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Speculating against an overconfident market*

Jordi Caballé** and József Sákovics***

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Abstract

We distinguish two components of self-confidence in a financial market: *private confidence* measures the self-confidence level of speculators, while *public confidence* measures the confidence level they attribute to their competitors. We then study how independent changes in these components affect the equilibrium trading strategies. We conduct the analysis in a financial market with imperfect competition where investors submit limit orders. We calculate the unique linear symmetric equilibrium as well as the major indicators of the market. In addition to providing a partial explanation for the excess volatility of asset prices as well as for trading volume unexplained by the arrival of new information, our model highlights the differences between the effects of public versus private confidence.

Keywords: overconfidence, financial markets, imperfect competition.

JEL classification codes: D84, G14, G12.

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1. Introduction

Investor sentiment, that is, changes in trading strategies not fully based on the arrival of hard information about the fundamentals, is known to affect the performance of financial markets (see, for example, Black, 1986). An important class of such not fully rational behavior is that of overconfident traders. This paper provides a formal analysis of a financial market with such speculators. Our main contribution to the literature is that we are able to separate the effects of "private overconfidence" –traders being overconfident–, from those of "public overconfidence" –the traders believing that the market (that is, the rest of the traders) is overconfident. This distinction is relevant since these two indicators –while positively correlated– need not vary together: just because there is the perception of a "bull" market, a given trader need not become more confident.

We have ample experimental and empirical evidence documenting that overconfidence pervades everyday life, and therefore its analysis is pertinent. For instance, Svenson (1981) considers overconfidence concerning the ability to drive a car and he estimates that over 80% believe they rank among the best 30%. Focusing on pre-arbitration negotiation, Neale and Bazerman (1983) find that 68% of negotiators believe the arbitrator will favor their offer. Another typical setting refers to the entrepreneurs' decision about entering into a market. Here, Camerer and Lovallo (1999) find that excess entry leads to negative industry profits in more than 70% of the experiments they perform, while Dunne et al. (1988) estimate that 60% of real businesses fail in the first five years.[1]

Most of the evidence in a competitive setting is consistent with a hypothesis slightly different from the presence of overconfident traders. For example, the entrepreneurs' decision to enter a market in the presence of excess entry may be rational even if they have the right amount of self-confidence, if they believe (in this case, mistakenly) that their competitors are overconfident. Note that the very evidence that there are many overconfident people substantiates a generalized belief that on average the market participants are overconfident. This belief needs not vary together with the actual realizations of the agents' self-confidence.

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[1] For further evidence, see also Alpert and Raiffa (1982), Griffin and Tversky (1992), Heath and Tversky (1991), Lichtenstein et al. (1982), Oskamp (1965), and Wagenaar and Keren (1986).

Recently, there has been a boom of papers that study the effects of overconfidence in a financial market. Most of these address the issue using as a benchmark the model of Kyle (1985) and Admati and Pfleiderer (1988) with informed traders and noise traders submitting market orders to a fair-pricing market maker. Benos (1998) assumes an extreme form of overconfidence, where traders believe that their signal is void of noise. He also shows that overconfident traders can survive in an evolutionary model.[2] Kyle and Wang (1997) do parametrize the level of overconfidence at the cost of considering two traders only. Odean (1998) assumes a single insider who is overconfident. He also analyzes the effects of overconfidence in competitive markets. All these authors obtain that trading volume, price volatility and price informativeness increase with overconfidence. Daniel et al. (1998) and Gervais and Odean (1997) present dynamic models with an endogenous level of trader self-confidence. They assume that the updating process of these beliefs is asymmetric: traders attribute good results to their own ability and their self-confidence rises, while they blame bad results on external factors and barely correct their self-confidence.

Our analysis differs from the previous ones in two main respects. First, we do not impose common knowledge of the investors' level of self-confidence and this allows us to differentiate between the effects of changes in the traders' beliefs about the level of self-confidence of the others and the effects of changes in the level of their own self-confidence. Therefore, our analysis can show how the perception about the self-confidence of the others affects both the behavior of investors and the corresponding equilibrium.[3]

Second, our analysis considers a richer space of strategies, since we allow price dependent demands. This modeling choice also simplifies the belief structure, since we need not be concerned about the market maker's beliefs, which are crucial in the market order setting. We thus study overconfidence in the framework of Kyle's (1989) model of an imperfectly competitive financial market with informed speculators submitting limit orders and noise traders. The non-competitiveness of the market has a special

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[2] Wang (1997) finds similar results in the context of overconfident fund managers.

[3] Note that Kyle and Wang (1997) also model a version of public and private confidence. Since, in their model the first order beliefs are common knowledge, they do this by varying the traders' beliefs about the precision of the other's signal. However, in the market order setting they use, this parameter does not affect the equilibrium strategies, since the traders cannot condition on the price. But this is the only avenue through which those beliefs could matter.

importance in our case, since one of the main concerns of a speculator that is affected by her overconfidence is how to conceal her information from her competitors. To obtain explicit solutions for the equilibrium strategies, we assume that traders are risk neutral, just as in Caballé (1992). In the absence of risk aversion the calculations become tractable and with imperfect competition we still have existence of equilibrium. A standard consequence of overconfidence is that traders bear more risk than they would if they were not overconfident (see Odean, 1998). Thus, in our analysis we assume away this effect and concentrate on additional patterns of the resulting behavior instead.

A straightforward way to model overconfidence could be to hypothesize biased prior beliefs.[4] However, in the case of informed speculation, there is a more innocuous assumption. Since investors receive private information before they act, overconfidence can be a result of overvaluing this information or the ability in interpreting it and, hence, there is no need to assume any biased predictor. This way, the bias appears endogenously in the posteriors. We thus model overconfidence via erroneous, optimistic beliefs about the precision of the private signal received. An overconfident trader receives an independent draw from the underlying distribution, just as a "normal" speculator would. However, he mistakenly thinks that the draw comes from a distribution with the same mean but with lower variance than the true distribution.

Now, the others do not necessarily know an individual's degree of self-confidence, that is, his belief about the precision of his signal. This leads us to build a model where the individual confidence levels are private information and traders have subjective beliefs about each other's level of self-confidence. We do not impose any correlation between a trader's confidence and the beliefs he entertains about the others'. In fact, we consider that one of the most important characteristics of an overconfident trader is that he thinks that he is not overconfident even when he believes that everybody else is. Of course, the belief hierarchies do not stop at this level. The traders also form beliefs about what the others think they think about the others' level of self-confidence, and so on. In order to maintain tractability, we cut short the description of this hierarchy by assuming that the second order beliefs (formed about the self-confidence level of the other traders) are common knowledge. The strategies used by

[4] This is the approach taken, for example, by De Long et al. (1990) and Palomino (1996).

the agents will thus depend on their own self-confidence and on their beliefs about the self-confidence of the others.

We compute the equilibrium in two steps. First we derive the hypothetical equilibrium that would result if the traders' beliefs about public confidence were correct. This would be the final result, if we followed the standard literature. Instead, we assume that each trader uses this equilibrium only to anticipate the strategies of the competitors he thinks that he is facing, and he plays the best response (based on his true beliefs) to these. For reasons of tractability, we assume that traders entertain degenerate beliefs, that is, they will assign probability one to a single type (which, in fact, may be non-existent). Moreover, in order to obtain closed form analytical solutions, we assume that the beliefs are symmetric across individuals.

After finding the equilibrium strategies of the model, we calculate several market indicators. We consider our results on price volatility and trading volume the most empirically relevant. Based on an empirical study of the orange juice futures market, Roll (1984) was the first to point out one of the most puzzling characteristics of financial markets: they exhibit patterns of trading volumes and price volatilities that cannot be fully explained by the information flow about the fundamental values of the assets. As expected, through higher sensitivity to private information and greater disparity of posterior beliefs, overconfidence does partially justify these phenomena. However, we show that price volatility may be decreasing in the level of self-confidence when traders are not very self-confident. On the other hand, the relationship between price volatility and public confidence is also non-monotonic. However, price volatility is decreasing in public confidence when traders believe that the market is very self-confident.

In principle, one could think that both private and public confidence have qualitatively the same effect on individual trading behavior. After all, the best response to aggressive trading is aggressive trading. We show, however, that the similarity is far from total. In fact, the speculators' trading intensity as a function of public confidence changes in a non-monotonic way: for low values of public confidence level, trading intensity decreases with public confidence. As a consequence, the weight of the two types of biased beliefs in the aggregate effect of overconfidence is not constant. Public confidence dominates at its extreme values, while private confidence is more relevant at intermediate values to explain the trading volume.

A further consequence of the distinct effects of private and public confidence is that they also differ from those of the standard treatment, where public and private confidence is assumed to coincide (since private confidence is assumed to be common knowledge). As a result, we identify three different definitions of self-confidence in a market, each of which leads to somewhat different comparative statics.

The rest of the paper continues as follows: We give a detailed description of our model in Section 2. In Section 3 we find the equilibrium. In Section 4 we discuss some properties of the market equilibrium. Finally, Section 5 concludes. The proofs of our results are presented in the Appendix.

2. The Model

As mentioned above, we use Kyle's (1989) model as our basic framework. However, for the sake of tractability we make the simplifying assumptions that: (i) there are no uninformed speculators, and (ii) traders are risk neutral. That is, we consider a market where a single risky asset is traded between two types of investors: liquidity (or noise) traders and informed speculators who submit limit orders (or demand schedules). The random payoff of the asset is denoted by $\tilde{v}$. The aggregate trading quantity of liquidity traders is described by the exogenous random variable $\tilde{z}$. There are *N* speculators, each of whom is in the possession of a private signal, i_n for speculator *n*, which is the realization of the random variable $\tilde{i}_n$.[5] We make the following standard assumption about the true distributions of the exogenous random variables of the model:

Assumption DA *The aggregate demand of noise traders is normally distributed with mean zero and variance* σ_z^2, *that is,* $\tilde{z} \sim N\left(0, \sigma_z^2\right)$. *Similarly,* $\tilde{v} \sim N(0, 1/\tau_v)$, *so that* τ_v *is the precision of* $\tilde{v}$. *The signal that speculators receive can be written as the sum of the random payoff and some noise:* $\tilde{i}_n = \tilde{v} + \tilde{e}_n$, *where* $\tilde{e}_n \sim N(0, 1/\tau_e)$, $n = 1, \ldots, N$. *That is, all the speculators receive information of the same quality. Finally,* $\tilde{v}$, $\tilde{z}$ *and the* $\tilde{e}_n$, $n = 1, \ldots, N$, *are distributed independently.*

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[5] Throughout this paper, we will omit the tilde when we refer to the realization of a random variable.

Except for the precision of $\tilde{e}_n$, $n = 1,\ldots,N$, all the above distributional characteristics are common knowledge. The vector of private precisions, on the other hand, is not only not common knowledge but it is not even known to any of the traders. Consequently, the beliefs of the agents about the reliability of their own and of each other's information are crucial in determining their behavior. First of all, agents entertain first-order beliefs, that is, beliefs about the precisions of the private signals in the economy. We do not impose common knowledge of all these first order beliefs. This leads us to the explicit consideration of the second-order beliefs: the beliefs the players hold about the others' first-order beliefs.[6] To keep the analysis tractable, we assume symmetry and that the second-order beliefs (and therefore all the remaining levels of the belief hierarchies) are common knowledge among the speculators. Given symmetry, the latter assumption simply amounts to saying that the general opinion about investor confidence in the market is common knowledge. We next describe our specific assumptions on the actual beliefs held by the players:

(i) *The first order beliefs of the investors about the precision of the random vector of signal noises* $\{\tilde{e}_n\}_{n=1,\ldots,N}$ *are described as follows: it is common knowledge that each trader believes that the precision of each of his competitors' noise is* τ_e. *Moreover, each trader puts probability one on his own precision being* $q\tau_e$ *(instead of the true precision* τ_e*), where* q, *the coefficient multiplying the true precision* τ_e, *is the private confidence level of the traders. A speculator is then said to be overconfident if and only if* $q > 1$. *Recall that* q *is not common knowledge.*

(ii) *The second order beliefs are the beliefs of each investor about the first order beliefs of the others. Each trader believes almost surely that the public confidence level is s, that is, she believes that her opponents think that the precision of the noise of their own signals is* $s\tau_e$. *Moreover, as a consequence of the assumed first order beliefs, each trader puts probability one on all the other*

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6 Note that the second order beliefs should be defined in general as a joint distribution over the vector of precisions and the first order beliefs. However, we assume that the vector of precisions and the first order beliefs are statistically independent, and so second order beliefs are fully described by the corresponding marginal distributions.

traders believing that the precision of her noise term is τ_e. *These second order beliefs are common knowledge. Note that a trader thinks that the rest are overconfident if and only if* $s > 1$.

Since second order beliefs are common knowledge, the *types* of investors are composed of the signals and the first and second order beliefs about the coefficient of the signal precision. Given the above assumptions, the profile of *relevant* investor types is $\{(i_n, c^1, c^2)\}_{n=1,\ldots,N} \in (\Re \times \{q,s\} \times \{s\})^N$. The following table summarizes the corresponding hierarchy of beliefs:

(INSERT TABLE 1)

Each investor *n* submits a demand schedule (or generalized limit order) X_n, which is a mapping from the asset price *p* into the number x_n of shares he desires to trade at this price. Let the set of such mappings be $\aleph$. The strategy of each trader *n*, φ_n, is a mapping from the three-dimensional space of types into demand schedules, $\varphi_n : \Re \times \{q,s\} \times \{s\} \to \aleph$. For simplicity, we assume that all traders use the same strategy, so we can drop the sub-index of φ.

The price of the asset is formed according to market clearing, that is,

$$\sum_{n=1}^{N} X_n(p) + z = 0 \ . \tag{1}$$

Note that equation (1) implicitly defines the equilibrium price *p* as a function of the profile of the speculators' strategies and noise trading. Thus, we can write $p = p(\varphi(i_1, c^1, c^2),\ldots,\varphi(i_N, c^1, c^2), z)$. Obviously, the quantity x_n of asset traded by speculator *n* is also a function of the speculators' strategies and noise trading, and so we write $x_n = x_n(\varphi(i_1, c^1, c^2),\ldots,\varphi(i_N, c^1, c^2), z)$.

The speculators are assumed to be risk neutral and to maximize expected profits. The random profits of speculator *n* for a given vector of private and public confidence are thus given by

$$\tilde{\pi}_n = \left(\tilde{v} - p\left(\varphi(\tilde{i}_1, c^1, c^2),\ldots,\varphi(\tilde{i}_N, c^1, c^2), \tilde{z}\right)\right) \cdot x_n\left(\varphi(\tilde{i}_1, c^1, c^2),\ldots,\varphi(\tilde{i}_N, c^1, c^2), \tilde{z}\right) .$$

We look for a symmetric Bayesian Nash equilibrium. As in Kyle (1989), for reasons of tractability we restrict attention to strategies that are linear in the signal, and to demand schedules that are linear in the price.[7] Then strategies take the following functional form:

$$\varphi\left(i_n, c^1, c^2\right) \equiv \alpha\left(c^1, c^2\right) + \beta\left(c^1, c^2\right) i_n - \gamma\left(c^1, c^2\right) p. \qquad (2)$$

Given our assumption that beliefs are point beliefs, the equilibrium has an unusual feature, which allows us to obtain the explicit solution, but nevertheless is perfectly compatible with Harsányi's original definition. Note that, by assumption, $c^1 \equiv q$ and thus there exists no type with $c^1 = s$. However, just as in the case of non-degenerate beliefs, we need to model the behavior of each type, which is attached positive probability in the *beliefs* of someone. The difference is that in our model, some types *only* exist in the beliefs of others. The operational advantage of our approach comes from the fact that we can easily separate the calculation of equilibrium strategies into two steps. In the first step, we calculate the equilibrium strategies of the *imaginary* types. A profile of these strategies could be interpreted as an equilibrium of the market that traders believe the rest of the traders believe to trade in. This equilibrium would also be the one obtained in our model under the standard assumption that the first order beliefs are common knowledge and equal to s. Therefore, the profile of strategies $\{\varphi_n\}_{n=1,\ldots,N}$ satisfies

$$E_n^s\left[\left(\tilde{v} - p\left(\varphi\left(\tilde{i}_1, s, s\right), \ldots, \varphi\left(\tilde{i}_n, s, s\right), \ldots, \varphi\left(\tilde{i}_N, s, s\right), \tilde{z}\right)\right) \cdot x_n\left(\varphi\left(\tilde{i}_1, s, s\right), \ldots, \varphi\left(\tilde{i}_n, s, s\right), \ldots, \varphi\left(\tilde{i}_N, s, s\right), \tilde{z}\right)\right] \geq$$

$$E_n^s\left[\left(\tilde{v} - p\left(\varphi\left(\tilde{i}_1, s, s\right), \ldots, \hat{\varphi}\left(\tilde{i}_n, s, s\right), \ldots, \varphi\left(\tilde{i}_N, s, s\right), \tilde{z}\right)\right) \cdot x_n\left(\varphi\left(\tilde{i}_1, s, s\right), \ldots, \hat{\varphi}\left(\tilde{i}_n, s, s\right), \ldots, \varphi\left(\tilde{i}_N, s, s\right), \tilde{z}\right)\right],$$

for all $n = 1,\ldots,N$ and for every alternative strategy $\hat{\varphi}$, where the operator $E_n^s(\cdot)$ is the expectation computed using the distributional assumption DA, except that the precision of the noise $\tilde{e}_n$ is $s\tau_e$, whereas the precision of $\tilde{e}_j$ (for all $j \neq n$) is τ_e.[8]

In our case, just as in the usual Bayesian equilibrium, players must play a best response to the strategies they attribute to their competitors, weighted by their beliefs. Since in our model these beliefs are concentrated, a speculator simply plays a best response against a market where the rest of the players play according to the strategies of the imaginary market. Therefore, the profile of strategies $\{\varphi_n\}_{n=1,\ldots,N}$ must also satisfy

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[7] Rochet and Vila (1994) analyze the existence of non-linear equilibria in the context of Kyle games.

[8] On the other hand, the standard expectation and variance operators $E(\cdot)$ and $Var(\cdot)$ will be computed

$$E_n^q\left[\left(\tilde{v}-p\left(\varphi\left(\tilde{i}_1,s,s,\right),\ldots,\varphi\left(\tilde{i}_n,q,s\right),\ldots,\varphi\left(\tilde{i}_N,s,s\right),\tilde{z}\right)\right)\cdot x_n\left(\varphi\left(\tilde{i}_1,s,s\right),\ldots,\varphi\left(\tilde{i}_n,q,s\right),\ldots,\varphi\left(\tilde{i}_N,s,s\right),\tilde{z}\right)\right]\geq$$

$$E_n^q\left[\left(\tilde{v}-p\left(\varphi\left(\tilde{i}_1,s,s,\right),\ldots,\hat{\varphi}\left(\tilde{i}_n,q,s\right),\ldots,\varphi\left(\tilde{i}_N,s,s\right),\tilde{z}\right)\right)\cdot x_n\left(\varphi\left(\tilde{i}_1,s,s\right),\ldots,\hat{\varphi}\left(\tilde{i}_n,q,s\right),\ldots,\varphi\left(\tilde{i}_N,s,s\right),\tilde{z}\right)\right],$$

for all n = 1,...,N and for every alternative strategy $\hat{\varphi}$, where the operator $E_n^q(\cdot)$ is the expectation computed using the distributional assumption DA, except that the precision of the noise $\tilde{e}_n$ is $q\tau_e$, whereas the precision of $\tilde{e}_j$ (for all $j \neq n$) is τ_e.

Finally, note that if $q \equiv s \neq 1$ we recover the equilibrium concept discussed in other papers on overconfidence, which assumed common knowledge of the private self-confidence levels. On the other hand, if $q \equiv s \equiv 1$ we recover the standard concept of noisy rational expectations equilibrium with imperfect competition introduced in Kyle (1989).

3. The Equilibrium

The following proposition provides the explicit equations describing the equilibrium defined in the previous section:

Proposition 3.1 *Let $N > 2$ and $s < \dfrac{2(N-1)}{N-2}$. Then the following is the unique symmetric linear equilibrium strategy:*

$$\varphi(i_n,q,s)=\beta(q,s)\,i_n-\gamma(q,s)\,p,$$

where

$$\beta(q,s)=\frac{(2\tau_v+sN\tau_e)q}{2(N-1)\tau_v+(q+s(N-2))N\tau_e}\sqrt{\frac{(N-1)(N-2)\sigma_z^2\tau_e}{(N+(N-2)(1-s))s}} \qquad (3)$$

and

$$\gamma(q,s)=\left(\frac{2\tau_v+Nq\tau_e}{Nq\tau_e}\right)\beta(q,s)\,. \qquad (4)$$

Note that the equilibrium strategy can be rewritten as

using the true distributions given in assumption DA.

$$\varphi(i_n, q, s) = \beta(q,s)\left(i_n - \left(\frac{2\tau_v + Nq\tau_e}{Nq\tau_e}\right)p\right). \tag{5}$$

That is, the traders' strategy can be described by the parameter of trading intensity, $\beta(q,s)$, and the parameter of *relative* price sensitivity, $\delta(q) = \frac{2\tau_v + Nq\tau_e}{Nq\tau_e}$. The first thing to note is that the relative weight a trader attaches to the information revealed through price as compared to weight he attaches to her private signal, $\delta(q)$, is independent of the level of public confidence. This is not surprising. Recall that each trader believes that she knows the self-confidence level of the other traders and therefore, in equilibrium, she knows their strategies. Consequently, independently of their level of public confidence, she can invert these to infer their information, the quality of which is common knowledge.

Let us now investigate how do the equilibrium coefficients change as a function of the different confidence levels. There are three relevant cases to study: a common change in both the public and the private confidence (corresponding to the standard models on overconfidence), or a variation in either one of these, holding the other constant.

Proposition 3.2

(i) $\frac{\partial \beta(q,s)}{\partial q} > 0$, $\frac{\partial \gamma(q,s)}{\partial q} > 0$ *and* $\frac{\partial \delta(q)}{\partial q} < 0$. *Moreover,*

$$\lim_{q\to 0} \beta(q,s) = 0, \quad \lim_{q\to\infty} \beta(q,s) = \lim_{q\to\infty} \gamma(q,s) = \overline{\gamma}(s) < \infty, \text{ and}$$

$$\lim_{q\to 0} \gamma(q,s) = \underline{\gamma}(s) > 0 \text{ for all } s > 0.$$

(ii) *There exist a value of the level of public confidence* $\overline{s} \in \left(0, \frac{2(N-1)}{N-2}\right)$ *such that* $\frac{\partial \beta(q,s)}{\partial s} > (<) 0$ *and* $\frac{\partial \gamma(q,s)}{\partial s} > (<) 0$ *for all* $s > (<) \overline{s}$. *Moreover,*

$$\lim_{s\to 0} \beta(q,s) = \lim_{s\to \frac{2(N-1)}{N-2}} \beta(q,s) = \lim_{s\to 0} \gamma(q,s) = \lim_{s\to \frac{2(N-1)}{N-2}} \gamma(q,s) = \infty \text{ for all } q > 0.$$

. (iii) *Let* $\hat{\beta}(x) = \beta(x,x)$ and $\hat{\gamma}(x) = \gamma(x,x)$ *for* $x \in \left(0, \frac{2(N-1)}{N-2}\right)$, *then* $\frac{d\hat{\beta}(x)}{dx} > 0$, *whereas there exists a value* $\bar{x} \in \left(0, \frac{2(N-1)}{N-2}\right)$ *such that* $\frac{d\hat{\gamma}(x)}{dx} > (<) 0$ *for all* $x > (<) \bar{x}$. *Moreover,*

$$\lim_{x \to 0} \hat{\gamma}(x) = \lim_{x \to \frac{2(N-1)}{N-2}} \hat{\gamma}(x) = \lim_{x \to \frac{2(N-1)}{N-2}} \hat{\beta}(x) = \infty \quad and \quad \lim_{x \to 0} \hat{\beta}(x) = 0 .$$

Figures 1 and 2 give an exhaustive qualitative picture of these comparative statics exercises for the functions $\beta(x,1)$, $\beta(1,x)$ and $\hat{\beta}(x)$, and $\gamma(x,1)$, $\gamma(1,x)$ and $\hat{\gamma}(x)$, respectively.

(INSERT FIGURES 1 AND 2)

Part (i) of the previous proposition tells us that, as the level of private confidence increases, the sensitivity of trades both to information and to price increases, while the relative sensitivity to price decreases. The fact that, as q increases, speculators put more weight (both in absolute and in relative terms) on their information is an obvious manifestation of their increased confidence. Note however, that the trading intensity does not increase without bound: even if a trader believes that she knows the value of the asset, she cuts back on her order in order not to make the price move too much against her. This is a standard consequence of the market power associated with imperfect competition. Finally, the higher absolute sensitivity to price is due to the fact that they want to increase market depth (which is proportional to γ) so as to reduce the amount of their information revealed through the price. Note that as q changes, our trader thinks that the strategies of the rest of the traders stay constant, since she believes that they depend on *s*, not on *q*. Therefore, since each speculator can influence the quality of the price as a signal of his private information, an investor who thinks that he owns better information increases his coefficient γ so as to reduce the informativeness of the price by making it less sensitive to private information. The limiting values of γ are finite at both extreme values of private confidence. As q increases without bound, the relative sensitivity to price, δ, decreases to one[9] and thus γ converges to the same value as β. On the other hand, as the level of private confidence vanishes, γ still remains positive. This means that the possession of the public signal makes it possible that the gains from trading against the noise traders compensate the losses from trading against the informed traders.

Turning to the level of public confidence considered in part (ii) of the previous proposition, observe that the trading intensity β is a U-shaped function of *s*. First, let us look at what happens when *s* is small. In this case, each trader thinks that the market is under-confident and, as we have seen above, this results in a high γ, making the market very deep. In this scenario our trader thinks that the information leakage through prices is very weak, so he will behave almost competitively, that is, choosing a high β. As we have seen before, this automatically implies that γ will be high also, since he will want to provide more depth to hide his trade. If *s* is large, our trader thinks that the market is behaving in an overconfident way, revealing a lot of information through the price. Consequently he believes that the price is very informative and, therefore, he will choose a high γ so as to capture the information embodied in the price. Again, since he believes the market to be deep, this leads to a high β as well.[10] Note also that an unbounded limit is reached for a finite level of public overconfidence. If it is common knowledge that all the speculators think that the others have a level of confidence equal to $\frac{2(N-1)}{N-2}$, prices become fully revealing.[11] That is why the above equilibrium only exists for $s < \frac{2N-2}{N-2}$. In the interim region he believes the market to be thin, so he restricts his sensitivity to information leading to lower values of both β and γ.

Finally, let us see how does the equilibrium strategy vary as we change public and private confidence at the same time (part (iii) of Proposition 3.2). This is the type of comparative statics that has been done in the literature. The behavior of $\hat{\beta}(x)$ is

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[9] Note that the relative price sensitivity is bounded from below by one, since in the limiting case of perfect information it is equal to one.

[10] Recall that δ is independent of *s*.

[11] Note that, substituting *s* in the formula for τ_n (see Corollary 4.7) by its upper bound, we obtain that the information revealed by prices is the total precision of private information as perceived by each speculator, $q\tau_e + (N-1)\tau_e$.

straightforward: the better information traders believe that they have at their disposal, the more they use it. In this case, however, the trading intensity is not bounded: since all traders trade more and more they need not worry as much about hiding their information. On the other hand, even if a trader believes that the market has as bad information as he does, as the precision of information vanishes, he still prefers not to trade.

The behavior of absolute price sensitivity $\hat{\gamma}(x)$ is similar for high values of confidence: it is monotonically increasing, without bound. However, unlike in the case where we only moved the private confidence level, for low values of confidence the investors increase their $\hat{\gamma}(x)$ as well. This has a straightforward explanation. As x decreases, the weight they put on their private information decreases to zero, while even with uninformative prices they want to sell when the price is positive and buy when the price is negative, and this leads to extreme price sensitivity.

4. Properties of the Equilibrium

Next, we calculate the major indicators of our market and pay special attention to price volatility and expected volume of trade. We start by computing price volatility:

Corollary 4.1 $V(q,s) \equiv Var(\tilde{p}) = \frac{1}{(\delta(q))^2}\left(\frac{1}{\tau_v} + \frac{1}{N\tau_e}\right) + \frac{\sigma_z^2}{N^2[\gamma(q,s)]^2}$.

The first term describes the amount of variance of the public and private signals that is transmitted to the price through the speculators' trading. It is increasing in q, since speculators put higher weight on their information as their confidence increases. At the same time it is independent of the level of public confidence, since that only affects the trading intensity, but not the relative weights on price and information. The second term describes the effect of noise trade. It is decreasing in q, since as the speculators' confidence increases, market depth also increases, making noise trade less influential in the determination of the market price. Thus, there is a trade-off, depending on the parameters. Price volatility may either increase or decrease with the speculators' level of private confidence. When the level of private confidence is initially very low, and thus the market is very thin, an increase in q is not translated into a much more aggressive trading by the informed speculators, since the price is going to reveal much of the perceived improvement in private precision. Therefore, in such a circumstance, the negative effect on volatility due to the increase in the depth will outweigh the positive effect due to the increase in the size of informed trading. However, when q, and therefore the depth of the market, is high, the converse argument applies and we obtain higher volatility as the level of private confidence increases. In fact, the following can be shown:

Corollary 4.2

(i) *There exists a threshold level of private confidence* q^* *such that* $\frac{\partial V(q,s)}{\partial q} > (<) 0$ *whenever* $q > (<) q^*$.

(ii) *In the limit, as N tends to infinity,* q^* *converges to* $\frac{s(2-s)}{2}$.

(iii) *There exists a threshold level of public confidence* $s^* \in \left(0, \frac{2(N-1)}{N-2}\right)$ *such that* $\frac{\partial V(q,s)}{\partial s} < (>) 0$ *whenever* $s > (<) s^*$.

(iv) Let $\hat{V}(x) \equiv V(x,x)$ *for* $x \in \left(0, \frac{2(N-1)}{N-2}\right)$, then $\frac{d\hat{V}(x)}{dx} > 0$.

Since $\frac{s(2-s)}{2} \le \frac{1}{2}$ for all $s > 0$, an implication of part (ii) of the previous corollary is that the volatility of prices is increasing in q in a large market whenever the speculators do not undervalue their information by more than a factor of 2. In fact, even if the previous explicit result is found only for large markets, we have not been able to find any example for which volatility drops as already overconfident speculators (that is, speculators having $q > 1$) become more overconfident.[12] As we have seen, public confidence only affects the behavior of volatility through the absolute price sensitivity

[12] We have simulated the threshold level q^* using an exhaustive grid for the values of the parameters appearing in the model. In particular, for the number of investors we have considered all the integer

and, thus, it is inversely U-shaped. As a consequence, the effects of public and private confidence on volatility appear as opposed.

The combined effect of moving the public and private confidence level together is markedly different. As shown in part (iv) of the previous proposition, volatility is strictly increasing in the level of confidence, just as it is predicted by the literature. Figure 3 depicts the three aforementioned volatility curves.

(INSERT FIGURE 3)

We can also evaluate the quantitative contribution of the different types of overconfidence on price volatility. To this end we consider three cases: (i) when $s=1$ and q varies, (ii) when $q=1$ and s varies, and (iii) when $q \equiv s$. The first case corresponds to a situation in which the investors believe that their competitors are rational, while each investor is possibly wrong about his own precision. The second case corresponds to a situation where every investor has correct beliefs about the precision of his own signal, whereas the perceived degree of confidence of the their competitors differs from the true one. Finally, the third case is homomorphic to a case where the levels of self-confidence are common knowledge. The following corollary provides the exact comparison:

Corollary 4.3

(i) *There exists a value* $x^* \in \left(0, \frac{2(N-1)}{N-2}\right)$ *such that*

$$V(x,1) > \hat{V}(x) > V(1,x) \text{ for all } x \in \left(x^*, \frac{2(N-1)}{N-2}\right).$$

(ii) *There exists a value* $x^{**} \in \left(0, \frac{2(N-1)}{N-2}\right)$ *such that*

$$V(x,1) > V(1,x) > \hat{V}(x) \text{ for all } x \in \left(0, x^{**}\right).$$

(iii) *There exists an open interval* (x_1, x_2) *with* $x_1 > x^*$ and $x_2 < x^{**}$ *such that*

$$V(1,x) > V(x,1) \text{ for all } x \in (x_1, x_2).$$

numbers from 3 to 100.

(iv) $\lim_{x\to 0} V(1,x) = \lim_{x\to \frac{2(N-1)}{N-2}} V(1,x) = \left(\frac{N\tau_e}{2\tau_v + N\tau_e}\right)^2 \left(\frac{1}{\tau_v} + \frac{1}{N\tau_e}\right).$

(v) $$\lim_{x\to 0} V(x,1) = \frac{\sigma_z^2}{N^2 [\underline{\gamma}(1)]^2} < \infty \qquad \text{and}$$

$$\lim_{x\to \infty} V(x,1) = \left(\frac{1}{\tau_v} + \frac{1}{\tau_e}\right) + \frac{\sigma_z^2}{N^2 [\overline{\gamma}(1)]^2} < \infty .$$

Note that parts (i), (ii) and (iii) of the previous corollary imply that private confidence contributes more to total price volatility for extreme values of *x*, whereas the contribution of public confidence is more relevant for intermediate values. Note also that part (v) implies that price volatility is always bounded.

Finally, we can calculate the expected trading volume of a speculator, which is defined as the mathematical expectation of the absolute value of his demand $E(|\tilde{x}_n|)$:

Corollary 4.4 *The expected volume traded by a speculator is*

$$Q(q,s) \equiv E(|\tilde{x}_n|) = \sqrt{\left(\frac{2}{\pi}\right)\left(\frac{(N-1)(N-2)[\beta(q,s)]^2}{N^2\tau_e} + \frac{\sigma_z^2}{N^2}\right)} .$$

In the expression for trading volume we can see how the volume is divided into transactions among speculators and transactions of speculators with noise traders. Since the expected trading volume is monotonically increasing in $\beta(q,s)$, the behavior of the expected volume as a function of both private and public confidence mimic the one of the function β. Therefore, we get the following corollary:

Corollary 4.5

(i) $\frac{\partial Q(q,s)}{\partial q} > 0$. *Moreover,* $\lim_{q\to 0} Q(q,s) = \sqrt{\left(\frac{2}{\pi}\right)\left(\frac{\sigma_z^2}{N^2}\right)}$ *and*

$$\lim_{q\to\infty} Q(q,s) = \sqrt{\left(\frac{2}{\pi}\right)\left(\frac{(N-1)(N-2)[\bar{\gamma}(s)]^2}{N^2\tau_e} + \frac{\sigma_z^2}{N^2}\right)} < \infty .$$

(ii) *There exist a value of the level of public confidence* $\bar{s} \in \left(0, \frac{2(N-1)}{N-2}\right)$ *such that* $\frac{\partial Q(q,s)}{\partial s} > (<) 0$ *whenever* $s > (<) \bar{s}$ *. Moreover,*

$$\lim_{s\to 0} Q(q,s) = \lim_{s\to\frac{2(N-1)}{N-2}} Q(q,s) = \infty, \text{ for all } q > 0 .$$

(iii) *Let* $\hat{Q}(x) \equiv Q(x,x)$ *for* $x \in \left(0, \frac{2(N-1)}{N-2}\right)$*, then* $\hat{Q}(x)$ is independent of both τ_v *and* τ_e *. Moreover,* $\frac{d\hat{Q}(x)}{dx} > 0$,

$$\lim_{s\to 0} \hat{Q}(x) = 0 \quad \text{and} \quad \lim_{s\to\frac{2(N-1)}{N-2}} \hat{Q}(x) = \infty .$$

Not surprisingly, since the volume of liquidity trading is exogenously given, all the increase in volume as private overconfidence increases, comes from transactions among the speculators, and it is explained by the greater dispersion in their posterior beliefs. Note however that private overconfidence can increase only up to a finite limit the expected trading volume.

The equilibrium volume's dependence on public confidence is characterized by the fact that, when speculators think that the rest are mistaken (s is far from one), they try to exploit this fact by trading harder. In this case, the expected trading volume can become arbitrarily large. As a result, high volumes cannot be explained exclusively by the presence of overconfident traders. It is also necessary that they believe that the public confidence is either very high or very low.

It is also worth noting, that, when second-order beliefs are correct ($s \equiv q$), the expected volume traded is independent of the qualities of public and private information (see part (iii)). This means, for example, that if the precision of public information τ_v is increased, the equilibrium price distribution will adjust in such a way that the amount of trading does not change. Finally, we can perform an exercise similar to the one contained in Corollary 4.3 to evaluate the contribution of the different types of self-confidence on the expected volume of trade.

Corollary 4.6

(i) *There exists a value* $x' \in \left(0, \frac{2(N-1)}{N-2}\right)$ *such that*

$$\hat{Q}(x) > Q(1,x) > Q(x,1) \text{ \textit{for all} } x \in \left(x', \frac{2(N-1)}{N-2}\right).$$

(ii) *There exists a value* $x'' \in \left(0, \frac{2(N-1)}{N-2}\right)$ *such that*

$$Q(1,x) > \hat{Q}(x) > Q(x,1) \text{ \textit{for all} } x \in (0, x'').$$

(iii) *There exists an open interval* (x_3, x_4) *with* $x_3 > x'$ and $x_4 < x''$ *such that*

$$Q(x,1) > Q(1,x) \text{ \textit{for all} } x \in (x_3, x_4).$$

The previous corollary reinforces our previous argument. We see that for extreme values of *x*, public confidence is more relevant to explain a high volume of trading since traders believe that their competitors have very erroneous beliefs, and they react to this by trading very intensively so as to exploit the others' misperception. However, for intermediate values of *x*, the contribution of private confidence to trading volume becomes more important.

We conclude this section with a brief discussion of two additional market indicators: price informativeness and expected profits of speculators. In order to check the informational efficiency of this market, let us define the information content of prices as $\tau_u(q,s) = \left[Var(\tilde{v}|\tilde{p})\right]^{-1} - \left[Var(\tilde{v})\right]^{-1}$, that is, the increase in precision of the beliefs of an outside observer (who knows both the true precision of private signals and the equilibrium strategies) about the realization of the random return, due to the observation of the equilibrium price. Similarly, $\tau_n(q,s) = \left[Var_n^q(\tilde{v}|\tilde{p}, \tilde{i}_n)\right]^{-1} - \left[Var(\tilde{v})\right]^{-1}$ is the increase in precision of the beliefs about the realization of the random return that a speculator expects (wrongly, unless $q = s = 1$) from the observation of the equilibrium price and his private signal.[13]

Corollary 4.7

(i) *The information content of prices for an observer who knows both the true precision of the private signals and the beliefs of the players is*

$$\tau_u(q,s) = \frac{N^2[\beta(q,s)]^2\tau_e}{N^2[\beta(q,s)]^2 + \sigma_z^2\tau_e} \;,$$

which is increasing in the level of private confidence and it is non-monotone (U-shaped) in the level of public confidence.

(ii) *Let* $\hat{\tau}_u(x) \equiv \tau_u(x,x)$*, then* $\hat{\tau}_u(x)$ *is strictly increasing.*

(iii) *The increase in the precision of his estimate of the random return perceived by a speculator upon observing the price and his private signal is*

$$\tau_n(q,s) = \left(q + \frac{(N-2)s}{2}\right)\tau_e \;,$$

which is increasing in both parameters.

(iv) *Let* $\hat{\tau}_n(x) \equiv \tau_n(x,x)$*, then* $\hat{\tau}_n(x)$ *is strictly increasing.*

As the level of private confidence increases, speculators will overreact to their information. As a consequence, overconfident speculators actually reveal more of their private information than it would be optimal for them and thus make prices more informative. As we have seen before, when the level of public confidence is either low or high, speculators expect a deep market and therefore they react strongly to their private information. Of course, the speculators perceive it differently, since when *s* is low, they expect the rest to put small weight on their information, making prices little revealing.

Turning to profits, note first that in the current scenario there are two kinds to consider: the profits expected by an overconfident – and therefore mistaken –

[13] Obviously, the variance operator $Var_n^q(\cdot)$ is defined as $Var_n^q(\tilde{y}) = E_n^q\left(\tilde{y} - E_n^q(\tilde{y})\right)^2$. Note also that $Var_n^q(\tilde{v}) \equiv Var(\tilde{v})$.

speculator, and the average profits such a speculator actually makes in this market. The first quantity, despite the complexity of its calculation, does not provide any additional insight so we restrict our attention to the second one.

Corollary 4.8

(i) *The average profit of speculators is* $\Pi(q,s) = E(\tilde{\pi}_n) = \dfrac{\sigma_z^2}{N^2\gamma(q,s)}$, *which is decreasing in the level of private confidence and it is non-monotone (inverted U-shaped) in the level of public confidence.*

(ii) *Let* $\hat{\Pi}(x) \equiv \Pi(x,x)$, *then* $\hat{\Pi}(x)$ *is inverted U-shaped.*

Since in a model such as ours, the profits of informed speculators are financed by the losses of noise traders, speculators' profits are increasing in the amount (variance) of liquidity trading and decreasing in the number of speculators. Note that the coefficient γ is proportional to the depth of the market, and liquidity traders are better off trading in deep markets. Therefore, as it is easily seen through γ, private under-confidence in the market increases, while private overconfidence decreases, average profits of speculators. The first of these results is at first blush surprising. How can sub-optimal behavior increase expected profits? Does this mean that in a standard context, speculators could gain by under-reacting to their information? The answer is, obviously, not. What happens in our model is that, when speculators are under-confident, noise traders are exploited more and this surplus is distributed evenly among speculators. However, in the standard case with common knowledge of the first order beliefs, this would not be an equilibrium, since by reacting more to his information, a speculator would decrease the surplus only by a little, while he could change its distribution in his favor. In our case, if we had only one overconfident trader, by the same argument, he would lose on the modified sharing of the surplus more than he would gain by increasing the losses of liquidity traders. Finally, observe that, agreeing with intuition, average profits plunge when the level of public confidence takes extreme values and they peak when the speculators' perception of the market is close to the truth.

5. Conclusion

In this paper we have presented a model of trading where the underlying beliefs are not only different but also not common knowledge. This environment seems to be most adequate to analyze the consequences of speculator overconfidence. We believe that in addition to its intrinsic theoretical interest, it should be considered as another step towards a better description and understanding of human behavior in the economic sphere.

Our benchmark results generalize to the context of limit-order markets the consensus of the literature: when over/under confidence is common knowledge, it increases/decreases both price volatility and trading volume. At the same time, we show that if the traders' perception of the level of confidence in the market and their own level of confidence may vary independently, these results become significantly different.

The first interesting conclusion we can draw is that generalized private overconfidence of the speculators can increase only up to a finite limit both trading volume and the depth of the market. That is, to explain very high volume or liquidity with overconfidence it is necessary that public overconfidence be high. In addition, public under-confidence also leads to high volume. Thus, to explain very low volume with public under-confidence it is necessary that private confidence be very low.

The effects of overconfidence on the volatility of prices are less straightforward. In our model, the variance of price is decreasing in the level of private confidence up to a threshold value, and from then on it is increasing in it. In large markets the value of private confidence at which price volatility reaches its minimum is strictly below one though, and thus we can say that private overconfidence does increase the variance of price. On the other hand, public confidence has the opposite effect on volatility: for low values of public confidence, volatility is increasing, while for high values it is decreasing.

We have thus arrived at the – testable – conclusion, that the effects of private versus public overconfidence are markedly different, which should make a more in depth empirical analysis possible. We leave that for the experts.

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Appendix

Proof of Proposition 3.1

We prove Proposition 3.1 in two steps. First, we compute the equilibrium strategy of the type (i_n, s, s) having the functional form given in (2). Second, we compute the strategy actually played by the existing type (i_n, q, s) as a best response to the strategies played by the non-existing types $\{(i_j, s, s)\}_{j \neq n}$.

Claim 1 *Let N > 2 and* $s < \dfrac{2(N-1)}{N-2}$. *There exists a unique symmetric linear equilibrium where type* (i_n, *s, s*) *follows the strategy with the functional form given in equation* (2). *The equilibrium values of the parameters defining the strategies are*

$$\alpha(s,s) = 0\,, \tag{A.1}$$

$$\beta(s,s) = \sqrt{\frac{s(N-2)\sigma_z^2\tau_e}{(N-1)\big(N+(N-2)(1-s)\big)}}\ , \tag{A.2}$$

$$\gamma(s,s) = \left(\frac{2\tau_v + Ns\tau_e}{Ns\tau_e}\right)\beta(s,s)\ . \tag{A.3}$$

Proof: According to the conjectured linearity of the demand schedules, the market clearing condition, $\sum_{n=1}^{N} X_n(p) + z = 0$, takes the form of

$$N\alpha(s,s) + \beta(s,s)\sum_{n=1}^{N} i_n - N\gamma(s,s)p + z = 0\ .$$

This implies that the random equilibrium price satisfies

$$\tilde{p} = \frac{N\alpha(s,s) + \beta(s,s)\sum_{n=1}^{N}\tilde{i}_n + \tilde{z}}{N\gamma(s,s)}. \tag{A.4}$$

Since each informed trader n considers the others' strategies as given and consistent with equation (2), he is facing the following residual demand:

$$p = \frac{(N-1)\alpha(s,s) + \beta(s,s)\sum_{j\neq n} i_j + z}{(N-1)\gamma(s,s)} + \frac{x_n}{(N-1)\gamma(s,s)}\ . \tag{A.5}$$

Therefore, speculators solve the following maximization problem (recall that E_n^c

denotes the expectation taken according to the distributional assumption DA, except that the precision of the noise $\tilde{e}_n$ is $c\tau_e$, whereas the precision of $\tilde{e}_j$ is assumed to be τ_e, for all $j \neq n$):

$$\underset{x_n \in \Re}{Max}\, E_n^s\left((\tilde{v}-\tilde{p})x_n \middle| i_n, p\right) = \underset{x_n \in \Re}{Max}\, E_n^s\left[\left(\tilde{v} - \frac{(N-1)\alpha(s,s) + \sum_{j\neq n} \tilde{i}_j + \tilde{z}}{(N-1)\gamma(s,s)} - \frac{x_n}{(N-1)\gamma(s,s)}\right)x_n \middle| i_n, p\right].$$

The first order condition for this problem is

$$E_n^s(\tilde{v}|i_n, p) - \frac{2x_n}{(N-1)\gamma(s,s)} - E_n^s\left(\frac{(N-1)\alpha(s,s) + \beta(s,s)\sum_{j\neq n}\tilde{i}_j + \tilde{z}}{(N-1)\gamma(s,s)} \middle| i_n, p\right) = 0. \quad \text{(A.6)}$$

Because of (A.5), (A.6) may be written as

$$E_n^s\left(\tilde{v}|i_n, p\right) - \frac{x_n}{(N-1)\gamma(s,s)} - p = 0,$$

and this implies that

$$x_n = (N-1)\gamma(s,s)[E_n^s(\tilde{v}|i_n, p) - p] = X_n(p). \quad \text{(A.7)}$$

The second order sufficient condition for the maximization problem is $\frac{-2}{(N-1)\gamma(s,s)} < 0,$ that is, $\gamma(s,s)$ must be strictly positive.

Next, note that to observe the random variables $\tilde{p}$ and $\tilde{i}_n$ is informationally equivalent to observing $\tilde{i}_n$ and the following random variable:

$$\frac{N\gamma(s,s)\,\tilde{p} - N\alpha(s,s) - \beta(s,s)\,\tilde{i}_n}{(N-1)\beta(s,s)} = \tilde{v} + \tilde{y},$$

where the random variable

$$\tilde{y} = \frac{\sum_{j\neq n}\tilde{e}_j}{N-1} + \frac{\tilde{z}}{(N-1)\beta(s,s)}, \quad \text{(A.8)}$$

is clearly independent of $\tilde{i}_n$. The precision of $\tilde{y}$ is

$$\tau_y = \frac{(N-1)^2[\beta(s,s)]^2\tau_e}{(N-1)[\beta(s,s)]^2 + \sigma_z^2\tau_e}, \quad \text{(A.9)}$$

since trader n correctly believes that the noises of the signals of the other traders have precision τ_e. Applying the projection theorem for normally distributed random variables, we can compute the following expectation:

$$E_n^s\left(\tilde{v}\middle|i_n, p\right) = \frac{s\tau_e i_n + \tau_y\left(\frac{N\gamma(s,s)p - N\alpha(s,s) - \beta(s,s)i_n}{(N-1)\beta(s,s)}\right)}{\tau_v + s\tau_e + \tau_y}, \tag{A.10}$$

since each trader of type (i_n, s, s) believes that the noise of her own signal has precision $s\tau_e$. Substituting (A.10) in (A.7), making the conjecture that

$$X_n(p) = \alpha(s,s) + \beta(s,s)\, i_n - \gamma(s,s)\; p\,,$$

and equating coefficients, we obtain the following system of equations:

$$\alpha(s,s) = (N-1)\gamma(s,s)\left[\frac{-\tau_y\left(\frac{N\alpha(s,s)_s}{(N-1)\beta(s,s)}\right)}{\tau_v + s\tau_e + \tau_y}\right], \tag{A.11}$$

$$\beta(s,s) = (N-1)\gamma(s,s)\left[\frac{s\tau_e - \frac{\tau_y}{(N-1)}}{\tau_v + s\tau_e + \tau_y}\right], \tag{A.12}$$

$$\gamma(s,s) = (N-1)\gamma(s,s)\left[1 - \frac{\tau_y\left(\frac{N\gamma(s,s)}{(N-1)\beta(s,s)}\right)}{\tau_v + s\tau_e + \tau_y}\right]. \tag{A.13}$$

The solution for $\alpha(s,s)$ is clearly zero. We then substitute (A.9) in (A.12) and (A.13). Under the assumptions of this Claim, we can find the unique solution of this two-equation system which satisfies the second order condition, yielding the values of $\beta(s,s)$ and $\gamma(s,s)$ given in the statement of the Claim. *Q.E.D.*

Claim 2 *Let* $N > 2$ *and* $s < \frac{2(N-1)}{N-2}$. *There exists a unique symmetric linear equilibrium where the type* (i_n, q, s) *follows the strategy with the functional form given in equation* (2). *The equilibrium values of the parameters defining the strategies are given in the statement of Proposition 3.1.*

Proof: Since investor n of type (i_n, q, s) believes that her opponents have a level of confidence equal to s, we must compute the best response to the strategies obtained in Claim 1. The residual demand as perceived by individual n is (A.5), where

$\alpha(s,s)$, $\beta(s,s)$ and $\gamma(s,s)$ are given by Claim 1. As in the proof of Claim 1, and after changing the expectation operator, we arrive at the following optimal quantity x_n demanded by trader *n*:

$$x_n = (N-1)\gamma_s [E_n^q(\tilde{v} | i_n, p) - p] = X_n(p). \tag{A.14}$$

The equilibrium price, as perceived by a trader *n* of type (i_n, *q*, *s*), is

$$\tilde{p} = \frac{\alpha(q,s) + \beta(s,s)\sum_{j\neq n}\tilde{i}_j + \beta(q,s)\tilde{i}_n + \tilde{z}}{(N-1)\gamma(s,s) + \gamma(q,s)},$$

where we have used the fact that $\alpha(s,s) = 0$ as follows from Claim 1. Thus, we see that to observe $\tilde{p}$ and $\tilde{i}_n$ is observationally equivalent to observing $\tilde{i}_n$ and the following random variable:

$$\frac{((n-1)\gamma(s,s) + \gamma(q,s))\tilde{p} - \alpha(q,s) - \beta(q,s)\tilde{i}_n}{(N-1)\beta(s,s)} = \tilde{v} + \tilde{y},$$

where the random variable $\tilde{y}$ is defined in (A.8). Following again the steps of the proof of Claim 1, we arrive at the formula for the conditional expectation,

$$E_n^q(\tilde{v}|i_n, p) = \frac{q\tau_e i_n + \tau_y\left(\frac{((N-1)\gamma(s,s) + \gamma(q,s))p - \alpha(q,s) - \beta(q,s)i_n}{(N-1)\beta(s,s)}\right)}{\tau_v + q\tau_e + \tau_y}, \tag{A.15}$$

where τ_y is given in (A.9), since speculator *n* correctly believes that the precision of the others' noise terms is τ_e.

Substituting (A.15) in (A.14), making the conjecture that

$$X_n(\tilde{p}) = \alpha(q,s) + \beta(q,s)\, i_n - \gamma(q,s)\, p,$$

and equating coefficients, we get the following system of equations:

$$\alpha(q,s) = (N-1)\gamma(s,s)\left[\frac{\tau_y\left(\frac{\alpha(q,s)}{(N-1)\beta(s,s)}\right)}{\tau_v + q\tau_e + \tau_y}\right], \tag{A.16}$$

$$\beta(q,s) = (N-1)\gamma(s,s)\left[\frac{q\tau_e - \frac{\tau_y\beta(q,s)}{(N-1)\beta(s,s)}}{\tau_v + q\tau_e + \tau_y}\right], \tag{A.17}$$

$$\gamma(q,s)=(N-1)\gamma(s,s)\left[1-\frac{\tau_y\left(\dfrac{(N-1)\gamma(s,s)+\gamma(q,s)}{(N-1)\beta(s,s)}\right)}{\tau_v+q\tau_e+\tau_y}\right]. \qquad \text{(A.18)}$$

From (A.16), $\alpha(q,s)$ is clearly equal to zero. Substituting in the values of $\beta(s,s)$, $\gamma(s,s)$ and τ_y found in Claim 1, and after some tedious algebra to solve the system (A.17)-(A.18), we obtain the equilibrium values of $\beta(q,s)$ and $\gamma(q,s)$ given in the statement of Proposition 3.1. *Q.E.D.*

Proof of Proposition 3.2

The sign of the derivative of $\delta(q)$ is straightforward.

Concerning the properties of $\beta(q,s)$ in parts (i) and (ii), we just have to notice that the term $\frac{(2\tau_v+sN\tau_e)q}{2(N-1)\tau_v+(q+s(N-2))N\tau_e}$ appearing in the function $\beta(q,s)$ (see (3)) is strictly increasing in both *q* and *s*, and it converges to a strictly positive limit as *s* tends to zero for $q>0$. On the other hand, the term $\sqrt{\frac{(N-1)(N-2)\sigma_z^2\tau_e}{(N+(N-2)(1-s))s}}$ is a function of *s* that has a unique critical point and it tends to infinity as *s* tends to either zero or to $\frac{2(N-1)}{N-2}$. It is also obvious that $\lim_{q\to 0}\beta(q,s)=0$,

$$\lim_{q\to\infty}\beta(q,s)=\frac{2\tau_v+sN\tau_e}{N\tau_e}\sqrt{\frac{(N-1)(N-2)\sigma_z^2\tau_e}{(N+(N-2)(1-s))s}}\equiv\bar{\gamma}(s)<\infty,$$

and $\lim_{s\to 0}\beta(q,s)=\lim_{s\to\frac{2(N-1)}{N-2}}\beta(q,s)=\infty$.

For the properties of $\gamma(q,s)$ in parts (i) and (ii), we see from (4) that the behavior of $\gamma(q,s)$ as a function of *s* replicates the one of $\beta(q,s)$ since the term $\frac{2\tau_v+Nq\tau_e}{Nq\tau_e}$ is independent of *s*, and it converges to 1 as *q* tends to infinity. The sign of the partial derivative of $\gamma(q,s)$ with respect to *q* comes from straightforward differentiation. We also have that $\lim_{q\to\infty}\gamma(q,s)=\bar{\gamma}(s)<\infty$ and

$$\lim_{q\to 0}\gamma(q,s)=\left(\frac{2\tau_v}{N\tau_e}\right)\frac{(2\tau_v+sN\tau_e)}{2(N-1)\tau_v+(q+s(N-2))N\tau_e}\sqrt{\frac{(N-1)(N-2)\sigma_z^2\tau_e}{(N+(N-2)(1-s))s}}\equiv\underline{\gamma}(s)>0,$$

Finally, for the properties of $\hat{\beta}(x)$ and $\hat{\gamma}(x)$, we can evaluate

$$\frac{d\hat{\beta}(x)}{dx}=\sqrt{\frac{(N-1)(N-2)\tau_e\sigma_z^2}{x[2(N-1)-x(N-2)]^3}}\ ,$$

which is strictly positive for all $x\in\left(0,\frac{2(N-1)}{N-2}\right)$ and $N>2$. After some algebra, it can be checked that $\hat{\gamma}(x)$ has a unique critical point at

$$\bar{x}=\frac{2(N-1)\tau_v}{2(N-2)\tau_v+N(N-1)\tau_e}.$$

The limiting behavior of $\hat{\gamma}(x)$ can be easily checked. *Q.E.D*

Proof of Corollary 4.1

The equilibrium random price is given by

$$\tilde{p}=\frac{\beta(q,s)\sum_{n=1}^{N}\tilde{i}_n+\tilde{z}}{N\gamma(q,s)}. \tag{A.20}$$

Then, after substituting the equilibrium values of $\beta(q,s)$ and $\gamma(q,s)$, we obtain that $Var(\tilde{p})=\left(\frac{\beta(q,s)}{\gamma(q,s)}\right)^2\left(\frac{1}{\tau_v}+\frac{1}{N\tau_e}\right)+\frac{\sigma_z^2}{N^2[\gamma(q,s)]^2}$. Since $\frac{\beta(q,s)}{\gamma(q,s)}=\frac{1}{\delta(q,s)}$, the result follows. *Q.E.D.*

Proof of Corollary 4.2

Parts (i) and (ii) are the result of some omitted tedious computations. Part (iii) comes directly from part (ii) of Proposition 3.2. Finally, for part (iv) we can explicitly compute

$$\frac{d\hat{V}(x)}{dx}=\frac{2\tau_e\left[Ns\tau_e\left(N^2-2N-1\right)+2\tau_v\left(N^2+(s-2)+1\right)\right]}{(N-2)(2\tau_v+Ns\tau_e)^3}>0,$$

where the inequality follows since $s < \frac{2(N-1)}{N-2} < 2$ and N is an integer greater than 2.

Q.E.D.

Proof of Corollary 4.3

(i) To see that $V(x,1)) > \hat{V}(x)$ in the proposed interval we only have to notice that the first summand in the expression for $Var(\tilde{p}(q,s))$ in the statement of Corollary 4.1 is independent of s whereas the second is decreasing in s for a value of public confidence sufficiently close to $\frac{2(N-1)}{N-2}$ as dictated by part (ii) of Proposition 3.2. Therefore, $V(x,1) > V(x,x)$ for x close enough to $\frac{2(N-1)}{N-2}$. For the inequality $\hat{V}(x)) > V(1,x)$ in the proposed interval we use the fact that the second summand in the expression for $Var(q,s)$ tends to zero as s approaches $\frac{2(N-1)}{N-2}$, whereas the first term is strictly increasing in q as follows from part (i) of Proposition 3.2.

(ii) For the first inequality we just evaluate the finite values $V(0,1) = \frac{\sigma_z^2}{N^2\left[\underline{\gamma}(0,1)\right]^2}$ and $V(1,0) = \left(\frac{N\tau_e}{2\tau_v + N\tau_e}\right)^2\left(\frac{1}{\tau_v} + \frac{1}{N\tau_e}\right)$ and make the corresponding straightforward comparison. For the second inequality just notice that $\hat{V}(0) = 0$.

(iii) Since $V(x,1) = V(1,x)$ at $x = 1$, the result immediately follows from the properties of the functions $\gamma(x,1)$ and $\gamma(1,x)$.

(iv) and (v) The proofs are immediate. *Q.E.D.*

Proof of Corollary 4.4

Since the random quantity of asset $\tilde{x}_n$ demanded by trader n is normally distributed with zero mean, we have that $E\left(\left|\tilde{x}_n\right|\right) = \left(\frac{2}{\pi}\right)^{1/2}\left(Var\left(\tilde{x}_n\right)\right)^{1/2}$, and we only need to compute $Var\left(\tilde{x}_n\right)$. To this end, we replace $\tilde{p}$ in the demand $\tilde{x}_n = X_n(\tilde{p}) = \beta(q,s)\tilde{i}_n - \gamma(q,s)\tilde{p}$ by the formula given in (A.20), and then perform the computation of the variance of the individual demand $\tilde{x}_n$, that turns out to be equal to $\dfrac{(N-1)(N-2)[\beta(q,s)]^2}{N^2\tau_e}+\dfrac{\sigma_z^2}{N^2}$. *Q.E.D.*

Proof of Corollary 4.5

From inspection, we see that the qualitative behavior of $Q(q,s)$ replicates that of $\beta(q,s)$. Therefore, all the properties appearing in the statement of this corollary follow directly from the characterization of the functions $\beta(q,s)$ and $\hat{\beta}(x)$ given in Proposition 3.2 and from direct computation. In particular,

$$\hat{Q}(x)=\frac{1}{N}\sqrt{\left(\frac{2}{\pi}\right)\left[1+\frac{(N-2)^2}{\left(N+(N-2)(1-x)\right)x}\right]\sigma_z^2}\ ,$$

that is independent of both τ_v and τ_{e}. *Q.E.D.*

Proof of Corollary 4.6

(i) The first inequality is a consequence of the fact that $\beta(x,x)>\beta(1,x)$ for $x\in\left(1,\dfrac{2(N-1)}{N-2}\right)$ since $\beta(q,s)$ is increasing in its first argument as follows from part (i) of Proposition 3.2. The second inequality follows since $\lim\limits_{x\to\frac{2(N-1)}{N-2}}\beta(1,x)=\infty$ (see part (ii) of Proposition 3.2) while $\beta\left(\dfrac{2(N-1)}{N-2},1\right)$ is finite (see part (i) of Proposition 3.2).

(ii) For the first inequality observe that $\lim\limits_{x\to 0}\beta(1,x)=\infty$, whereas $\lim\limits_{x\to 0}\hat{\beta}(x)=0$ as follows from parts (ii) and (iii) of Proposition 3.2. Finally, the second inequality holds since $\beta(q,s)$ is decreasing for low values of the public confidence s (see part (ii) of Proposition 3.2) and, thus, $\beta(x,x)>\beta(x,1)$ for x sufficiently close to zero.

(iii) Since $Q(x,1)=Q(1,x)$ at $x=1$, the result immediately follows from the properties of the functions $\beta(x,1)$ and $\beta(1,x)$. *Q.E.D.*

Proof of Corollary 4.7

(i) Note that the random variable $\tilde{p}$ is informationally equivalent to $\delta(q,s)=\tilde{p}=\tilde{v}+\tilde{\xi}$, where $\tilde{\xi}=\dfrac{\sum_{n=1}^{N}\tilde{e}_n}{N}+\dfrac{\tilde{z}}{N\beta(q,s)}$. Since the precision of $\tilde{\xi}$ is

$\frac{N^2[\beta(q,s)]^2\tau_e}{N[\beta(q,s)]^2+\sigma_z^2\tau_e}$, we obtain

$$[Var(\tilde{v}|\tilde{p})]^{-1}=\tau_v+\frac{N^2[\beta(q,s)]^2\tau_e}{N[\beta(q,s)]^2+\sigma_z^2\tau_e} \tag{A.19}$$

The result immediately follows after subtracting $[Var(\tilde{v})]^{-1}=\tau_v$. Since τ_u depends on q and s only through $\beta(q,s)$, it directly follows that it is increasing in the level of private confidence, while its dependence on the level of public confidence is non-monotone, just as in Figure 1.

(ii) Obvious from part (iii) of Proposition 3.2.

(iii) Similarly, we must note that to observe $\tilde{i}_n$ and $\tilde{p}$ (as defined in (A.4)) is informationally equivalent to observe $\tilde{i}_n$ and $\tilde{y}$, where $\tilde{y}$ is defined in (A.8). Therefore, $\left[Var_n^q\left(\tilde{v}\middle|\tilde{i}_n,\tilde{p}\right)\right]^{-1}=\left[Var_n^q\left(\tilde{v}\middle|\tilde{i}_n,\tilde{y}\right)\right]^{-1}=\tau_v+q\tau_e+\tau_y$, where τ_y is given in (A.9). Substituting the equilibrium value $\beta(s,s)$ given in the proof of Proposition 3.1, and subtracting τ_v, we immediately obtain τ_n.

(iv) It is also obvious from part (iii) of Proposition 3.2. *Q.E.D.*

Proof of Corollary 4.8

Since the expected total cost of trading for the noise traders is $-E((\tilde{v}-\tilde{p})\tilde{z})=\frac{\sigma_z^2}{N\gamma(q,s)}$, we only need to divide by the number N of insiders so as to obtain the average profits of an insider. The rest of the corollary follows directly from Proposition 3.2.

Precision of the Noise of the Signals	$\tilde{e}_n$	$\tilde{e}_j \quad (j \neq n)$
First Order Beliefs	Belief of trader *n* about the precision of $\tilde{e}_n$: $q\tau_e$ (Not Common Knowledge)	Belief of trader *n* about the precision of $\tilde{e}_j$: τ_e (Common Knowledge)
Second Order Beliefs (Common Knowledge)	Belief of trader *j* about the belief of trader *n* about the precision of $\tilde{e}_n$: $s\tau_e$	Belief of trader *j* about the belief of trader *n* about the precision of $\tilde{e}_j$: τ_e

Table 1: Beliefs Structure for all n and $j \neq n$)

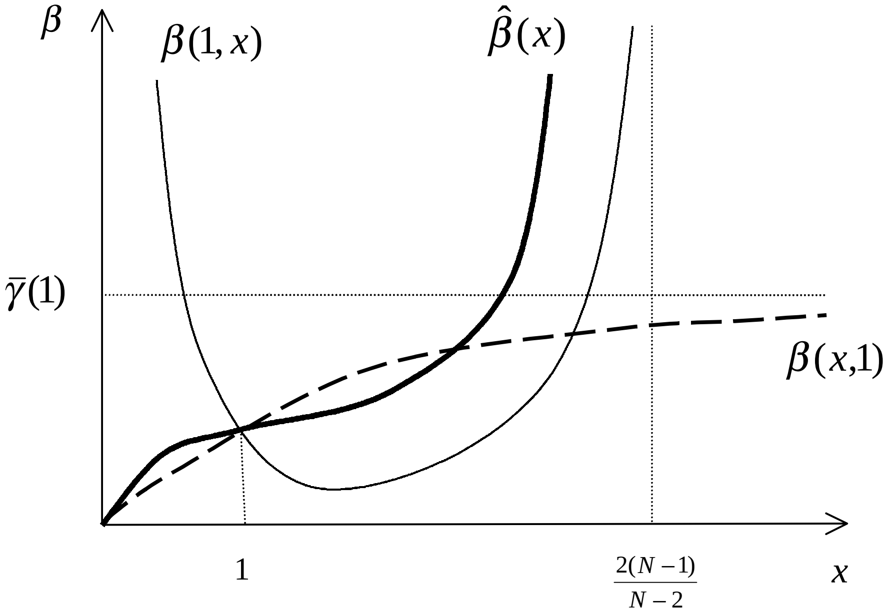

Figure 1: The function $\beta(q,s)$ of trading intensity.

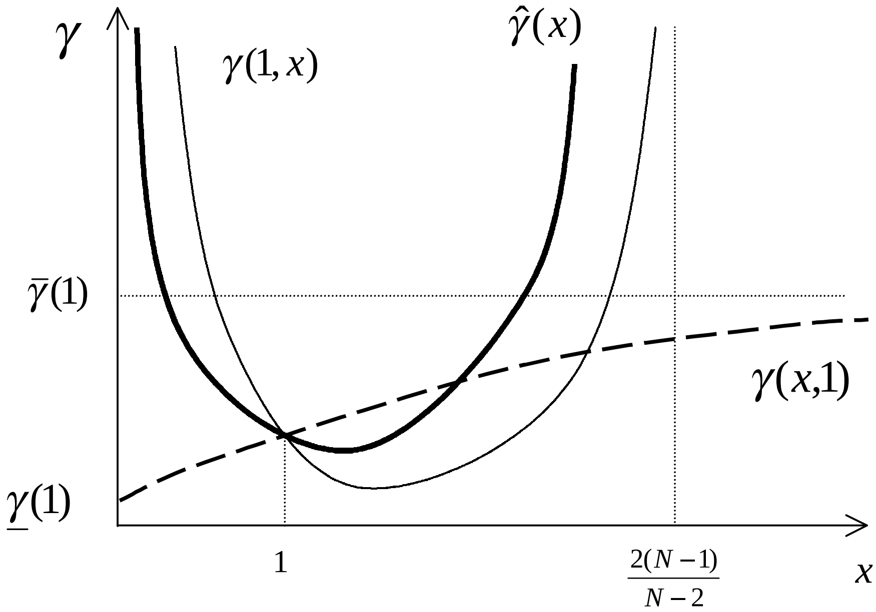

Figure 2: The function $\gamma(q,s)$ of absolute price sensitivity.

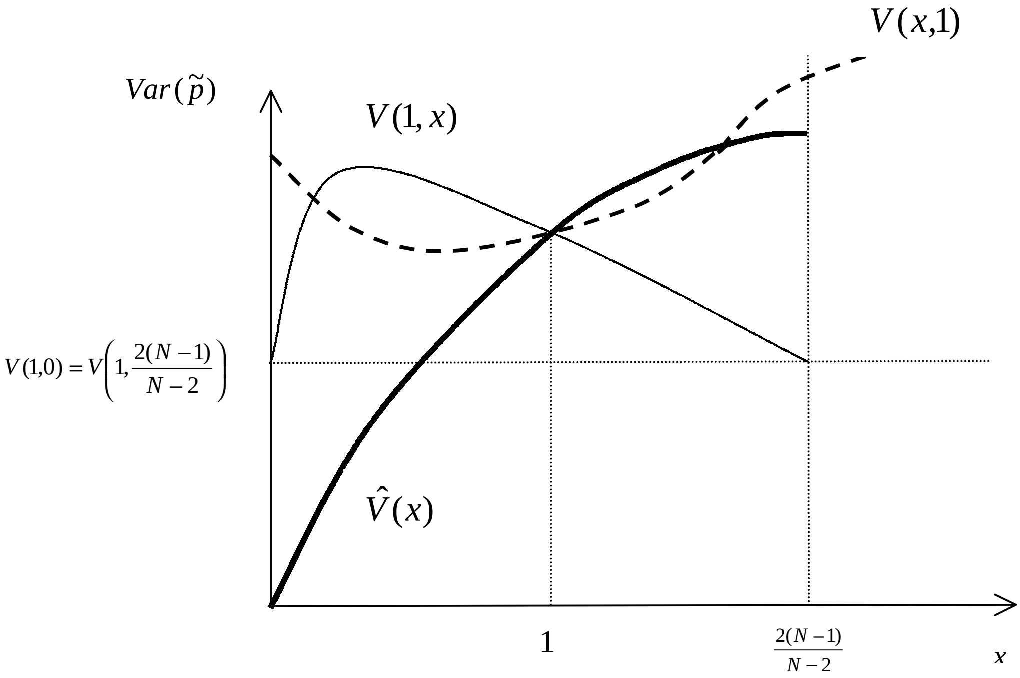

Figure 3: The variance of price.