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Walras Retrouvé: Decentralized Trading Mechanisms and the Competitive Price[¤]

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Abstract

We extend the standard analysis of decentralized markets allowing for the possibility that traders interact simultaneously with more than one trader on the opposite side of the market. In contrast to the literature, we are able to reconcile the Walrasian equilibrium with the outcome of decentralized strategic trade: we show that there exist generic matching mechanisms which determine local market conditions such that, as market frictions vanish, the expected equilibrium price is the one that would result in the static Walrasian market. At the same time, we highlight the relevance of local market conditions in the determination of equilibrium prices.

Keywords: Decentralized markets, Price formation, Walrasian equilibrium.

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1 Introduction

One of the most celebrated issues in economics is the theory of price formation. In a competitive market for a homogeneous good only the Walrasian price –where demand and supply meet– can constitute an equilibrium, and yet there are no satisfactory descriptions of how this price comes to be established. The fictitious Walrasian auctioneer has received much attention, despite its notable absence from most real-life markets. Two important foundations of a trading mechanism that captures the essential features of real markets are the realization that traders behave strategically (Dubey and Shubik, 1978), and the recognition that trade is decentralized: in most markets, a buyer comes into "direct contact" with only a limited number of sellers, even when there are many traders on each side of the market. These observations have led to the identification of two conceptually separate stages in the functioning of a market: matching and bargaining. In the typical model, buyers and sellers are randomly "matched," viz assigned to a local market. After the matching stage, the traders in each local market engage in strategic bargaining, following some explicit procedural rules. The agents who reach an agreement trade and exit the market; those who do not trade are assigned to local markets after a costly delay, possibly together with new entrants.

Following the early lead of Diamond (1981)[1], this literature[2] has analyzed extensively the "pair-wise" matching model: in each local market there are exactly one buyer and one seller. However, the results obtained are disappointingly non-Walrasian: the main conclusion of this literature is that, even as frictions vanish, the equilibrium price does not equate demand and supply, but rather the aggregate surpluses of sellers and buyers. In the simple case of homogeneous valuations, this entails the equilibrium price to be a strictly monotonic function of the relative size of the two sides of the market, when, on the contrary, the Walrasian price depends only on which side of the market is the shorter (see Rubinstein and Wolinsky, 1985, and Binmore and Herrero, 1988). In defense of the competitive market paradigm, Gale (1986, 1987) argued that if there are new entrants in each period the Walrasian price should be calculated in terms of the incoming flow of the new traders, not of the aggregate stock of demand and supply at a given date (as Ru-

[1] See also Diamond and Maskin (1979) and Mortensen (1982a, 1982b).

[2] See Osborne and Rubinstein (1990) for an excellent survey. Note that the search model (see McMillan and Rothschild (1994) for an overview) can also be interpreted in this framework: the matching mechanism is determined by the search probability of the traders who search, and the bargaining is of the take-it-or-leave-it type: the traders who do not search post a price, and the traders on the other side of the market can accept this price or continue to search.

binstein and Wolinsky did). However, Gale's clarification did not eliminate the non-Walrasian feature of the Rubinstein-Wolinsky market equilibrium constituted by the presence of permanent unsatisfied excess demand.

Our paper reconciles the model of decentralized trade with the Walrasian paradigm: we show that there exist decentralized trading mechanisms which lead to the Walrasian outcome both in the stock and in the flow sense. We obtain this result by relaxing the restriction to one-to-one matching: we assume that, with positive probability, there is more than one trader on one side of each local market. This captures two important stylized facts. First, the seller, of, say, a house, may well be in the position to negotiate with several potential buyers at the same time. Second, conditions usually vary across local markets: in a certain region there might be no job vacancies, while, at the same time, in other regions there is unemployment; a geographical area may experience a housing shortage, another a glut; and so on.

We show that the local market structures are crucial in the determination of equilibrium prices. When they are "aligned" with the global market conditions we are able to restore the Walrasian outcome. On the other hand, we also show that, for the same global conditions, there are matching mechanisms which determine local market conditions such that the outcome is distinctly non-Walrasian. In highlighting the importance of local market conditions for price determination, our analysis sheds some light on the source of the non-Walrasian outcome of the Rubinstein-Wolinsky model: in the presence of decentralized trade, both local and global market conditions matter, and the equilibrium price resulting in the Rubinstein-Wolinsky model is affected by the bilateral monopoly nature of the local market they hypothesize.

Our analysis also puts the Gale critique of the non-Walrasian interpretation of Rubinstein and Wolinksy's result into perspective. In our model, entry is endogenous, as the incoming flow of new traders is a function of the market price which, in turn, depends on the local conditions determined by the matching mechanism. It follows that the Walrasian flow price cannot be defined in terms of the global market parameters alone. If, on the other hand, we define the Walrasian flow outcome in terms of the equilibrium flow of entry, the claim that as frictions disappear the market equilibrium converges to the Walrasian flow outcome reduces to a tautology.

The rest of the paper is organized as follows: the model is presented in Section 2. In Section 3, we obtain the market equilibrium. In Section 4 we provide comparative statics for the case when the valuations are uniformly distributed, while in Section 5 we analyze the relationship between the local market conditions and the Walrasian price. Finally, Section 6 contains a brief conclusion and relates our analysis to the existing literature.

2 The model

We follow closely the standard model of decentralized strategic trade (Rubinstein and Wolinsky, 1985). There are large numbers of two types of traders: the sellers, who own one indivisible unit of a homogeneous good each, and the buyers, each of whom wishes to consume one unit of the good. For expositional clarity, sellers are female, buyers male. All sellers have the same valuation for the good, normalized to 0. The buyers' valuations, $v \in [0,1]$, are independent, random draws[3] from a known atomless distribution, $G(v)$, with positive density, $g(v)$. The time horizon is infinite and divided in periods, indexed by t, $t = 1, 2, \ldots$. Traders are risk neutral and discount future payoffs by a common factor of $\delta \in (0,1)$ per period. In time period 1, B potential buyers and S potential sellers are present in the market. The potential traders who are in the market in the beginning of a period decide whether to enter the trading stage. We assume that they enter if and only if they expect a strictly positive payoff from trade.[4] In each period, the traders who have entered are matched according to a random matching process, which we discuss below. Matching is followed by a negotiation phase, where the matched agents negotiate the terms of trade; if two agents reach an agreement, they trade, leave the market and are replaced by identical[5] agents in the next period. Consequently, the number of traders on each side of the market, as well as the distribution of their valuations, are constant over time, and the market is in steady state. Finally, the traders who do not trade can re-enter the market in the next period.

Figure 1 sketches the sequence of events within each time period. The following subsections specify it in detail.

Insert Figure 1 about here

[3] This is a generalization of the Rubinstein-Wolinsky model, who have homogeneous valuations on both sides of the market.

[4] Notice that we cannot have an explicit entry cost. As Fudenberg and Tirole (1991, p. 414) point out, even with an infinitesimal entry cost, the market would not exist. Gale (1987) avoids this problem by assuming that there are only a finite number of possible valuations, and therefore the lowest type makes, generically, positive profits in equilibrium.

[5] This assumption is realistic if the buyers have actually multi-unit demand, but either only require at most one unit per period or they are constrained to purchase sequentially (newspapers, bread, holidays, and post-docs are examples). In these cases, the leaving and the entering agent is actually the same. Admittedly, however, our main motivation for this assumption is that only in a stationary environment can we derive the explicit solution of our model.

2.1 The matching process

The interaction between buyers and sellers is governed by a stationary matching process. This is a probability distribution over the possible assignments of buyers to sellers. The novelty of our approach is that we introduce the possibility of one-to-many matching: a seller can be matched simultaneously with more than one buyer. However, for simplicity, we do not assume the converse: each buyer is assigned to at most one seller.

Since the number of possible assignments is very large, in order to retain tractability, we impose some natural anonymity restrictions. For every k, firstly, the probability to be matched with k, buyers is the same for all sellers. We denote this probability with q_k; secondly, the probability that a given buyer is matched to a seller who is matched with k buyers altogether is independent of their identities and of the buyer's valuation; we denote this probability by p_k. The following lemma describes the implications of these two anonymity restrictions; let B_0 be the number of buyers who choose to enter the matching stage.

Lemma 1 $q_k = \frac{B_0 p_k}{kS}$ for $k = 1, \ldots, B_0$, and $q_0 = 1 - \sum_{j=1}^{B_0} q_j = 1 - \frac{B_0}{S} \sum_{j=1}^{B_0} \frac{p_j}{j}$.

Proof. For any $k = 1, \ldots, B_0$, there are, on average, $\frac{B_0 p_k}{k}$ sellers matched with exactly k buyers, and the anonymity restrictions require that a given seller has probability $\frac{1}{S}$ of being one of them. ■

Therefore, the anonymous decentralized matching mechanism with B_0 buyers and S sellers is fully described by B_0 parameters, either p_k or q_k, for $k = 1, \ldots, B_0$. Since we would like to control local and global market conditions independently, we want to keep the description of the matching process exogenous (independent of endogenous entry). To this end, we restrict further the matching technology, by ruling out all but one-to-one and two-to-one matches[6]. In our set-up, restricting the degrees of freedom not only facilitates

[6] An imaginary chemical example might illustrate the matching technology implied by our model. The Rubinstein and Wolinsky market can be visualized as a sealed box where two types of atoms (S and B) are introduced, which can form a molecule by bonding with each other in a one-to-one bond. The matching phase ends after the box is shaken. The probability of an atom being matched depends on the ratio between the number of B atoms and the number of S atoms. In the case considered here, after a one-to-one bond is formed, creating a (B-S) molecule, a further B atom could attach itself to this (B-S) molecule to form a (B-B-S) molecule. The hypothesis $p_k = 0$ for $k > 2$ implies that no further atoms could bond with a (B-B-S) molecule. The ease with which a B atom bonds with a loose S atom relative to the ease with which it bonds with a B-S molecule, will determine the relative frequency of the two types of molecules, B-S and B-B-S, at the end of the shaking (matching) phase.

the calculations, but it also strengthens our conclusions, since they would a *fortiori* remain true if we considered matching technologies with more parameters, such as a generic one-to-many or even many-to-many matching.

Assumption 1 *There are at most two buyers matched to a seller:* $p_k = q_k = 0$ *for* $k > 2$*.*

2.2 Negotiations

After the matching process is completed, the matched traders in each local market carry out their negotiations. This takes place according to the standard rules, adapted to the case studied here. The buyers are selected to make an offer with probability $\beta \in (0;1)$, the seller with probability $1 - \beta$. For the sake of definiteness, in any given match, the buyers do not observe the number of their local competitors: as it will become clear in the proof of Proposition 1, the converse hypothesis would not alter the qualitative features of our results.

If the buyers are selected to make the offers, negotiations take place according to a sealed-bid, first-price auction: each buyer simultaneously makes the seller an offer, among which the seller can either choose the highest (in which case she and the selected buyer trade at the offered price and leave the market), or reject them all. If the seller is chosen to make an offer, we follow the existing literature (for example, Gale 1987) in assuming that she observes the valuations of all the buyers she is matched with, and that she then makes a price offer to one of the buyers, which he can accept or reject.

This mechanism defines a local market and a global market. Local market conditions are determined by the actual number of buyers matched with a given seller (governed by the q_k), and can clearly differ among local markets. Global market conditions are the same for all markets, and are characterized by three parameters: impatience, δ, bargaining power, β, and the aggregate ratio of buyers to sellers, $\lambda = \frac{B}{S}$.

2.3 An Example

The familiar example of the academic job market may flesh out the situation we have in mind: the good in question is academic labor; there are B university departments (buyers) and S, *ex ante* identical, fresh PhD's, each looking for an academic appointment (sellers). Each department has a vacancy with some exogenous probability (determined, say, by the allocation of new academic posts across departments in each university). If a department has a vacancy, then it selects one candidate, and, on the day after the

AEA conference, posts a letter with a job offer. A department clearly cannot make more than one offer, because it is legally bound to appoint all those who accept. A few days later, all candidates receive their offers, if any. In practice, as captured by our model, there is nothing to prevent a candidate from receiving more than one offer, though this possibility is ruled out in the standard one-to-one matching model. If departments chose randomly among candidates, then the probability distribution would be binomial. However, there are other strategies available to departments: for example, suppose a proportion of departments are " poachers ". They make an offer to a candidate only if that candidate has already received an offer. The probability of a seller to receive more than one offer depends on the proportion of buyers who imitate, and on the ease with which information about offers is divulged.

3 The market equilibrium

In this section we characterize the expected equilibrium payoffs of each trader; from these, it is immediate to derive the traders' equilibrium strategies. The solution concept is Perfect Bayesian Equilibrium: each trader behaves optimally given the equilibrium strategies of the rest of the traders and given his/her information.

We begin by noting that, given the equilibrium strategies of the active traders, there exists a buyer valuation, v_0, which is the highest valuation such that a buyer would decide not to enter the matching stage: a buyer with valuation v_0 would expect zero profits in the market. Consequently, in each period, the number of buyers who enter the matching stage (the buyers who are active in the market) is given by:

$$B_0 = B \int_{v_0}^{1} g(v)\,dv = [1 - G(v_0)]\,B. \tag{1}$$

This is, of course, a function of the equilibrium payoffs, which we calculate next. Propositions 1 and 2 characterize the expected equilibrium payoff of buyers and sellers, respectively.

Proposition 1 The expected equilibrium payoff of a buyer with valuation $v > v_0$, denoted by $V_B(v)$, is given by:

$$V_B(v) = \int_{v_0}^{v} \frac{\beta(p_1 + p_2 G_0(x))}{1 - \delta(1 - \beta(p_1 + p_2 G_0(x)))}dx; \tag{2}$$

where $G_0(x)$ is the probability distribution induced by the truncation of the original distribution at v_0: $G_0(x) \equiv \frac{G(x) - G(v_0)}{1 - G(v_0)}$ for $x \in [v_0, 1]$.

Proof. When the seller is selected to offer, she will make an offer which makes a buyer indifferent between acceptance and rejection. Consequently, all the matched buyers, whether chosen by the seller or not, will earn $\delta V_B(v)$. When it is the buyers who can submit bids, we can use the revenue equivalence theorem for auctions with a stochastic number of bidders and independent private valuations (see McAfee and McMillan, 1987, and Harstad, Kagel and Levin, 1990). In equilibrium a (risk neutral) buyer's expected payoff is given by what this buyer would expect to receive if he bid optimally knowing the number of bidders.[7] Consequently, the buyers' value function, $V_B(v)$, satisfies:

$$\begin{aligned} V_B(v) &= \beta p_1 (v - b_1(v)) \\ &\quad + \beta p_2 \left[G_0(v)(v - b_2(v)) + (1 - G_0(v))\, \delta V_B(v)\right] \\ &\quad + (1 - \beta(1 - p_0))\, \delta V_B(v), \end{aligned} \tag{3}$$

where $b_i(v)$ is the optimal bid for a buyer who has valuation v, when there are altogether i bidders, $i = 1, 2$. The bidding functions, $b_1(.)$ and $b_2(.)$, are standard, and can be determined as follows. When there is only one buyer, the bid is given by the reservation value of the seller: $b_1(v) = \delta V_S$. If there are two bidders, the equilibrium bid $b_2(v)$ must be incentive compatible for a buyer of valuation v, in the sense that he should prefer to bid $b_2(v)$ in preference to bidding any other bid $b_2(x)$ for some $x \in (v_0, 1]$, whenever he expects the other bidder to follow $b_2(.)$. This implies:

$$v = \arg\max_x \,(v - b_2(x))\, G_0(x) + \delta V_B(v)\,(1 - G_0(x)). \tag{4}$$

The first order condition for (4) yields a differential equation in $b_2(v)$:

$$(v - b_2(v))\, g_0(v) - b_2'(v)\, G_0(v) - \delta V_B(v)\, g_0(v) = 0.$$

Rearrange it and add the initial condition (observe that no buyer would bid above his valuation) :

$$b_2'(v) = \frac{g_0(v)}{G_0(v)} (v - b_2(v) - \delta V_B(v)) \quad \text{s.t.:} \quad b_2(v_0) = v_0.$$

Its solution is:

$$b_2(v) = v - \delta V_B(v) - \frac{1}{G_0(v)} \int_{v_0}^{v} \left(1 - \delta V_B'(x)\right) G_0(x)\, dx.$$

Substituting $b_1(v)$ and $b_2(v)$ into (3) and rearranging:

$$\beta p_2 \int_{v_0}^{v} \left(1 - \delta V_B'(x)\right) G_0(x)\, dx \equiv V_B(v)\,(1 - \delta(1 - \beta p_1)) - \beta p_1 (v - \delta V_S).$$

[7] For this equilibrium to be implementable by a first-price auction it is necessary that the uncertainty about the number of bidders be symmetric across bidders. This is natural given our anonymity assumptions on the matching process.

Since this holds for every $v \in (v_0; 1]$, we can differentiate with respect to v to obtain:

$$V_B'(v) = \frac{\beta (p_1 + p_2 G_0(v))}{1 - \delta (1 - \beta (p_1 + p_2 G_0(v)))}.$$

Integrate and use the initial condition, which implies $V_B(v_0) = 0$, to obtain the Proposition. ■

Proposition 2 The expected equilibrium payoff of a seller, denoted by V_S, is given by:

$$V_S = \frac{\int_{v_0}^{1} (v - \delta V_B(v)) \left[(1 - \beta) q_1 + 2\beta q_2 + 2 G_0(v) q_2 (1 - 2\beta)\right] g_0(v)\, dv}{1 - \delta (q_0 + \beta q_1)}. \tag{5}$$

Proof. We have

$$\begin{aligned} V_S \;=\; & \delta V_S (q_0 + \beta q_1) + \beta q_2 \int_{v_0}^{1} 2 (v - \delta V_B(v)) (1 - G_0(v)) g_0(v)\, dv \qquad (6) \\ & + q_2 (1 - \beta) \int_{v_0}^{1} 2 (v - \delta V_B(v)) G_0(v) g_0(v)\, dv \\ & + q_1 (1 - \beta) \int_{v_0}^{1} (v - \delta V_B(v)) g_0(v)\, dv. \end{aligned}$$

This is because, with probability q_0 the seller is unmatched, so she goes to the next period; this gives her a payoff δV_S. With probability βq_1, she is matched to a single buyer, but the buyer's side is selected to make an offer, so she receives only her continuation payoff δV_S. With probability βq_2, the seller has two buyers and they make the offers. In this case, by the revenue equivalence theorem, the seller's expected revenue is the expected willingness to pay of the lower valuation buyer. The willingness to pay of the buyer with valuation v is $(v - \delta V_B(v))$, and we take the expected value of the second order statistic of the willingness to pay: this is the first integral in (6). With probability $(1 - \beta) q_2$, the seller has two buyers and she gets to make the offer: in this case, she chooses the higher valuation buyer, and her payoff is the expected value of the first order statistic of the willingness to pay, namely the second integral. Finally, with probability $(1 - \beta) q_1$, the seller has one buyer and she makes the offer, leaving the buyer at his continuation payoff. Rearranging (6), (5) is obtained. ■

Propositions 1 and 2 determine the traders' payoffs as a function of the valuation of the marginal buyer not entering the market. The additional equation we need for a full characterization of the equilibrium payoffs relates this valuation to the seller's payoff.

Proposition 3 The market equilibrium is characterized by, (1), (2), (5), and

$$v_0 = \delta V_S. \tag{7}$$

This equilibrium exists if

$$v_0 \geq G^{-1}\left(1 - \frac{1}{\lambda}\frac{2}{2p_1 + p_2}\right). \tag{8}$$

Proof. Ignore (8) for the time being. Recall that v_0 is the marginal buyer and, consequently, all buyers with valuation strictly above v_0 must expect a strictly positive payoff. Therefore, it must be the case that with strictly positive probability they buy the good at a price strictly below their valuation. However, the minimum price that the seller is willing to accept is δV_S; and (7) follows.

Now, consider the following function, defined in $[0,1]$.

$$\phi(y) = \frac{\int_y^1 \left(v - \delta \int_y^v \frac{\beta(p_1 + p_2 G_0(x))}{1-\delta(1-\beta(p_1+p_2G_0(x)))} dx \right) [(1-\beta)q_1 + 2\beta q_2 + 2G_0(v) q_2 (1-2\beta)] g_0(v) dv}{1-\delta(q_0 + \beta q_1)} - \frac{y}{\delta}.$$

The first term on the RHS is obtained substituting (2) into (5). Clearly, (7) is equivalent to $\phi(v_0) = 0$. We need to check that there is an admissible solution to this equation. To compact the notation, let $a(y) = 1 - \delta(q_0(y) + \beta q_1(y))$, $\xi(x;y) = \delta\beta(p_1 + p_2 G_0(x))$, $\sigma(x;y) = \frac{\xi(x;y)}{1-\delta+\xi(x;y)}$, $h(v;y) = \int_y^v \sigma(x;y)\, dx$ and $Q(v;y) = \left[(1-\beta)q_1(y) + 2\beta q_2(y) + 2\frac{G(v)-G(y)}{1-G(y)} q_2(y)(1-2\beta)\right]$. Then $\phi(y)$ becomes:

$$\phi(y) = \frac{1}{a(y)}\left\{\int_y^1 (v - h(v;y))\, Q(v;y)\, g_0(v)\, dv\right\} - \frac{y}{\delta}.$$

We need to show that there exists $y \in [0,1]$; such that $\phi(y) = 0$. From Lemma 1, $\lim_{y \to 1} q_k(y) = 0$; $k = 1,2$, and therefore $\lim_{y\to 1} a(y) = 1 - \delta$. It follows that, for $\delta < 1$, $\frac{1}{a(y)}$ is finite, and therefore, $\lim_{y\to 1}\phi(y) = -\frac{1}{\delta} < 0$. Next consider $\phi(0)$. Use $v = \int_0^v dx$ to write the term $(v - h(v;0))$ as $\int_0^v (1 - \sigma(x;y))\, dx$. Now note that

$$1 - \sigma(x;y) = \frac{1-\delta}{1-\delta+\xi(x;y)} > 0 \tag{9}$$

and therefore

$$\phi(0) = \frac{1}{a(0)}\left\{\int_0^1 \int_0^v (1 - \sigma(x;0))\, dx\, Q(v;0)\, g_0(v)\, dv\right\} > 0. \tag{10}$$

Since ϕ is continuous, it is 0 for some y. Therefore, the existence of a market equilibrium is guaranteed.

Now consider (8). A requirement of perfect bayesian equilibrium is that the post entry matching probabilities be consistent with the beliefs held by the buyers when they take their entry decision. For this to be the case, there must be enough sellers around. At the end of the matching stage B_0p_1 buyers are alone, B_0p_2 are in a couple. Consequently, there must be at least $B_0p_1 + \frac{B_0p_2}{2}$ sellers. This implies $\frac{B_0}{S} \leq \frac{2}{2p_1+p_2}$, which, using (1) gives (8). ■

Note that the market equilibrium is not necessarily unique. As in other models with traders of heterogeneous valuations and with endogenous entry (see, for example, Bester, 1988), it is possible to find a distribution of the valuations and parameter configurations which lead to multiple equilibria. However, conditional on entry (in our case, v_0) the equilibrium is always unique.

By Proposition 3, v_0 is the seller's reservation price: under no circumstances will the seller sell for less than v_0; knowing this, no buyer with valuation v_0 or below will bother to enter the market. Note that the necessary condition for existence (8) has a natural economic interpretation: it indicates the extent to which the local and global market conditions can be independent. While (8) holds, any change in the global market conditions, λ, can be accommodated by changing the sellers' matching probability, $1-q_0$, leaving the p_k unchanged. Conversely, we can vary this relative probability keeping the global market conditions constant, if and only if (8) is satisfied.

Given the sellers' expected payoff, we can derive the average price in the market.

Corollary 1 The average equilibrium price is given by: $P = \frac{v_0(1-\delta q_0)}{\delta(1-q_0)} = v_0\left(1 + \frac{1-\delta}{\delta\lambda(1-G(v_0))\left(p_1+\frac{p_2}{2}\right)}\right)$.

Proof. In equilibrium all matched sellers trade; therefore, the seller's expected payoff, which is $\frac{v_0}{\delta}$; must be given by the average price, weighted by the probability of trade, $(1-q_0)$, and v_0, the payoff for going to the next period, weighted by its probability, q_0. The second equality follows from Lemma 1. ■

Proposition 3 and Corollary 1 give a general characterization: to obtain an explicit analytical description of the equilibrium, we consider the special case of a uniform distribution of the buyers' valuations: $G(v) \equiv v$.

Proposition 4 Let the buyer valuations be uniformly distributed. Then the seller's reservation price is given by:

$$v_0 = 1 - \frac{1-\sqrt{(1-4\lambda f(\delta;\beta;p_1;p_0))}}{2\lambda f(\delta;\beta;p_1;p_0)};$$

where

$$f = \frac{\frac{1-\delta(1-\beta(1-p_0))}{\delta\beta p_2}\left(1+p_0+\frac{1-2\beta-\delta}{\delta\beta}\right)\ln\frac{1-\delta(1-\beta(1-p_0))}{1-\delta(1-\beta p_1)}+\left(\frac{1}{2}+\beta\right)p_2+\left(p_1+\frac{(2\beta-1)(1-\delta)}{\delta\beta}\right)}{2\beta p_2}. \quad (11)$$

Proof. In the Mathematical Appendix, available on request from the authors, or at http://www-users.york.ac.uk/~gd4/WalrasAppendix.tex. ■

4 Comparative statics

In this section, we study how the equilibrium derived in Proposition 4 changes as the parameters vary. We begin with an increase in local competition. To distinguish more intense competition from increased search intensity, we define a local increase in competition as an increase in p_2, keeping p_0 constant:[8] the same number of matched buyers are distributed less evenly and, therefore, the sellers' side suffers a decrease in the probability of being matched $(1-q_0)$.

Corollary 2 For any given p_0, v_0 is increasing in p_2 if $\beta \geq \frac{1}{2}$; if $\beta < \frac{1}{2}$, v_0 is increasing (respectively constant, respectively decreasing) in p_2 for $\delta > \frac{1-2\beta}{1-\beta(1+p_0)}$ (respectively $\delta = \frac{1-2\beta}{1-\beta(1+p_0)}$, respectively $\delta < \frac{1-2\beta}{1-\beta(1+p_0)}$).

Proof. In the Mathematical Appendix. ■

Thus, for some parameter combinations (low values of β and δ), an increase in competition on the buyer's side may reduce the seller's payoff. To understand this counter-intuitive result, note that an increase in p_2 has two separate effects on the sellers' payoff. A direct effect is the increase in the frequency of two-to-one matches, where the price is higher (in expectation). The importance of this effect is an increasing function of β: the difference in the seller's payoff between a two-to-one and a one-to-one match is higher when she has low bargaining power, as her payoff in a two-to-one match is less sensitive to which side of the market is chosen to make the offers. But there is also an indirect effect, which works in the opposite direction: when local (buyer) competition is increased, for fixed p_0, the probability of a seller being matched in any given period is reduced. This provokes a delay in the expected time of trade, and has a negative effect on the seller's payoff. This

[8] In terms of our chemical example, (footnote 6), this entails that for a loose B atom it becomes easier to bond with a B-S molecule and more difficult to bond with a loose S atom. In terms of our job market example, in Section 2.3, this might happen if more departments become poachers, or if information about job offers by other departments becomes more diffuse.

effect becomes weaker as β increases; to understand why this is the case, consider how the sellers' matching probabilities change with p_2: From Lemma 1, $\frac{dq_0}{dp_2} = \frac{dq_2}{dp_2} = -2\frac{dq_1}{dp_2}$: an increase in local buyer competition (an increase in p_2, keeping p_0 constant) distributes probability mass from one-to-one matches towards two-to-one matches and no matches in equal proportion. Now, recall that when the buyers make offers in a one-to-one match, the sellers just earn their continuation value, which is what they would expect to get if they were unmatched. Consequently, for high β the payoff loss due to an increase in q_0 and a corresponding decrease in q_1 is small, and it is swamped by the positive effect of an increase in the probability of bidding competition. Therefore, when β is sufficiently high the direct (price) effect dominates the indirect (matching) effect for any δ: The specific critical value, $\beta = \frac{1}{2}$, is due to the uniform distribution of the buyers' valuations, and does not generalize. Since the indirect effect is due to a delay cost, it disappears when the cost of delay vanishes: in the limit, as $\delta \to 1$, the seller's payoff is always increasing in p_2 for any $\beta > 0$.

Next, we consider changes in the global market conditions: they affect the price in the expected manner.

Corollary 3 v_0 is decreasing in β, and increasing in λ and in δ.

Proof. In the Mathematical Appendix. ■

In words, given the local market structure, the sellers' expected payoff increases with their bargaining power and decreases with their aggregate proportion in the market. At the same time, as frictions decrease, the minimal valuation of the buyers who trade increases.

We end this section with the study of the role played by β at the extreme values of its admissible range.

Corollary 4 $\lim_{\beta \to 0} v_0(\cdot) = 1$, $\lim_{\beta \to 1} v_0(\cdot) > 0$.

If all the local bargaining power is with the sellers ($\beta \to 0$) they get the whole surplus; however, the converse is not true: the sellers' payoff is strictly positive as $\beta \to 1$. This is because, even if the seller has no local bargaining power, she can always rely on the beneficial effect of (bidding) competition among buyers; unless, of course, p_1 also tends to 1 (see below, Proposition 5), in which case local competition becomes negligible, and her payoff is zero.

5 Local conditions and the Walrasian outcome

We can now come to the study of the relationship between local market structure in decentralized trade mechanisms and the market outcome in the limit as frictions disappear. Specifically, we ask the following question: Are there matching mechanisms which determine local market conditions such that the (expected) outcome of decentralized trade, under that mechanism, is the same as would be obtained in a Walrasian global market, namely the intersection of the current global demand and supply curves? We are able to answer this question in the affirmative. This is important, in view of the negative answer that Rubinstein and Wolinsky (1985) and Wolinsky (1988) obtain with their local trading mechanisms.

We begin by defining the (static) Walrasian outcome. Supply is the vertical line S. With a uniform distribution of valuations, (inverse) demand is $D(v) = B(1 - v)$. Their intersection gives the Walrasian price, $\max\left\{0; \frac{B-S}{B}\right\} = \max\left\{0; \frac{\lambda-1}{\lambda}\right\}$, which is also the sellers' payoff. We consider separately the cases of low and high λ: low λ implies that the sellers are on the long side of the market, that is, global and local conditions are not aligned. In this case, we obtain a generalization of the Rubinstein and Wolinsky (1985) result:

Proposition 5 Let the sellers be on the long side of the global market ($\lambda \leq 1$). The static Walrasian price obtains if and only if the buyers' probability of two-to-one matching, p_2, tends to 0, and either their bargaining power, β, or the discount rate, δ, tend to 1.

Proof. When $\lambda \leq 1$, the Walrasian price is 0. By Corollaries 2 and 3, we know that v_0 is decreasing in β and that for high β it is increasing in p_2: Consequently, it only reaches its minimum when the conditions of the proposition are satisfied. Thus, all we need to show is that these conditions are sufficient to yield the price of zero. Now, observe that $P = \frac{v_0(1-\delta q_0)}{\delta(1-q_0)}$ tends to 0 only when v_0 tends to 0; while v_0 tends to 0 only as $f \to 0$. Using (11), we obtain:

$$\lim_{p_1 \to 1 - p_0} f(\delta;\beta;p_1;p_0) = -\frac{1}{2}\frac{\delta(1-\beta)}{1-\delta(1-\beta)}$$

and the result follows. ∎

That is, for the static Walrasian price to obtain, it must be the case that the sellers have no market power at the local level. In addition, unless agents are infinitely patient, sellers must not have any bargaining power either.[9] To interpret this conclusion, note that, when $\lambda \leq 1$, the buyers are on the short side of the global market. However, the matching mechanism that we have set up is such that the buyers are, by construction, on the (weakly) long side of the local markets. Proposition 5 can therefore be interpreted as saying that the static Walrasian outcome cannot obtain if there is a conflict between local and global market conditions, in the sense that the long side of the global market is the short side of the local markets. Indeed, as the following Proposition shows, in the extreme case, where the long side of the global market has the full market power in the local markets, they receive the entire surplus from trade.

Proposition 6 $\lim\limits_{\delta \to 1} \lim\limits_{p_1 \to 0} v_0(\cdot) = 1$.

Proof. Follows immediately from (11). ∎

Note that this result holds independently of both the global market conditions, λ, and the distribution of bargaining power, β. Proposition 6 therefore shows that the local market structure can be more relevant in determining the market outcome than either of those indicators. And yet, it has been neglected by the existing literature, which has focused on the role of the global market conditions and the bargaining power.

We now turn to the case $\lambda > 1$, which we view as more natural, because, in this case, the local conditions reflect the global market conditions. For example, if there exists unemployment nationally, viz if there are few buyers, then, in any given locality, excess supply of labor seems more likely than excess demand.[10]

The next result shows that the equilibrium price determined by the decentralized trading mechanism studied in this paper can equal the static Walrasian price, provided the globally long side of the market has sufficient bargaining power.

Proposition 7 Fix $p_0 < 1$: In the limit as $\delta \to 1$, for every $\lambda > 1$, and for every $\beta \in \left(\frac{1}{2\lambda - 1}; 1\right)$, there exists $p_1 \in (0; 1)$ such that the expected average market price coincides with the static Walrasian price.

[9] Recall that Rubinstein and Wolinsky (1985) is a special case: they assumed that the market power is evenly distributed (one-to-one matches), and showed that the globally long side can have no bargaining power to obtain the Walrasian outcome.

[10] Note also that, given our assumptions on the distribution of the traders' valuations, only when $\lambda > 1$ the static Walrasian price is strictly between the sellers' valuation and the highest possible buyer valuation, implying that traders on both sides of the market receive a strictly positive payoff.

Proof. The proof requires that there exists a value of p_1, say p^*, such that $v_0 = \frac{-1}{\lambda}$, and that this p^* is feasible, in the sense that it satisfies (8). Given that $v_0 = 1 - \frac{1-\sqrt{(1-4\lambda f(\delta;\beta;p_1;p_0))}}{2\lambda f(\delta;\beta;p_1;p_0)}$; solving $1 - \frac{1-\sqrt{(1-4\lambda f(\delta;\beta;p_1;p_0))}}{2\lambda f(\delta;\beta;p_1;p_0)} = \frac{-1}{\lambda}$ for $f(\delta;\beta;p_1;0)$ (we set $p_0 = 0$, as this is the most stringent case, since the locus (8) is increasing in p_0), we obtain that p^* must satisfy: $f(\delta;\beta;p^*;0) = 1-\lambda$. If such p^* does exists, then it always satisfies (8). This is because, using $f(\delta;\beta;p_1;0) = 1-\lambda$, (8) can be written as $\lambda - \frac{2(1+p_1)-4(1-\lambda)}{(1+p_1)^2} < 0$, or $\lambda > \frac{2}{p_1+3}$, which is always satisfied as $\lambda > 1$.

Because f is a continuous and, for $\delta \to 1$, monotonically increasing function of p_1, and because $\lim_{\delta\to 1}(\lim_{p_1\to 0} f(\delta;\beta;p_1;p_0)) = -\infty$, the proposition obtains if $\lim_{\delta\to 1}(\lim_{p_1\to\infty} f(\delta;\beta;p_1;p_0)) > 1-\lambda$. Evaluating the limit, the above becomes (again considering the most stringent case, $p_0 = 0$): $-\frac{1-\beta}{2} > 1-\lambda$. Whenever this holds, then, by the Weierstrass theorem, there exists p^* such that $\lim_{\delta\to 1} f(\delta;\beta;p^*;p_0) = 1-\lambda$. This establishes the result. ■

Note that, as λ increases, the range of values of β such that the static Walrasian outcome is feasible also increases. Loosely speaking, this suggests that there is a trade-off between local and global market conditions. A high value of λ indicates poor global market conditions, whereas a high value of β indicates good local conditions, from the buyer's point of view.

Both in Wolinsky's (1988) analysis and in Proposition 7 the local market conditions reflect the global market conditions. However, in contrast to our conclusion, Wolinsky (1988) obtains a strongly non-Walrasian outcome. In his main result (Proposition 2), he shows that converting aggregate excess supply into local competition cannot restore the Walrasian outcome, and can in fact result in a price which is even farther from the Walrasian price than that obtained in Rubinstein and Wolinsky (1985); in particular, as frictions disappear, the long side (both locally and globally) of the market receives all the surplus from trade. The reason for this counterintuitive conclusion rests in the way in which Wolinsky models the traders' valuations. He assumes that traders draw a new valuation in each period. This creates a direct beneficial effect of waiting: as frictions disappear, it becomes worthwhile for a buyer to wait until he draws a high valuation and, consequently, his incentive to bid seriously is weakened: in the limit this destroys instantaneous competition. In our set-up, delaying trade is justified only as a way to obtain better terms in the future, which is consistent with the rest of the literature on strategic bargaining, as well as with economic intuition.

6 Concluding remarks

A fundamental component of the non-cooperative game theory program is the description of extensive form games –that is the formal sequences of moves and countermoves by the agents– which can be applied to economic situations. The next step consists in verifying that the appropriate equilibrium of the proposed game corresponds to what economists have believed the equilibrium to be in the economic situation considered. The contribution of our paper can also be interpreted in this vein: the most classical economic problem, the determination of the equilibrium price in the market for a homogeneous good, is clearly a very natural candidate for the application of this game-theoretic program. Since Marshall (1890, p. 333), we all teach our students that the equilibrium price is given by the intersection of demand and supply; but we would be unable to give a satisfactory answer to an innocent audience question as to why this should be the case. Indeed, the literature starting with Rubinstein and Wolinsky (1985) appeared to have dealt a fatal blow to the possibility of …nding an extensive form game whose equilibrium corresponds to the accepted notion of equilibrium. They propose a very natural decentralized trade mechanism which does not lead to the Walrasian outcome. However, we generalize the Rubinstein-Wolinsky mechanism and we both obtain a more natural mechanism –allowing as it does conditions to vary across local markets–, and show that decentralized trade may lead to the Walrasian outcome.

As far as we are aware, the matching mechanism introduced by Wolinsky (1988) –also used in a di¤erent context by Coles and Muthoo (1998)– is the only one not restricting matching to be one-to-one. However, apart from his counterintuitive result discussed above, his model is not well suited to the study of the interplay between local and global market conditions, because, by virtue of his assumption that the matching process is governed by the binomial distribution, the global conditions (the ratio of buyers to sellers) uniquely determine the local conditions (the probability of one-to-j matching, for the various admissible values of j).

Gale (1988) presents a model with one buyer and two sellers, where in each period it is random whether the sellers bid competitively (ex ante pricing) or one of them is selected to make an o¤er (ex post pricing). This set-up might be seen as a parsimonious version of our market game with many-to-one matching; however, the loss of generality is signi…cant. His main result is that independently of how small the probability of competition is, as frictions disappear, the outcome becomes fully competitive (price equals the sellers' valuation). But, this result hinges on the assumption that there is a single buyer and therefore the sellers cannot freely decide to delay trade, while the buyer can. As we show here, in a market context, the fully competitive result only obtains if instantaneous competition is guaranteed.

Another related paper suggests a di¤erent interpretation of our results: Taylor (1995) assumes that the aggregate market conditions are governed by an exogenous random variable. He observes that the current global conditions are not su¢ cient to determine the market price. This is because traders expect the global short and long sides to switch around, and this allows the forward looking equilibria to be markedly di¤erent in each period from the static Walrasian price. In our paper, it is also the case that knowledge of the aggregate demand and supply in a large decentralized market is not su¢ cient to generate predictions about the market equilibrium, not even about the ‡ow of agents in and out of the market. Consequently, –paralleling Gale's critique of the static concept of Walrasian equilibrium– we are led to the conclusion that, in a decentralized market, the description of the global conditions is not su¢ cient for an unambiguous de…nition of the Walrasian equilibrium: it is necessary to include the description the micro-structure, that is of the local market conditions.

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