Edinburgh School of Economics

Discussion Paper Series

Number 57

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Richard W P Holt (University of Edinburgh)

Date
December 2000

Published by
School of Economics
University of Edinburgh
30 -31 Buccleuch Place
Edinburgh EH8 9JT
+44 (0)131 650 8361
http://www.ed.ac.uk/schools-departments/economics

THE UNIVERSITY *of* EDINBURGH

The Manchester School Supplement 2000
1463 6786 51 74

NON-CONVEXITIES, ASYMMETRIES AND AGGREGATE INVESTMENT ACTIVITY: EVIDENCE FOR THE UK*

by
RICHARD W. P. HOLT
University of Edinburgh

This paper documents cyclical asymmetries in the aggregate investment activity of UK industrial and commercial companies. The ability of a model of aggregate activity based on heterogeneous actions under non convex adjustment at the individual level to account for this feature is then investigated. Aggregate activity is found to be con sistent with non convex adjustment at the individual level. Asymmetric features of microeconomic adjustment technology do not appear crucial for understanding aggregate non linearities.

1 Introduction

The existence of asymmetries in economic data has important ramifications for models of fluctuations. In particular, following Blatt (1980), to account for cyclical asymmetries, models must either be non-linear or involve non-Gaussian disturbances or both. If cyclical asymmetries are present, the next stage is to identify the proximate causes this requires a structural modelling framework. The presence of cyclical asymmetries provides some guidance as to the structural model. In this paper I examine the ability of non-convex, asymmetric adjustment costs at the individual level to account for asymmetries in aggregate investment activity.

A variety of cyclical asymmetric features have been considered in the recent literature (see Sichel, 1993; Verbrugge, 1997). One simple form of asymmetry is captured through the concept of *steepness*: an economic series exhibits negative (positive) steepness when its contractions (expansions) are sharper than its expansions (contractions). Another form of asymmetry which features regularly in the literature is captured by the concept of *deepness*, in which troughs involve a larger departure from the mean than do peaks (or vice versa). Thus, evidence of skewness in the distribution of first differences of the detrended series is consistent with

* This paper is a substantially revised version of Chapter 4 of my PhD thesis. This research was funded in part by the Royal Economic Society. I would like to thank Simon Burgess, John Driffill, Gary Koop, Mark Steel, David Winter, an anonymous referee, seminar participants at the Royal Economic Society Meeting, Nottingham, and the ESRC Money, Macro and Finance Meeting, Oxford, for helpful comments and discussions. Any errors remain my responsibility.

Published by Blackwell Publishers Ltd, 108 Cowley Road, Oxford OX4 1JF, UK, and 350 Main Street, Malden, MA 02148, USA

steepness, whilst deepness asymmetries are present if the distribution of the level of the detrended series exhibits skewness. Empirical studies (primarily for US data) indicate the presence of particular forms of cyclical asymmetries in different series. Whereas deepness appears to be commonplace, steepness is most clearly associated with labour market and price index data but is absent from output data (Sichel, 1993; Verbrugge, 1997). Hamermesh and Pfann (1996) note that factor inputs appear to exhibit greater cyclical asymmetries than are present in output data.

Once asymmetric features are identified, the emphasis shifts towards finding a *behavioural* explanation for these aggregate outcomes. This involves empirical implementation of an economic model consistent with Blatt's criteria; attention tends to focus on non-linear models rather than non-Gaussian disturbances. Given the flexibility inherent in non-linear models, some structural account (of microfoundations together with an account of aggregation) appears essential in establishing the economic as well as statistical credibility of the model. One fruitful avenue of research introduces non-linear adjustment technology into agents' optimization problems.[1] Some researchers have suggested that asymmetric adjustment costs may generate cyclical asymmetries in aggregate data; Palm and Pfann (1993) argue that asymmetric adjustment costs produce labour market behaviour consistent with costs of hiring and firing. Such costs typically introduce non-convexities into adjustment technologies, in the manner found in irreversible investment problems (Dixit and Pindyck, 1994). By this reasoning (partial) irreversibility could be argued to be the proximate cause of asymmetric convex adjustment costs, and hence of cyclical asymmetries at the aggregate level. However, this justification renders the microfoundations inconsistent with other aspects of investment behaviour, such as inaction and lumpy adjustments as presented by Nilsen and Schiantarelli (1997). Inaction and lumpiness are precisely the sort of features which would be present in micro-level investment data if an establishment were subject to non-convex adjustment technology. This suggests that the asymmetric convex cost model, while providing a plausible *statistical* description of the aggregate data, gives an inappropriate *structural* account of the features responsible for generating those data. Indeed, since symmetric non-convex adjustment costs introduce substantial non-linearities, it is unclear that asymmetric features of the adjustment

[1]In the recent empirical literature on UK investment, Arden *et al.* (1997) and Price (1996) outline the implications of irreversibility through a structural theoretical microfounda tion, but then implement and test reduced form asymmetric error correction and threshold autoregression models, respectively, to account for interesting non linear features of investment activity (asymmetric speed of adjustment and the impact of uncertainty) which are simply *ascribed to* irreversibility, without direct aggregation and implementation of the structural microeconomic model.

technology (i.e. irreversibility) are required to generate the cyclical asymmetries present in aggregate data.[2]

The difficulty with specifying microfoundations with non-convex adjustment costs has been that, unlike individual investment data, aggregate investment dynamics are smooth and serially correlated; therefore a representative agent approach is unsuitable. In a series of papers, Bertola and Caballero (1990, 1994) and Caballero and Engel (1993, 1999) have developed an elegant solution for aggregate dynamic behaviour under non-convex costs of microeconomic adjustment and tractable procedures for recovering the behavioural parameters at the individual level direct from aggregate data. The main focus of these authors has been to demonstrate the superiority of the aggregated (S, s) approach over the (representative agent) partial adjustment alternative, rather than to account for specific cyclical asymmetries. Moreover, the literature is ambiguous over the role of irreversibility, with Bertola and Caballero (1994) showing that a model based on completely irreversible investment accounts for aggregate investment fluctuations better than a frictionless model, and Caballero and Engel (1999) showing that a model with symmetric fixed costs is statistically superior to the partial adjustment model, but no study comparing the two.[3]

In this paper I address three questions. First, are there cyclical asymmetries in UK investment data? I document evidence that booms are deeper than recessions; there is no evidence of steepness. Second, to what extent do non-convex adjustment costs account for investment fluctuations in general, and cyclical asymmetries in particular? I find that the aggregated (S, s) model accounts well for both. Finally, to what extent can irreversibility (asymmetry) be considered a proximate cause of cyclical asymmetries? Here the evidence is more mixed. It is not possible to distinguish statistically between the symmetric and asymmetric forms of the aggregated (S, s) rule despite the fact that the point estimates of the microeconomic parameters strongly favour irreversibility in the asymmetric cost version.

[2]This is not the same as the argument in Caballero (1992), where it is shown that asymmetric non convex microeconomic adjustment technology does not produce the observed asymmetries in the volatilities of *gross* (job) creation and destruction flows described by Davis *et al.* (1996) unless *asymmetric shocks* are also present. Here the focus is on asymmetry in the *net* flow series.

[3]Other non linear models, such as models of regime switching and threshold autoregressions, appear consistent with the existence of action/inaction regimes at the individual level. Elements of regime switching exist in the aggregated (S, s) framework, since the extent of aggregate activity depends on the degree of synchronization of individual activity. If a large fraction of agents adjust, as may occur following a series of large aggregate shocks, then aggregate investment is high, because of the synchronization of individual activity. In the aggregated (S, s) framework, the degree of synchronization is endogenously determined by the history of shocks and adjustments. The main advantage of the aggregated (S, s) framework over these other non linear models is thus its explicit aggregation which permits direct recovery of microeconomic behaviour.

In the next section I provide evidence of cyclical asymmetries in UK investment data. In Section 3 I outline an aggregated (S, s) model of investment, following Caballero and Engel (1999). Section 4 describes the econometric approach and Section 5 presents the results. Section 6 contains a discussion of the implications of the paper, qualifications and suggestions for further work.

2 Evidence of Cyclical Asymmetries in UK Investment

Many tests of non-linearity cannot shed light on the form of non-linearity present in the data, and provide no information to restrict the set of feasible non-linear models. By contrast, evidence of asymmetries provides specific information about the nature of the non-linearities which behavioural models must account for. Unfortunately, until recently, the standard moment-based tests, based on the skewness coefficient of (detrended and differenced detrended) series, lacked statistical power (Verbrugge, 1997). This has led to distinctly weak empirical evidence on the existence of asymmetric features in economic series (De Long and Summers, 1986; Sichel, 1993).

Verbrugge (1997) introduced into the economics literature the non-parametric triples test of Randles *et al.* (1980). He showed that this test has greater power than moment-based tests and, since it is not dominated by outliers, is not subject to such severe small-sample problems. The triples test examines the skewness of all the possible triples (X_p, X_q, X_r) from a sample of size T. Any given triple is skewed if the intermediate valued observation X_q lies closer to one of the extreme observations X_p or X_r. A triple is left-skewed if the intermediate valued observation lies closer to the largest valued observation than to the smallest valued observation. Details of the triples test are provided in Appendix A. The test statistic is asymptotically (standard) normally distributed under the assumption that the data series are independent draws (from a given probability distribution), but given the serial correlation present in most aggregate economic data this result is not useful. The appropriate null hypothesis of the test in this environment is that the data-generating process is a linear autoregressive moving-average (ARMA) process with well-behaved symmetric independently and identically distributed (i.i.d.) disturbances. Verbrugge (1997) advocates Monte Carlo methods to ascertain the critical values for the triples test under serial correlation.[4] In fact he finds that correcting for serial correlation does not greatly affect the critical values.

[4]The procedure used here is (i) estimate an ARMA model for the investment series (here an AR(1) is used because this is the structure of the partial adjustment model: the standard linear approach to stock adjustment); (ii) construct the symmetrized residuals (estimated residuals and their additive inverse); (iii) perform 500 replications of the estimated process by drawing with replacement from the symmetrized distribution of residuals, calculating the test statistic for each replication.

TABLE 1
TRIPLES TEST STATISTICS FOR STEEPNESS AND DEEPNESS

	Linear detrending	*Hodrick Prescott detrending*
Deepness	3.552 (0.000)	2.447 (0.000)
Steepness	0.708 (0.246)	0.193 (0.434)

Notes: Number of observations 133 for deepness statistics, 132 for steepness statistics. *p* values in parentheses.

The data used in this study are the investment capital ratio I/K for UK industrial and commercial companies (ICC). The investment series, from the Office of National Statistics (ONS) database, is provided on a seasonally adjusted quarterly basis. The capital stock series is constructed from these investment data; details are provided in Appendix B. To use the triples test the series must be stationary and trendless. Dickey Fuller test statistics, augmented with four- and eight-lagged differenced terms, to correct for serial correlation in I/K and allowing for a constant and time trend in the underlying data-generating process, take values (with associated p values) -2.910 (0.055) and -2.934 (0.05) respectively. If the series were stationary, linear detrending would be appropriate. However, these statistics are inconclusive, so I use two detrending techniques: linear detrending and Hodrick Prescott filtering.[5] Due to the linearity of these filters, detrending has no impact on the skewness and kurtosis of the cyclical component of the series (see Canova, 1998). Table 1 shows the test statistics and p values for the triples test of steepness and deepness asymmetries in the ICC I/K ratio. This evidence indicates that expansions are not significantly sharper than corresponding slowdowns (or vice versa), but expansions in I/K are deeper than are contractions. That is, I/K exhibits positive deepness but no evidence of significant steepness asymmetry. These results are interesting in themselves in that they concur with the absence of steepness in non-labour-market data identified by Verbrugge, but also because of their consequences for modelling co-movement within economic fluctuations: the deepness in investment data arises from peaks of expansions, whereas the deepness in GDP data is

[5] Canova (1998) finds that, while different filtering procedures appear to alter the serial and cross correlation properties of the data, the higher moments are little altered by choice of detrending methodology. The smoothing parameter λ is set to 1600, which is standard for quarterly data. I do not use first differencing to obtain a stationary series, because this would compromise attempts to measure deepness.

associated with the troughs of recessions.[6] It is clear that the form of detrending does not alter these insights.

3 Model

In this section, a model of aggregate activity consistent with non-convex adjustment costs is outlined. The basis of this model is the standard decomposition of capital stock into long-run target and short-run dynamic components. The main features are the explicit treatment of heterogeneity, the characterization of the microeconomic adjustment technology and the aggregation procedure.

3.1 Microeconomic Framework

As is common in the literature on adjustment costs in factor demand (Hamermesh and Pfann, 1996), the dynamic behaviour of capital stock K is decomposed into a *target* capital stock K^* and an *imbalance* variable Θ. In logarithms[7]

$$\theta \equiv \ln \Theta \equiv \ln K - \ln K^* \tag{1}$$

Fluctuations in investment activity depend on the behaviour of both the target and the departure. The target level reflects the long-run response of capital stocks to current innovations in the exogenous variables, once the short-run effect of the frictions and rigidities (in this paper, adjustment costs) have died away. The elements of this decomposition can be computed directly once a firm's optimization problem is specified.

Suppose a firm has a Cobb Douglas production function in which output Q depends on the quasi-fixed factor, capital, K and the fully and costlessly flexible factor, labour, L, and technology A:

$$Q_t = A_t K_t^{\alpha} L_t^{1-\alpha} \qquad \alpha \in (0, 1)$$

Assume the firm faces a downward-sloping demand curve, in which output price P is a constant elasticity function of output and an index of economic conditions, Z:

$$P_t = Z_t^{1/\eta} Q_t^{-1/\eta} \qquad \eta \in (1, \infty)$$

It follows that current net operating revenues are

$$\pi(K_t, L_t, P_t, Q_t, w_t) = \max_{K,L}[P_t Q_t - w_t L_t - (r+\delta)K_t - \Omega]$$

[6]The form of the asymmetry is consistent with irreversible investment: I/K will decline only slowly if irreversibility is present, as this makes downward adjustment of capital stock more costly at the level of the individual firm. But, as argued above, irreversibility is not *necessarily* the cause of such a feature.

[7]In what follows, to avoid cumbersome language I refer to the *log imbalance* variable θ as the *imbalance*.

where w is the wage rate, r is the interest rate, δ the rate of depreciation and Ω represents the fixed cost incurred by adjusting capital stock: $I_t = 0 \Rightarrow \Omega = 0$, $I_t \neq 0 \Rightarrow \Omega > 0$.

The target capital stock K^* is obtained by setting adjustment costs to zero and maximizing operating revenues with respect to K and L. The target can be expressed as a function of the exogenous variables A_t, Z_t and w_t:

$$K_t^* = \left[\frac{\alpha}{(1-\alpha)(r+\delta)}\right]^{1+\alpha(\eta-1)} \left[(1-\alpha)\frac{\eta-1}{\eta}\right]^{\eta} A_t^{(\eta-1)} w_t^{(1-\alpha)(1-\eta)} Z_t$$

This expression makes clear that long-run target capital stock is decreasing in interest rates, wages and the rate of depreciation but increasing in technology A and demand Z. Assumptions about the processes driving the forcing variables A, Z, w carry through to K^*. Specifically, if these variables follow geometric random walks then K_t^* inherits this property. It is assumed that innovations in K^* can be decomposed into statistically independent aggregate and idiosyncratic components: v_t and ε_t respectively. Then for any firm the evolution of target capital stock is given by

$$K_t^* = K_{t-1}^* \exp(v_t + \varepsilon_t)$$

In the absence of adjustment, actual capital falls at the rate of depreciation, $K_t = K_{t-1}\exp(-\delta)$, so the evolution of the imbalance variable is given as $\Delta\theta_t = \Delta \ln(K_t/K_t^*) = -(\delta + v_t) - \varepsilon_t$.

3.2 *Characterizing Partially Synchronized Discrete Adjustment*

The intuition in modelling aggregate dynamic behaviour under non-convex adjustment costs is straightforward. At the individual level investment does not occur continuously but intermittently and in a lumpy fashion. Yet aggregate investment activity is comparatively smooth. Smooth aggregate activity can be reconciled with behaviour at the individual level only when firms exhibit heterogeneity that leads their investment decisions to be imperfectly synchronized. Capturing these features requires a model of discrete adjustment at the individual level, explicit treatment of aggregation and a formal statistical methodology. In this paper I use the approach developed by Caballero and Engel (1993, 1999).

The consequence of admitting individual heterogeneity and imperfect synchronization is that the representative agent approach must be ditched. The aggregate model of (S, s)-type behaviour uses the higher moments of the cross-section density function of imbalances across agents to track the fraction of units adjusting and the magnitude of those adjustments. This density function, $f(\theta, t)$, represents the fraction of firms at time t, with accumulated shocks θ, in respect of which adjustment has yet to occur. In other words, it incorporates the history of such shocks. When uncertainty is Markovian, the impact of history is entirely captured in the current realization of this density function. A key element of the non-representative agent approach is that the history of shocks to desired capital is mapped onto acquisitions and disposals in a non-linear and time-varying manner as a direct result of the endogenous synchronization of the heterogeneous responses to shocks to θ this permits the elasticity of investment to shocks to vary over time.

The approach used here is to infer the parameters of the microeconomic problem directly from aggregate data, rather than constructing aggregate dynamics from microeconomic data. Because of spatial and temporal aggregation problems, use of aggregate data restricts one's ability to identify microeconomic parameters, in that individual behaviour will only be recoverable at all to the extent that (i) a convenient analytical form is specified for the parameterization of individual heterogeneity, and (ii) aggregation is performed explicitly (see the discussion in Stoker, 1993).

A convenient simplification adopted by Bertola and Caballero (1990) and other early studies is to assume that all agents face identical adjustment costs, and thus have identical threshold rules. Then the only source of individual heterogeneity is the idiosyncratic component of shocks to θ (which results in imperfect synchronization of adjustment activity). Using establishment level data, Eberly (1994) and Caballero *et al.* (1995) show that the magnitude of adjustments is a further significant source of heterogeneity. Caballero and Engel (1993) develop an aggregated (S, s) model which admits both sources of heterogeneity. They assume that the probability that a unit i adjusts during a given time period t depends on the magnitude of the imbalance θ_{it}. This structure is represented by an adjustment function $\Lambda_i \equiv \Lambda_i(\theta) \in (0, 1)$ (for each unit $i \in (0, 1)$), which determines the probability that unit i adjusts, in a given time interval, as a function of the imbalance (see Fig. 1). Amongst other interpretations, this allows different firms to adjust by different amounts, and a given firm to adjust by different amounts at different points in time consistent with microeconomic evidence. This adjustment function also nests the standard linear partial adjustment model (constant adjustment function independent of imbalance) and the common threshold (S, s) model (discontinuous adjustment function with unit (zero) probability of adjustment at deviations greater (less) than the adjustment threshold) as special cases, but in general the probability of adjustment increases smoothly in the absolute value of the deviation.[8] When all firms face the same adjustment function

[8]Caballero and Engel (1999) develop explicit microfoundations for the adjustment function. They generalize the standard (S, s) framework to encompass adjustment costs which are i.i.d. for individual firms across time, according to a gamma distribution. They show that the adjustment function for an individual firm, $\Lambda_i(\theta)$, is differentiable and increasing and that $\lim_{|\theta| \to \infty} \Lambda_i(\theta) = 1$. By assuming that adjustment costs are i.i.d. across firms, the adjustment function can be viewed as an aggregate construct, describing the fraction of firms, at any level of deviation, who adjust (to the target) over a period of time.

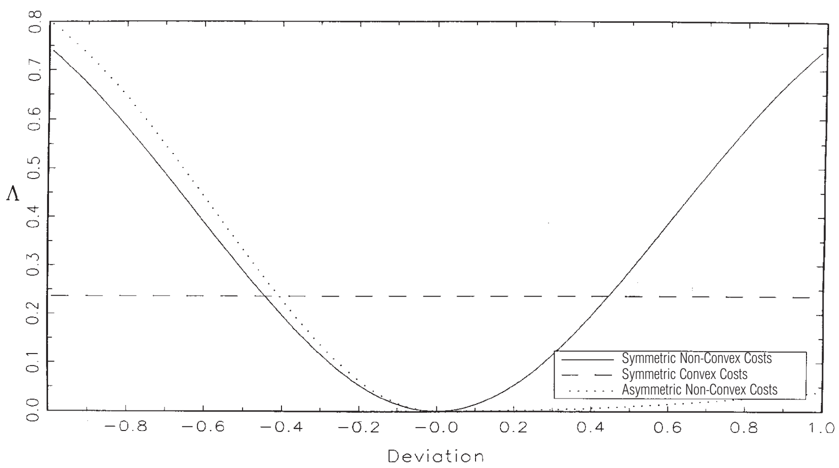

FIG. 1 Adjustment Function

$\Lambda_i(\theta) \equiv \Lambda(\theta), \forall i \in (0, 1)$, the latter represents the fraction of firms (in the economy) which adjust as a function of the imbalance.[9] In what follows it is assumed that all firms face the same adjustment function. Using this construct and the cross-section distribution of departures, $f(\theta, t)$, it is possible to model aggregate investment fluctuations.

It is convenient to assume that when a firm adjusts it completely eliminates any capital imbalance: following adjustment $\theta = 0$. Then, when firm i with imbalance $\theta_i = \bar{\theta}$ adjusts, its capital stock changes by

$$I_{i,t}(\bar{\theta}) = K^*_{i,t} - K_{i,t}(\bar{\theta}) = [\exp(-\bar{\theta}) - 1]K_{i,t}(\bar{\theta})$$

Expected investment by firm i with imbalance $\bar{\theta}$ is

$$\begin{aligned} E[I_{i,t}(\bar{\theta})] &= \{I_{i,t}(\bar{\theta}) \mid \text{adjustment}\} \Pr(\text{adjustment}) \\ &= [\exp(-\bar{\theta}) - 1]K_{i,t}(\bar{\theta})\Lambda_i(\bar{\theta}) \end{aligned}$$

Since the adjustment function is assumed to be identical across agents, statements about probability of action at the individual level translate into statements about the fraction of agents undertaking action at the aggregate level. Total investment by those firms $i \in \mho \subseteq (0, 1)$ with imbalance $\theta_i = \bar{\theta}$ is

[9] Implicit in this set up is the assumption that there are no interaction effects between firms. This simplifies the aggregation framework, since firms may be modelled, in probabilistic terms, as independent exchangeable agents. This is the case if neither entry nor exit (in the product and factor markets), nor bankruptcy, nor any congestion effects occur to create interaction effects between firms.

$$\bar{I}_t(\bar{\theta}) = \int_{i \in \Im} [\exp(-\bar{\theta}) - 1] K_{i,t}(\bar{\theta}) \Lambda_i(\bar{\theta})\, \mathrm{d}i = [\exp(-\bar{\theta}) - 1] \bar{K}_t(\bar{\theta}) \Lambda(\bar{\theta})$$

where $\bar{K}_t(\bar{\theta})$ is the average capital stock of firms with imbalance $\bar{\theta}$ and $\Lambda(\bar{\theta})$ is the fraction of those firms with imbalance $\bar{\theta}$ that adjust. Aggregate investment activity (for the entire economy) I_t^A is computed using the adjustments $\bar{I}_t(\theta)$ undertaken by the fraction of agents with capital imbalance θ by summing over all possible imbalances, $\theta \in (-\infty, \infty)$. Denote the proportion of agents at any θ, immediately prior to adjustment, at time t, by the cross-section density function $\tilde{f}(\theta, t)$. So

$$I_t^{\mathrm{A}} = \int_{\theta\ -\infty}^{\infty} [\exp(-\theta) - 1] \bar{K}_t(\theta) \Lambda(\theta) \tilde{f}(\theta, t)\, \mathrm{d}\theta$$

Assuming that the average capital stock is independent of the imbalance, the aggregate investment capital ratio is given by the approximation[10]

$$\frac{I_t^{\mathrm{A}}}{K_t^{\mathrm{A}}} \approx \int_{\theta\ -\infty}^{\infty} [\exp(-\theta) - 1] \Lambda(\theta) \tilde{f}(\theta, t)\, \mathrm{d}\theta \tag{2}$$

3.3 Modelling Investment Dynamics

Equation (2) highlights the key role played by the cross-section distribution of capital imbalances, $f(\theta, t)$, which represents the cross-section of accumulated shocks to target capital stock to which firms have not yet adjusted.[11] The behaviour of the density function depends on the shocks μ_t and the magnitude and direction of adjustment as prescribed by $\Lambda(\theta)$.

To track the evolution of the density function and fluctuations of the investment capital ratio in a discrete-time framework, the following timing convention is used to describe the cycle of idiosyncratic and aggregate shocks and (non-) adjustment *within* time period t. Define the cross-section density at the end of period $t-1$ as $f(\theta, t-1)$. The first event during period t is taken to be the effect of depreciation and the aggregate shock, v_t. These common effects alter the imbalance of all firms by $-(\delta + v_t)$. Define the density function following these events and

[10]Using a panel of firms from the Longitudinal Research Database, Caballero *et al.* (1995) find that this is the case: the average capital imbalance $\bar{K}_t(\theta)$ is largely independent of the extent of the imbalance, θ.

[11]Unless the adjustment function $\Lambda(\theta)$ is independent of the level of imbalance θ (which is only the case under symmetric quadratic costs), the level of investment depends on the higher moments of this distribution in a non linear and potentially time varying manner. Capturing the dynamic impact of partially synchronized discrete adjustment on aggregate investment requires knowledge of the evolution of this density function.

immediately preceding adjustment as $\tilde{f}(\theta, t)$. The next period t event is the adjustment decision. Adjustments are determined by the adjustment function $\Lambda(\theta)$. The final event in period t is that each unit is subjected to an idiosyncratic shock $\varepsilon_{i,t}$ drawn from the density function $g_\varepsilon(\cdot)$. The end of period t density is defined as $f(\theta, t)$. Those units that end period t with imbalance θ can arrive at that point whether or not they adjust during period t. In particular, since adjustment results in the complete elimination of the imbalance, the density $f(\theta, t)$ consists of the fraction of units (across all θ) which do adjust during period t *and* subsequently receive an idiosyncratic shock of $-\theta$, *plus* the fraction of those agents which do not adjust at imbalance $\theta + \varepsilon$ *and* subsequently receive a shock of exactly $-\varepsilon$ to leave them with an end of period t imbalance of θ, summed over all such realizations of shock ε. More formally, the timing convention results in the following sequence of events:

$$\tilde{f}(\theta, t) = f(\theta + \delta + v_t, t - 1) \tag{3}$$

$$\begin{aligned} f(\theta, t) = {} & \left[\int \Lambda(\phi)\tilde{f}(\phi, t)\mathrm{d}\phi\right] g_\varepsilon(-\theta) \\ & + \int [1 - \Lambda(\theta + \varepsilon)]\tilde{f}(\theta + \varepsilon, t) g_\varepsilon(-\varepsilon)\,\mathrm{d}\varepsilon \end{aligned} \tag{4}$$

Putting equations (3) and (4) together, the evolution of the cross-section density from one period to another and the expression for the aggregate investment rate are, respectively,

$$\begin{aligned} f(\theta, t) = {} & \left[\int \Lambda(\phi) f(\phi + \delta + v_t, t - 1)\mathrm{d}\phi\right] g_\varepsilon(-\theta) \\ & + \int [1 - \Lambda(\theta + \varepsilon)] f(\theta + \delta + v_t + \varepsilon, t - 1) g_\varepsilon(-\varepsilon)\,\mathrm{d}\varepsilon \end{aligned} \tag{5}$$

$$\frac{I_t^{\mathrm{A}}}{K_t^{\mathrm{A}}} = \int [\exp(-\theta) - 1]\Lambda(\theta) f(\theta + \delta + v_t, t - 1)\,\mathrm{d}\theta \tag{6}$$

or $I_t^{\mathrm{A}}/K_t^{\mathrm{A}} = y_t = y_t[v_t, f(\theta, t-1)]$. Applying these formulae recursively, it follows that, conditional on knowing the initial cross-section density of imbalances $f(\theta, 0)$, both $\{f(\theta, t)\}$ and $\{I_t^{\mathrm{A}}/K_t^{\mathrm{A}}\}$ can be computed as a function of the aggregate and idiosyncratic shocks and the adjustment function. Aggregate investment dynamics are governed by a non-linear stochastic difference equation in y_t. The solution can be expressed, implicitly, as a function of the aggregate shocks conditional on the initial distribution $y_t = y_t[v_t, \ldots, v_1, f(\theta, 0)]$. Alternatively, provided the relationship in equation (6) between aggregate investment and aggregate shocks is invertible, and assuming the initial cross-section distribution is known, both $f(\theta, t)$ and v_t can be computed as a function of y_t, for $t = 1, \ldots, T$.

4 Econometrics

Estimation of the model exploits the correlation patterns of aggregate I/K data to recover the parameters of the adjustment function, using a maximum likelihood approach, based on the second strategy outlined above of identifying v_t and $f(\theta, t)$ given y_t and $f(\theta, 0)$. This requires the structure of the model to be specified more explicitly. Section 4.1 outlines the assumptions made about the form of the adjustment function, the distributions of idiosyncratic and aggregate shocks and the initial cross-section distribution. Computational aspects of the estimation procedure are discussed in Section 4.2.

4.1 A Structural Framework

The ergodic density function $\lim_{t\to\infty} f(\theta, t)$ is used to approximate the initial cross-section distribution $f(\theta, 0)$. Caballero and Engel (1999) show that the Markovian nature of the driving processes ensures the existence of an ergodic density function. They also show that the choice of initial density function has little effect on the results, as it washes away within three periods. I therefore exclude the first three periods from the computation of the likelihood function. It is assumed that idiosyncratic shocks are $\mathrm{NIID}\left(0, \sigma_\varepsilon^2\right)$ across firms and time (the density function of idiosyncratic shocks is denoted $g_\varepsilon(\cdot)$), and that idiosyncratic and aggregate shocks are independent. I assume that the adjustment function takes the form

$$\Lambda(\theta) = \begin{cases} 1 - \exp(-\lambda_0 - \lambda_2^- \theta^2) & \theta < 0 \\ \\ 1 - \exp(-\lambda_0 - \lambda_2^+ \theta^2) & \theta \geq 0 \end{cases} \qquad \lambda_0, \lambda_2^-, \lambda_2^+ > 0 \tag{7}$$

This specification nests models of adjustment with convex *and* non-convex costs. Note that $\Lambda(\theta) \in (0, 1)$, $\forall\theta$, and is increasing in the absolute imbalance $|\theta|$. Aggregate shocks v_t, $t = 1, \ldots, T$, are assumed $\mathrm{NIID}\left(\mu_v, \sigma_v^2\right)$ with joint probability density function

$$q(v_1, v_2, \ldots, v_T) = \frac{1}{\left(2\pi\sigma_v^2\right)^{T/2}} \exp\left[-\sum_{t\ 1}^{T} \frac{(v_t - \mu_v)^2}{2\sigma_v^2}\right] \tag{8}$$

The joint density function of the aggregate investment rate series can be derived from a change of variables argument using equations (6) and (7). Caballero and Engel (1999) show that, if y_t, $\Lambda(\theta)$ and v_t are defined as in equations (6), (7) and (8) respectively, then $\partial y_t/\partial v_t \geq 0, \forall v_t$. It follows that the transformation (6) is one to one and hence invertible. Defining this inverse as $v_t = h(y_t)$ the density function of the aggregate investment series is

$$j(y_1, y_2, \ldots, y_T) = \frac{1}{\left(2\pi\sigma_v^2\right)^{T/2}} \exp\left\{ -\sum_{t=1}^{T} \frac{\left[h(y_t) - \mu_v\right]^2}{2\sigma_v^2} \right\} |J|$$

where J is the Jacobian of the transformation. Since v_t is independent of y_{t+i} for all $i > 0$ and for all t, the absolute value of the Jacobian can be written $\prod_{t=1}^{T} \left|\partial v_t / \partial y_t\right|$. The log-likelihood function $\mathcal{L}$ can be written as

$$-\mathcal{L} = \frac{T}{2} \ln 2\pi + \sum_{t=1}^{T} \ln\left|\frac{\partial y_t}{\partial v_t}\right| + \frac{T}{2} \ln \sigma_v^2 + \frac{1}{2} \sum_{t=1}^{T} \left(\frac{v_t - \mu_v}{\sigma_v}\right)^2$$

The estimation problem is further simplified by concentrating the likelihood function with respect to σ_v^2 and μ_v, using $\hat{\mu}_v = (1/T) \sum_{t=1}^{T} v_t$ and $\hat{\sigma}_v^2 = (1/T) \sum_{t=1}^{T} (v_t - \hat{\mu}_t)^2$, yielding

$$-\mathcal{L}^{\mathrm{c}} = \frac{T}{2}(1 + \ln 2\pi) + \sum_{t=1}^{T} \ln\left|\frac{\partial y_t}{\partial v_t}\right| + \frac{T}{2} \ln \sum_{t=1}^{T} \frac{(v_t - \hat{\mu}_v)^2}{T} \tag{9}$$

This simplifies the problem to the computation of maximum likelihood estimates of the parameters of the adjustment function Λ as functions of the aggregate innovations v_t (and the initial cross-section distribution $f(\theta, 0)$).

4.2 Implementation

Grid search over the parameters of the adjustment function is used for estimation. Given a set of parameter values $\{\lambda_0, \lambda_2^-, \lambda_2^+\}$, elements of the likelihood function are computed directly. This procedure makes extensive use of equations (5) and (6) in updating the density function following each sequence of shocks and adjustment. Numerical methods are used throughout.[12]

The state space of imbalances θ is divided into a grid of 101 points. Because estimation requires that the evolution of the cross-section density be modelled explicitly (using the updating equations), and the location of the density function depends upon the size and direction of aggregate shocks, it is vital that the state space is sufficiently broad that mass does not 'escape' from the implemented cross-section density. The interval $[-20\sigma_\varepsilon, 20\sigma_\varepsilon]$ was used, with σ_ε set to 0.1, following Caballero and Engel (1999). This was adequate to prevent loss of mass from the density function for values of the adjustment function parameters consistent with those of Caballero and Engel (1999) and Caballero *et al.* (1995).

For a given realization $\{\lambda_0, \lambda_2^-, \lambda_2^+\}$, the first step is to compute the ergodic density function, which is used to approximate the initial cross-section density function of imbalances $f(\theta, 0)$. This is done using equations

[12] All computations are undertaken using Gauss 3.2.31.

(5) and (6). It is assumed that the ergodic density function results from an infinite constant I/K sequence at the level determined by the sample mean: $y_s = \bar{y} \equiv (1/T)\sum_{t=1}^{T} y_t, \forall s \in -\infty, \ldots, 0$. Then, starting from a prespecified imbalance density $f(\theta, s)$ (for simplicity assumed to be standard normal), I use equation (5) to compute, by grid search, the aggregate shock that best approximates y_s to $\bar{y}$.[13] The estimated aggregate shock v_s can be used to update the density function through equation (6). Iterating forwards the density function is found to converge after some 10–15 iterations.

Next, given this estimate of the initial cross-section density function, the series of aggregate shocks are computed, individually, from equation (5) by grid search over v_t as described in the previous paragraph. For each period the aggregate shock estimate $\hat{v}_t$ is used to update the density function, through equation (6), and to compute the corresponding Jacobian $\partial y_t/\partial v_t$. For any $\{\lambda_0, \lambda_2^-, \lambda_2^+\}$, these computations yield the elements of the likelihood function.[14]

5 Results

This section outlines the results obtained for various linear and non-linear specifications arising from the model, and presents evidence on the extent of misspecification in the models.

Table 2 displays parameter estimates obtained for symmetric and asymmetric versions of the model.[15] Results for the standard linear (partial adjustment) model are in column 1. Results for two sets of aggregated (S, s) models are presented: (i) with the restriction $\lambda_0 = 0$, which do not nest the partial adjustment model (columns 2 and 3); (ii) with $\lambda_0 \geq 0$, which nests the partial adjustment model (column 4).[16] Judging from the values of the log-likelihood statistic for the different models, it seems that the linear (partial adjustment) model is dominated by the aggregated (S, s) model in either its symmetric or asymmetric forms; however, to establish this formal statistical criteria must be used. These differ for nested and non-nested models.

Following Caballero and Engel (1999), the test due to Rivers and

[13]The aggregate shock v_t is allowed to vary with step size one hundredth of a grid point of the state space of imbalances, i.e. $\Delta v = 0.01 \times \Delta\theta$.

[14]As discussed, the first three observations for the computed I/K series are omitted from the likelihood function.

[15]Standard errors for the parameter estimates are computed from the second derivative of the likelihood function. These are obtained by fitting a polynomial (quadratic) approximation to the numerical values of the likelihood function over the grid of parameter values.

[16]This framework $(\lambda_0, \lambda_2 \geq 0)$ can be thought of as incorporating some limited (small scale) replacement investment through the term in λ_0, with the term in λ_2 capturing lumpy adjustment.

TABLE 2
ADJUSTMENT FUNCTION PARAMETER ESTIMATES

Parameter	1 *Symmetric convex*	2 *Symmetric non convex*	3 *Asymmetric non convex*	4 *Symmetric non convex*
λ_0	0.27 (0.13)			0.04 (0.117)
λ_2		1.37 (0.34)		1.70 (1.07)
λ_2^+			0.04 (2.26)	
λ_2			1.62 (0.40)	
ln likelihood	793.37	819.52	821.08	817.75

Notes: Number of observations 130. Standard errors in parentheses.

Vuong (1991) is used to distinguish between non-nested models (details of this test are provided in Appendix C). The results are displayed in Table 3. The test statistic is asymptotically standard normally distributed. Significant negative values provide evidence in favour of the (S, s) model at the head of the respective column in Table 3, whereas significant positive values provide evidence in favour of the model in the corresponding row: the convex adjustment cost model. According to the Rivers Vuong statistic, the convex adjustment cost model, $\lambda_0 > 0$, $\lambda_2 = \lambda_2^- = \lambda_2^+ = 0$, is rejected in favour of the non-convex models (of Table 2, columns 2 and 3), $\lambda_2, \lambda_2^-, \lambda_2^+ \geq 0$, $\lambda_0 = 0$. This suggests, consistent with US evidence (Caballero *et al.*, 1995; Caballero and Engel, 1999) that the aggregated (S, s) framework provides a convenient representation of UK ICC aggregate I/K dynamics. However, the parameters of the adjustment function (see Table 2) are smaller than the estimates obtained by Caballero and Engel (1999). This is because Λ represents the probability of adjusting

TABLE 3
RIVERS VUONG STATISTICS FOR NON NESTED MODELS

	Symmetric non convex [2]	*Asymmetric non convex* [3]
Symmetric convex [1]	1.82 (0.07)	2.08 (0.046)

Notes: Compares the partial adjustment model (Table 2, column 1) with the (S, s) models in columns 2 and 3 of Table 2. Large negative values provide evidence for the (S, s) models.
Numbers in square brackets indicate the column number in Table 2. Numbers in parentheses are p values.

Table 4
Likelihood Ratio Statistics for Nested Models

	Unrestricted model	
Restricted model	*Asymmetric non convex* [3]	*Symmetric non convex* [4]
Symmetric convex [1]		52.47 (0.00)
Symmetric non convex [2]	3.12 (0.08)	0.19 (0.74)

Notes: Numbers in square brackets indicate the column number in Table 2. Numbers in parentheses are p values.

in any given period, which is one quarter here and one year in their study.[17,18]

The next question to address is whether allowing some combination of 'small' investments, characterized by partial adjustment, and 'major' investments, exhibiting lumpiness of an (S, s) form, improves our ability to explain aggregate fluctuations. The answer to this lies in the model of column 4 in Table 2, which nests the column 1 and column 2 models. Table 4 presents a likelihood ratio statistic for column 4 of Table 2 versus column 1, which rejects the restriction that $\lambda_2 = 0$. Table 4 contains a series of likelihood ratio statistics. The unrestricted model is given at the head of the column and the restricted model in the corresponding row. Comparing models 2 and 4 of Table 2, it is not possible to reject the null that $\lambda_0 = 0$, leaving column 2 of Table 2 as the preferred symmetric specification on grounds of parsimony. For this specification, the fraction of firms adjusting increases in the absolute value of the capital imbalance: some 5 per cent of firms adjust when the absolute imbalance is 0.2, while around half the firms adjust when the imbalance is 0.7.

If microeconomic asymmetries (e.g. (pure) irreversibility) are important features in explaining aggregate investment fluctuations, one would expect to be able to distinguish between the symmetric and asymmetric versions of the aggregated (S, s) model. The parameter estimates suggest irreversibility: in column 3, λ_2^+ is statistically indistinguishable from zero. Since column 3 nests column 2, a likelihood ratio statistic is used to analyse the impact of the restriction $\lambda_2^+ = \lambda_2^-$. As is clear from Table 4 the

[17]The high rate of adjustment may be due to the choice of idiosyncratic uncertainty. It would be useful to examine the relative magnitudes of idiosyncratic and aggregate uncertainty building directly from microeconomic data.

[18]I have considered the sensitivity of the results generated using the model to the assumptions made in constructing the I/K series, by setting $\delta = 0.08$. It appears that the model is robust to such variation. The results are not reported here for brevity.

TABLE 5
MISSPECIFICATION TESTS USING AGGREGATE SHOCKS

	Symmetric non convex [2]	*Asymmetric non convex* [3]
Deepness	1.39	1.38
	(0.082)	(0.084)
Steepness	0.04	0.04
	(0.484)	(0.484)
$\rho(1)$	0.028	0.056
	(0.374)	(0.261)

Notes: Number of observations 130; p values in parentheses. Square brackets indicate the corresponding column number in Table 2. Triples statistics are (standard) normally distributed under the null hypothesis.

evidence against the symmetric model is weak.[19] This is consistent with existing microeconomic evidence (Caballero *et al.*, 1995), which shows only mild evidence of asymmetry in the adjustment function.

Before we can conclude that the models with non-convex adjustment costs provide an appropriate vehicle for modelling aggregate UK investment behaviour, it is necessary to test for potential misspecification. Implicit in the models analysed in this paper are a number of restrictions on the aggregate shocks. Specifically, the aggregate shock process was assumed to display normality and temporal independence.[20] In addition, it is possible to undertake a number of traditional residual-based diagnostic tests.

The triples test is used to examine asymmetry of the aggregate shock process (as discussed in Section 2, moment-based tests have lower power to distinguish departures from normality), and the first-order serial correlation coefficient is computed. The results are shown in Table 5 where, for brevity, the models of column 1 and column 4 in Table 2 are omitted. The evidence on normality is not entirely conclusive, but to a first approximation the models of non-convex adjustment do not require non-normality in the aggregate shock process to generate asymmetric features of aggregate I/K. Considering temporal independence, the sample first-order autocorrelation coefficient of the aggregate shock series is statistically insignificant at conventional levels for both the symmetric non-convex cost case and the asymmetric non-convex cost case. These results suggest

[19]This appears to be because the likelihood function is fairly flat in λ_2^+. Since disposals would typically be a small fraction of acquisitions, it is likely to be difficult to identify a role for irreversibility from aggregate data alone. Note that this is consistent with the data used: see Appendix B.

[20]For example, rejection of temporal independence might lead one to reformulate the likelihood function allowing for a low order AR process in v_t.

TABLE 6
MISSPECIFICATION TESTS USING RESIDUALS

	Symmetric non convex [2]	*Asymmetric non convex* [3]
Q_{levels} (4)	4.05 (0.399)	2.29 (0.69)
Q_{levels} (8)	10.88 (0.209)	3.49 (0.899)
Q_{squares} (4)	1.11 (0.891)	4.46 (0.347)
Q_{squares} (8)	3.27 (0.916)	12.52 (0.129)

Notes: p values in parentheses. Numbers in square brackets correspond to the columns of Table 2. Ljung Box $Q(k)$ statistics follow a $\chi^2(k)$ distribution.

that neither the assumption of normality nor that of independence is violated for the aggregate shock series.[21]

Turning to the properties of the residuals, Table 6 presents Ljung Box statistics as tests of time dependence in the levels and squares of the residuals; the latter test for the existence of autoregressive conditional heteroscedasticity effects. These residuals-based tests can also be interpreted as tests of model specification. It is clear that neither the symmetric nor the asymmetric form of the non-convex cost model displays temporal dependence using these criteria.

Finally, since the non-convex adjustment cost models do not display evidence of misspecification, the residuals can also be used to display the extent to which the different models of non-convex adjustment capture the cyclical asymmetries present in UK aggregate ICC investment data: how well they match the stylized facts. The triples tests of steepness and deepness in the residuals presented in Table 7 suggest that both aggregated (S, s) models do a very good job of accounting for the deepness asymmetries present in aggregate UK ICC data.

6 DISCUSSION AND CONCLUSION

There are three sets of findings in this paper. First, UK ICC investment exhibits asymmetries: booms are deeper than recessions. Second, these features appear consistent with an aggregated (S, s) model. Finally, how-

[21]In an earlier version of the paper I found that the symmetric convex cost model did require non normal (deep) shocks to generate asymmetric features of the UK ICC I/K series, and that the sample first order autocorrelation coefficient was statistically significant at the 2 per cent level.

TABLE 7
ASYMMETRY TESTS OF THE RESIDUALS

	Symmetric non convex [2]	*Asymmetric non convex* [3]
Deepness	1.31 (0.095)	1.08 (0.852)
Steepness	0.63 (0.264)	0.02 (0.492)

Notes: p values in parentheses. Numbers in square brackets correspond to the columns of Table 2. Steepness and deepness triples statistics follow a standard normal distribution.

ever, irreversible investment technology does not offer any further explanatory power to a model of aggregate investment activity.

The form of asymmetries present in UK investment data is qualitatively different from the deep recessions in output data (and steepness in labour market data) found in other studies, e.g. Verbrugge (1997). This is likely to be important in (general equilibrium) models of co-movements in economic series during business cycles. At face value these features appear consistent with an irreversibility-based story. But a convex asymmetric adjustment cost framework, following Palm and Pfann (1993), is clearly inconsistent in spirit with lumpy and intermittent microeconomic behaviour. An aggregated (S, s) model provides a good explanation of investment fluctuations and accounts for cyclical asymmetries in investment activity. While an aggregated (S, s) framework with asymmetric microeconomic features explains investment fluctuations almost as well as a symmetric version, there is little evidence, using aggregate data, that irreversibility *per se* is important for understanding investment asymmetries. One way to think of this is that symmetric fixed costs capture the important features arising under irreversibility: (a zone of) inaction in response to a negative shock to desired capital stock.

There are a number of issues which require further attention. First, the entire analysis uses aggregate data. This restricts one's ability to recover the parameters governing microeconomic behaviour. Inference over features of the economic/technological environment facing individuals can only be undertaken by imposing a significant amount of structure which is not separately tested. Comfortingly, results are broadly consistent with existing studies using US establishment data (Caballero *et al.*, 1995), which lends credence to the aggregate model. Nevertheless, analysis with UK micro data is an obvious next step. Another unfortunate feature of this model is that price variables play no role. This may be a reasonable restriction as a starting point since, as Chirinko (1993) notes, quantity variables dominate price variables in investment equations. However, price variables ought to play some role, at least in the long run.

One way to capture this would be to allow general equilibrium effects to produce variation in the adjustment function parameters over the economic cycle.[22] A third interesting extension to the model would be to allow time-varying uncertainty. This would facilitate analysis of the relation between investment and uncertainty, following Dixit and Pindyck (1994), using a structural aggregate framework consistent with the non-convex adjustment technology underlying such models, whereas at present this important issue tends to be addressed in reduced form (Guiso and Parigi, 1999).

In sum, this paper shows that an aggregated (S, s) model of investment provides a plausible structural account of UK ICC investment fluctuations in general and the cyclical asymmetries in particular. This suggests the need to rethink the linear approach to modelling economic variables in the light of evidence on the nature of microeconomic behaviour.

Appendix A

This section contains details of the triples test for symmetry versus asymmetry; a fuller discussion is provided in Verbrugge (1997). Define

$$f^*(X_p, X_q, X_r) \equiv \frac{1}{3}\left\{\begin{array}{l}\text{signum}(X_p + X_q \quad 2X_r)+ \\ \text{signum}(X_q + X_r \quad 2X_p)+ \\ \text{signum}(X_p + X_r \quad 2X_q)\end{array}\right\}$$

Then $f^* \in \{\ \frac{1}{3}, 0, \frac{1}{3}\}$. The triples test statistic, which is asymptotically normally distributed, is defined as

$$\text{Tr} \quad \frac{\hat{\eta} \quad \eta}{(\hat{\sigma}^2_{\hat{\eta}}/T)^{1/2}}$$

where

$$\hat{\eta} \equiv \frac{1}{\binom{T}{3}} \sum_{p<q<r} f^*(X_p, X_q, X_r)$$

$$\frac{\hat{\sigma}^2_{\hat{\eta}}}{T} \equiv \sum_{j \quad 1}^{3} \binom{3}{j} \binom{T \quad 3}{3 \quad j} \hat{\varsigma}_j$$

The $\hat{\varsigma}_j$ terms are

$$\hat{\varsigma}_1 \equiv \frac{1}{T} \sum_{p \quad 1}^{T} [f_1^*(X_p) \quad \hat{\eta}]^2 \qquad \hat{\varsigma}_3 \equiv \frac{1}{9} \quad \hat{\eta}^2$$

and

[22]However, there are major computational (dimensionality) problems associated with the general equilibrium treatment of the aggregated (S, s) model.

$$\hat{\varsigma}_2 \equiv \frac{1}{\binom{T}{2}} \sum_{1<r<T} \sum_{q<r} \left[f_2^*(X_q, X_r) - \hat{\eta}\right]^2$$

where

$$f_1^*(X_p) \equiv \frac{1}{\binom{T-1}{2}} \sum_{\substack{1<r<T \\ r \neq p}} \sum_{\substack{q<r \\ q \neq p}} f^*(X_p, X_q, X_r)$$

$$f_1^*(X_q, X_r) \equiv \frac{1}{T-2} \sum_{\substack{1<p<T \\ p \neq q \neq r}} f^*(X_p, X_q, X_r)$$

Under the null hypothesis that the series displays no asymmetries, $\eta = 0$.

Appendix B

The investment data of UK ICC are taken from the Annual Supplement to Economic Trends 1997 (series AAUS). The data span the period 1963:1–1996:1; the sample length, 133 observations, is restricted by the availability of data. The investment data are seasonally adjusted, quarterly and are reported in current prices. To obtain a series for real ICC investment, the implicit investment deflator from private sector investment expenditures is used.

Since quarterly capital stock data are unavailable, it is necessary to construct a series directly from the investment data. This raises issues over the treatment of replacement investment and the choice of the initial value of private sector capital stock. The former is addressed by assuming a constant proportional rate of depreciation, at an annual rate of 10 per cent. Geometric depreciation is consistent with the theoretical model while empirical evidence suggests that depreciation is accelerated in the early years of an asset's life (Jorgenson, 1996). The initial capital stock is constructed from the investment data. The first step in this procedure is to identify the trend rate of growth of investment by estimating the regression

$$\ln I_t^{\mathrm{Ag}} = \beta_{I1} + \beta_{I2} t + \varepsilon_t$$

using the entire sample of investment data. Then the initial capital stock is computed as the cumulative sum of an infinite sequence of investments at this trend rate (allowing for depreciation) using the relation

$$K_{1963:1}^{\mathrm{Ag}} = \sum_{j=0}^{\infty} (1-\delta)^j I_{t-j}^{\mathrm{Ag}} \approx \exp(\hat{\beta}_{I1}) \sum_{j=0}^{\infty} (1-\delta)^j \exp(-\hat{\beta}_{I2}\, j)$$

The remaining elements of the capital stock series are computed recursively from the capital accumulation identity $K_t^{\mathrm{Ag}} = \exp(-\delta) K_{t-1}^{\mathrm{Ag}} + I_t^{\mathrm{Ag}} \approx (1-\delta) K_{t-1}^{\mathrm{Ag}} + I_t^{\mathrm{Ag}}$ using the investment series $\{I_t\}_{1963:1}^{1996:1}$. The resulting I/K series is pictured in Fig. 2.

It should be noted that capital stock computed in this way makes no allowance for disposals. Official UK capital stock data are estimated on an annual basis by the ONS; they adopt the permanent inventory model as above but impose straight line depreciation with retirements distributed randomly around the mean asset life. Without access to asset level data on acquisitions by ICC over the sample period, computation of a disposals sequence in this way is infeasible. Thus the data are inconsistent with the real world unless investment is completely irreversible. However, this may not be too extreme an assumption since resale of equipment to

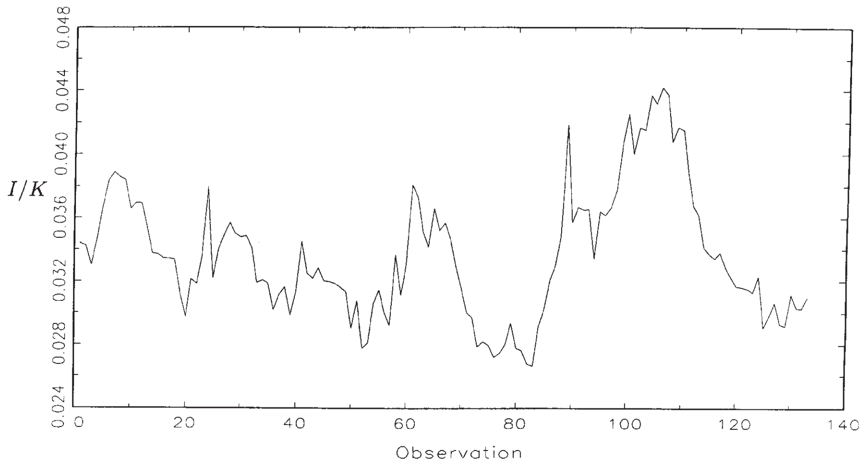

FIG. 2 UK ICC Investment Capital Ratio

other firms within the ICC grouping are not counted, and the data record purchases of *new* equipment. Thus disposals would only comprise sales of equip ment to non ICC firms and scrappings. It is not clear that either of these flows is sizeable, since asset specificity may reduce cross sector sales and scrap values are so low as to deter large scale scrapping activity. A full treatment of this issue would require a wealth of data at a considerably lower level of aggregation, which are not available to the author. Two solutions, given the available data, are (i) proceed as if investments are completely irreversible, and (ii) admit the possibility of (and model) disposals and assume that the investment data available comprise acquisitions less disposals (excluding depreciation). Both these solutions leave the model misspecified, but for standard parameterizations of the strength of growth (and depreciation) relative to uncertainty, the disposals series is small (less than one tenth of the acquisitions series), suggesting that this modelling approximation is of second order importance compared with the existence of non convex adjustment costs.

APPENDIX C

This section documents the Vuong statistic used to compare two non nested models α and β. Define the log likelihoods for models α and β as $\mathcal{L}_\alpha$ and $\mathcal{L}_\beta$, and the number of parameters in each model as k_α, k_β. If T denotes the number of obser vations, then the test statistic is

$$V_{\alpha,\beta} \quad \frac{\mathcal{L}_\alpha \quad \mathcal{L}_\beta \quad \frac{1}{2}\left(k_\alpha \quad k_\beta\right)\ln T}{(\hat{S}T)^{1/2}} \sim \mathrm{N}(0, 1)$$

where $\hat{S}$ is the Newey West estimate of the variance of the time series of likelihood differences $d_t \equiv \mathcal{L}_{\alpha,t} \quad \mathcal{L}_{\beta,t}$

$$\hat{S}(T, q) \quad \rho_0 + 2\sum_{i \;\, 1}^{q}\left(1 \quad \frac{i}{q+1}\right)\rho_i$$

and ρ_i is the ith order sample autocorrelation of the d_t series. q is set to 4, since $\hat{S}(T, q)$ does not vary much for larger q. The numerator of the Vuong statistic corrects for differences in the number of degrees of freedom in each model.

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