Law, Property, and Marital Dissolution

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This paper challenges the view that legal rights are not important in affecting whether people divorce, but it puts as much emphasis on property rights (given, for example, by the law on alimony) as on dissolution rights. The paper sets out two stylised models of marriage and examines the consequences of fuller compensation for economic sacrifices made during marriage. If the dominant economic issue in a marriage is who undertakes household tasks then a law giving fuller compensation makes divorce more likely. If the dominant issue is child custody, divorce is less likely.

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What is the effect of the law on the incidence of divorce, and to what extent has the liberalisation of divorce laws over the last thirty years been responsible for the dramatic rise in marital dissolution? As some US states have begun to consider repealing the no-fault legislation of the 1970s, and as the UK continues to change its divorce laws, these questions lie at the heart of the public policy debate on the effects and desirability of divorce reform.

The economic approach to this problem starts by recognising that divorce law confers certain rights regarding marital dissolution, and it treats these as property rights in the marriage; see, for example, Becker (1991), Becker, Landes, and Michael (1977), and Peters (1986). A law that allows unilateral divorce thus has a very different disposition of rights than one requiring mutual consent. But whether this makes any difference to the incidence of divorce has been challenged by the following argument. Suppose it is the husband who wishes to divorce; if his gain from divorce is large then under a mutual consent law he may be able to compensate his wife so that after a divorce they would both be better off. In effect the wife has sold her right to the marriage. Thus divorce occurs under mutual consent if there is a potential for both partners to be better off, perhaps after a suitable transfer that would form part of the divorce settlement. Under a unilateral law, if the husband's gain from divorce is small and the wife's loss is great, then she may be able to compensate her husband for not seeking a divorce. They would then both be better off than had they divorced. In effect the husband has sold his right to divorce, his compensation being some change in the way the marriage is conducted. The conclusion is that whether divorce occurs or not depends not on the law, but on the relative magnitude of the gains and losses of divorce.

Attempts to test whether divorce law affects divorce rates have produced mixed results. Using U.S. cross section data, Peters (1986, 1992) finds the law to be neutral. Her results have been criticised by Allen (1990, 1992), mainly on the grounds that she misclassified some states as having fault-based laws. Using U.S. panel data, Zelder (1993) and Friedberg (1997) find a positive impact on divorce rates of the change to a unilateral law,
whereas Smith (1997), using U.K. time series data, finds that rules of legal procedure are important, but not the change from fault to no-fault. Overall, the empirical evidence has not been decisive.

This paper looks at the theoretical basis for a relationship between divorce law and divorce incidence. I argue that there is no basis for the argument that the law necessarily has no effect on the incidence of divorce, but this does not imply that couples are missing mutually beneficial trades or that economic efficiency is compromised. I also emphasise other aspects of the law apart from the right to dissolve a marriage. How assets and resources are allocated within a marriage, and on dissolution, plays a central role in the analysis. These two branches of the law define resource rights that determine the gains and losses from divorce, and these rights are also important in determining whether divorce occurs.

The plan of the remainder of the paper is as follows: Section I provides a brief discussion of how family law defines some of the property rights alluded to above, with some concrete examples, drawn mainly from Scots law. Section II sets up a formal economic model of property rights and distribution within marriage and after divorce. In Section III two stylised models of marriage are used to analyse compensation for economic sacrifices made in marriage, and child custody. Section IV concludes.

I. PROPERTY RIGHTS AND FAMILY LAW

I.1 Dissolution Rights

Scots law has allowed divorce since the Reformation, but until 1976, it was based almost exclusively on some idea of fault: adultery, desertion, or, since 1938, cruelty, sodomy, or bestiality. Since the Divorce (Scotland) Act of 1976, the only ground for divorce has been the irretrievable breakdown of the marriage, which must be proved by at least one of five "facts": adultery; unreasonable behaviour; desertion; two years separation if both partners
consent to divorce; five years separation if one partner wishes to divorce. The last fact allows for unilateral divorce, the main innovation of the 1976 act.

Fault-based divorce is often held to be identical with divorce requiring mutual consent, on the grounds that in a fault-based system divorce can only happen if one spouse commits a fault (presumably in full knowledge of the possible legal consequences) and the other brings an action at law. Similarly, the absence of a fault requirement is almost invariably identified with unilateral divorce, although there is no reason why a dissolution law should be unilateral if and only if it is a no-fault law. For our purposes the important issue is whether one partner can withhold consent to a divorce or not, and this is the criterion by which the paper assesses past and prospective legal reforms, even if they are couched in the language of fault.

I.2 Resource Rights Within Marriage

The title to a physical or financial asset may belong to one or both partners, and it can be transferred from one partner to another. Who has the right to such property when a couple disagree on how it should be used depends on the current state of the law relating to marital property. This law differs between countries, and has changed over time. In Scotland before the Married Women's Property (Scotland) Act of 1881, a wife had no right to her own moveable property, such as money or furniture, even if she had earned it or inherited it (an exception was so-called paraphernalia such as dresses and jewellery). Until the passing of the Married Women's Property (Scotland) Act of 1920, heritable property such as land or a house, even though owned by the wife, was controlled by the husband. Broadly speaking, Scots law now applies the same property law to spouses as to any two people, so that a wife's earnings are her own property. If the title to the matrimonial home is in the husband's name, then he owns the home, and can sell it. His wife cannot prevent the sale, although she may have a claim on the proceeds and may still have a right to occupy the house. Indeed, the husband can be excluded
from his own house if there is a risk of danger to his wife or a child of the family (Matrimonial Homes (Family Protection) (Scotland) Act 1981).

These examples of property law show dramatic changes in the disposition of property rights within marriage over the course of a century: from having virtually absolute control over family resources, a husband can now be excluded from his own house.

I.3 Resource Rights at Divorce

How property, financial assets, and income are divided up at a divorce is a large and complex part of family law. In Scotland, the Family Law (Scotland) Act of 1985 sets out the principles by which this should be done. Before 1985, financial provision at divorce was left mainly to the judges' discretion. My intention here is not to give an account of the history of the law, but to identify some important issues which have a bearing on the model of rights and bargaining set out in this paper.

(i) Will one partner be compensated for any economic sacrifices they have made during a marriage? An affirmative answer to this is one of the central principles of the 1985 Act, the intention being to protect women who have devoted time and effort to housekeeping and rearing children, and in doing so have possibly given up a well paid job or damaged their future career prospects.

(ii) Who will bear the cost of caring for dependent children of a marriage? The 1985 Act lays down that this is to be shared equally between the husband and wife, although the courts' jurisdiction has largely been superseded by the Child Support Act of 1991, which gives powers to officers of the Child Support Agency to make maintenance assessments.

(iii) What means are at a court's disposal for reallocating property and income? In Scotland, a court can make awards of a capital sum, of periodical allowances, and the transfer of property. It can set aside or vary the terms of prenuptial settlements, transfer property or money to trustees, and make orders regarding the division of benefits from life policies and pensions. In
short, Scottish courts currently have a very wide range of powers, so that, as Thomson has remarked, "[a] court can make orders for financial provision which can be tailor-made for the particular couple concerned" (1996, p126).

The law concerning child custody defines another forms of right, which might not fall within a narrow definition of property rights but which is certainly important in determining the division of the pie at divorce, both because resources have to be devoted to children, and because children can be considered as 'assets' of the marriage i.e. as part of the pie itself. In England, for example, before the Child Custody Act of 1839 a wife, even if separated, had no custody or visiting rights. Since then the law has changed significantly. Both in England and Scotland, custody law is now theoretically neutral between husband and wife.

II. THE MODELLING FRAMEWORK

The previous section illustrated how property rights and family law have changed dramatically in the United Kingdom over the last 150 years. Broadly speaking, property rights have been transferred from men to women. Historically this legal emancipation has been accompanied by profound changes in the political and economic status of women. In order to isolate the effect of legal changes on the incidence of divorce, I now set up a simple model of property rights and marital dissolution.

I consider a husband and wife, living together as a married couple. They may already have children. Following Dorothy Parker, we might think of them as seven years into their marriage. Each has a certain earning capacity, which may depend on their human capital, including their previous labour market history. They have also accumulated physical and financial assets such as real estate, bank accounts, stocks, and perhaps pensions rights.

II.1 Utility under Marriage and Divorce
The combined resources of the couple define a utility possibility set, labelled $M$, giving the feasible levels of utility that can be achieved through marriage. Although the position of $M$ does not depend on the disposition of marital resource rights, to achieve some points in $M$ may mean that one partner has to give up rights, or at least not to enforce them, and perhaps to transfer resources. Movements within $M$ may involve not just changes in consumption expenditure, but also changes in who goes out to work, who looks after any children, who does the housework; all this will depend on the precise specification of household members’ tastes, resources, assets, and employment opportunities.

The combined resources of the couple also define a utility possibility set, labelled $D$, giving the feasible levels of utility that can be achieved if the couple divorce. Again, the position of $D$ does not depend on resource rights on divorce: it is a collection of possibilities. But it will be affected by any opportunities that follow from divorce, for example the ability to find happiness and wealth with another partner. It will also reflect the downside of divorce, especially that arising from the splitting of the household, such as the loss of economies of scale, and the reduced access to children suffered by one of the parents.

I make a free disposal assumption about $M$ and $D$ that their boundaries are their Pareto frontiers, denoted by $B_M$ and $B_D$, which I also assume to be continuous and nowhere positively sloped. I further assume that $D$ and $M$ are not affected by the disposition, enforcement, or transfer of legal or resource rights (in effect, there are no transaction costs).

II.2 Dissolution Rights, Outside Options, and Efficiency

To analyse the decision to dissolve a marriage I rely on two principles. Firstly, I assume that dissolution rights define, for each partner, an outside option which, once exercised, determines whether the couple will separate or stay together. In the case of a unilateral dissolution law, the outside option is divorce: either partner can, without the consent of the other, force a
dissolution of the marriage. With a mutual consent law, either partner can refuse to divorce and, without the consent of the other, force a continuation of the marriage. For the outside option principle to have power, it is important that exercising the option is irrevocable. This may seem a rather stark assumption, since it appears to rule out reconciliation, or changing one's mind. These are interesting problems, briefly considered in Section 4.

The second principle is efficiency: no Pareto improvement is left unexploited. In the absence of informational asymmetries and transaction costs this assumption may seem quite mild, but it implies a collective rationality by the couple even if, for example, one of them has been spurned in favour of another lover.

If the outside option is exercised, the couple will reach an efficient agreement on the allocation of resources, giving a utility pair denoted (in obvious notation) either by \( m^* \) or \( d^* \). The agreement they come to depends on a number of things, but most importantly on property law.\(^1\) Conditional on the marital state, the law defines what each partner is entitled to. If marriage is effectively a zero-sum game (so that there is no gain to cooperation within marriage) the law on marital property defines directly the point \( m^* \) on the Pareto frontier of \( M \). If marriage is a positive-sum game, the law defines entitlements (e.g. the right to one's own earnings) but the relationship between such entitlements and the agreed point \( m^* \) on the Pareto frontier of \( M \) depends on how couples reach agreement within marriage. A change in marital property law changes these entitlements, and we can expect this to change the agreement \( m^* \) to some other point \( B_M \). A similar argument applies to the relationship between alimony law and the point \( d^* \) on \( B_D \) when one partner has exercised the right to unilateral divorce.

Consider now the situation prior to the exercise of any outside option. With a dissolution law requiring mutual consent, there is no reason for one partner, e.g. the wife, to agree to a divorce unless she gets at least \( m_{w^*} \), her utility level if she refuses consent; similarly for the husband. Consequently

\(^1\) To use the phrase coined by Mnookin and Kornhauser (1979), the couple "bargain in the shadow of the law".
divorce will not occur if \( m^* \not\in D \). If \( m^* \in D \) then divorce offers a potential Pareto improvement over marriage. It might be that only the husband, say, gains by a move to \( d' \); but if \( m^* \in D \) then by giving up post-divorce property rights he can compensate his wife for consenting to the divorce yet remain better off than at \( m^* \). If there is a range of divorce allocations that make both partners better off compared to \( m^* \) which of these is chosen requires further detail on how a divorcing couple reach agreement. Nevertheless under a law requiring mutual consent, divorce occurs if and only if \( m^* \in D \).

With a dissolution law allowing unilateral divorce, a similar argument applies. Whatever agreement the couple come to, the elements of \( d' \) are the minima that the wife and husband would accept. Therefore if \( d' \not\in M \) the marriage cannot be saved and the outcome is \( d' \). If \( d' \in M \), then although one partner might benefit by a unilateral divorce, there will generally exist a range of marital allocations that make both of them better off compared to \( d' \), although further structure is required to specify which is chosen. Nevertheless, under a unilateral law, divorce occurs if and only if \( d' \not\in M \).

The discussion so far may be summarised as follows:

<table>
<thead>
<tr>
<th>LAW</th>
<th>DIVORCE OCCURS IF AND ONLY IF</th>
<th>OUTCOME IF</th>
<th>OUTCOME IF</th>
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<tbody>
<tr>
<td>unilateral</td>
<td>( d' \not\in M )</td>
<td>( d' )</td>
<td>( m' \text{ where } m' \geq d' )</td>
</tr>
<tr>
<td>mutual consent</td>
<td>( m^* \in D )</td>
<td>( d' \text{ where } d' \geq m^* )</td>
<td>( m^* )</td>
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**TABLE 1**

This formulation enables us to be precise about the conditions under which dissolution and property law matter. For arbitrarily given sets \( M \) and \( D \) and points \( m^* \) and \( d' \) the conditions \( d' \not\in M \) and \( m^* \in D \) are not equivalent, and neither necessarily implies the other. It is only in the special case in which \( M \) is wholly contained within \( D \) or vice versa that the law does not matter. We
can now interpret the claim of Becker and others that legal rules make no difference to divorce rates. A result of Becker's model of the household is that each partner's utility is positively related to their share of "commodity wealth", an aggregate which may increase or decrease when the couple divorce. Because commodity wealth is transferable between spouses at a rate of one-for-one whether they are married or divorced, the two frontiers in utility space cannot cross. Then the law on dissolution rights makes no difference to whether the couple divorce, nor do the laws on marital property and divorce settlements.

If $M$ and $D$ intersect, the outcome depends on both dissolution and property law. Consider Figures 1 to 4. In Figure 1, the marriage survives whatever the law on dissolution, and in Figure 4 divorce occurs, whatever the law on dissolution. This isolates the potential importance of property law (which determines the positions of $m^*$ and $d^*$) in affecting divorce. Models that explore this further are considered in Section 3. In Figures 2 and 3, reminiscent of diagrams showing the Scitovsky paradox, dissolution law does matter. In Figure 2 a move from mutual consent to a unilateral law changes the outcome from marriage to divorce. In Figure 3, divorce occurs under mutual consent, but with a more "liberal" unilateral law the marriage survives.

III TWO STYLISED MODELS OF MARRIAGE AND DIVORCE

Dissolution and property law matter if the sets $M$ and $D$ intersect. I now present two stylised models with this characteristic. In Model 1 efficient marriage involves cooperation. In Model 2 efficient divorce also involves cooperation. To resolve the indeterminacy of how gains from cooperation are distributed, I use a bargaining solution with minimal structure. Both models use Cobb-Douglas utility functions. Details of mathematical workings are given in the Appendix.

III.2 Model 1: who spends time at home?
I assume that if the couple are married their utility functions are given by
\[ u_i = (x_i + x_j)^{0.5} y_i^{0.5} \quad i = h, w. \]
Here, \( x_i \) is \( i \)'s output of some local public good such as housework. I assume that one unit of housework takes one unit of time. Partner \( i \) has an endowment of time, \( T_i \), which can also be used to earn money at a wage \( w_i \). Earned income can be used to buy the private good \( y_i \) at a price \( p = 1 \). If the couple cooperate in marriage, we can think of earned income being pooled, so that household activity satisfies \( w_i x_i + w_h x_h + y_i + y_w = w_i T_i + w_h T_h \). The frontier \( B_M \) is generated by maximising \( u_h \) subject to a given level of \( u_w \), the pooled budget constraint and \( 0 \leq x_i \leq T_i, 0 \leq x_w \leq T_h \). Then along \( B_M, (u_w)^2 + (u_h)^2 = \max\{w_w, w_h\} T_e^2 \).

To identify the point \( m^* \) (the outside option under mutual consent) I first analyse the Nash equilibrium of the non-cooperative marriage game. If the couple do not cooperate, they spend the money that they are entitled to, but each takes no account of the effect that his/her production of the public good has on the other. Partner \( i \) chooses \( x_i \) and \( y_i \) to maximise \( u_i \) subject to \( 0 \leq x_i \leq T_i \) and \( w_i x_i + y_i = Y_i \), given \( x_j \), where \( Y_i = w_i T_i + S_i \). The sums \( S_h \) and \( S_w \) are transfers that reflect legal entitlements and obligations, so \( S_h + S_w = 0 \). I assume any legal obligation is feasible, i.e. \( Y_i \geq 0 \). Partner \( i \)’s choices satisfy \( x_i = \min \left[ \max \left[ 0, \frac{1}{2} \left( \frac{Y_i}{w_i} - x_j \right) \right], T_i \right] \) and the budget constraint \( y_i = Y_i - w_i x_i \). Similarly for partner \( j \). The Nash equilibrium uniquely solves these four equations and the resulting utility pair achieved is denoted by \( m^0 \).

I now assume that the point \( m^0 \) depends on \( m^0 \) in a simple way: I posit a vector-valued function \( m^* = f(m^0, M) \) such that (i) \( m^* \) is on \( B_M \); (ii) \( m^* \geq m^0 \); and (iii) the matrix of first derivatives, \( \partial f/\partial m^0 \), always has positive diagonal elements and negative off-diagonal elements. The first condition repeats the principle of Pareto efficiency; the second is one of individual rationality - no agreement can make anyone worse off than disagreement; the third condition is a monotonicity requirement that if \( m^0 \) moves in favour of \( i \) then \( m^* \) moves in favour of \( i \) and against \( j \). One function satisfying (i), (ii) and (iii) is the Nash bargaining solution \( m^* = \arg\max_{m^0} M(m_h - m^0_h)(m_w - m^0_w) \), so my approach is consistent with that taken in Ulph (1988), Kanbur and Haddad (1994),
Lundberg and Pollack (1993), and Bergstrom (1996), and is in contrast to the models of Manser and Brown (1980) and McElroy and Horney (1981), who adopt the Nash bargaining solution but take as the disagreement point the outcome if the couple divorce. The function $f$ implies that $m^*$ depends via $m^0$ on the legal entitlements given by matrimonial property law.

Divorce has two effects. Firstly, by separating the couple, it destroys the advantages of living together i.e. the sharing of public goods. Secondly, it creates new opportunities for wealth and happiness. I shall model this second effect by a common shock, $\varepsilon$, to each utility function. We might think of this as a measure of the mismatch of the marriage, which disappears on divorce. A couple that are badly matched therefore have a positive shock when they separate. After divorce $i$'s utility is given by $u_i = (x_i^{0.5}y_i^{0.5}) \exp(\varepsilon)$ which is maximised subject to $w_i x_i + y_i = Y_i$, and $0 \leq x_i \leq T$, where $Y_i = w_i T + A_i$ and $A_i$ is a transfer payment (or alimony) to $i$ such that $A_h + A_w = 0$ and $Y_i \geq 0$. This specification of post-divorce utility assumes that there are no cooperative gains to be had after marriage, so the utilities achieved directly define the point $d^*$. As alimony law varies, $A_h$ and $A_w$ change and the frontier $B_D$ is traced out. Let $w_h \geq w_w$ (as in the numerical example below); then along $B_D$

$$w_h^{0.5}u_h + w_w^{0.5}u_w = \frac{1}{2}(w_h + w_w)\exp(\varepsilon)T$$

if $A_w < w_w T$

$$2w_h^{0.5}\exp(\varepsilon)Tu_h + w_w^2 = w_h\exp(2\varepsilon)T^2$$

if $A_w \geq w_w T$.

Suppose that at the beginning of the marriage the two partners were equally well qualified, and had identical employment opportunities. During the early part of their marriage the wife withdrew from the labour market to undertake "domestic production". Suppose further that this had a damaging effect on the wife's employment prospects, so that now, seven years on, her potential wage is significantly less than the husband's. To be specific, let us

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2 Although I do not provide an extensive form justification for the function $f$, my approach is also consistent with Binmore, Rubinstein, and Wolinsky's (1986) non-cooperative underpinning of the Nash bargaining solution, the essence of which is that the players receive $m^0$ while bargaining i.e. before agreement is reached.
assume that \( w_h = 4 \), and \( w_w = 1 \). To complete the model, assume that \( T = 12 \) and \( \exp(\varepsilon) = 1.5 \) Then the frontiers \( B_M \) and \( B_D \) are as shown in Figure 5.

For both marriage and divorce, let us now look at two possible stances that the law might take on property rights. Firstly, the law might specify that each partner is entitled to his or her own earned income, whatever the marital state. Then \( S_w = S_h = A_w = A_h = 0 \), and whether married or divorced the wife is entitled to a full income \( (w_wT) \) of 12, and the husband to a full income \( (w_hT) \) of 48, the value of their respective time endowments. This generates the ordered pair \( (u_w, u_h) = (8, 16) \), labelled \( a \) in Figure 5, if the couple play a non-cooperative game within marriage, and the pair \( b = (9, 18) \) if they divorce; thus \( m^0 = a \) and \( d^* = b \). The point \( m^* \) must satisfy \( m^* \geq m^0 \), so \( m^* \notin D \); but \( d^* \in M \). Hence the marriage survives whatever the law on dissolution. Alternatively, marital and alimony law might both specify that the wife is entitled to be compensated for her loss of earning potential and to be rewarded for the contribution she has made to any increase in her husband's earnings. Suppose this entitles the wife to an equal share of their joint full income of 60, implying a value of 18 for \( S_w \) and \( A_w \), and of -18 for \( S_h \) and \( A_h \). This generates the point \( c = (14.70, 18.97) \) if the couple play a non-cooperative game within marriage, and the point \( d = (22.05, 11.25) \) if they divorce.\(^3\) Now \( m^* \notin D \) and \( d^* \notin M \), so the marriage survives only if divorce requires mutual consent.

However, it can readily be seen that further increases in \( S_w \) would eventually place \( m^* \) inside the set \( D \). This hints at a deeper message in Figure 5. If the husband's utility is zero, the wife would prefer divorce \( (u_w = 36) \) over marriage \( (u_w = 24) \). In this sense divorce favours the wife; similarly marriage favours the husband. The underlying reason is that after a divorce the husband has a much higher opportunity cost of acquiring the good \( x \); he must give up time at work, and suffers a greater loss of earnings. Consequently in situations where an important issue is who spends time at home, divorce is

\(^3\) Note that at \( c \) the outcome is efficient. Also, both parties are better off at \( c \) than at \( a \). This reflects an aspect of public good games with differing production costs that has been noticed by a number of writers, especially Buchholz and Konrad (1995). It implies that a change in the law that increases the transfer to the wife does not necessarily benefit her.
more likely if the relevant property law (marital property law under mutual consent, alimony law under unilateral divorce) moves in favour of the partner with the lower wage rate.

III.3 Model 2: who gets child custody?

In this section I modify Model 1 so that it becomes a special case of a model of child custody due to Weiss and Willis (1985). The public good is now expenditure on a single child, which benefits both parents. In a marriage, cooperative or not, the child lives with both parents, and either parent can spend on the child. After a divorce, child expenditure is still a (local) public goods, but the couple live separately. One parent has custody (e.g. the mother), and only she spends on the child. I assume that it is impossible for the father to monitor this expenditure, and so there is no mechanism whereby the mother can internalise the impact of her child expenditures on the father. Hence \( B_0 \) is constrained rather than fully efficient, the constraint being one of behavioural feasibility, not resources.

The utility functions in marriage are as in Model 1, \( x_i \) now being \( i \)'s expenditure on the child. There is no decision about housework and labour supply, and \( i \)'s earned income is exogenously given as \( w_i T \). Child and adult consumption have a common price of 1, so a cooperative marriage has a pooled budget constraint of \( x_h + x_w + y_h + y_w = w_x T + w_y T \). Then along \( B_M, (u_h)^2 + (u_i)^2 = (0.5(w_h + w_i)T)^2 \). As in Model 1, the disagreement point in marriage is the Nash equilibrium of a game in which \( i \) chooses \( x_i \) and \( y_i \) given \( x_j \) and \( Y_i = w_i T + S_i \) (earned income, plus any transfers as defined by marital property law). Partner \( i \)'s choices satisfy \( x_i = \max [0, \frac{1}{2}(Y_i - x_j)] \) and the budget constraint \( y_i = Y_i - w_x x_i \). Similarly for partner \( j \). In this model the Nash equilibrium, \( m^0 \), has the property, characteristic of models of private provision of a public good with identical opportunity costs, that as long as each partner spends something on the child then their individual consumption levels and the total expenditure on the child are invariant to the distribution of total
income \(w_w T + w_h T\). The locus of all possible non-cooperative equilibria is symmetric around the 45° line, but unless \(Y_w\) and \(Y_h\) are very unequal the point \(m^0\) will be on the 45° line. Given that \(M\) is symmetric this limits how different the elements of \(m^*\) can be.

If the couple divorce, there are two effects. As before, there is a shock to the utility functions. But only the parent with custody of the child makes child expenditures, even though both parents continue to benefit. If \(i\) has custody, \(u_i = (x_i^{0.5} y_i^{0.5}) \exp(\varepsilon)\) which is maximised subject to \(x_i + y_i = Y_i\), where \(Y_i = w_i T + A_i\). The non-custodial parent, \(j\), spends total income \(Y_j\) on \(y_j\) and gets utility \(u_j = (x_j^{0.5} y_j^{0.5}) \exp(\varepsilon)\). The frontier relating \(u_w\) and \(u_h\) when \(i\) has custody, denoted by \(C_i\), is given by \((u^2_j)^2 = u_i((w_i + w_j) T \exp(\varepsilon) - 2u_i )\). The feasible set bounded by \(C_i\) I label \(D_i\).

Despite any inability to enforce agreements about child expenditure, there is still room for cooperation after divorce. Legal rights assign custody and alimony, isolating a point on \(C_w\) or \(C_h\) which we can identify as \(d^0\), the divorce disagreement point. But if the custodial parent, \(i\), has a low total income, \(w_i T + A_i\), then \(d^0\) is in the interior of \(D_i\). A cooperative divorce would involve an agreement to transfer custody to \(j\) and adjust alimony. Then \(B_D\) is the outer envelope of \(C_w\) and \(C_h\) and \(D\) is the union of \(D_i\) and \(D_j\).

As in Model 1, \(M\) is convex and symmetric around the 45° line. \(D\) is the union of two intersecting convex sets and is thus not convex; but even if \(w_w\) and \(w_h\) differ, \(D\) is symmetric around the 45° line. This is because the wage rates (the only way in which the husband and wife might differ) only enter the model via the terms \(w_w T\) and \(w_h T\), the sum of which is redistributed by the divorce settlement. In particular, \(w_i\) is not \(i\)'s post-divorce opportunity cost of the public good, as in Model 1. It can be shown that (i) if \(\varepsilon < 0\) \(D\) lies within \(M\).

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4 For a general analysis of the invariance property, see Bergstrom, Blume, and Varian (1986).

5 With the current parameterisation, \(m^0\) will be on the 45° line if \(0.5 Y_h < Y_w < 2 Y_h\).

6 However there is no loss of utility from loss of custody per se.
so the marriage will survive, regardless of the law; (ii) if $\varepsilon > \log_e(3/\delta G)$ $M$ lies within $D$, so divorce occurs, regardless of the law; (iii) if $0 < \varepsilon < \log_e(3/\delta G)$ that $M$ and $D$ intersect twice. In case (iii), $B_M$ lies beyond $B_D$ in the neighbourhood of the $45^0$ line. Figure 6 illustrates. The implications of this are that under a dissolution law requiring mutual consent, if marital property law, through its effect on $m^0$, brings about approximate equality in the elements of $m^*$ then $m^* \notin D$ and the marriage will survive. Under a unilateral dissolution law the marriage survives if $d^* \in M$; this requires that the law on alimony and custody combine to bring about approximate equality in the elements $d^*$. The point where $C_w$ and $C_h$ intersect is exactly on the $45^0$ line, so $d^* \in M$ can be achieved by approximate equality in the elements of $d^*$. With equal wage rates this requires that the parent with custody receive alimony (to pay for child expenditures); but a custodial spouse with a lower wage should be compensated with higher alimony; a non-custodial spouse with a lower wage should be compensated by paying less alimony.\(^7\) The law can place $d^0$ well away from the $45^0$ line, and hence increase the likelihood of divorce, if it grossly favours one partner, perhaps by excessive or insufficient compensation for prior economic sacrifices. Alternatively, the custodial parent might receive excessive or insufficient child maintenance. If, for whatever reason, $d^* \notin M$, then the favoured partner cannot be dissuaded from seeking a divorce, and in an efficient divorce settlement will retain or gain custody. To put this another way, where custody is the central economic issue in a marriage, a trend in the law to awarding more equitable divorce settlements will reduce the likelihood of divorce.

\section*{IV CONCLUSION}

This paper analyses how dissolution law and property law interact to determine divorce incidence. In addition to assuming that legal rights can be costlessly enforced and transferred, the paper relies on a limited number of

\footnote{With the current parameterisation, if $i$ has custody, $d^0$ is on the $45^0$ line if $A_i = (2w_j - w_i)T/3$. This implies $w_iT + A_i = 2(w_jT + A_j)$}
simplifying principles: (i) that dissolution rights define an outside option; (ii) that the final allocation is Pareto efficient; (iii) that neither partner will not agree to a settlement (within marriage or after a divorce) that gives him or her less than in the outside option; (iv) that what a partner gets in any agreed settlement is positively related to what he or she can get by refusing to agree.

As in all models, these results are more or less sensitive to variations in the assumptions. In particular, the outside option principle appears to go against the spirit of recent legislation in the UK (the Family Law Act 1996) which seeks to help couples draw back from the brink of divorce. One way in which this could be included is to allow bargaining not over either the set $M$ or $D$, but over their union. Exercising the right to divorce unilaterally might then be modelled by a switch of disagreement point from $m^0$ to $d^0$. One difficulty with this is that in general the union of $M$ or $D$ will not be convex, so even the use of the Nash Bargaining solution is problematic. This is an interesting area for future research, although early work suggests that it qualifies rather than reverses the main conclusions.

Nevertheless, these principles take the analysis a long way: I show that in general both dissolution rights and property rights are important in affecting whether people divorce, with the consequent policy implication that divorce reform and property law reform must be considered together. If the law on dissolution changes from one of mutual consent to unilateral divorce, then it is essential also to reform or update the law on the post-divorce division of property. Otherwise, spouses who are disadvantaged by an unreformed property law may suffer, either within their marriage or after a divorce, in a way unintended by legislators. Even if the couple remain married, the option of divorce might affect the way they conduct their marriage. Hence an important side effect of changes in alimony law that improve the lot of

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8 The standard approach to a non-convex feasible set is to recognise that the payoffs are von Neumann-Morgenstern expected utilities. Consequently, a lottery over any pair of feasible points is itself feasible, so we should consider the convex hull of the union of $M$ and $D$. This implies that that a couple will throw dice in order to decide whether to divorce or not! A number of authors have proposed solutions to non-convex problems which do not permit convexification; for example, Conley and Wilkie (1996), Herrero (1989), Kaneko (1980), Zhou (1996).
divorced women is that they also tend to strengthen the bargaining position of married women.
APPENDIX

MODEL 1

1. The equation for $B_M$

The frontier $B_M$ can be generated by varying $l$ in the problem: choose $x_h$, $x_w$, $y_h$ and $y_w$ to maximise

$$W = u_h^{1/2} u_w^{1/2} = (x_h + x_w)^{0.5} y_h^{0.5} y_w^{0.5}$$

subject to $w_h x_h + w_w x_w + y_h + y_w = (w_h + w_w) T$, $0 \leq x_h \leq T$, $0 \leq x_w \leq T$, $y_h \geq 0$, $y_w \geq 0$. Suppose, as in the numerical examples in the paper, $w_h > w_w$. Then clearly for efficiency $x_h = 0$ if $x_w < T$ and $x_h > 0$ only if $x_w = T$.

Suppose $x_h = 0$ and consider the first order condition for $x_w$. Satisfied as an equality it yields

$$x_w = \frac{1}{2}(w_h + w_w) T / w_w.$$  

Since $w_h > w_w$, the R.H.S. is greater than $T$. Hence $x_w < T$ cannot be optimal. Suppose now $x_w = T$, and consider the first order condition for $x_h$. Satisfied as an equality it yields

$$x_h = \frac{1}{2}(w_h + w_w) T / w_h - T.$$  

Since $w_h > w_w$, the R.H.S. is negative so $x_h > 0$ cannot be optimal. Thus any solution to (1) must have $x_w = T$ and $x_h = 0$. If we now consider the simplified problem: choose $y_h$ and $y_w$ to maximise

$$W = u_h^{1/2} u_w^{1/2} = T^{0.5} y_h^{0.5} y_w^{0.5}$$

subject to $y_h + y_w = w_h T$, then $y_h = lw_h T$ and $y_w = (1-l)w_h T$. Together with $x_w = T$ and $x_h = 0$, this yields $u_h = T^{0.5} (lw_h T)^{0.5}$ and $u_w = T^{0.5} ((1-l)w_h T)^{0.5}$. Hence

$$(u_h)^2 + (u_w)^2 = w_h T^2.$$  

A similar result holds if $w_w > w_h$. If $w_h = w_w$ then a simple variant of the argument shows $x_h + x_w = T$ is optimal. Thus
\[(u_i)^2 + (u_j)^2 = \max\{w_{hi}, w_{hj}\}T^2.\]

N.B. This rather neat result depends on the private and public goods having equal weight in the utility functions.

2. Nash equilibrium in non-cooperative marriage

Partner $i$ chooses $x_i$ and $y_i$ to maximise $u_i = (x_i + x_j)^{0.5} y_i^{0.5}$ given $x_j$ and subject to $w_i x_i + y_i = w_i T + S_i = Y_i$, $0 \leq x_i \leq T$, $y_i \geq 0$. Substituting the budget constraint into the maximand generates a first order condition for $x_i$ which satisfied as an equality yields

\[x_i = \frac{1}{2} (Y_i/w_i - x_i)\]

The R.H.S. of (3) might be negative for high $x_i$ or for negative $S_i$ (partner $i$ has to work all the time to support partner $j$). Similarly the R.H.S. of (3) might be greater than $T$. Hence the optimal choices are: $x_i = \min \left[ \max \{0, \frac{1}{2} (Y_i/w_i - x_i)\}, T \right]$ and $y_i = Y_i - w_i x_i$. Similarly $x_j = \min \left[ \max \{0, \frac{1}{2} (Y_j/w_j - x_j)\}, T \right]$ and $y_j = Y_j - w_j x_j$.

The two equations for $x_i$ and $x_j$ can be considered as reaction functions $x_i = x_i(x_j)$ and $x_j = x_j(x_i)$. They are both continuous, non-increasing, bounded below by 0 and above by $T$, and with slopes, where defined, less than one in absolute value. Hence they have a unique solution (giving a unique Nash equilibrium) in the non-negative quadrant. Note that $x_i = x_j = 0$ can be ruled out since $Y_i + Y_j = (w_i + w_j) T > 0$. If $S_i = - S_j = 0$, then $x_i + x_j = \frac{2}{3} T$, which is less than the efficient level of $T$ (see the analysis of $B_{hi}$ above). At this point, $d(x_i + x_j)/dS_i = \frac{1}{3} (1/w_i - 1/w_j)$, so a transfer to the partner with the strictly lower wage increases $(x_i + x_j)$, but it will never be greater than $T$.

3. The equations for $B_{hi}$

Partner $i$ chooses $x_i$ and $y_i$ to maximise $u_i = x_i^{0.5} y_i^{0.5} \exp(e)$ subject to $w_i x_i + y_i = w_i T + A_i$, $0 \leq x_i \leq T$, $y_i \geq 0$. If $A_i < w_i T$ then $x_i < T$ (i works), and $u_i = \frac{1}{2} (T + A_i/w_i) w_i^{0.5} \exp(e)$. If $A_i \geq w_i T$ then $x_i = T$ and $u_i = (TA_i)^{0.5} \exp(e)$. Suppose $w_h \geq w_w$. Then the assumption that legal obligations are fesasible, $w_w T + A_{hi} \geq 0$, implies $A_h \leq w_h T$. Thus

\[u_h = \frac{1}{2} (T + A_{hi}/w_h) w_h^{0.5} \exp(e)\]

\[u_w = \begin{cases} \frac{1}{2} (T + A_{hi}/w_h) w_h^{0.5} \exp(e) & \text{if } A_{hi} < w_w T \\ (TA_{hi})^{0.5} \exp(e) & \text{if } A_{hi} \geq w_w T. \end{cases}\]
Since $A_w = - A_h$

\[
\begin{align*}
\frac{w_h^{0.5}}{u_h} + \frac{w_w^{0.5}}{u_w} &= \frac{1}{2}(w_h + w_w)\exp(e) T & \text{if } A_w < w_w T \\
2 \frac{w_h^{0.5}}{u_h} \exp(e) T u_h + u_w^2 &= w_h \exp(2e) T^2 & \text{if } A_w \geq w_w T.
\end{align*}
\]

4. The numerical example in Figure 5

We take $w_h = 4, w_w = 1, T = 12, \exp(e) = 1.5$. Along $B_{lh}$ $(u_w)^2 + (u_h)^2 = 576$. The equations for $x_h$ and $x_w$ in a non-cooperative marriage are:

\[
x_h = \min \left[ \max \left[ 0, 6 + \frac{S_h}{8} - \frac{x_w}{2} \right], 12 \right]:
\]

\[
x_w = \min \left[ \max \left[ 0, 6 + \frac{S_w}{2} - \frac{x_h}{2} \right], 12 \right].
\]

Along $B_0$

\[
2 u_h + u_w = 45 & \quad \text{if } A_w < 12,
\]

\[
72 u_h + u_w^2 = 1296 & \quad \text{if } A_w \geq 12.
\]

If $S_w = S_h = 0$, then in a noncooperative marriage $x_w = x_h = 4$ solves the reaction functions above, with $y_w = 8$ and $y_h = 32$, giving the point $a = (8, 16)$, which is within the frontier $(u_w)^2 + (u_h)^2 = 576$. If $A_w = A_h = 0$, then after divorce we get the point $b = (9, 18)$, again within the frontier $B_{lh}$.

If $S_w = - S_h = 18$, then in a noncooperative marriage $x_w = 12$ and $x_h = 0$ solves the reaction functions, with $y_w = 18$ and $y_h = 320$ giving the point $c = (14.70, 18.97)$, which is on the frontier $(u_w)^2 + (u_h)^2 = 576$. If $A_w = - A_h = 18$, then after divorce $x_w = 12$, $x_h = 3.75$, $y_w = 18$, $y_h = 15$, giving the point $d = (22.05, 11.25)$, outside the frontier $B_{lh}$.
MODEL 2

5. The equation for $B_M$

The frontier $B_M$ can be generated by varying $l$ in the problem: choose $x_h$, $x_w$, $y_h$ and $y_w$ to maximise

$$ W = u_h l^{1/2} (x_h + x_w)^{0.5} y_h^{1/2} y_w^{1/2} $$

subject to $x_h + x_w + y_h + y_w = (w_h + w_w) T$, $x_h, x_w, y_h, y_w > 0$. This yields total child expenditure $x_h + x_w$ of $1/2 (w_h + w_w) T$, whatever the value of $l$. Utility levels are

$$ u_h = \frac{1}{2} l^{0.5} (w_h + w_w) T $$
$$ u_w = \frac{1}{2} (1-l)^{0.5} (w_h + w_w) T. $$

Hence

$$ (u_h)^2 + (u_w)^2 = (\frac{1}{2}(w_h + w_w) T)^2. $$

6. Nash equilibrium in non-cooperative marriage

Partner $i$ chooses $x_i$ and $y_i$ to maximise $u_i = (x_i + x_j)^{0.5} y_i^{0.5}$ given $x_j$ and subject to $x_i + y_i = w_i T + S_i$, $x_j$, $y_i > 0$. This problem is the same as that in Section 2 above, but without an upper bound on $x_i$. Thus $x_i = \max [0, \frac{1}{2}(Y_i - x_j)]$ and $y_i = Y_i - x_i$. Similarly $x_j = \max [0, \frac{1}{2}(Y_j - x_i)]$ and $y_j = Y_j - x_j$.

The reaction functions $x_i = x_i(x_j)$ and $x_j = x_j(x_i)$ are both continuous, non-increasing, bounded below by 0, and with slopes, where defined, less than one in absolute value. Hence they have a unique solution (giving a unique Nash equilibrium) in the non-negative quadrant. Note that the reaction functions have the same functional form, so that the locus of all possible Nash equilibria, generated by varying the division of $(w_i + w_j) T$, is symmetric (in utility space) around the 45° line. Note that $x_i = x_j = 0$ can be ruled out since $Y_i + Y_j = (w_i + w_j) T > 0$. If we ignore the non-negativity constraint in the reaction functions we get $x_i = \frac{2}{3} Y_i - \frac{1}{3} Y_j$ and $x_j = \frac{2}{3} Y_j - \frac{1}{3} Y_i$, implying that if $Y_i > 2 Y_j$ then $x_i = 0$ and if $Y_j > 2 Y_i$ then $x_i = 0$. But if both $x_i$ and $x_j$ are positive then $x_i + x_j = \frac{1}{3}(w_h + w_w) T$, which is less than the efficient level of $T$ (see the analysis of $B_M$ above). At this point, $d(x_i + x_j)/dS_i = 0$, so a marginal transfer between partners, whatever their respective wages, has no effect on total child expenditure, nor on either partner's private expenditure.

7. The equations for $C_w$, $C_m$ and $B_D$
If \( i \) has custody, \( u_i = (x_i^{0.5} y_i^{0.5}) \exp(e) \) which is maximised subject to \( x_i + y_i = w_i + A_i \). Thus

\[
u_i = \frac{1}{2} (w_i + A_i) \exp(e).
\]

The non-custodial parent spends total income \( w_j + A_j \) on \( y_j \) and so gets

\[
u_j = \left( \frac{1}{2} (w_i + A_i) \right)^{0.5} (w_j + A_j)^{0.5} \exp(e).
\]

Putting \( A_i = -A_j \) we get a relationship between \( u_i \) and \( u_j \) when \( i \) has custody, denoted by \( C_i \), given by

\[
(u_j)^2 = u_j((w_i + w_j) \exp(e) - 2u_i).
\]

This gives the frontier \( C_i \). Clearly, as \( u_i \) increases from zero, \( u_j \) increases from zero, and then decreases. Setting \( u_i = u_j \) gives a quadratic equation with solutions at zero and \( (w_i + w_j) \exp(e)/3 \), so on \( C_i \) \( u_i > u_j \) if \( u_i < (w_i + w_j) \exp(e)/3 \). Since \( C_i \) and \( C_j \) are reflections of each other around the 45\(^\circ\) line, this implies that the intersection of the two sets with respective frontiers \( C_i \) and \( C_j \) is non-empty. It also implies that if \( i \) has custody and \( u_i \) is less than \( (w_i + w_j) \exp(e)/3 \) then the point on \( C_i \) is in the interior of \( D \), the union of the sets with frontiers \( C_i \) and \( C_j \), i.e. there is a potential Pareto improvement to be had by transferring custody. A custodial parent \( i \) has utility less than \( (w_i + A_i) \exp(e)/3 \) if \( A_i \) is less than \( (2w_j - w_i) / 3 \). If this is so, then \( w_i + A_i \) is less than \( 2(w_j - A_j) \). In other words, in an efficient divorce the custodial parent will have an overall income at least twice that of the other parent. This line of argument also justifies the statement in Footnote 7.

8 The intersection of \( M \) and \( D \)

To determine whether \( M \) and \( D \) intersect, or whether one lies wholly within the other, consider the equations for \( B_M \) and \( C_i \):

\[
(u_i)^2 + (u_j)^2 = z^2.
\]

\[
(u_j)^2 = 2u(z \exp(e) - u_i).
\]

where \( z = \frac{1}{2} (w_i + w_j) T \).
If we treat these as simultaneous equations and solve we find that if $\exp(e) < 1$, these have no real solutions. If $\exp(e) = 1$, there is a unique solution $u_i = z$, $u_j = 0$, but all other points on $C_i$ lie within $M$.

If $1 < \exp(e) < 3/\sqrt{8}$ there is a single real solution in the positive quadrant, with $u_i$ greater than $u_j$. Given the symmetry of $C_i$ and $C_j$ this point must be on the frontier $B_0$. There is a second point where $C_i$ and $B_M$ intersect. Between these two intersections, points on both $C_i$ and $C_j$ lie within $M$.

If $\exp(e) = 3/\sqrt{8}$, $u_i = u_j = z/\sqrt{2}$, so the intersection of $C_i$ and $C_j$ just lies on $B_M$, but elsewhere points on the outer envelope of $C_i$ and $C_j$ lie beyond it. If $\exp(e) > 3/\sqrt{8}$, then at the intersection of $B_M$ and $C_i$, $u_i$ is less than $u_j$, and all points on the outer envelope of $C_i$ and $C_j$ lie beyond $B_M$. 

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