Hahn and A.S. Tay (1999), "Multivariate Density Forecast Evaluation and Calibration in Financial Risk Management: High Frequency Returns on Foreign Exchange," *Review of Economics and Statistics*, 81, 661-673.

[12] Draper, D. (1990), "Assessment and Propagation of Model Uncertainty," *Journal of Royal Statistical Society Series B,* 58, 45-97.

[13] Fair, R.C. (1980), "Estimating the Expected Predictive Accuracy of Econometric Models," *International Economic Review*, 21, 355-378.

[14] Fair, R.C. (1993), "Estimating Event Probabilities form Macroeconometric Models Using Stochastic Simulations," Chapter 3 in *Business Cycles, Indicators, and Forecasting,* eds. by J.H. Stock and M.W. Watson. National Bureau of Economic Research, Studies in Business Cycles Volume 28. University of Chicago Press: Chicago.

[15] Garratt, A., K. Lee, M.H. Pesaran, and Y. Shin (2000), "A Structural Cointegrating VAR Approach to Macroeconometric Modelling," Chapter ? in *Econometric Modelling: Techniques and Applications*, eds. by S. Holly and M. Weale. Cambridge University Press: Cambridge.

[16] Garratt, A., K. Lee, M.H. Pesaran, and Y. Shin (2001), "A Long Run Structural Macroeconometric Model of the UK," Revised version of University of Cambridge, *DAE Working Paper* No. 9812. Downloadable at http://www.econ.cam.ac.uk/faculty/pesaran.

[17] Garratt, A., K. Lee, M.H. Pesaran, and Y. Shin (2002), *A Structural Cointegrating Macroeconomic Model of the UK* (a monograph under preparation).

[18] Granger, C.W.J. and M.H. Pesaran (2000a), "A Decision Theoretic Approach to Forecast Evaluation," forthcoming in *Statistics and Finance: An Interface*, eds. by W.S. Chan, W.K. Li and H. Tong. Imperial College Press: London.

[19] Granger, C.W.J. and M.H. Pesaran (2000b), "Economic and Statistical Measures of Forecast Accuracy," *Journal of Forecasting*, 19, 537-560.

[20] Hall, P. (1992), *The Bootstrap and Edgeworth Expansion.* New York: Springer-Verlag.

[21] Harding, D. and A. Pagan (2000), "Dissecting The Cycle: A Methodological Investigation," *mimeo.*

[22] Harris, I.R. (1989), "Predictive Fit for Natural Exponential Functions," *Biometrika,* 76, 675-684.

[23] Levy, M.S. and S.K. Perng (1986), "An Optimal Prediction Function for the Normal Linear Model," *Journal of the American Statistical Association, 81*, 196-98

[24] Neave, H.R. and P.L. Worthington (1992), *Distribution-free Tests*, Routledge, London.

[25] Peel, D.A., and R.A. Nobay (1998), "Optimal Monetary Policy in A Model of Asymmetric Central Bank Preferences," *mimeo.*

[26] Pesaran, M.H. and B. Pesaran (1997), *Working with Microfit 4.0: An Interactive Introduction to Econometrics.* Oxford University Press: Oxford.

[27] Pesaran, M.H. and Y. Shin (2001), "Long-Run Structural Modelling," forthcoming in *Econometric Reviews.* http://www.econ.cam.ac.uk/faculty/pesaran/lrs.pdf.

[28] Pesaran, M.H., Y. Shin and R.J. Smith (2000), "Structural Analysis of Vector Error Correction Models with Exogenous I(1) Variables,"*Journal of Econometrics*, 97, 293-343.

[29] Pesaran, M.H. and S. Skouras (2001), "Decison-Based Methods For Forecast Evaluation," in *A Companion to Economic Forecasting*, eds. by M.P. Clements and D.F. Hendry. Blackwell: Oxford.

[30] Pesaran, M.H. and A. Timmermann (1992), "A Simple Nonparametric Test of Predictive Performance," *Journal of Business and Economic Statistics, 10, 461-465.*

[31] Poulizac, D., M. Weale and G. Young (1996),"The Performance of National Institute Economic Forecasts," *National Institute Economic Review*, 55-62.

[32] Rosenblatt, M. (1952), "Remarks on a Multivariate Transformation," *Annals of Mathematical Statistics*, 23, 470–472.

[33] Wallis, K.F. (1999), "Asymmetric Density Forecasts of Inflation and the Bank of England's Fan Chart," *National Institute Economic Review*, 106-112.

[34] West, K.D. (1996), "Asymptotic Inference About Predictive Ability," *Econometrica*, 64, 1067-1084.

[35] Yates, A., (1995), "On the Design of Inflation Targets," in *Targeting Inflation*, ed. by A.G. Haldane. Bank of England, 135-169.