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*Investment, Irreversibility, and Financial
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INVESTMENT, IRREVERSIBILITY, AND FINANCIAL IMPERFECTIONS[¶]

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Abstract

Research finds that firms' investment decisions are distorted by irreversibility and finance constraints. Whereas the existing literature examines the effects of these features separately, this paper studies their interaction. The impact of these constraints on a firm's incentive to invest is characterised using option pricing techniques. Financial constraints reduce the initial capacity, raise the marginal value product of capital and the value of the option to invest.

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1 Introduction.

The consensus view is that the frictionless neoclassical model of investment behaviour is flawed. Recent research focuses on two themes, technological and financial constraints, to explain observed investment behaviour, see Dixit and Pindyck (1994) and Hubbard (1998) for surveys. However, while much progress has been made, virtually all the existing literature considers the impact of one such constraint in isolation.¹ This is problematic because each constraint is used to explain similar features of investment activity,² yet the causes of each and hence policy recommendations to which each leads are substantially different. In this paper I take a step towards integrating these two features of investment behaviour. I introduce a financial constraint into a model of irreversible investment under uncertainty and characterise the impact on the firm's investment policy using both a q-type approach and an option pricing framework. This does not permit analysis of how the constraints themselves might interact, but does allow us to consider how the presence of both constraints affects the incentive to invest.³

The basic insight of the option pricing approach is that, when investment is (completely) irreversible, involves sunk costs and may be postponed until some uncertainty has been resolved, the standard net present value rule becomes invalid. The firm must take account of the fact that by undertaking investment, it exercises an option, since this option is valuable, it must be incorporated into capital budgeting calculations. Increases in the level of uncertainty raise the value of this option to invest, and so reduce the incentive to invest. This literature has developed to allow for features such as alternative market structures etc. see inter alia Dixit and Pindyck

¹ Two recent empirical studies formally control for the interaction of the two constraints: Scaramozzino (1997) shows, for a panel of UK firms, that q-theory holds only for the subset of firms for which both irreversibility and financial constraints are unlikely to be present. Guiso and Parigi (1999), using cross-section data on Italian manufacturing firms, find that the impact of uncertainty on investment, is consistent with irreversibility, controlling for the existence of financial constraints.

² Irreversibility and financial constraints may each account for several empirical regularities: the history dependence of investment decisions; the existence of hurdle rates for investment and periods of inactivity (threshold effects and nonlinearities); the dominance of quantity variables over price variables in investment equations; the volatility of aggregate investment (both why it is so high and why it is lower than that predicted by a frictionless model).

³ In the real options approach, with its focus on the dynamic resolution of uncertainty, constraints are imposed exogenously. Even though informational asymmetry, in the form of a lemons problem in secondary markets, is a frequently cited potential source of irreversibility, Dixit and Pindyck (1994), it is standard practice to take the irreversibility as given and concentrate on its consequences rather than model the role of informational asymmetries explicitly. Financial constraints are also seen as arising from informational asymmetries, Hubbard (1998), but as the focus here is on the consequences of the resolution of uncertainty, it is more convenient to avoid explicit treatment of the informational asymmetries, take the constraint as given.

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(1994). Many of the results in this literature have been established in the context of continuous time models, however the key insights can be conveyed and generalised more conveniently in a much simpler setting. For instance Abel et al. (1996) use a two period model to demonstrate the relation between option pricing and q theoretic approaches, in a paper which extends the canonical model to allow for costly expandability as well as incomplete reversibility. Below I adopt a two-period framework as a convenient means of extending the canonical irreversible investment model to allow for financial constraints.

One paper related to these issues is by Vercammen (2000). He considers the impact of a particular form of financial constraint, the threat of bankruptcy, on an irreversible investment decision. He assumes that investment must be financed by debt, and that the lender may foreclose if the value of debt is greater than the value of the firm's assets. He shows that, if the contribution to the probability of bankruptcy arising from additional investment is increasing in the level of debt held, the firm will prefer to delay investment when the (initial) probability of bankruptcy is high. This behaviour has a natural option value interpretation. Whereas Vercammen (2000) considers the decision to invest in a discrete project, this paper focusses on the more realistic and elaborate problem of optimal capacity choice. Vercammen deliberately removes the standard features of the option pricing model so as to establish the separate option value arising from the probability of bankruptcy. Yet this makes direct comparison of the results difficult. The integrated treatment of the two features of investment behaviour provided below allows optimal investment policy to be characterised analytically and, importantly, facilitates direct comparison with canonical model of irreversible investment.

A simple two-period model of firm investment behaviour is outlined in the next section and the impact of introducing financial constraints into irreversible decisions is analysed using q_i theoretic and option pricing frameworks. The next Section considers the impact on the option value multiple. The impact of changes in the distribution of returns on the incentive to invest is discussed in the penultimate Section. The final Section contains a conclusion and suggestions for future research.

2 Optimal Capacity Choice.

This section illustrates the distinct roles played by irreversibility and financial constraints in a dynamic model of investment under uncertainty. I use a simple two period framework that incorporates only the necessary features: second period returns are stochastic and investment is irreversible but may be postponed, the firm's decisions may be affected by the availability of funds. Behaviour is characterised first in terms of a q_i type measure, and then using option pricing techniques.

2.1 The Model

Suppose that a firm exists for two periods. It inherits cash (wealth) with value X_1 in period 1, but is unable to raise external finance in period 1. This may be due to the presence of informational asymmetries between the firm and potential lenders⁴, but for current purposes, rather than model the origins of the financial constraint, it is more convenient to take the existence of this constraint as given and discuss its consequences. In period 1 the firm installs capital K_1 irreversibly, at unit cost P , subject to the constraint $X_1 \geq P K_1$, and receives total return $r(K_1)$, where $r'(K_1) > 0$; $r''(K_1) < 0$. In period 2 the firm's return to capital is given by $R(K; e)$, where e is stochastic, and represents the level of demand (and other factors affecting the profitability of the firm). It is assumed that all uncertainty is captured in e . Thus once e is known all uncertainty is resolved. For simplicity it is assumed that any informational asymmetry is also resolved by period 2 so that financial constraints only bind in period 1.⁵ It is assumed that the period 2 marginal revenue product of capital, $R_K(K; e) \geq 0$, continuous and strictly decreasing in K and continuous strictly increasing in e . Finally assume that the price of capital equipment remains constant through time.⁶

Given these assumptions, optimal investment policy in the second period is characterised by a threshold value of $e = \hat{e}$, such that

$$R_K(K_1; \hat{e}) = P; \quad (1)$$

⁴ For instance, Hellwig and Stiglitz (2000) demonstrate conditions under which a firm may be jointly equity and credit constrained.

⁵ It would be perfectly possible to extend the financial constraint to the second period, for example by imposing the additional constraint $P(K_2 \leq K_1) \leq X_2 - P K_1 + r(K_1)$. It isn't clear that this adds insight.

⁶ Allowing for imperfect expandability, time varying P , while not difficult, would add little to the main results.

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where $\delta e > \hat{e}$, investment occurs at unit cost P , until a new optimal capacity level $K_2(e)$ is attained, such that $R_K(K_2(e); e) = P$, and $\delta e \leq \hat{e}$ no investment occurs: $K_2(e) = K_1$:

The value of the firm, $V(K_1)$, with capacity K_1 in period 1 is the net present value of expected revenues as given by equation (2)

$$V(K_1) = r(K_1) + \int_{\hat{e}}^{\infty} R(K_1; e) dF(e) + \int_{\hat{e}}^{\infty} [R(K_2(e); e) - P] dF(e); \quad (2)$$

where δ is a discount factor. The firm's period 1 problem is to maximise this value through (irreversible) choice of the initial capacity, subject to the finance constraint

$$\max_{K_1} V(K_1) - \lambda (X - P K_1) \geq 0; \quad (3)$$

There are two cases, depending on whether or not the finance constraint binds.

Case 1. $X > P K_1$: This is the standard irreversible investment problem, where the financial constraint does not bind and is a special case of the model analysed by Abel et al. (1996). The optimal capital stock in period 1 under the pure irreversibility constraint alone, denote this as K_1^I , is given by the condition

$$V'(K_1^I) = r'(K_1^I) + \int_{\hat{e}_1}^{\infty} R'(K_1^I; e) dF(e) - P [1 - F(\hat{e}_1)] = 0 \quad (4)$$

where \hat{e}_1 is the threshold value of e at which investment occurs under the pure irreversibility constraint. This provides a baseline against which the combination of irreversibility and financial constraints can be compared.

Case 2. $X = P K_1$: Here the financial constraint binds. Write the firm's problem as

$$\max_{K_1} V(K_1) - \lambda (X - P K_1) \geq 0$$

where λ is the multiplier associated with the inequality constraint $X - P K_1 \geq 0$. Denote the optimal period 1 capital stock when the financial constraint binds as K_1^{FI} . This level is determined by the first order condition

$$V'(K_1^{FI}) = r'(K_1^{FI}) + \int_{\hat{e}^{FI}}^{\infty} R'(K_1^{FI}; e) dF(e) - P + \lambda = 0 \quad (5)$$

Where \hat{e}^{FI} is the threshold value of e at which period 2 investment occurs, when period 1 activity has been financially constrained.

2.2 An Interpretation

One way to understand the impact of the financial constraints is to re-write $V^0 i_{K_1^{F1}} \text{€}$ in terms of $V^0 i_{K_1^I} \text{€}$ and other terms. Since the impact of the financial constraint is over and above that of the irreversibility constraint $K_1^{F1} < K_1^I$. It follows that $R_K i_{K_1^{F1}} \text{€} > R_K i_{K_1^I} \text{€}$ $\forall e$. Since the threshold value of e at which period 2 investment occurs is $R_K (K_1^j; e) = P, j = 2, F1; I$, and as $R_K (K; e)$ is continuous and increasing in e it follows that $e_{F1} < e_I$. Using this information and equation (4) rewrite equation (5) as

$$V^0 i_{K_1^{F1}} \text{€} = r^0 i_{K_1^I} \text{€} + \int_{e_{F1}}^{e_I} (R_K i_{K_1^{F1}} \text{€} - R_K i_{K_1^I} \text{€}) dF(e) + \int_{e_I}^{\infty} R_K i_{K_1^I} \text{€} dF(e) + P \int_{e_{F1}}^{e_I} dF(e) + P \int_{e_{F1}}^{\infty} dF(e)$$

Hence

$$V^0 i_{K_1^{F1}} \text{€} - V^0 i_{K_1^I} \text{€} = \int_{e_{F1}}^{e_I} (R_K i_{K_1^{F1}} \text{€} - R_K i_{K_1^I} \text{€}) dF(e) + P \int_{e_{F1}}^{e_I} dF(e)$$

The difference in the marginal value product of capital in case 1 and case 2 arises from differences in the marginal revenues over the lifetime of the firm. The marginal revenue product of capital is greater in each period when the firm is financially constrained.

To see this note that since $K_1^{F1} < K_1^I$ it follows that $r^0 i_{K_1^{F1}} \text{€} - r^0 i_{K_1^I} \text{€} > 0$. The difference between the expected period 2 marginal revenue product of capital in the two cases consists of three elements, depending on whether the realisation e would trigger investment in both cases, neither case or only one case. Since $e_{F1} < e_I$ then for $e < e_{F1}$ the firm would choose not to invest irrespective of the presence or absence of financial constraints. Moreover since $R_K (K; e)$ is decreasing in K , so $\int_{e_{F1}}^{e_I} (R_K i_{K_1^{F1}} \text{€} - R_K i_{K_1^I} \text{€}) dF(e) > 0$. For $e > e_I$ the firm would invest choosing $K_2(e)$ such that $R_K (K_2(e); e) = P$, regardless of whether or not it had previously been financially constrained. Therefore the difference in the expected marginal revenue product of capital is zero for $e > e_I$. Finally when $e \in [e_{F1}; e_I)$, if the firm had been financially constrained it sets $K_2(e)$ such that $R_K (K_2(e); e) = P$; but otherwise leaves capital stock unchanged at K_1^I .

Since $R_K(t; \epsilon)$ is increasing in ϵ and $R_K(K_1^j; \epsilon_1) = P$; it follows that the difference in the case 2 and case 1 expected marginal revenue products over this interval, $\int_{\epsilon_1}^{\epsilon_2} [R_K(K_1^j; \epsilon) - P] dF(\epsilon)$ is also positive. As the sum of three non-negative terms, the overall difference between the expected period 2 marginal revenue product of capital in the financially constrained and unconstrained cases is positive.⁷

3 A Real Option Perspective

A different perspective on the impact of financial constraints can be obtained using the real option approach. The value of the firm can be decomposed into two components: i) the present value of profits accruing from current capital stock, ii) the present value of profits accruing from the option to invest once uncertainty has been resolved.

Define the value of the firm with capital stock K_1^j , $j = 2, \dots, J$, assuming that no further investment is permitted in period 2, as $G(K_1^j)$. That is,

$$G(K_1^j) = \int_0^{\infty} \frac{R(K_1^j; \epsilon)}{1+r} dF(\epsilon); \quad j = 2, \dots, J$$

Next define the (call) option to invest in each case as

$$C(K_1^j) = \int_0^{\infty} \max\{R(K_2(\epsilon); \epsilon) - P, 0\} \frac{dF(\epsilon)}{1+r}; \quad j = 2, \dots, J$$

The value of the firm is then $V(K_1^j) = G(K_1^j) + C(K_1^j)$; $j = 2, \dots, J$: Hence the marginal value product of capital, the incentive to invest, is

$$V'(K_1^j) = G'(K_1^j) + C'(K_1^j); \quad j = 2, \dots, J$$

Note that $G'(K_1^j)$ can be written in terms of $G(K_1^j)$ as

$$G'(K_1^j) = G(K_1^j) \left[\frac{R'(K_1^j; \epsilon)}{R(K_1^j; \epsilon)} \right] + \int_0^{\infty} \frac{R(K_1^j; \epsilon)}{1+r} \left[\frac{R'(K_1^j; \epsilon)}{R(K_1^j; \epsilon)} - \frac{R'(K_1^j; \epsilon)}{R(K_1^j; \epsilon)} \right] dF(\epsilon)$$

where the last two terms are positive, so that $G'(K_1^j) > G(K_1^j)$. Also $C'(K_1^j)$ can be written in terms of $C(K_1^j)$

$$\begin{aligned} C'(K_1^j) &= \int_0^{\infty} \frac{R'(K_1^j; \epsilon)}{1+r} dF(\epsilon) \\ &= C(K_1^j) \left[\frac{R'(K_1^j; \epsilon)}{R(K_1^j; \epsilon)} \right] + \int_0^{\infty} \frac{R(K_1^j; \epsilon)}{1+r} \left[\frac{R'(K_1^j; \epsilon)}{R(K_1^j; \epsilon)} - \frac{R'(K_1^j; \epsilon)}{R(K_1^j; \epsilon)} \right] dF(\epsilon) \end{aligned}$$

⁷ It is a simple matter to show that $\frac{\partial G}{\partial K_1} < 0$, $\frac{\partial C}{\partial K_1} > 0$ and $\frac{\partial V}{\partial P} > 0$.

Again the last two terms are positive. It follows that

$$V^0 i_{K_1^{FI}} = V^0 i_{K_1^I} + \int_{\hat{e}_1}^{\infty} \left[\frac{R_{i,1}}{r} \frac{\partial}{\partial e} i_{K_1^{FI}} + \frac{R_{i,1}}{r} \frac{\partial}{\partial e} i_{K_1^I} + \frac{R_{\hat{e}_1}}{\hat{e}_{F1}} \frac{\partial}{\partial e} i_{K_1^{FI}} \right] P^\alpha dF(e)$$

The incentive to invest (irreversibly) in period one is higher when the firm also faces financial constraints because of two effects. First, financial constraints increase in the marginal value product of capital installed, assuming that no future alterations in capital stock are undertaken. This is captured by $\frac{R_{i,1}}{r} \frac{\partial}{\partial e} i_{K_1^{FI}} + \frac{R_{i,1}}{r} \frac{\partial}{\partial e} i_{K_1^I}$, and arises because $K^{FI} < K^I$ when the financial constraint binds. Second, imposition of the additional burden of a financial constraint raises the value of the marginal call option to invest, which reduces the incentive to invest by increasing the payoff to delaying investment until the uncertainty and financial constraints are resolved. To see this note that, since $K^{FI} < K^I$, so for all $e > \hat{e}_1$ at which the firm facing just irreversibility constraints has the incentive to invest in period 2, it has an even greater incentive to invest when it faces the additional burden of financial constraints, $\frac{R_{\hat{e}_1}}{\hat{e}_{F1}} \frac{\partial}{\partial e} i_{K_1^{FI}} > 0$. Also, for values of $e \in [\hat{e}_{F1}, \hat{e}_1)$, the previously financially constrained firm has an incentive to invest which is not present if it had not earlier been subject to financial constraints, as captured in the term $\frac{R_{\hat{e}_1}}{\hat{e}_{F1}} \frac{\partial}{\partial e} i_{K_1^{FI}} > 0$.

The option pricing approach reveals that, the financial constraint, when binding, reduces the period 1 capital stock, this raises the incentive to invest by raising the marginal productivity of capital installed, although this effect partially offset by a rise in the marginal value of the call option to wait and invest in period 2.

3.1 The Option Value Multiple

A standard metric used to illustrate the impact of irreversibility on optimal investment decisions, under uncertainty, is the option value multiple. This hurdle rate feature determines the extent to which the marginal value product of capital installed (permanently) in period 1 must rise above the purchase price of new equipment. Under the additional burden of financial constraints, the

marginal (call) option to invest in period 2 rises, that is the marginal value of waiting increases, therefore to justify investment, the marginal value of (period 1) installed capital must be higher than P by an even greater multiple than under pure irreversibility constraints. The algebra is straightforward. Define the option value multiple for a firm with capital stock K to be $\hat{A} = \frac{G^0(K)}{P}$.⁸

Then

$$\begin{aligned} \hat{A}^{FI} &= \frac{G^0(K_1^{FI})}{P} = \frac{V^0(K_1^{FI}) + C^0(K_1^{FI})}{P} = 1 + \frac{C^0(K_1^{FI})}{P} + \frac{V^0(K_1^{FI})}{P} \\ &= 1 + \frac{C^0(K_1^I)}{P} + \frac{G^0(K_1^{FI}) - G^0(K_1^I)}{P} > 1 + \frac{C^0(K_1^I)}{P} = \hat{A}^I \end{aligned}$$

Under the additional burden of financial constraints, the option value multiple exceeds that under pure irreversibility precisely because the restriction on period 1 investment means that the marginal value of capital currently (and permanently) installed is higher.

4 Changes in The Distribution of Returns

One particular area of interest is the impact of changes in the distribution of e on the incentive to invest. First consider a rise in the mean value of e (a first order increase in the distribution of e).

Write $V^0(K_1)$ as

$$V^0(K_1) = r^0(K_1) + \int_{\hat{e}}^{\bar{e}} R_K(K_1; e) dF(e) + P \int_{\hat{e}}^{\bar{e}} dF(e):$$

An increase in the mean value of e produces the standard "bad news" effect, Bernanke (1983): unless the shift of the distribution is entirely confined to the ranges $[\hat{e}; \bar{e}]$, the incentive to invest will increase. The impact of the extra burden of financial constraints is straightforward: since $\hat{e}_{FI} < \hat{e}_I$, good news is a more likely phenomenon when uncertainty is resolved, but good news, doesn't affect the incentive to invest. In other words, there is an interval $(\hat{e}_{FI}; \hat{e}_I)$ of the support of $F(e)$ such that if the first order change in the distribution of e is entirely due to re-arrangement of probability mass within that interval, a firm facing only irreversibility constraints would have an increased incentive to invest, whereas a firm under irreversibility and financial constraints would face no such incentive.

Finally consider a second order change in the distribution of e , a mean-preserving spread, which

⁸ This is the "value of assets in place" version of q , see Dixit and Pindyck (1994), p. 147.

provides insight into the role of uncertainty in determining the level of investment. In general this change will have an ambiguous effect under irreversibility. The marginal value of capital already installed may rise or fall depending on whether $R_K(K_1; e)$ is a convex or concave function of e . The marginal value of capital installed rises if this function is convex in e and declines if it is concave. The option to invest will rise in value under such a shift, leaving the net effect ambiguous. Provided the point of crossing between the old and new distributions lies below \hat{e} , the value of the marginal call option increases. Since this decreases the incentive to invest in period 1 decreases. When the firm faces financial constraints these same effects are at work, and it remains indeterminate whether or not investment will rise.

5 Conclusion

This paper provides a simple analytical characterisation of a firm's investment policy under irreversibility and financial constraints in terms of real options, and illustrates the linkage to standard q_j theoretic treatments. Financial constraints accentuate irreversibility constraints, with the result that the firm is more wary of undertaking investment. The additional financial constraint leads the firm to adopt a higher option value multiple (hurdle rate) in investment decisions than a pure irreversibility constraint alone. Prior to the resolution of uncertainty, this reduces the capital stock that a firm is willing to hold, so that the marginal value product of capital is higher for such a firm. This was explained i) directly in terms of differences in the expected marginal revenue product of capital throughout the firm's life and also in terms of the option to invest. In the latter case, the presence of an additional finance constraint raises the value of the option to delay investment until uncertainty is resolved, although this effect is outweighed by the increase in the marginal value of capital currently installed (since the capacity is lower when the two constraints occur simultaneously).

Here the two constraints are imposed exogenously, and one could consider them as arising independently. However, it is clear that the constraints may well act to complement each other. Financial constraints can exacerbate the irreversibility constraint by making the firm more wary of undertaking investment, while irreversibility could exacerbate any tendency for firms to face

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...nancial constraints, since one would imagine that lenders would be less willing to lend to those firms who wished to undertake irreversible investments. This suggests that it would be fruitful to consider the impact of irreversibility of investment in a model of informational asymmetries in finance in order to allow the interaction of irreversibility and financial constraints to enter the analysis.

References

- [1] Abel, A. B., A. K. Dixit, J. C. Eberly and R. S. Pindyck, 1996, Options, the Value of Capital and Investment, *Quarterly Journal of Economics*, 111, 753-777.
- [2] Dixit, A. K. and R. S. Pindyck, 1994, *Investment Under Uncertainty*, Princeton University Press, Princeton, NJ.
- [3] Guiso, L. and G. Parigi, 1999, Investment and Demand Uncertainty, *Quarterly Journal of Economics*, 114, 185-227.
- [4] Hellmann, T. and J. Stiglitz, 2000, Credit and Equity Rationing in Markets with Adverse Selection, *European Economic Review*, 44, 281-304.
- [5] Hubbard, R. G., 1998, Capital Market Imperfections and Investment, *Journal of Economic Literature*, 36, 193-225.
- [6] Scaramozzino, P., 1997, Investment Irreversibility and Financial Constraints, *Oxford Bulletin of Economics and Statistics*, 59, 89-108.
- [7] Vercammen, J., 2000, Irreversible Investment Under Uncertainty and the Threat of Bankruptcy, *Economics Letters*, 66, 319-325.