Edinburgh School of Economics
Discussion Paper Series
Number 47

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Date
January 1999

Published by
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http://www.ed.ac.uk/schools-departments/economics

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Keywords: Excess returns, Trading rule, Efficient markets, Present value model, Stock prices.
JEL classification: G12; G14

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ABSTRACT

We contribute to the debate over whether forecastable stock returns reflect an unexploited profit opportunity or rationally reflect risk differentials. We test whether agents could earn excess returns by selecting stocks which have a low market price compared to an estimate of the fundamental value obtained from an econometric model. The criterion for stock picking is one which could actually have been implemented by agents operating in real time. We show that statistically significant, and quantitatively substantial, excess returns are delivered by portfolios of stocks which are cheap relative to our estimate of fundamental value. There is no evidence that the under priced stocks are relatively risky and hence excess returns cannot easily be interpreted as an equilibrium compensation for risk.

I Introduction

It is now widely accepted that cross-section stock returns can be forecast by the ratio of the current stock price to a number of accounting variables. For example it has been shown that returns can be forecast by the ratio of the market value to the book value of assets, Fama and French (1992), the price dividend ratio, Elton et al (1983), and the ratio of cash flow to market value of equity, Lakonishok et al (1994). Even the well known size effect, Banz (1981), falls into this class, since size is usually measured by stock price multiplied by the number of outstanding shares. However what remains unsettled is the question of whether the evidence that price scaled accounting variables can forecast returns indicates a rejection of efficient markets. Is this a genuine profit opportunity or do the return differentials instead rationally reflect risk differences between stocks?

This latter interpretation follows from the fact that risky stocks, which must offer higher expected returns, will inevitably have relatively low market prices relative to accounting variables like book value, current dividends, current earnings and the number of outstanding shares. In the case of the price dividend ratio this follows immediately from the Gordon growth model. Berk (1996) develops a formal model which demonstrates that in an efficient market both size and market to book will be proxies for risk and hence will forecast returns. Fama and French (1995) report empirical evidence for the efficient markets interpretation by showing that fluctuations in book to market are rational since they are correlated with subsequent earnings growth.

The interpretation that forecastable returns represent an economic profit opportunity follows from the hypothesis that there are irrational swings in market sentiment for stocks, see for example Shiller (1984). Although Shiller formulated a model of aggregate stock mispricing, the ideas easily generalise to individual stocks. In this model we would expect current accounting variables to serve as a proxy for fundamental value and hence we can identify under priced stocks as those with a low price relative to these stocks. Lakonishok et al (1994) develop this argument and present evidence to suggest that this is indeed the reason why these price scaled variables have forecasting power. Dechow and Sloan (1997) test a corollary of this model by showing that stock prices reflect analysts' earnings forecasts which have been shown to be irrational by LaPorta (1995) and Bulkley and Harris (1996).

The problem that two very different models of stock pricing imply the same empirical relationships in the data arise because current price is measured relative to a proxy for the fundamental which is independent of company risk. It is inevitable in this approach that risky companies will on average appear relatively under priced. In this paper we attempt to overcome the problem of observational equivalence by constructing an estimate of the fundamental value which reflects the risk of the stock. We test whether returns can be forecast by the ratio of current price, not an accounting variable, to an estimate of the fundamental value for the stock price calculated from an econometric model which explicitly allows for risk. This implies that it is no longer necessarily the case that risky stocks will on average have a low market price relative to the benchmark.

The fundamental value for a stock is estimated as follows. We assume the dividend process for individual companies is stationary in first differences. The present value model then implies that prices and dividends are co-integrated (Campbell and Shiller (1987)). We estimate the parameters of this cointegrating regression for each company in our sample. Our estimate of the fundamental stock price at any date is then obtained by substituting the stock's current dividend into the estimated co-integrating regression. Intuitively, co-integration means that in the long run there is a stable relationship between the dividend and the stock price for each company. Substituting the current dividend into the co-integrating relationship gives what one might informally describe as the stock price which one would expect to see in the long-run for that company, conditional on the current dividend.

The reason that the stock's risk premium is reflected in our measure of fundamental value is that stocks with high required rates of return will have on average a low price relative to their dividend. This will be captured in the parameters of the co-integrating regression, as shown in the next section. Thus our estimate of fundamental value, obtained from this co-integrating regression, will reflect the risk adjusted discount rate which the market is actually observed to apply to each stock over the sample period. There is therefore no need to assume a particular model of risk pricing. All that needs to be assumed is that however risk is measured and priced, the *relative* risk of a company is fixed within the sample period. It is possible for both the risk free rate and the average equity risk premium to be time varying, since these aggregate fluctuations will not affect the relative ranking of stocks using our mispricing measure. There is thus no reason to expect that risky stocks will necessarily appear on our criterion and therefore if under priced stocks are found to deliver higher returns this cannot so easily be explained away as an equilibrium compensation for risk.

This technique for identifying under priced stocks may be compared to the conventional criterion of the current dividend yield. This latter approach tests for excess returns on portfolios of stocks which have a high dividend yield relative to other stocks at the same date. Our approach will tend to select stocks which have a high yield not relative to other stocks but relative to their own past dividend yield. Our method is not exactly that, because we allow for a constant in the relationship between prices and dividends, but this description gives the comparative flavour of the two approaches.

In discussing returns forecasting it is important to ensure that the explanatory variables are public information at the beginning of the period over which returns are measured. We use a measure of fundamental value obtained from a co-integrating regression which is estimated using only the data available up to that same date that the forecast is made. In this way we are testing for the existence of a profitable trading rule which could have actually been employed by agents trading in real time. The same could not be said if the measure of the fundamental were obtained from a co-integrating regression where the parameters were estimated with the econometrician's benefit of hindsight of the whole data series. Our approach implies that the implicit discount rate used to price a stock is the historic average which has implicitly been applied to it.

We work with a sample of stocks listed on the New York Stock Exchange and test whether portfolios of the "under priced" stocks selected on the above criterion deliver excess returns over a one to ten year time horizon. There are two reasons for working with such a long returns period. First is the prior, which is confirmed in the results, that mispricing, which is the source of excess returns, is eliminated over years rather than months. Secondly, this long horizon allows us to study the whole life cycle of mispricing and thus relate our results to the evidence of negative serial correlation in long term returns reported by DeBondt and Thaler (1985).

We test for a correlation between subsequent returns and our mispricing measure in a number of ways. We first compare the absolute size of returns over one to ten years on portfolios of stocks which are overpriced to returns on portfolios of underpriced stocks. Finding a substantial difference in the size of the realized returns on these portfolios we next report tests of statistical significance. We use both parametric and non-parametric methods to evaluate the statistical significance of the out-performance of the under priced stocks. Finally, and notwithstanding the above arguments that our benchmark ratio should not be correlated with risk we examine whether there is any evidence that the profits which the stock picking criterion can be explained away by differences in risk.

In the next section we explain how we obtain an estimate of the fundamental value of individual companies. In Section III we describe the data and report our results, and in Section IV examine whether there is any evidence that the excess returns which we find simply reflect increased risk.

II An Estimate of the Fundamental Value of Stocks

We first show how the present value model implies prices and dividends are co-integrated, under the assumption that dividends are integrated of order one, or I(1).

The fundamental value for each firm at date *t*, $P_{i,t}^{*}$, is the expected value of future dividends, $d_{i,t+j}$, discounted by the company-specific discount rate d_i. Thus suppressing the i subscript:

$$P_t^{*} = \sum_{j=0}^{\infty} \mathrm{d}^{\,j+1} E_t\left[d_{t+j}\right] \qquad (1)$$

Subtract $\mathrm{d}P_t^{*}$ from both sides of this expression to obtain

$$P_t^{*} = \frac{\mathrm{d}}{(1-\mathrm{d})} E_t\left[d_t\right] + \frac{\mathrm{d}}{(1-\mathrm{d})} \sum_{j=1}^{\infty} \mathrm{d}^{\,j} E_t\left[\Delta d_{t+j}\right] \qquad (2)$$

If dividends are I(1) then:

$$\Delta d_{t+j} \equiv \mathrm{m} + \Delta \tilde{\tilde{d}}_{t+j} \qquad (3)$$

where m is the unconditional mean and $\Delta\tilde{\tilde{d}}_{t+j}$ is the deviation of dividend growth from the unconditional mean.

Then we can write:

$$P_t^* = \Theta d_t + \Theta^2 m + \Theta \sum_{j=1}^{\infty} d^{\,j} E_t\left[\Delta\tilde{\tilde{d}}_{t+j}\right] \tag{4}$$

where $\Theta = d/(1-d)$

$\Theta^2 m$ is the constant in the co-integrating regression, and the third term on the right hand side of (4) is the stationary residual. The co-integrating vector for each company, $\lfloor -1, \Theta^2 m, \Theta \rfloor$ can be seen to reflect both the company specific discount rate and its long term dividend growth rate. This ensures, as we indicated in the introduction, that when we calculate our estimate of the fundamental value appropriate allowance for risk is made.

We estimate (4) at each date when we wish to calculate a fundamental value for a company, using only price and dividend data up to and including that same date. The reason we do not simply estimate a single co-integrating regression for each company using the whole data set available to us is that this would implicitly give us the information advantage of hindsight not available to the actual investor in real time. As we explained in the introduction our aim is to test whether stocks are mispriced conditional on the information sets of agents who do not know the true parameters of the model, but have to estimate them from public data which is available at the date when the fundamental value must be calculated.

Our estimate of the fundamental value of a stock, $\hat{P}_{i,t}$, is then the co-integrating price obtained by substituting the current dividend into the co-integrating relationship estimated at that date. i.e. we calculate:

$$\hat{P}_{i,t} = \hat{\Theta}_{i,t} d_{i,t} + \left(\hat{\Theta}^2_{i,t} \hat{m}_{i,t}\right),$$

The subscript i,t denotes a parameter estimated at date t for a stock i.

In an efficient market price will differ from the co-integrating price as a consequence of rationally forecast temporary differences between dividend growth in the short-run and its long-run value, measured by the third term on the right hand side of (4). But this difference is public information and hence cannot forecast returns. Under the alternative hypothesis the fad will be an additional term on the right hand side of (7). The ratio of market price to co-integrating price will then be a measure of the fad or stock mispricing and hence should forecast returns if mispricing is temporary.

III Data and Results

Data on prices, dividends, Beta, and size were taken from CRSP (Centre for Research in Security Prices, University of Chicago). The dividend yield is measured by the total annual dividend payment in the calendar year ending December 31st divided by the stock price on December 31st. Size is market capitalisation measured in $10m. The co-integrating regressions are estimated using a price and dividend series for each stock which is adjusted for all forms of capital re-organisations, for example stock splits, using data provided by CRSP (coded FACPR).

The sample of companies studied in any year is all those which have been continuously listed on the New York Stock Exchange from December 1945 to that same date and paid dividends in at least 10 of these years. This sample choice reflects a trade-off between the number of companies included in the sample and the length of data available on each to estimate the co-integrating regression. The earlier the start date the more degrees of freedom for estimating the regression but the fewer the number of companies. We also wanted a “natural” definition of the sample to avoid a suspicion that the results were obtained by data mining. This suggested a start date of either December 1925, the beginning of data collected by CRSP, or beginning after World War II. Choosing the latter gives a much larger sample of firms, and still allows portfolios to be constructed at twenty three different dates and returns traced over a decade. We judged this to be preferable to estimating the co-integrating regression on a longer data set for those companies which were included , but working with the much smaller sample of companies. The final sample consisted of 580 firms in 1960, which had fallen to 432 in 1970, and 348 in 1980.

In view of the large number of companies, and data sets of different spans, we do not report test statistics for the time series properties of the individual dividend series, or for the company co-integrating regressions. The assumption that for the typical firm dividends are first difference stationary and that prices and dividends are co-integrated is a necessary condition for our procedure to successfully identify under priced stocks on average. Of course we will always make some mistakes in identifying “under priced” stocks and one reason for this may be that these assumptions are false for some companies. The critical issue is whether there is systematic mispricing which can be profitably identified by a strategy which requires these assumptions to be satisfied for the average company.

We measure mispricing by the ratio of current market price to the estimated fundamental price. We work with the ratio rather than the absolute difference between the two, because of the substantial variation across companies in the absolute size of their stock prices. All stocks are ranked by this ratio each year and then are assigned to Decile portfolios. The top and bottom Deciles are sub-divided into two. All companies in a particular portfolio are initially given equal weighting and thereafter there is no rebalancing of portfolios. Cumulative returns for individual stocks are calculated assuming dividends are re-invested in the same stock. Cumulative portfolio returns are the average of the cumulative returns of the individual stocks. Returns are everywhere real returns.

There is no problem of survivorship bias since we trace returns on all companies irrespective of their survival history after the sample selection year. In the case where companies disappeared from the data set for any reason, for example bankruptcy or merger, we assume any final payments to stockholders are reinvested in the same portfolio in the same proportions as the remaining companies occupy at that date.

We first report returns on five portfolios consisting of the full sample of firms, FS; the 5% most under-valued firms, UN1; the next 5% most under-valued firms, UN2; the 5% most over-valued firms, OV1; and the next 5% most over-valued, OV2. Portfolios are constructed, giving equal weight to all companies, for twenty three dates starting December 31st 1960 and then annually on the same date until December 31st 1982. For each of these twenty three start dates, the returns on portfolios were calculated for the subsequent decade. In Table 1 we report the average returns across the twenty three start dates, the returns are calculated for the subsequent decade.

In Table I it can be seen that on average the UN1 and UN2 portfolios both beat the full sample for all horizons, and UN1 also beats UN2 for all horizons. The expected excess returns are substantial. For example, the UN1 portfolio delivers an average cumulative excess return of 36.1% over five years, compared to the full sample. The UN1 portfolio beats the OV1 portfolio on average by 45.7% cumulative over five years, or approximately 9% per annum for five consecutive years.

Both of the over-valued portfolios, OV1 and OV2, on average under-perform the under-valued portfolios for all horizons. They also under-perform the full sample average for all horizons, except the first year in the case of OV1. The one somewhat surprising result is that OV1 delivers higher returns than OV2 for the first six years. We conjectured that this might be a result of the fact that the most over-valued portfolio contained a very large number of zero dividend companies. It is often argued that zero dividends are a signal of financial distress so that the well documented fact that zero dividend companies deliver higher returns, Christie (1990), can be explained as a risk premium. We therefore repeated the whole exercise omitting any companies which paid no dividend in the twelve months preceding the date of portfolio formation. The results of this exercise are reported in Table II.

In Table II it can be seen that OV1 now under-performs OV2 so that the relative performance of all four portfolios is consistent with our mispricing measure monotonically forecasting returns. The deletion of zero-dividend companies increases the relative over-performance of the most under-valued portfolio compared to the most over-valued. Over 5 years the cumulative differential between the two portfolios is now 57.2%. If it is correct that zero dividends signal temporary distress and therefore command a risk premium, then we might conclude that the appropriate risk adjusted excess return comparison between OV1 and UN1 should be based on the sample which excludes zero dividend companies.

Tracing returns for a decade contributes evidence on the long run life cycle of mispricing and allows us to relate our results to the work which has reported negative serial correlation in long run returns, for example DeBondt and Thaler

(1985). In order to focus on the life cycle of returns we report in Table III average incremental returns from holding each portfolio for more than one year. Table III reports results for the full sample including zero dividend companies. The evidence in Table III is consistent with, but suggests a qualification to, the evidence reported by DeBondt and Thaler that performance over a five year horizon is negatively correlated with subsequent returns. It is consistent in that an overvalued stock must have previously delivered high returns in the course of becoming over-valued and in this case high past returns will be followed by low subsequent returns. However high returns which are the result of the unwinding of of earlier underpricing are not usually followed by unusually low returns. It can be seen in Table III that an undervalued portfolio, which consistently delivers substantial excess returns over a five year horizon, performs marginally better than the full sample over the following five years, with a small cumulative excess return of only 4.9%. Five years over performance is followed by five years of approximately average returns. This implies that the underlying factor which explains excess returns is not underperformance in the past but underpricing in the present. Past underperformance is an imperfect proxy for current under valuation.

Table III also highlights an interesting asymmetry in the time profile of the excess returns for under and over-valued portfolios. The UN1 portfolio delivers a cumulative excess return of 31.6% over the first five years. On the other hand the 5% most over-valued portfolio, OV1, only under-performs in the first five years by a cumulative 9.6%, (or 18.1% when zero dividend companies are deleted). However, in the second half of the decade it under-performs the full sample by a cumulative 30.3%, with particularly pronounced under-performance in the last three years of the decade, as can be seen from Table II. This suggests that the stock market is quicker at identifying under-valued stocks and returning them to their fundamental value, the source of the excess return, than it is at recognising that glamour stocks have been over-valued.

A further perspective on the difference in performance over the decade of the 5% most under-valued and 5% most over-valued portfolios is provided by the serial correlation in our mispricing measure. The average probability of a company which was in OV1 one year re-appearing OV1 in the following year was 43.5%. Mispricing is highly persistent, both for over and under-valued shares. However, the persistence of over valuation is rather greater than that of under-valuation. This is consistent with the evidence in Table III which we surveyed in the preceding paragraph. The greater persistence of over valuation means that the negative excess returns associated with it are less in the short run. Both the persistence evidence and excess returns evidence imply that under-valued stocks bounce back to fundamental values more quickly than glamour stocks are exposed for their true worth. The market is quicker to spot a bargain than it is to lose faith in a favourite.

On our best estimate it is possible to earn substantial profits using this trading rule. We turn next to the question of whether these profits are statistically significant. We start with a non-parametric test, which places no restrictions on the underlying distributions, the Sign Test. Under the efficient markets hypothesis $\hat{P}_{i,t} / P_{i,t}$ is in the agent's information set and hence the probability of any portfolio selected on this criterion beating any other is 0.5. In Table IV we report the number of times the portfolio of the 5% most under-valued shares delivered higher returns than the portfolio consisting of the 5% most over-valued shares over horizons of one to ten years. In our sample size of 23, the probability in a one sided test of 16 successes is 5%, of 17 successes is 2.5%, of 18 is 1%, with 1/2 % probability of 19 successes. We can reject the null hypothesis that our criterion does not forecast returns at the 95% significance level for holding periods of two years or more.

We next report regression results. We regress returns on the ratio of market price to theoretical stock price. We argued above that this ratio should not be correlated with the risk of the stock providing firm risk is constant over time. If this assumption is valid then a properly specified returns regression requires that we also include regressors which explicitly capture risk. We use three variables to measure risk. Under CAPM it is covariance with the market, i.e. Beta, which should measure risk, and this is the first variable which we include. However, there is now widespread doubt about the validity of this model, see for example Fama and French (1992). Fama and French, amongst many others, show that market capitalisation is important in explaining cross-section returns. Berk (1995) shows why size will serve as a risk measure. Finally, we include dividend yield as a measure of risk. Ball (1978) argued that dividend yield should serve as a "catch-all" measure of risk in an efficient market. Even if the researcher has not assumed the correct model of risk pricing then the variance in total returns due to risk will be proxied by the dividend yield, since the dividend yield is a component of the total return. The only assumption required for this argument is that if dividends are paid on a stock then the fraction of total returns which are delivered by dividends is not itself systematically related to risk. This gives the specification of our cross-section regression, estimated at date *t*:

$$R_{i,t} = \gamma_0 + \gamma_1 \log\left(\frac{P_{i,t}}{\hat{P}_{i,t}}\right) + \gamma_2 Beta_{i,t} + \gamma_3 Size_{i,t} + \gamma_4 \frac{d_{i,t}}{P_{i,t}} + u_{i,t} \qquad (5)$$

The null hypothesis of efficient markets implies that $\gamma_1 = 0$: deviations of market price from the theoretical price reflect expected temporary deviations in the dividend growth rate from its long run average and are uncorrelated with subsequent returns.

A problem in estimating regression (5) using cross-section data is that the 't' statistic will be biased because we would not expect $\operatorname{cov}(u_i, u_j) = 0$, $i \neq j$. Since firms are drawn from a single economy we would expect a macro component to unexpected returns common to all firms at the same date.

Fama and MacBeth (1973) proposed a solution to this problem which has subsequently been widely applied and which we adopt here. The estimates of the coefficients themselves are not biased despite the presence of cross-section dependence in the error term. Therefore regression (5) is estimated for *t* years and the sample distribution of $\hat{\gamma}_{k,t}$ is constructed for each coefficient, $k = 0,\ldots,4$. The sample standard deviation of the estimated coefficients provides an estimate of the standard error of the distribution of $\hat{\gamma}_k$. We then have the 't' statistic:

$$t(\hat{\gamma}_k) = \frac{\hat{\gamma}_k}{\hat{\sigma}_{\gamma_k}}, \quad k = 0,\ldots,4.$$

where

$$\hat{\gamma}_k = \frac{1}{T}\sum_{t=1}^{T} \hat{\gamma}_{k,t}$$

and

$$\hat{\sigma}^2_{\gamma_k} = \frac{1}{(T-1)}\sum_{t=1}^{T} \left(\hat{\gamma}_{k,t} - \hat{\gamma}_k\right)^2$$

where T is the number of estimated cross-section regressions.

The disadvantage of this approach is that we cannot study long horizon returns. Although techniques for handling overlapping data are available for conventional regression estimation, Fama and MacBeth estimation requires non-overlapping data. In order to run a reasonable number of independent cross-section regressions we only consider returns for horizons of up to three years. We retain the same start date, 1960, but since we now require a shorter run of returns after the stocks have been ranked on the mispricing index we are able to extend the estimation much further forward in time. The cross-section regression is estimated each year 1960-1991 for one year returns; every second year 1960-90 for two year returns; and every three years 1960-1989 for three year returns. This gave 15 two-year horizon regressions and 10 three-year horizon regressions.

In Table V we report the coefficients and 't' statistics using the Fama-MacBeth method to estimate (5). We can see from Table V that for return horizon of two and three years, we can reject the implication of the efficient markets hypothesis that $g_1 = 0$. The signs, and 't' statistics, on the remaining variables are all largely in line with results obtained in previous research. The size variable is significant and of the expected sign in our sample. The dividend yield coefficient is of the expected sign for most periods. Although this is not statistically significant we do not attach any importance to this in view of the few years available from which to calculate the 't' statistic. For the same reason the positive, but only marginally significant, coefficient on Beta is unsurprising.

The fact that we could run so few regressions to build up the sample distribution of the coefficients in the case of two and three-year returns implies that our test had low power to reject the null hypothesis. It is therefore the more striking that we were able to do so. It is interesting to compare the 't' statistic on our mispricing variable with the evidence which our test delivers on the role of size. It is widely accepted that size is an important determinant of returns over this same time period, see for example Fama and French (1992). However, the 't' statistic on size in our test is still smaller than that on the mispricing variable for both two and three year returns.

IV Is Our Criterion Simply Selecting Riskier Shares?

The excess returns on the under priced portfolios appear to be both large and statistically significant. However, it might yet be argued that the above assumption, that companies' perceived relative risk at any time is the historic average up to that date in the sample, is invalid. In this case stocks which are of temporarily high risk will appear under priced. One answer to this is that in the above regression we controlled for risk using contemporaneously measured size, dividend yield and Beta measured over the previous five years. Another perspective is provided by looking at the average value of size and Beta for the under and over-valued portfolios and these statistics are reported in Table VI.

It can be seen in Table VI that although the average value of Beta for the 5% most under-valued portfolio is larger than for the 5% most over-valued, the size of the difference does not seem large enough to account for all of the excess return. If the equity premium over the safe rate of interest is taken to be the historic average of approximately 8%, then the difference in Beta between UN1 and OV1 requires a yield difference under CAPM of only 1.4% per annum against the average measured difference of approximately 9% per annum over the first 5 years. Furthermore the average value of Beta for UN2 is less than for OV1, and yet on average for every horizon UN2 beats OV1.

The average company size in both of the under-valued portfolios is greater than in both of the over-valued portfolios. Since there is overwhelming evidence, Banz (1981), Fama and French (1992), that on average smaller companies deliver higher returns it is striking that UN1 and UN2 both beat OV1 and OV2, despite the fact that the latter are comprised on average of significantly smaller companies.

However many measures of risk we report it can still be argued that excess returns may be rationalised by some other model of risk. An alternative approach is to not focus on ex ante risk measures but to examine instead whether as a matter of fact the under priced shares were, when combined into portfolios, riskier ex post. That is rather than comparing just the mean of the resulting returns distribution for under-priced / over-priced portfolios, as in the previous section, we compare the whole distributions. We report results on the whole distribution of returns for three and five year returns in Table VII and Table VIII.

We test whether a risk-averse individual would strictly prefer the distribution of returns from the under priced portfolio to the distribution of returns from the over priced portfolio. The general criterion which requires no restriction on the distribution functions or investor's utility functions, other than they are not risk-loving, is the criterion of second order stochastic dominance (SSD). A probability density function, *f*, is said to exhibit SSD over a density *g* if:

$$\int_a^y F(x)dx \leq \int_a^y G(x)dx \qquad \forall y,$$

with strict inequality for some y. F and G denote cumulative density functions for *f* and g respectively.

If f exhibits SSD with respect to *g*, then *f* is preferred to *g* by all risk-averters. See Rothschild and Stiglitz (1972).

Using this criterion we compute the distribution of returns on the 5% most over-valued portfolio with the 5% most under-valued portfolio. We evaluate these distributions for 3 and 5 year returns. To do this we need to approximate the true continuous distribution by our discrete observations. For each return horizon there are 23 distinct observations for each portfolio, each occurring with frequency $1/23$. We denote a particular observation by x_i, and first rank the data from Table VII and Table VIII in ascending order. $F(x_i)$ is obtained by summing the sample frequencies for x_j, $j \leq i$. We found that for both the 3 and 5 years return horizons the decile of most under-valued shares exhibited second order stochastic dominance over the decile portfolio of most over-valued shares. Indeed for five year returns the under valued stocks only fail to exhibit first order stochastic dominance by a tiny margin. It can be seen from Table VIII that for every start date the five year returns of the most under-valued decile were greater than the five-year returns on the most over-valued decile, with the trivial exception of 1964 where the cumulative difference between the two was 0.3% over 5 years. In order to interpret the dominance of our under priced portfolios over five years as consistent with rational asset pricing theory one would have to argue that the excess return on the under priced stocks reflected a stronger positive covariance with shocks to aggregate wealth of a kind that was not observed in over thirty years.

It is particularly interesting to see the relative protection offered by the under and over priced portfolios to crashes. The largest annual fall in our sample, with average twelve month returns of –29.7%, was in 1974. In all cases which overlap this period the most under valued portfolio 5 year returns can still beat the most over valued portfolio. In the case of 3 year returns the under priced portfolio bought in December 1971 lost, by the end of 1974, 41.6% against a loss of 48.1% on the corresponding over priced portfolio. In the case of five year returns the under priced portfolio bought in December 1969 lost, by the end of 1974, 34.2% against a loss of 49.7% On the corresponding over-priced portfolio.

We conclude that it seems hard to account for the profits delivered by the trading rule as equilibrium rewards to holding riskier stocks.

V Summary and Conclusions

We found support for the hypothesis that there is a component of forecastable stock returns which is driven by irrational swings of market sentiment. We employed a number of distinct approaches to the question of whether the returns forecastability we uncovered could be rationalised as an equilibrium reward to risk. Firstly, and most importantly, we worked with a benchmark price ratio which did not imply that risky shares would *necessarily* appear under priced, unlike those price scaled accounting variables which have previously been shown to forecast returns. Secondly we used a number of approaches to evaluate whether the portfolios of stocks which appear under priced were more risky. Each approach gave the same negative answer. Taking all the evidence we have reported together suggests that one would have to have a very strong prior belief in efficient markets to argue that the stock picking rule which we have tested does not generate economic profits

Furthermore the mispricing measure which we employed could have been used to earn excess returns by market participants trading in real calendar time. We conditioned stock selection strictly on an econometric model which was estimated using only data which would have been currently available to market participants at the trading date. The mispricing could have been exploited by rational investors who buy which are temporarily out of fashion and avoid over priced glamour stocks, thereby earning economic excess returns compared to the market portfolio.

Our results provide a framework in which the evidence of negative serial correlation in long run returns reported by DeBondt and Thaler (1985) can be interpreted. Undervalued stocks must have delivered negative returns in the course of becoming undervalued and hence the use of past returns will be a proxy for a direct measure of current mispricing. However it is an imperfect proxy since we showed that excess returns which resulted from the unravelling of earlier mispricing were not correlated with subsequent returns.

An interesting extension of this work would be to try to find variables to explain the mispricing of equities, as measured by $\hat{P}_{i,t} / P_{i,t}$.

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Table 1

Average Cumulative Portfolio Returns Expressed as Percentage,
Stocks ranked by $P_{i,t} \big/ \left(\hat{a}_{i,t} + \hat{b}_{i,t} d_{i,t}\right)$

Holding Period in Years	Full Sample	UN1	UN2	OV2	OV1
1	9.7	15.3	11.9	7.5	9.9
2	18.9	32.1	26.1	12.1	15.2
3	29.1	46.9	37.1	18.3	22.2
4	41.1	69.0	51.3	25.5	31.7
5	55.9	92.0	69.9	38.5	46.3

6	70.0	108.2	85.5	51.2	58.4
7	86.6	130.0	104.6	67.5	67.4
8	100.4	154.9	124.5	76.8	69.3
9	114.8	178.1	148.4	86.6	70.5
10	133.4	197.0	175.3	96.9	74.7

Table II

Average Cumulative Portfolio Returns Expressed as Percentage,

Stocks ranked by $P_{i,t} / \left(\hat{a}_{i,t} + \hat{b}_{i,t} d_{i,t}\right)$

Zero Dividend Companies Removed

Holding Period in Years	FS	UN1	UN2	OV2	OV1
1	7.4	13.4	11.3	8.2	6.5
2	15.5	30.2	25.2	14.5	9.5
3	25.3	45.0	36.4	21.0	14.3
4	34.2	63.7	50.8	26.9	18.9
5	46.1	85.2	67.5	39.6	28.0

6	59.3	102.6	79.1	49.7	37.9
7	74.1	122.1	94.2	62.1	44.2
8	84.8	147.6	109.8	70.0	42.9
9	100.1	170.6	134.5	73.8	49.3
10	144.8	189.7	163.3	84.9	57.3

Table III

Incremental Portfolio Returns
Average Incremental Returns Expressed as Percentage,
Stocks ranked by $P_{i,t} \big/ \left(\hat{\alpha}_{i,t} + \hat{\beta}_{i,t} d_{i,t} \right)$

Holding Period in Years	Full Sample	UN1	UN2	OV2	OV1
1	9.7	15.3	11.9	7.5	9.9
2	8.4	14.6	12.7	4.3	4.8
3	8.6	11.2	8.2	5.5	6.1
4	9.3	15.0	10.4	6.1	7.7

5	10.5	13.6	12.3	10.4	11.1
6	9.1	8.4	9.2	9.2	8.3
7	9.8	10.5	10.3	10.8	5.7
8	7.6	10.8	9.7	5.6	1.1
9	7.2	9.1	10.6	5.5	0.7
10	8.7	6.7	10.8	5.5	2.5

N.B. Incremental Returns from year 5 to year 10 =54.6 on UN1
= 49.7 on FS
=19.4 on OV1

Table IV

A Sign Test of Whether UN1 beats OV1

Holding Period in Years	1	2	3	4	5	6	7	8	9	10
No of times UN1 beats OV1	13	16	17	18	22	23	20	21	22	22

Table V

$$R_{i,t} = \gamma_{0,t} + \gamma_{1,t} \log\left(\frac{\hat{P}_{i,t}}{P_{i,t}}\right) + \gamma_{2,t} Beta_{i,t} + \gamma_{3,t} Size_{i,t} + \gamma_{4,t} \frac{d_{i,t}}{P_{i,t}} + u_{i,t}$$

Return Horizon in Years	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$
1	0.049 (1.76)	-0.003 (-0.34)	0.059 (1.60)	-0.016 (-2.25)	0.048 (0.21)
2	0.156 (2.74)	-0.053 (-2.58)	0.038 (0.66)	-0.030 (-1.99)	-0.110 (-0.17)
3	0.226 (3.71)	-0.106 (-2.39)	0.045 (0.94)	-0.037 (-1.45)	0.623 (1.05)

Table VI
Average Values of Beta and Company Size

	FS	UN1	UN2	OV2	OV1
Beta	1.17	1.46	1.25	1.2	1.29
Size	21,072	15,427	17,636	15,000	9,960

Table VII
3 Year Returns on Decile Portfolios
Ranked by $P_{i,t} / \left(\hat{\alpha}_{i,t} + \hat{\beta}_{i,t} d_{i,t}\right)$

Start Date	Most Under									Most Over-

	- value d Decile									value d Decile
1960	58.1	37.7	48.5	43.1	35.6	38.7	40.3	30.9	29.2	26.5
1961	40.4	43.8	43.0	29.7	24.1	26.8	28.2	22.4	26.2	10.9
1962	96.0	92.2	87.0	80.4	75.4	90.2	84.7	64.4	77.4	89.3
1963	44.7	39.8	30.7	24.3	15.8	28.8	24.9	21.0	57.0	55.2
1964	88.0	57.1	46.1	47.8	45.6	35.5	54.0	67.9	75.4	72.6
1965	51.2	37.9	36.2	51.1	63.1	56.8	88.6	58.9	70.3	58.3
1966	48.6	29.5	28.5	39.3	54.3	46.9	40.0	32.9	60.3	22.3
1967	10.7	11.3	-1.4	6.5	-1.5	-0.9	-9.4	-3.6	-24.1	-34.1
1968	-9.8	0.1	-5.2	-4.4	-9.8	-21.2	-9.3	-19.9	-20.4	-24.8
1969	15.5	19.5	19.4	18.6	17.1	17.7	30.6	13.9	-6.9	6.2
1970	-10.1	-7.9	-10.9	1.9	-11.2	-9.9	6.6	3.6	-15.7	-5.8
1971	-41.6	-36.2	-38.9	-34.5	-37.6	-34.6	-49.0	-42.5	-45.9	-48.1
1972	-25.5	-29.7	-22.4	-27.5	-19.8	-30.6	-20.4	-33.0	-31.2	-38.1
1973	46.6	46.5	28.0	42.8	36.6	40.2	22.2	-0.1	-3.4	-11.5
1974	129.4	121.5	96.9	69.0	64.2	58.3	58.1	41.5	46.7	27.8
1975	73.5	40.0	41.0	39.2	36.3	26.6	23.4	20.7	18.0	15.2
1976	30.5	17.7	7.3	2.3	27.1	14.1	10.2	27.7	15.3	23.1
1977	27.2	13.4	3.9	17.1	28.1	36.7	53.1	26.6	57.8	53.0
1978	5.4	1.3	14.7	19.2	14.7	16.4	32.1	19.2	43.4	37.5
1979	48.6	32.4	28.1	24.6	6.7	19.5	10.7	16.6	18.0	5.3
1980	92.1	80.8	80.9	59.8	46.0	41.6	47.3	33.1	23.3	26.4
1981	94.8	73.8	78.8	68.7	55.6	45.1	39.7	46.2	52.1	44.4
1982	61.1	83.5	82.1	72.2	84.3	69.9	54.8	55.3	83.0	66.4

Table VIII

Five Year Returns on Decile Porfolios

Ranked by $P_{i,t} \big/ \left(\hat{a}_{i,t} + \hat{b}_{i,t} d_{i,t}\right)$

Start Date	Most Under - value d Decile									Most Over- value d Decile

1960	177.0	116.0	134.2	101.9	103.3	100.0	97.8	86.3	90.3	92.8
1961	51.9	59.9	59.1	40.0	38.3	35.1	37.0	37.3	54.0	32.3
1962	184.2	130.9	138.5	117.7	114.7	133.6	115.6	103.4	115.5	150.9
1963	171.2	151.4	116.6	86.6	107.0	133.9	153.9	133.2	163.2	154.6
1964	70.2	63.6	36.6	57.2	46.9	32.6	61.3	62.1	81.7	70.5
1965	26.2	15.9	13.5	16.4	34.9	19.4	54.1	5.6	7.9	0.4
1966	86.8	47.8	50.0	44.8	61.2	67.4	47.3	28.0	64.1	28.9
1967	14.1	32.2	23.8	27.0	20.7	18.7	8.4	15.0	5.9	-21.7
1968	-24.3	-15.9	-21.6	-15.5	-17.1	-37.8	-16.3	-32.6	-39.2	-51.4
1969	-34.2	-34.7	-33.7	-31.4	-34.7	-33.7	-24.0	-36.5	-56.1	-49.7
1970	-6.8	-11.4	-17.8	-7.9	-19.3	-16.4	-12.6	-11.7	-14.6	-8.3
1971	16.5	0.6	10.1	11.9	5.2	-0.8	-16.3	3.2	2.4	-16.0
1972	0.1	-5.3	-4.3	-9.1	-1.4	-8.9	3.4	-22.2	-16.5	-10.7
1973	71.4	48.6	40.8	37.7	34.0	42.4	17.9	-10.2	-13.7	-14.7
1974	169.8	146.4	142.4	103.1	85.0	84.5	97.8	84.4	85.6	74.1
1975	98.8	71.5	77.8	67.5	82.1	77.6	47.0	44.1	52.2	62.1
1976	26.1	22.2	9.5	6.2	28.0	16.3	16.2	29.9	16.7	23.6
1977	73.1	34.5	23.7	62.6	41.9	57.1	27.3	23.6	69.0	57.1
1978	120.0	56.9	87.1	105.0	64.0	91.9	101.4	91.4	88.8	97.8
1979	107.6	68.0	70.1	71.0	35.8	73.1	51.3	53.9	56.5	37.5
1980	156.8	144.7	148.6	101.5	88.3	89.8	96.0	47.3	34.2	54.9
1981	208.8	193.5	179.4	186.7	142.0	126.9	90.0	127.8	102.3	99.8
1982	117.3	148.6	161.7	155.6	148.9	125.7	101.9	123.5	111.9	114.2