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Two Cheers for the Aggregated (S, s) Model!

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Abstract

Aggregated $(S; s)$ models purport to provide a structural, microfounded and statistically robust explanation of aggregate investment fluctuations. In this paper I analyse these claims, present several empirical puzzles arising from the model and discuss how the model might be extended to account for these puzzles.

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Two Cheers for the Aggregated (S; s) Model !

1 Introduction.

The analysis of economic fluctuations has been an important and enduring field of research in macroeconomics. In particular, economists have searched for structural explanations of differences between the short-run and long-run responses of economic aggregates to exogenous impulses. Costly adjustment is one feature to which researchers commonly appeal; it provides a structural propagation mechanism through which realistic sluggish responses may be generated. Recent advances have extended the analysis (of aggregate dynamics) to incorporate non-convex adjustment costs at the individual level, leading to (S; s)-type adjustment rules, Caplin (1985), Caballero and Engel (hereafter CE) (1991). The empirical models developed in this literature typically involve estimating microeconomic parameters from aggregate data, Bertola and Caballero (hereafter BC) (1990), CE (1993) and (1999).

Interest in this structural approach reflects the outstanding empirical performance of the model, which stands in stark contrast to the ongoing failure of earlier models. This is nowhere more true than in the literature on investment - the focus of this paper - see for example Chirinko (1993). In particular, the aggregated (S; s) framework a) provides a structural account of the non-linearities in investment expenditures; b) captures a channel through which individual heterogeneity matters for aggregate dynamics; c) reconciles lumpy, intermittent stock adjustment patterns at the individual level with smooth aggregate activity; d) explains the presence of lagged quantity variables in investment equations.

Most recently, CE (1999) embed the aggregated (S; s) model in a formal statistical framework. They impose sufficient structure on the microeconomic problem to obtain an aggregate model in which there is an invertible relationship between aggregate investment and the aggregate (forcing) shocks. By making explicit distributional assumptions about these shocks, they are able to write down a likelihood function for the model. The structure of the likelihood function is close to that which would be obtained for a linear autoregressive model, augmented with terms re‡ecting the nonlinear features of the economic model. Thus CE (1999) estimate and make inference about the model parameters directly from the correlation structure of aggregate investment data. They argue that these statistical foundations dominate those of earlier models such as BC (1990), CE (1993), since the nonlinear univariate scheme circumvents problems of constructing reliable measures of the cost of capital.[1]

In short, the aggregated $(S;s)$-model appears to be an unusually successful contribution to the study of economic ‡uctuations with sound economic structure and statistical foundations. Any structural model is only as good as its underlying economic and statistical assumptions. However, analysis of the speci…cation of the aggregated $(S;s)$ model has been limited. Until CE (1999) provided a formal statistical methodology there had been no attempt to clarify and test the statistical assumptions; moreover, their analysis of the such aspects of model speci…cation is incomplete. Analysis of economic aspects of the model speci…cation has until now been limited to discussion of the plausibility of the parameter estimates of the microeconomic adjustment problem; other auxilliary assumptions made in constructing the model are not considered. In particular, CE (1999) make assumptions about the nature of …rms' labour inputs; I therefore proceed to examine the behaviour of the labour input resulting from the estimated aggregate $(S;s)$ model of investment, and document a number labour market puzzles that need to be solved. In short it appears that other frictions, besides non-convex adjustment costs may have an important role to play in explaining aggregate investment ‡ucuations.

The rest of the paper is organised as follows. In Sections 2 and 3 I outline a model of a …rm and the aggregation framework. In Section 4 I replicate CE (1999)'s results with a

1 Below I document further statistical problems of the earlier empirical work on aggregated $(S;s)$ rules.

modified data set, and analyse the statistical specification of the model. In section 5 I derive analytic expressions for labour input variables, and use these to analyse the labour market performance of the model. Section 6 concludes.

2 Model.

Below I outline an aggregated (S,s) model. This follows CE (1993), (1999).

2.1 Microeconomic Framework.

In common with much of the literature on adjustment costs in factor demand, see Hamermesh and Pfann (1996), the dynamic behaviour of capital stock, K, can be decomposed into a target capital stock, K^*, and an imbalance variable, Ω. In logarithms[2]

$$\theta \equiv \ln \Omega \equiv \ln \frac{K}{K^*}.$$

Fluctuations in investment activity depend on the behaviour of both the target and the imbalance. The target level reflects the optimal long-run response of capital stocks to current innovations - once the short run effect of the frictions and rigidities such as adjustment costs have died away. The elements of this decomposition can be computed from a firm's optimisation problem and under certain assumptions can be identified in the data.

Suppose a firm has a Cobb Douglas production function in which output, Q, depends on the quasi-fixed factor, capital, K, and the fully and costlessly flexible factor, labour, L, and technology, A,

$$Q_t = A_t K_t^{\alpha} L_t^{1-\alpha} \quad ; \alpha \in (0,1).$$

Assume the firm faces a downward sloping demand curve, in which output price, P, is a constant elasticity function of output and an index of economic conditions, Z:

$$P_t = \left(\frac{Z_t}{Q_t} \right)^{\frac{1}{\eta}} \quad ; \eta \in (1, \infty).$$

[2] To avoid cumbersome wording, I call the log-imbalance variable, θ, the 'imbalance'.

Then current net operating revenues are given by

$$\pi(A_t; K_t; L_t; Z_t; w_t) = \max_{K;L} \{P_t Q_t - w_t L_t - (r + \delta) K_t - \Omega\} \tag{1}$$

where w represents wages, r is the interest rate, δ is the rate of depreciation, Ω represents the fixed cost incurred by adjusting capital stock: $I_t = 0 \Rightarrow \Omega = 0$, $I_t \neq 0 \Rightarrow \Omega > 0$.[3]

The target capital stock, K^*, can be derived by setting adjustment costs at zero and maximising the operating revenues with respect to L & K. The target can be expressed in terms of the exogenous variables $A_t; Z_t$ and w_t and the parameters r, δ, α, η:

$$K_t^* = \left[\frac{\alpha}{(1-\alpha)(r+\delta)}\right]^{1+\alpha(\eta-1)} \left[(1-\alpha)\frac{(\eta-1)}{\eta}\right]^{\eta} A_t^{(1-\alpha)(\eta-1)} w_t^{-(1-\alpha)(\eta-1)} Z_t. \tag{2}$$

In this expression long-run target capital stock is decreasing in interest rates, wages, and the rate of depreciation and increasing in technology and demand. Assumptions about the processes driving the forcing variables, A, Z, w will carry through to K^*. Specifically, if these variables follow geometric random walks then K_t^* inherits this property.

In principle the effects of individual driving processes on the target variable could be estimated directly - this is the strategy in BC (1990) and CE (1993). In contrast, CE (1999) argue that their univariate framework is superior to the earlier two-step approach to estimating aggregate (S; s) models since no measure of the cost of capital need be constructed to estimate the shocks. Moreover, this univariate framework only requires that a distinction be drawn between aggregate and idiosyncratic shocks to K^*- the precise origin of the shocks (to technology, wages etc.) is of no consequence. Denote the statistically independent aggregate and idiosyncratic shocks to target capital stock as v_t and ε_t respectively. For any firm the

[3] CE (1999) use a reduced form for net operating revenues (absent adjustment costs): $R(K; \Theta) = \Theta K^{\beta} - (r + \delta) K$: $\beta < 1$; their Equation (1). Revenues are determined by capital stocks and a variable, Θ, representing the combined effect of technology, demand and wages. The functional forms assumed in the text generate CE's Equation (1) and are given in their footnote 7. Under such assumptions their variable θ (here Θ) equals $\left[\frac{\eta}{(1-\alpha)(\eta-1)}\right]^{\frac{-\eta}{1+\alpha(\eta-1)}} \left[\frac{1+\alpha(\eta-1)}{(1-\alpha)(\eta-1)}\right] A_t^{\frac{(1-\alpha)(\eta-1)}{1+\alpha(\eta-1)}} w_t^{\frac{-(1-\alpha)(\eta-1)}{1+\alpha(\eta-1)}} Z_t^{\frac{1}{1+\alpha(\eta-1)}}$, while $\beta = \frac{\alpha(\eta-1)}{1+(\eta-1)}$.

target capital stock is given by $K_t^* = K_{t-1}^* e^{v_t + \varepsilon_t}$. In the absence of adjustment actual capital stock falls at the rate of depreciation, $K_t = K_{t-1}e^{-\delta}$, so the evolution of the imbalance variable is given as $\Delta\theta_t = \Delta \ln \frac{K_t}{K_t^*} = -(\delta + v_t) - \varepsilon_t$.

2.2 Aggregating (S,s) Rules.

The implications for modelling aggregate dynamic behaviour under non-convex adjustment costs are straightforward. Smooth aggregate activity can be reconciled with observed behaviour at the individual level only when firms exhibit heterogeneity that leads their investment decisions to be imperfectly synchronised. Consequently the empirical framework must abandon the representative agent approach. In the aggregated (S,s) model this is captured through a) heterogeneous imbalances across firms, and b) heterogeneous inaction bands across firms. These are consistent with microeconomic evidence, see e.g. Eberly (1994).

Heterogeneous imbalances are captured through the cross-section density function, $f(\theta, t)$. This represents the fraction of firms, with accumulated shocks θ, in respect of which adjustment has yet to occur. In other words it incorporates the history of such shocks. When uncertainty is Markovian, the impact of history is entirely captured in the current realisation of this density function. A tractable representation of heterogeneous adjustment triggers was introduced by CE (1993), who assume that the probability that a unit, i, adjusts during a given time period depends on the magnitude of the imbalance . This structure is represented by an adjustment function, $\Lambda_j \equiv \Lambda_j(\theta)$, (for all units $j \in (0,1)$), which determines the probability that unit i adjusts, during a given time interval, as a function of the imbalance, see Figure (1), where, the probability of adjustment increases smoothly in the absolute value of the deviation.[4] When all firms face the same adjustment function $\Lambda_j(\theta) \equiv \Lambda(\theta); \forall j \in (0,1)$,

[4] CE (1999) develop explicit microfoundations for the adjustment function. They generalise the standard (S,s) framework to encompass adjustment costs which are i.i.d. for individual firms across time, according to a gamma distribution. They show that the adjustment function for an individual firm, $\Lambda_j(\theta)$ is differentiable and increasing and $\lim_{|\theta|\to\infty} \Lambda_j(\theta) = 1$.

the latter represents the fraction of firms (in the economy) which adjust as a function of the imbalance. In what follows it is assumed that all firms face the same adjustment function.[5]

Suppose that when a firm adjusts it completely eliminates any capital imbalance: $\theta = 0$ after any adjustment. Then, when a firm, j, with imbalance $\theta_j = \bar\theta$, adjusts at time t, its capital stock changes by $I_{j,t}\left(\bar\theta\right) = K^*_{j,t} - K_{j,t}\left(\bar\theta\right) = \left(e^{-\bar\theta} - 1\right) K_{j,t}\left(\bar\theta\right)$. Expected investment by firm j, with imbalance $\bar\theta$ is

$$E\left[I_{j,t}\left(\bar\theta\right)\right] = \left\{I_{j,t}\left(\bar\theta\right) \mid \text{Adjustment}\right\} \Pr(\text{Adjustment}) = \left(e^{-\bar\theta} - 1\right) K_{j,t}\left(\bar\theta\right) \Lambda_j\left(\bar\theta\right)$$

Industry level investment, in industry $i \in \{1,\ldots,N\}$, by those firms, $j \in (0,1)$ with imbalance $\theta_j = \bar\theta$, is $\bar I_{i,t}\left(\bar\theta\right) = \int_{j\in} \left(e^{-\bar\theta} - 1\right) K_{j,t}\left(\bar\theta\right) \Lambda_j\left(\bar\theta\right) dj = \left(e^{-\bar\theta} - 1\right) \bar K_{i,t}\left(\bar\theta\right) \Lambda\left(\bar\theta\right)$ where $\bar K_{i,t}\left(\bar\theta\right)$ is the average capital stock of those firms with imbalance $\bar\theta$. Industry level investment activity is computed using the adjustments undertaken by the proportion of agents at capital imbalance, θ, by summing over all possible imbalances, $\theta \in (-\infty,\infty)$. Denote the proportion of agents at any θ, at time t, immediately prior to adjustment by the industry cross-section density function, $\tilde f_i(\theta,t)$. Total investment in industry i is

$$I_{i,t} = \int_{\theta=-\infty}^{\infty} \left(e^{-\theta} - 1\right) \bar K_{i,t}(\theta) \Lambda(\theta) \tilde f_i(\theta,t)\, d\theta$$

Assuming that the average capital stock is independent of the imbalance,[6] the investment capital ratio is given by the approximation

$$\frac{I_{i,t}}{K_{i,t}} \approx \int_{\theta=-\infty}^{\infty} \left(e^{-\theta} - 1\right) \Lambda(\theta) \tilde f_i(\theta,t)\, d\theta. \tag{3}$$

2.3 Investment Dynamics

Equation (3) highlights the key role played by the cross-section distribution of capital imbalances, $f(\theta,t)$, in determining the investment-capital ratio. As Froote et al. (2000) stress,

[5] Since the adjustment function is assumed to be identical across agents, statements about probability of action at the individual level translate into statements about fraction of agents undertaking action at the aggregate level.

[6] Using American firm level data, Caballero, Engel and Haltiwanger (1996) confirm that the average capital imbalance, $\bar K_t(\theta)$, is largely independent of the extent of the imbalance, θ.

non-convex adjustment costs and individual heterogeneity only affect aggregate dynamics to the extent that there is some mechanism to co-ordinate individuals' actions. In the aggregated $(S;s)$ model, aggregate shocks perform this role by shifting the cross-section density, $f(\theta;t)$; with respect to the adjustment function, $\Lambda(\theta)$ thereby changing the fraction of agents who adjust.[7] Fluctuations in investment depend on the evolution of this density function, which in turn depends on the shocks, $v_{i;t}+\varepsilon_t$ and the magnitude and direction of adjustment as prescribed by $\Lambda(\theta)$.

To track the evolution of the density function and fluctuations of the investment-capital ratio in a discrete time framework, the following timing convention is adopted describing the 3 events that occur within time period t. Define the end of period $t-1$ cross-section density for industry i as $f_i(\theta;t-1)$. The first event during period t is taken to be the effect of depreciation and the industry aggregate shock, $v_{i;t}$. This alters the imbalance of all firms in the industry by $-(\delta+v_{i;t})$. Define the density function following these events and immediately preceding adjustment as $\tilde{f}_i(\theta;t)$. The next period t event is the adjustment decision. The fraction of adjustments at each imbalance is determined by the adjustment function $\Lambda(\theta)$. The final event in period t is that each unit is subject to an idiosyncratic shock, ε, drawn from the density function $g_\varepsilon(\cdot)$. The end of period t density is defined as $f_i(\theta;t)$. Those units in industry i that end period t with imbalance θ can 'arrive' at that location regardless of whether or not they adjust during period t. In particular, since adjustment results in the complete elimination of the imbalance, the density $f_i(\theta;t)$ consists of a) the fraction of units in the industry (across all θ) which do adjust during period t and subsequently receive an idiosyncratic shock of $-\theta$, plus b) the fraction of those agents which do not adjust at imbalance $\theta+\varepsilon$ and subsequently receive a shock of exactly $-\varepsilon$ to leave

[7] Generally this feature only makes a noticeable difference to aggregate behaviour for large aggregate shocks, or around turing points. This explains why the aggregated $(S;s)$ model outperforms linear models in brisk expansions and contractions.

them with end of period t imbalance of θ (summed over all realisations of shock ε). This timing convention is summarised in equations (4) and (5).

$$\tilde{f}_i(\theta,t) = f_i(\theta+\delta+v_{i,t},t-1) \tag{4}$$

$$f_i(\theta,t) = \left[\int \Lambda(\psi)\tilde{f}_i(\psi,t)\,d\psi\right] g_\varepsilon(-\theta) + \int [1-\Lambda(\theta+\varepsilon)]\tilde{f}_i(\theta+\varepsilon,t)\,g_\varepsilon(-\varepsilon)\,d\varepsilon \tag{5}$$

Putting equations (4) and (5) together, the evolution of the cross-section density from one period to another and the expression for the industry level investment rate are, respectively

$$f_i(\theta,t) = \begin{array}{l} \left[\int \Lambda(\psi) f_i(\psi+\delta+v_{i,t},t-1)\,d\psi\right] g_\varepsilon(-\theta) + \\ \int [1-\Lambda(\theta+\varepsilon)] f_i(\theta+\delta+v_{i,t}+\varepsilon,t-1)\,g_\varepsilon(-\varepsilon)\,d\varepsilon \end{array} \tag{6}$$

$$\frac{I_{i,t}}{K_{i,t}} = \int \left(e^{-\theta}-1\right)\Lambda(\theta) f_i(\theta+\delta+v_{i,t},t-1)\,d\theta. \tag{7}$$

or $\frac{I_{i,t}}{K_{i,t}} = y_{i,t} = y_{i,t}(v_{i,t}, f_i(\theta,t-1))$. So conditional on knowing the initial cross-section density of imbalances, $f_i(\theta,0)$, both $\{f_i(\theta,t)\}$ and $\{y_{i,t}\}$ for industry i can be computed as a function of the aggregate and idiosyncratic shocks and the adjustment function: $y_{i,t} = y_{i,t}(v_{i,t},\ldots v_{i,1}, f_i(\theta,0))$.

3 The Econometrics of Aggregate (S,s) Rules.

Estimation of the model exploits the correlation patterns of aggregate investment rate data to infer the parameters of the adjustment function using a maximum likelihood procedure. In this section I outline assumptions that lead to a tractable likelihood function and discuss noteworthy features of the estimation procedure. An appendix describes computational aspects of the implementation.

Following the literature, the ergodic density function $\lim_{t\to\infty} f_i(\theta,t)$ is used to approximate the initial cross-section distribution, $f_i(\theta,0)$.[8] Assume that idiosyncratic shocks are

[8] CE (1999) show that the Markovian nature of the process driving the density function ensures the existence

$NIID\left(0;\sigma_{\epsilon}^2\right)$ across firms and time. Idiosyncratic and aggregate shocks are assumed to be independent: Let the adjustment function take the form[9]

$$\Lambda(\theta) = 1 - e^{-\lambda_2\theta^2} \quad ; \in (0;1) \quad ; \text{ such that } \lambda_2 > 0: \tag{8}$$

Aggregate shocks $v_{i;t}$, $i = 1;...;N$, $t = 1;...;T$ are assumed $NIID(\mu_V;C)$, where C is the cross-industry covariance matrix and μ_V is the $N \times 1$ vector of industry average shocks, of the column vector V: $(v_{1;1};v_{1;2};....;v_{1;T};....v_{N;1};.....;v_{N;T})'$. Defining l_T as a $(T \times 1)$ vector of ones, the joint probability density function for V is

$$q(V) = \frac{1}{(2\pi)^{\frac{NT}{2}}|C|^{\frac{T}{2}}}\exp\left\{-\frac{1}{2}(V - \mu_V \otimes l_T)'\left(C^{-1} \otimes I_T\right)(V - \mu_V \otimes l_T)\right\} \tag{9}$$

The joint density function of the aggregate investment rate series can be derived from a change of variables argument using equations (7) and (8). CE (1999) show that, if $y_{i;t}$; $\Lambda(\theta)$ and $v_{i;t}$ are defined as in equations (7), (8) and (9) respectively then $\frac{\partial y_{i;t}}{\partial v_{i;t}} \geq 0; \forall v_{i;t}$. It follows that the transformation in equation (7) is one to one and hence invertible. The density function of the aggregate investment series is:

$$J(Y) = \frac{1}{(2\pi)^{\frac{NT}{2}}|C|^{\frac{T}{2}}}\exp\left\{-\frac{1}{2}(V - \mu_V \otimes l_T)'\left(C^{-1} \otimes I_T\right)(V - \mu_V \otimes l_T)\right\}|J|:$$

where J is the Jacobian of the transformation and $Y = (y_{1;1};y_{1;2};....;y_{N;T})$. The log-likelihood function, L, is given by

$$-L = \frac{NT}{2}\ln 2\pi + \frac{T}{2}\ln|C| + \frac{1}{2}(V - \mu_V \otimes l_T)'\left(C^{-1} \otimes I_T\right)(V - \mu_V \otimes l_T) + \sum_{i=1}\sum_{t=1}\ln\left|\frac{\partial y_{i;t}}{\partial v_{i;t}}\right|$$

The estimation problem is further simplified by concentrating the likelihood function with respect to C and μ_V yielding

$$-L = \frac{T}{2}(1 + \ln 2\pi) + \frac{T}{2}\ln\left|\frac{(V - \mu_V \otimes l_T)(V - \mu_V \otimes l_T)'}{T}\right| + \sum_{i=1}\sum_{t=1}\ln\left|\frac{\partial y_{i;t}}{\partial v_{i;t}}\right|: \tag{10}$$

of an ergodic density function. They also show that the choice of initial density function has little effect on the results as it washes away within 3 periods (years) in their data. Below, the first 3 periods are excluded from the computation of the likelihood function. This is done to allow the effects of imposing the ergodic density function as the initial value to wash away without impacting on the parameter estimates.

[9] CE (1999) assume $\Lambda(\theta) = 1 - e^{-\lambda_0 - \lambda_2\theta^2}$: $\lambda_0;\lambda_2 > 0$. I set $\lambda_0 = 0$ to reduce the computational burden of the grid search procedure.

Thus the log-likelihood function consists of three components, a constant, a term measuring the variance of the aggregate shocks, and the Jacobian terms. The latter can be viewed as measuring the sensitivity of current investment in industry i to current shocks in that industry - and thus varies over time with the history of shocks captured in $f(\mu; t)$.

A discussion of how the components of the likelihood function vary as λ varies is useful in what follows. This discussion is most easily understood using Figure (2) which illustrates the relationship between investment and aggregate shocks described by equation (7) when the cross-section density is in steady state. As λ rises the mean and variance of the aggregate shocks fall. This is because a rise in λ raises the probability of adjustment (at all levels of imbalance), therefore a) a lower (average) shock is required to generate a given level of investment and b) aggregate shocks have to be less volatile in order to generate a given degree of volatility in aggregate investment. Since the volatility of the shocks declines as λ rises, the variance term declines which raises the value of the likelihood function, while the sensitivity terms rise, which reduces the value of the likelihood function. This is illustrated in Figure (3).

4 Results For US Manufacturing Data.

In this section I replicate CE's (1999) study with revised capital stock data and consider statistical aspects of the model specification. The data used in this study are Bureau of Economic Analysis industry level annual capital stock data for 21 US manufacturing industries over the period 1947-97. A model-consistent treatment of depreciation is one of a number of features of the revised data that make it more attractive than that used in CE (1999). Further details of the dataset are described in the appendix.

Table (1) displays the maximum likelihood estimates of microeconomic adjustment parameters of the aggregated $(S; s)$ model. These results confirm the insight of the generalised

(S; s) model that adjustment is increasingly likely as the extent of the capital imbalance increases. Figure (1) illustrates the estimated adjustment function. Adjustment behaviour is clearly nonlinear: some 10% of firms adjust when the (absolute) imbalance reaches 0.225, 50% adjust when the imbalance reaches 0.55 and 90% adjust when the imbalance reaches 1.05.

Tests of statistical misspecification revolve around the assumed properties of the aggregate shocks, in particular the assumptions of normality and of temporal independence. The second property appears particularly important, since the Markovian property of the shocks was crucial in obtaining a convenient form for the empirical implementation of the model - otherwise it would have been necessary to use more information than is available in the current cross-section density, $f(\mu; t)$. Surprisingly, CE (1999) pay scant attention to temporal independence, and implicitly accord non-normality greater importance.[10]

With regard to the assumption of temporal independence, Table (2) reports industry level Box-Ljung Portmanteau Statistics for the aggregate shock data in levels and in squares. Statistically significant values for the latter would indicate presence of ARCH type effects in the aggregate shock data. Results are presented for correlations at four lags (years), other lags led to qualitatively similar results but are not reported for brevity. For only 3 of the 21 industries does aggregate shock data, in levels, display temporal dependence, at the 5% significance level. Five industries exhibit statistically significant temporal dependence in the squared values of the shocks at the 5% level. As Box-Ljung Portmanteau tests have notoriously low power, I also test directly for first order serial correlation of the shock series. It turns out that none of the 21 industry level first order serial correlation coefficients is statistically significant at the 5% level ($2/\sqrt{T} \simeq 0.283$) these results are documented in Table (3). Finally exploiting the panel nature of the shock data, I estimate the model

[10] They do make reference to an earlier working paper version of their article. There they document "very little" (sic !) serial correlation in the aggregate shock processes at industry level.

$v_{it} = \alpha_i + \phi_i v_{i,t-1} + \xi_{it}$ for each industry i and use the mean-group estimator of Pesaran et al. (1996). This estimator does not impose a common degree of serial correlation across industries.[11] No statistically significant evidence of temporal dependence is found using this approach either. The results are displayed in Table (4). Since temporal dependence is not widespread, this evidence is taken as broadly supportive of the aggregated (S,s) model.

Due to the presence of contemporaneous cross-industry correlation, tests of normality of the aggregate shocks may be better undertaken at the industry level, rather than presenting statistics which average across industries as CE (1999) do.[12] Industry level skewness and kurtosis statistics are presented in Table (5).[13] It is possible to reject the null hypothesis of an absence of skewness at the 5% level for 9 out of 21 industries, and to reject the null hypothesis of no excess kurtosis for 12 out of 21 industries. Moreover, if aggregate shocks for a particular industry exhibit skewness they are also likely to display excess kurtosis. One problem with moment based tests is that they have relatively poor power properties, Verbrugge (1997), yet this is likely to bias the results against rejecting normality. Clearly, this is compelling evidence of non-normality, but may not invalidate the model since, given the number of observations, desirable asymptotic properties of MLE may hold anyway.

5 The Economic Implications of (S,s) Rules

While a test of statistical misspecification is an important check on the internal consistency, it amounts to comparing the model against a rather arbitrary norm. It may be standard to assume that disturbances are normally distributed and temporally independent, but these

[11] The mean-group estimator ϕ_{MG} is obtained by runs OLS regressions $v_{it} = \alpha_i + \phi_i v_{i,t-1} + \xi_{it}$ separately for each $i \in N$ and averaging across groups, $\hat{\phi}_{MG} = \sum_i \frac{\hat{\phi}_i}{N}$. Under the assumption $\xi \sim NID\left(0, \sigma^2_\xi\right)$ the parameters ϕ_i are independently distributed across groups. The variance of this estimator is consistently estimated across groups as $\hat{V}\left(\hat{\phi}_{MG}\right) = \frac{1}{N(N-1)} \sum_i \left(\hat{\phi}_i - \hat{\phi}_{MG}\right)^2$.

[12] They reject the hypothesis that aggregate shocks are skewed (for equipment and structures separately), but are unable to reject the hypothesis that shocks exhibit excess kurtosis, at the standard 5% significance level.

[13] The skewness statistics are standardised skewness measures. The p-values are associated with the squares of these figures, which under the null hypothesis are distributed as $\chi^2(1)$, and indicate the probability that values greater than the square of the observed statistic could be obtained.

assumptions primarily ensure tractability of the likelihood function, while their violation principally affects the properties of the parameter estimates of adjustment costs, rather than casting doubt on the economic content of the model. The invest / wait / scrap behaviour at the individual level and the smoothing properties of aggregation would be unlikely to break down simply because the aggregate shocks fail to obey these distributional assumptions. In short, these distributional assumptions have little economic rationale. The most interesting specification issues surround the economic content of the model, which is the subject of this Section. Performance along two dimensions is considered: a) the model's account of aggregate investment fluctuations; and b) the implied labour market behaviour.

5.1 Aggregate Investment Dynamics

Consistent statistical specification and plausible microeconomic parameter estimates are encouraging, but the real content of the aggregated (S,s) model must come from its ability to account for aggregate fluctuations. Inability of the model to explain investment fluctuations would suggest a role for other frictions, beyond costs of adjusting capital equipment. This issue can be addressed using the (within-sample) forecast of investment.[14] With a regression equation for a linear AR(1) model this would involve computing $\hat{y}_{i,t} = \hat{a}_0 + \hat{a}_1 y_{i,t-1}$, since, as a condition of estimation, the residual terms sum to zero. As the aggregated (S,s) model is nonlinear, and has a non additive error term, estimation does not require that the aggregate shocks sum to zero. The shocks are chosen, without restriction as to their average value, by inverting equation (7). The within sample forecast of the aggregated (S,s) model is

$$\hat{y}_{i,t} = \int \left(e^{-\theta} - 1\right)\left(1 - e^{-\hat{\lambda}_2\theta^2}\right) f_i\left(\theta + \delta + E_{t-1}[v_{i,t}]; t-1\right) d\theta. \tag{11}$$

[14] CE (1999) find that the mean square error of the out-of-sample 1-step ahead forecast is lower for the aggregated (S,s) model than a linear (AR(2)) alternative. However, this is unsurprising since nonlinearities in aggregate investment are well documented. It seems more appropriate to gauge the absolute performance of the aggregate (S,s) model.

This says that the predicted value reflects information contained in the cross-section density function, which represents the history of shocks to which firms have yet to adjust. A large difference between y and $\hat{y}$ suggests that (the unexpected component of) aggregate innovations play the key role in the good performance of the model, small differences indicate that the history of accumulated shocks described by the cross-section density function, $f(\mu, t)$, are of principal importance in describing investment fluctuations. I use sample average of aggregate shocks, $\hat{v}_i = \sum_t \frac{v_{it}}{T}$, as an estimate of $E_{t-1}[v_{i,t}]$ in evaluating equation (11).[15]

With the predicted values in hand, I present two metrics of the model's performance. The correlation between actual and predicted investment capital ratio, $\rho_{y,\hat{y}}$, is presented by industry in Table (6) along with the correlation of actual and predicted growth rates of this ratio (ie the correlation of $(\hat{y}_{i,t} - y_{i,t-1})/y_{i,t-1}$ and $(y_{i,t} - y_{i,t-1})/y_{i,t-1}$), which I denote $\rho_{g_y, g_{\hat{y}}}$. The (industry level) correlations are not particularly high, at around 0.6 and those for growth rates is more variable suggesting that the model does not experience universal success in explaining the direction of movement of the investment capital ratio. Nonetheless, the results suggest that the aggregated (S, s) model goes a good deal of the way towards capturing investment fluctuations.[16]

The results of Table (6) put in perspective the extremely high goodness of fit measures which supported earlier studies, BC (1990) and CE (1993) and which initially fostered interest in the aggregated (S, s) model and serve to highlight the statistical problems of this earlier work. There the use of goodness of fit as the criterion for estimating the adjustment function parameters may have lead to overfitting of the data even as the estimators fail to possess

[15] An alternative definition of $E_{t-1}[v_{i,t}]$ is the estimate of the average shock corresponding, through equation (7), to the (sample) average value of $\hat{y}_i = \sum_t \frac{y_{it}}{T}$. This is not the same as the sample average of the aggregate shocks, $\hat{v}_i = \sum_t \frac{v_{it}}{T}$, both because the relationship between the investment capital ratio and the aggregate shock is nonlinear for a given cross-section density (this follows from Jensen's inequality), and because the cross-section density fluctuates over time. It is not clear that either of these definitions is wrong, but the sample average of the realised shocks is simple to compute.

[16] Even so, if the prediction errors were forecastable using lagged values of the investment capital ratio and other variables, this would suggest that fixed adjustment costs, through the aggregated (S, s) model provided an incomplete account investment behaviour. This is the subject of ongoing research.

desirable properties such as unbiasedness or efficiency. Related to this, the non-standard statistical framework, and in particular the lack of knowledge of the sampling distribution has prevented appropriate test statistics being devised and inhibited formal comparison with alternative models.[17]

The statistics in Table (6) also highlight the fact that for all the structure of the model, it is the unexpected component of the aggregate shocks which enable the model to provide a successful account of investment fluctuations. It appears then that one way to assess the economic assumptions in the model would be to analyse the determinants of aggregate shocks. Aggregate shocks are assumed to be a combination of innovations in technology, demand and wages: $v_{i;t} = b_0 + b_1 \cdot \ln A_t + b_2 \cdot \ln Z_t + b_3 \cdot \ln w_t$, which are assumed exogenous but left unmodelled. Unfortunately, it would be difficult to identify these components individually, since neither technology nor demand are directly observable.

5.2 Labour Market Puzzles.

There are other key economic assumptions that are more amenable to examination. To get some impression of how well the model captures firms' decisions one can examine its implications for labour input fluctuations. The issue here is that the success of the account of investment fluctuations may imply implausible labour input behaviour. To address this one can exploit some of CE's implicit auxilliary assumptions which were made explicit in

[17] An alternative approach, pursued by Caballero et al. (1996), is to construct aggregate dynamics directly from microeconomic evidence on individual adjustments. They show that microeconomic adjustment is non-linear in a manner which they argue is consistent with (S; s) rules and that this nonlinearity carries over to aggregate dynamics. Unfortunately, the data they use exhibits so much heterogeneity that they are forced to allow not only for heterogeneous trigger thresholds and heterogeneous imbalances, but also for heterogeneous return-points. They attribute this return-point heterogeneity to noise, yet it could reflect other frictions, rather than pure (S; s)-rule effects - for example under investment might be due to financial constraints. A more appropriate response, given the richness of microeconomic data, might have been to estimate the (S; s) model at the microeconomic level and test its implications. As they neither estimate nor test the implications of an (S; s) model using the microeconomic data, so it is unreasonable to attribute either individual or aggregate behaviour purely to (S; s) rules. In addition, they assume both that adjustment frictions matter only for investment, and not for other factors of production, and that the production and demand functions are multiplicatively separable (in technology, demand, capital and labour). Thus they are likely to face the same problems as outlined in Section (5.2).

Section 2. In particular, it was assumed that there are two factors of production, that the production function is of Cobb Douglas form and exhibits CRTS, that the demand function is of constant elasticity form, and that the parameters of demand and production functions are time invariant. These assumptions yield the convenient property that the expressions for price and quantity in the revenue function are multplicatively separable in technology, demand, capital and labour variables: this is required to generate CE's equation (1): $\pi(K;Y) = YK^{\beta} - (r+\delta)K$.

Using the structure of CE's model (which follows from their Equation (1)), there are two ways to proceed: either labour must be assumed to be a fixed factor of production, or it must be perfectly flexible. Since labour input self evidently is not fixed over time, perfectly flexible labour input is adopted.[18] This would be justified if i) in the annual data, observations are sufficiently infrequent that all necessary labour input adjustment occurs before the next observation is made (this will hold if costs of adjusting employment are small); ii) hours per worker can vary, so that even if there are costs of adjusting the number of workers, these do not need to be incurred to adjust labour input.[19]

With these assumptions, it is possible to compute the demand for labour implied by the model. From Equation (1) the first order condition for optimal labour input can be computed as:

$$L_t^* = \left[\left(\frac{\eta}{(\eta - 1)(1 - \alpha)} \right)^{-\eta} w_t^{-\eta} Z_t A_t^{\eta - 1} K_t^{\alpha(\eta - 1)} \right]^{\frac{1}{1+\alpha(\eta-1)}} : \tag{12}$$

Using equation (2) to eliminate the unobservable A and Z terms gives

$$L_t^* = \frac{(1-\alpha)}{\alpha} (r+\delta) \frac{\left[K_t^* K_t^{\alpha(\eta-1)} \right]^{\frac{1}{1+\alpha(\eta-1)}}}{w_t} : \tag{13}$$

---

[18] Were one modelling labour demand, it might be appropriate to consider capital stock fixed (at least as a first approximation). In our case, perfectly flexible labour input is the least unacceptable approximation.

[19] I do not argue that costless adjustment is a characteristic of labour markets, but merely highlight what is being assumed in the model in order the better to assess the model specification.

From the decomposition $K_t = K_t^* \Theta_t$, it follows that

$$L_t^* = \frac{(1-\alpha)}{\alpha}(r+\delta)\frac{K_t^*}{w_t}\Theta_t^{\frac{\alpha(\eta-1)}{1+\alpha(\eta-1)}}$$

Taking logs, lagging and differencing gives the following expression for the percentage change in labour input as a function of aggregate shocks, v_t and innovations in real wages $\Delta \ln w_t$:

$$\begin{aligned} \Delta l_t^* &= \Delta k_t^* + \frac{\alpha(\eta-1)}{1+\alpha(\eta-1)}\Delta\theta_t - \Delta \ln w_t; \\ &= \gamma_0 + \gamma_1 v_t + \gamma_2 \Delta \ln w_t \qquad (14) \end{aligned}$$

where $\gamma_0 = -\frac{\alpha(\eta-1)\delta}{1+\alpha(\eta-1)}$, $\gamma_1 = \frac{1}{1+\alpha(\eta-1)}$ and $\gamma_2 = -1$.

A formal test of the assumptions of the model is obtained by regressing changes in labour input on the aggregate shocks and wage innovations and testing the restrictions on γ_0, γ_1 and γ_2 implied by the theory.[20] Rather than employment, an appropriate measure of labour input is total hours worked (per annum), L_t^*. Industry level data on hours worked and on real wages, can be computed using the Bureau of Labour Statistics series for 19 industries. As for the parameters η, r I use the values adopted by CE (1999): $(6.5; 0.06)$. For α and δ, I use 0.3 and 0.8 respectively, reflecting the fact that the revised capital stock data refers to equipment and (lower depreciation rate) structures jointly. Since v_t is not directly observed, but computed as v_t^*, from equation (7), given an estimate of λ, a regression of Δl_t^* on v_t^* and $\Delta \ln w_t$ is subject to a generated regressors problem, which interferes with inference. To circumvent this I use an instrumental variables approach.

In Section 2 it was assumed that aggregate shocks, v_t, represented innovations in wages, technology and demand variables. Data is available on the first of these, but neither of the others is directly or separately observable. However, demand and technology shocks will be reflected in changes in industry output, Q_t, suggesting that output and wages are theory

[20] An alternative, and more complex, approach would be to write down the likelihood function for the multi-factor demand model, under the assumption that capital is quasi-fixed and labour input is flexible.

consistent instruments for aggregate shocks. Finally, note that aggregate shocks are linked to the investment capital ratio through equation (7). This suggests using i/k as an instrument. However, current i/k will presumably be correlated with the disurbance term that arises from the generated regressor problem. This, and the smoothness of aggregate investment data suggests that lagged i/k would be a more appropriate instrument. Thus the first stage regression of v^* on instruments is $v_t^* = a_0 + a_1 \Delta \ln w_t + a_2 \Delta \ln Q_t + a_3 i/k_{t-1} + \xi_t$. The predicted values from this regression $\hat{v}_t$ are used in the second stage regression equation of $\Delta l_t^* = \gamma_0 + \gamma_1 \hat{v}_t + \gamma_2 \Delta \ln w_t$. Results, by industry, are presented in Table (7). The first and second columns presents R^2 statistics and F statistics for the joint significance of the instruments in the first stage regression. The third and fourth columns presents R^2 statistics for the second stage regression and F statistics for the (joint) restrictions $\hat{\gamma}_0 = -\frac{\alpha(\eta-1)\delta}{1+\alpha(\eta-1)}$, $\hat{\gamma}_1 = \frac{1}{1+\alpha(\eta-1)}$ and $\hat{\gamma}_2 = -1$ The restrictions are rejected at the 5% level for all but five industries. This suggests that one can reject the structural assumptions of the aggregated $(S;s)$ model with regard to the labour market.[21]

In order to understand why the model fails this specification test, and to give direction to attempts to remedy these flaws, I compare the behaviour of actual labour market data with that implied by the model. In what follows I denote the actual labour input and labour input implied by the model as L_t and L_t^* respectively.

Note that the aggregated $(S;s)$ model implies that labour input is

$$L_t^* = \frac{(1-\alpha)}{\alpha}(r+\delta)\frac{K_t}{w_t}\Gamma_t^{\frac{-1}{1+\alpha(\eta-1)}}. \tag{15}$$

Equation (15) states that, under the (auxilliary) assumptions of the $(S;s)$ model, implied

[21] However five of the industries for which rejection of flexible labour market assumption occurs are those where the instruments are only weakly correlated with v_t^*. On the other hand inference from straightforward OLS estimates of Δl_t^* on v_t^* and $\Delta \ln w_t$ and a constant, overwhelmingly rejects the flexible labour market restrictions in all but one industry. Nonetheless, these issues suggest that there may be merit in adopting a systems approach such as estimating a multifactor demand model directly and testing the restrictions implied when capital is quasi-fixed and labour costlessly flexible.

labour input is an increasing function of captal employed (and interest rates and depreciation), but a decreasing function of wages and of the average capital imbalance. To understand the last feature note that £ > 0 and that a small value of £ corresponds to a signi…cant dearth of capital. In such a situation desired employment rises (to allow product demand to be satis…ed).[22] It is possible, therefore, to compare the properties of actual industry level data on labour input with those implied by the model through equation (15). The results of this approach suggest that the labour marlet behaviour implied by the model is widely at variance with reality in both the short-run and long-run.

First compare the behaviour of the trend components of actual and implied data. Figures (4) and (5) show that there is little or no trend in total hours in the actual labour input data, yet the series implied by the model displays high growth. This is con…rmed in Table (8) which documents the growth rate of actual and implied labour market data, L and L¤, for the manufacturing sector as a whole. From equation (15) the aggregated (S; s) model requires that labour demand is increasing in capital stock, and decreasing in wages and the average capital imbalance. The data reveal that, consistent with the theoretical assumptions, the latter is a stationary variable exhibiting no trend.[23] Thus growth in the real wage and capital stock drive growth in implied labour input. Since actual wages in US manufacturing industry have grown substantially less quickly than actual capital stock, see Table (8), implied labour demand has grown quickly. The features of this Table broadly carry across to individual industries.

One explanation for these problems lies with the functional forms used. The (assumed) CRTS Cobb-Douglas production function, with time-invariant parameters, implies constant factor shares of income. Yet actual labour share (for the whole manufacturing sector) de-

[22] The term $£_t$ is simply the (exponent of the industry level) average cross-section capital imbalance at time t. The density of imbalances is updated (in equation (6)) during estimation so $£_t$ is a by-product of estimation.
[23] In the data, the (time series) mean of the cross-section imbalance, $\bar{£}$ is 0:856. This varies between 0:839 and 0:878 across industries. Figures are available from the author on request.

creases over the sample period, while implied labour share increases, see Figure (6). In short, while this form of production function is analytically tractable it is unsuitable for use with industry level data. However, suitable functional forms require a respecification of the model since the absence of multiplicative separability will mean that CE's Equation (1) no longer holds. The severe analytic complication is likely to mean that estimates of microeconomic parameters are no longer recoverable from aggregate data.[24]

Next consider the cyclical components of the series. Implied labour input, L^* may capture the cyclical behaviour of the actual series, even though the average growth rates differ. To obtain comparable series, by industry, I compute the percentage change (log-difference) in actual and implied series, $L_{i;t}$ and $L^*_{i;t}$, at each point in time. Table (9) displays, by industry, the ratio of the standard deviations of these log-differenced series. These statistics show that L^*_i is, on average, less volatile than L_i. This is somewhat surprising, as one might expect the reverse result if absence of frictions allows L^* to absorb the full effect of innovations in the capital imbalance. This result appears to compound the problems of misspecification.

An alternative manifestation of the divergence of labour market assumptions in the model from reality is that the implied series L^* may react more quickly to shocks than does actual labour input L. This is consistent with the view that frictions prevent labour market adjustment. It is possible to get some evidence on this issue by considering the correlation structures of (the cyclical components of) both L^*_{t-i} and L_{t-i} with L_t.[25] Table (10) documents the relatively low contemporaneous correlation of the cyclical components of L and L^*. The cross-industry arithmetic average is 0.334. Figure (7) illustrates the positive serial correlation present in actual labour input using industry level data. Table (11) shows that

[24] One of the barriers to the adoption of non-convex adjustment costs has been the lack of analytical tractability. Since one major achievement of the aggregated (S; s) model lies in overcoming this hurdle, it seems inappropriate to use analytical tractability as an argument for using an inappropriate production function.

[25] I use a Hodrick-Prescott filter to decompose both actual and implied labour market data into trend and cycle components before comparing their correlation structure. The smoothing parameter is set to 100 for annual data.

implied labour input leads actual labour input by some two years - a time frame consistent with the impact of adjustment costs (or other frictions) on labour market input adjustment. Figure (8) indicates that this feature is present at the industry level. The evidence on cyclical behaviour suggests that economic mispecification might be corrected by incorporating frictions in adjusting labour input.

Taken together this evidence indicates that, there are substantive economic defects in the aggregated $(S; s)$ model as an explanation of US industry level factor demand dynamics, and indicates directions in which the model should be extended to remedy these defects.

6 Conclusion.

This paper has examined the economic and statistical foundations of the aggregated $(S; s)$ model. Although the model does not fail a range of tests for statistical misspecification, this amounts to meeting requirements of internal consistency against a conventional, if somewhat arbitrary statistical benchmark, selected with an eye to achieving a tractable computational structure for the likelihood function. The economic content of the model should be of greater interest in assessing its adequacy.

When economic aspects of the model are considered, its performance is less impressive. Although the model does quite well at forecasting aggregate investment dynamics, it is possible to reject the aggregated $(S; s)$ model's assumptions about the nature of the labour market. Two labour market puzzles were identified. The long-run trend component of implied labour, L^{*}, and of implied labour share are at variance with reality, suggesting that the functional form for the production and/or the demand function, the constant factor shares parameterisation or some other feature of the economic structure is inappropriate at the industry level. Turning to the short-run, features of the cyclical component of the implied labour series exhibit disrepancies from actual data. Since the aggregated $(S; s)$ model implicitly assumes flexible labour input, one might expect the volatility of the model generated data, to exceed that of actual data. Although this does not appear to be the case, the model-generated labour input leads actual labour input, which does appear consistent with the view that actual labour input exhibits greater rigidity than is present in the aggregated $(S; s)$ model. In addition formal tests reject the restrictions imposed by the assumption that the labour market can adjust costlessly. Extensions suggested by these defects are the subject of ongoing research.

References

[1] Bertola G. and R. J. Caballero, (1990), 'Kinked Adjustment Costs and Aggregate Dynamics', in O. J. Blanchard and S. Fischer, Eds., NBER Macroeconomics Annual, 237-88.

[2] Blinder A. S., (1981), 'Retail Inventory Behaviour and Business Fluctuations', Brookings Papers on Economic Activity, 443-505.

[3] Caballero R. J. and E. M. R. A. Engel, (1991), 'Dynamic (S; s) Economies', Econometrica, 59, 1659-86.

[4] Caballero R. J. and E. M. R. A. Engel, (1993), 'Microeconomic Adjustment Hazards and Aggregate Dynamics', Quarterly Journal of Economics, 108, 697-711.

[5] Caballero R. J. and E. M. R. A. Engel, (1994), 'Explaining Investment Dynamics in U.S. Manufacturing: A Generalised (S; s) Approach', NBER Working Paper 4487.

[6] Caballero R. J. and E. M. R. A. Engel, (1999), 'Nonlinear Aggregate Investment Dynamics: Theory and Evidence', Econometrica, 67.

[7] Caballero R. J., E. M. R. A. Engel and J. Haltiwanger, (1996), 'Plant Level Adjustment and Aggregate Investment Dynamics', Brookings Papers on Economic Activity.

[8] Caplin A. S., (1985), 'The Variability of Aggregate Demand with (S; s) Inventory Policies', Econometrica, 53, 1395-1410.

[9] Chirinko R. S., (1993), 'Business Fixed Investment Spending: A Critical Survey of Modelling Strategies, Empirical Results and Policy Implications', Journal of Economic Literature, 31, 1875-911.

[10] Doms M. and T. Dunne, (1998), 'Capital Adjustment Patterns in Manufacturing Plants', Review of Economic Dynamics, 1, 409-429.

[11] Eberly, J. C., (1994), 'Adjustment of Consumer Durable Stocks: Evidence From Automobile Purchases', Journal of Political Economy, 102, 403-36.

[12] Froote, C., E. Hurst, and J. Leahy (2000), 'Testing the (S; s) Model', American Economic Review, Papers and Procedings, 90, 116-9.

[13] Hamermesh D. S. and G. A. Pfann, (1996), 'Adjustment Costs in Factor Demand', Journal of Economic Literature, 34, 1264-92.

[14] Jorgenson, D. W., (1996), 'Empirical Studies of Depreciation', Economic Enquiry, 24, 24-42.

[15] Pesaran, M. H., R. Smith and K. S. Im, (1996), 'Dynamic Linear Models for Heterogeneous Panels', in L. Matyas and P. Sevestre, Eds., The Econometrics of Panel Data, Kluwer Academic, London.

[16] Verbrugge R. (1997), ' Investigating Cyclical Asymmetries', Studies in Nonlinear Dynamics and Econometrics, Vol. 2, pp. 15-22.

	Estimate
¸	2:469 (0:781)
Likelihood	3868.169

Table 1: Adjustment Function Parameters
Standard Errors in Parentheses,
computed by quadratic approximation.

Industry	Q_4^{levels}	$Q_8^{squares}$	Industry	Q_4^{levels}	$Q_4^{squares}$
Lumber	0:583	1:075	Food products	14:162¤	35:561¤
Furniture	5:097	5:850	Tobacco products	5:602	15:770¤
Stone, Glass	0:203	2:775	Textile mill products	1:402	11:073¤
Primary Metal	2:837	2:238	Apparel/textile prods.	0:344	8:312
Fabricated Metal	0:142	7:221	Paper	0:622	4:291
Industrial Machinery	0:018	3:238	Printing & Publishing	0:622	22:682¤
Electronic Equipment	0:265	5:756	Chemicals	4:464	1:819
Motor Vehicles	0:727	2:902	Petroleum & Coal	3:164	3:490
Transport Equipment	12:132¤	2:911	Rubber and Plastics	1:436	1:817
Instrumentation	4:835	3:817	Leather	22:919¤	23:419¤
Miscellaneous Durable	7:724	4:239	-	-	-

Table 2: Temporal Dependence of Shocks by Industry.
Under the null hypothesis of no temporal
dependence, these Box Ljung Portmanteau Statistics
follow a Chi-square distribution with 4 degrees of freedom.
¤ indicates statistical signi…cance at the 5 percent level.

Industry	Serial Correlation	Industry	Serial Correlation
Lumber	0.052	Food products	-0.206
Furniture	-0.140	Tobacco products	-0.140
Stone, Glass	0.0289	Textile mill products	-0.077
Primary Metal	0.114	Apparel/textile prods.	-0.036
Fabricated Metal	0.025	Paper	0.052
Industrial Machinery	0.009	Printing & Publishing	0.048
Electronic Equipment	0.035	Chemicals	0.147
Motor Vehicles	-0.059	Petroleum & Coal	0.121
Transport Equipment	0.234	Rubber and Plastics	-0.082
Instrumentation	0.149	Leather	-0.271
Miscellaneous Durable	-0.192	-	-

Table 3: Serial Correlation Statsistics by Industry.

Statistic	Estimate
ρ_{MG}	-0.00809 (0.0249)

Table 4: Dynamic Panel Model Estimates.
Standard errors in parentheses;
p-value for Hausman Statistic

Industry	Skewness	Excess Kurtosis	Industry	Skewness	Excess Kurtosis
Lumber	-1.924	0.224	Food prods.	-3.21^{*}	7.767^{*}
Furniture	-4.573^{*}	7.420^{*}	Tobacco prods.	-0.837	5.095^{*}
Stone, Glass	-1.198	-0.161	Textile mill prods.	-1.799	2.749^{*}
Primary Metal	2.042	1.720	Apparel prods.	-0.205	1.166
Fabricated Metal	0.229	0.864	Paper	-0.374	0.071
Industrial Machry.	-0.010	2.263^{*}	Printing & Publish	-3.851^{*}	4.135^{*}
Electronic Equipmt.	-3.925^{*}	7.528^{*}	Chemicals	1.394	1.076
Motor Vehicles	-4.321^{*}	5.112^{*}	Petroleum & Coal	2.638^{*}	3.927^{*}
Transport Equipmt.	0.399	0.370	Rubber & Plastics	-2.977^{*}	4.785^{*}
Intrumentation	-1.849	0.046	Leather	-2.897^{*}	6.394^{*}
Misc. Durable	3.236^{*}	7.445^{*}	-	-	-

Table 5: Normality Tests by Industry.
Standardised Skewness and Excess Kurtosis measures.
* indicates a statistically significant rejection of normality at the 5 percent level.

Industry	½$_{y;\hat{y}}$	½$_{g_y;g_{\hat{y}}}$	Industry	½$_{y;\hat{y}}$	½$_{g_y;g_{\hat{y}}}$
Lumber	0:614	0:462	Food products	0:542	0:849
Furniture	0:495	0:977	Tobacco products	0:599	0:886
Stone, Glass	0:651	0:478	Textile mill products	0:658	0:930
Primary Metal	0:656	0:351	Apparel/textile prods.	0:567	0:667
Fabricated Metal	0:610	0:441	Paper	0:633	0:489
Industrial Machinery	0:611	0:478	Printing & Publishing	0:643	0:681
Electronic Equipment	0:638	0:755	Chemicals	0:592	0:445
Motor Vehicles	0:561	0:999	Petroleum & Coal	0:571	0:434
Transport Equipment	0:701	0:466	Rubber and Plastics	0:591	0:676
Instrumentation	0:702	0:473	Leather	0:470	0:996
Miscellaneous Durable	0:346	0:758	-	-	-

Table 6: Correlation of Actual and Predicted Investment by Industry.
Industry level correlation coe¢cients for i) actual and predicted values of the investment capital ratio, ii) actual and predicted values of the growth rate of that ratio.

Industry	R^2	F	R^2	F
Lumber	0:434	10:980¤	0:587	13:004¤
Furniture	0:361	8:092¤	0:631	9:108¤
Stone, Glass	0:281	5:603¤	0:434	10:803¤
Primary Metal	0:008	0:119	0:200	0:921
Fabricated Metal	0:395	9:369¤	0:492	10:518¤
Industrial Machinery	0:116	1:881	0:731	0:864¤
Motor Vehicles	0:261	5:074¤	0:480	11:183¤
Transport Equipment	0:062	0:946	0:071	6:294¤
Miscellaneous Durable	0:467	12:563¤	0:032	25:461¤
Food products	0:277	5:497¤	0:272	70:703¤
Tobacco products	0:048	0:717	0:018	34:321¤
Textile mill products	0:065	0:991	0:496	28:229¤
Apparel/textile prods.	0:074	1:147	0:044	17:182¤
Paper	0:160	2:737	0:456	31:302¤
Printing & Publishing	0:330	7:050¤	0:228	22:766¤
Chemicals	0:080	1:253	0:096	21:588¤
Petroleum & Coal	0:175	3:036¤	0:126	44:591¤
Rubber and Plastics	0:043	0:643	0:419	4:204
Leather	0:109	1:760	0:083	5:955

Table 7: Tests of Labour Market Assumptions.
Columns 1 and 2 : R^2 and F-statistics, F(47 ¡ 1; 4), for joint signi…cance of instruments.
Columns 3 and 4: R^2 for second stage regression and F-Statistics, F(47 ¡ 1; 3),
of the restrictions $°_0 = \frac{¡±®(´¡1)}{1+®(´¡1)};\ °_1 = \frac{1}{1+®(´¡1)};\ °_2 = ¡1$ in the regresion
$¢ l_t = °_0 + °_1 + °_2 ¢ \ln w_t + u_t$,
¤ indicates signi…cance at the 5 per cent level.

Series	Growth Rate
L	0.0023
L^*	0.0258
Y	0.0176
K	0.0349
w	0.0087

Table 8: Annual Growth Rates for the Whole Manufacturing Sector

Industry	$\frac{\text{Implied}}{\text{Actual}}$	Industry	$\frac{\text{Implied}}{\text{Actual}}$
Lumber	0.448	Food products	1.880
Furniture	0.491	Tobacco products	1.096
Stone, Glass	0.601	Textile mill products	0.489
Primary Metal	0.417	Apparel/textile prods.	0.877
Fabricated Metal	0.431	Paper	0.946
Industrial Machinery	0.405	Printing & Publishing	1.220
Motor Vehicles	0.391	Chemicals	1.244
Transport Equipment	0.449	Petroleum & Coal	0.810
Miscellaneous Durable	0.859	Rubber and Plastics	0.569
Sector Average	0.747	Leather	0.572

Table 9: Relative Volatility of Implied and Actual Labour Input

Industry	$\rho(L_t, L^*_t)$	Industry	$\rho(L_t, L^*_t)$
Lumber	0.246	Food prods.	0.342
Furniture	0.580	Tobacco prods.	0.277
Stone, Glass	0.350	Textile mill prods.	0.226
Primary Metal	0.059	Apparel prods.	0.234
Fabricated Metal	0.514	Paper	0.234
Industrial Machry	0.330	Printing & Publishing	0.477
Motor Vehicles	-0.158	Chemicals	0.577
Transport Equipmt.	0.510	Petroleum & Coal	0.213
Misc. Durable	0.454	Rubber and Plastics	0.290
Sector Average	0.334	Leather	0.540

Table 10: Contemporaneous Correlations: Actual and Implied Labour Input Data

Lag	$\rho(L_{t-i}, L_t)$	$\rho\left(L^*_{t-i}, L_t\right)$
-3	-0.201	0.092
-2	-0.076	0.339
-1	0.413	0.462
0	1.000	0.324
1	0.389	-0.074
2	-0.061	-0.184
3	-0.176	-0.162

Table 11: Averaged Correlation Structure of Labour Input
Column 2 (3): Correlation of lag actual (implied)
labour input with current actual labour input.
Arithmetic averages of industry level correlation structures.

Appendix A

1 Computation of the Likelihood Function

Grid search over the parameters of the adjustment function is used for estimation.[26] Following CE (1999), for a given value of λ_2, the first step is to compute the ergodic distribution, which is used to approximate the initial cross-section density of imbalances, $f(\theta;0)$: This is achieved using equations (6) and (7) through the following steps. Assume that the ergodic density function results from an infinite investment rate sequence at the constant rate determined by the sample industry mean: $y_{i;\tau} = \bar{y}_i \left(\frac{1}{T} \sum_{t=1}^{T} y_{i;t} \right); \forall \tau \in -\infty;...;0$. Then starting from a prespecified imbalance density, $f(\theta;-\tau)$, (for simplicity assumed distributed $N(0;0.01)$), use equation (6) to compute, by grid search, the aggregate shock that gives the best approximation $\hat{y}_{-\tau} \simeq \bar{y}$. The aggregate shock, $v_{i;\tau}$ is allowed to vary with step size $0.01 \cdot \theta_j$. The estimated aggregate shock $v_{-\tau}$ can be used to update the density function through equation (7). Iterating forwards the density function is found to converge after some 10-15 iterations.

Finally, conditional on the estimate of the initial cross-section density function, the series of aggregate shocks are computed, individually, from equation (6) by grid search over $v_{i;t}$ as outlined in the previous paragraph. For each period the aggregate shock estimate v_t^* is used to update the density function, through equation (7), and also to compute the corresponding Jacobian through $\frac{\partial y_{i;t}}{\partial v_{i;t}}$.

[26] Given a parameter value, $\lambda_2 = \bar{\lambda}_2$; elements of the likelihood function are computed directly. This procedure makes extensive use of the equations (6) and (7) in updating the density function following each sequence of shocks and adjustment. All computations are undertaken using Gauss 3.2.31. The state space of imbalances, θ, is divided into a grid of 49 points. Because estimation requires that the evolution of the cross-section density be modelled explicitly (using the updating equations), and because the location of the density function depends upon the size and direction of aggregate shocks, it is vital that state space is sufficiently broad that mass does not 'escape' from the cross-section density. For the values estimated for the adjustment function parameters, the interval $[-40\sigma_\epsilon; 40\sigma_\epsilon]$ was used σ_ϵ is set to 0.1. This was found sufficient to prevent significant loss of mass at all but very low-values of adjustment cost. Note the grid used here is time invariant.

2 Data

The capital stock data used in this study are Bureau of Economic Analysis, (BEA), annual industry level data for US manufacturing (21 industries, since Motor Vehicles are separated from Transport Equipment) over the period 1947-97. A number of differences between this dataset and that used by CE (1999), are worthy of note. i) Unlike CE (1999), the industry level capital stock estimates are not decomposed into structures and capital equipment, this turns out to be useful in examining the implied labour market behaviour - see Section (4). ii) The extra five years data in the sample gives a further 105 more observations (1050 industry level observations in all). iii) Under the new BEA methodology, depreciation is assumed to occur at a constant geometric rate, whereas the data used in CE's study is constructed under the assumption that depreciation is of the straight line form with retirements distributed around the mean retirement date according to a Winfrey distribution. Thus this new methodology has the advantage that it is consistent with the theoretical treatment of depreciation adopted in the Section (2), and with the empirical evidence, which broadly favours accelerated depreciation over straight line form, Jorgenson (1995). The previous methodology captured disposals only through retirements, (estimated using the Winfrey distribution) whereas the new methodology ignores disposals altogether. Both approaches are approximations. The model will be data consistent only as long as disposals are small (in value and volume) in comparison with acquisitions. If this is the case, then recorded innovations in capital stock (acquisitions only in the new methodology; acquisitions less retirements in the old methodology), will approximately equal actual net investment (acquisitions less disposals).

Industry labour market and output data used in Section (5.2) are also taken from the BEA sources. Labour input used in empirical work is total hours worked per annum. This is constructed from average weekly hours data and annual employment data. The wage rate used is the hourly wage which is computed from the weekly hours and weekly wage data. The output and wage data are de‡ated using the (economy-wide) GDP de‡ator. Data is available for 19 industries, however data for Electronic and other electric equipment and Instruments and related products industries does not date back to 1947 and data for Other Transport Equipment industry must be computed from the whole Transport Equipment industry and Motor Vehicles Industry series using straightforward transformations.