Putting it off for later - Procrastination and: end of fiscal year spending spikes

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I never put off till tomorrow what I can possibly do the day after.

Oscar Wilde

Abstract

Many government departments around the world exhibit heightened end of fiscal year spending. The UK and Northern Ireland governments are no exception spending around 1.91 and 3.06 times more on capital in the final month of the fiscal year than the average month. These spending spikes present a problem for policy makers due to their tendency to result in lower quality spending.

A model is presented with procrastination as the driver of heightened end of year spending. Departmental performance is measured on a fiscal year basis and hence there is an incentive to delay spending effort in spending money until later in the fiscal year. A new technique of time variant budgetary taxes are suggested for disincentivising heightened end of year spending and increasing spending efficiency. The model is calibrated to Northern Ireland government spending data with the finding that time variant budgetary taxes can improve the value of government spending by more than 10%.

JEL Codes: H11, H50, H61
Keywords: Government spending; fiscal year distortions

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1 Introduction

Heightened spending at the end of the fiscal year is a salient feature of government spending. It has been observed in a variety of contexts including Canadian Military spending (Hurley, Brimberg & Fisher 2013), United States Government Procurement (Liebman & Mahoney 2013), West German local government job training programs (Fitzenberger, Furias, Orlanski & Sajons 2014) as well as in the Australian government (Mannheim 2012). It can also be seen in the U.K. where 16% of governmental capital spending occurs in the last month of the fiscal year. In total the heightened spending in the last month of the fiscal year represented an extra £9.5 billion of expenditure in 2011-2012 and £3.9 billion for the 2014-2015 fiscal year. To give an idea of how much £3.9 billion is, this is more than the cost of the UK’s education system for half a month (HM Treasury 2011) and more than the cost of the USS Ronald Reagan supercarrier (O’Rourke 2005).

This concentration of spending in the latter months of the financial year present a problem for policy makers due to the tendency for rushed spending to be of lower quality. This lower quality has been found by Liebman & Mahoney (2013) who analysed data on U.S. government I.T. procurement data and find that IT projects commissioned at the end of a fiscal year are 2.2 to 5.6 times more likely to receive a quality score of “low quality”. McPherson (2007) also investigated the quality of end of year spending by conducting several interviews with senior managers in the United States Department of Defense. In these interviews it was suggested that 24% of final two months spending goes to low priority projects and 8% is at least partially wasted. Taken in total the evidence tends to indicate that government spending does exhibit some level of diminishing returns.

The question that emerges is what causes government departments to defer spending until the end of the fiscal year. There are two putative explanations that are often offered but prove specious. The first is that in many countries (but not currently the UK) if the departments do not spend their whole budget they lose the unspent portion back to general treasury. The second is that if they do not spend their whole budget their budget for future fiscal years can be reduced. While both of these may be arguments for spending the entire budget in a fiscal year they are not arguments for deferring spending until the final months of the fiscal year. There must be some other mechanism that encourages departments to defer spending until the end of the year.

As of yet the most developed economic model to offer such a mechanism is that of Liebman & Mahoney (2013) who explained end of year spending through precautionary savings. Government departments build up a rainy
day fund to insure themselves against stochastic shocks. At the end of the fiscal year they cannot carry over this fund to the subsequent fiscal year and hence they spend it all. The clear policy implication is to allow the use of rollover policies where unspent appropriations can be saved to be spent in future years. As will be shown in section 4.1 however there are solid reasons for believing that this mechanism cannot fully explain the spending spikes seen in practice.

This paper offers an alternative explanation of spending spikes being caused by procrastination\(^1\). Departmental performance is primarily measured on a fiscal year basis and discounting provides an incentive for departments to delay spending money (and incurring disutility from the associated effort of spending it wisely) until later in the fiscal year. It is shown that the testable predictions of this model are more consistent with UK and Northern Irish experience than the precautionary savings model. This distinction between models is important as both models yield radically different policy prescriptions.

The diminishing returns of government spending suggests that measures to smooth spending over the fiscal year can increase the efficiency of government spending. This paper uses the procrastination model to suggest a new budgetary mechanism, time variant budgetary taxes, which are suitable for smoothing spending over the fiscal year. With this mechanism spending worth \(x\) pounds would cost \(x\) pounds for a department in month 1 and \(x\theta\) pounds in month 12 (with \(\theta > 1\)), thus making procrastination more costly for the department. It is shown that these taxes encourage the smoothing of spending and can lead to more efficient spending over the year.

This paper proceeds as follows. Section 2 highlights a few aspects of the UK budgetary system that are relevant for motivating model assumptions and evaluating testable predictions. This section also presents the dataset and the extent of spending spikes in Northern Ireland and for the UK central government. The procrastination model of government spending is then presented and calibrated to the data in section 3. Section 4 briefly outlines the precautionary savings model of Liebman & Mahoney (2013) before comparing the testable predictions of each model in the UK experience. Policy implications to smooth spending are then discussed and calibrated in section 5 before section 6 concludes.

\(^1\)The term “procrastination” is often taken to imply time inconsistency in economics. The model presented in this paper does not require time inconsistency. Two calibrations are done in section 3, one with time inconsistent discounting with the other using exponential discounting in a completely rational framework.
2 Government Spending in the United Kingdom

The United Kingdom operates on a fiscal year running from April to March. Budgets are first split by the purpose of the funds including current expenditure and capital expenditure. Current expenditure budgets are intended for the payment of operational expenses such as wages, rent and transfer payments. Capital expenditure budgets refer to money which is allocated for investment in durable assets that will benefit the department over an extended period. This may include computers, furniture, vehicles & building renovations.

The United Kingdom government traditionally budgeted on a fiscal year basis with unspent funds being withdrawn from departments at the end of each year. This was changed with the introduction of a rollover mechanism in the 1998-1999 fiscal year (Crawford, Emmerson & Tetlow 2009). Called the "End of Year Flexibility" (EYF) scheme, departments were able to retain an entitlement to all of their unspent appropriation in previous years. In HM Treasury’s budgetary guidance document encouragement was given for departments to make available EYF for lower level budget holders within an organisation (HM Treasury 2007, Chapter 15).

Concerned at the levels of “savings” that departments were accruing under the EYF scheme, the incoming coalition government overhauled this system in the 2010 spending review. The overhaul amounted to the implementation of a more restrictive rollover system as well as the loss of all savings accrued under the previous system (HM Treasury 2010, Section 1.17). The new “Budget Exchange System” (BES) came into force for the 2011-2012 financial year and included set limits of between 0.75% (for larger agencies) and 4% (for smaller agencies) on how much expenditure could be carried forward (HM Treasury 2013, Chapter 2). In addition the Budget Exchange System included measures to prevent savings being accumulated over long periods of time by only allowing money to be rolled over only once.

Both datasources used for this paper are based on accrual accounting meaning expenditure is recorded in the month when the underlying economic transaction took place rather than the month in which cash was exchanged.

\[^2\]Budgets are further split by the planning horizon of the funds. This division is less important for the purposes of this paper and discussion is deferred for appendix A.
Figure 1: Average monthly spending in Northern Ireland 2008 - 2013 (Where circles represent capital expenditure and triangles current expenditure)

2.1 Northern Ireland Spending

The primary dataset for this study consists of monthly spending for various departments of the Northern Ireland (NI) government for the fiscal years 2008-2009 to 2012-2013. This data is segmented by capital and current expenditure and is disaggregated for all Northern Irish government departments. In addition data is available showing how much each department carried between fiscal years.

The NI government is a good case study as it provides a wide range of governmental services and also consists of departments of varying size. In addition it is generally subject to the same budgetary restrictions as the wider UK government. Thus the NI dataset offers variation between departments without variation in budgetary framework or general environmental conditions.

Figure 1 shows average monthly spending (aggregated across all Northern Ireland departments and the fiscal years 2008 - 2013) divided by the annual average monthly spending level, a measure of in-year spending that will be termed as the spending moment. A month in which spending is typically equal to the average annual monthly expenditure would have a spending moment of 1. There is a small spending spike for current expenditure, the average final month spending moment being 1.38. This is not particularly surprising as wages, transfer payments and other routine expenditures are inherently regular expenses. Capital expenditure is generally more discre-
tionary and it can be seen that capital expenditures exhibits heightened spending at the end of the fiscal year with a final month spending moment of 3.06 over the years in the sample\textsuperscript{3}.

Over a 5 year period from 2008-2009 to 2012-2013 the smallest average final month spending moment was 1.1 for the Department for Employment and Learning’s current expenditure. The largest final month spending moment was 5.3 for the capital budget of the Office of First Minister and Deputy First Minister. This spending moment indicates that over the five years this office generally spent nearly half their capital budget in the last month of the fiscal year. There is some variation in the size of each years spending spike but there does not appear to be any consistent pattern governing this variation. There does not appear to be any cointegration between capital and current expenditure after they are both rescaled as a proportion of annual total spending.

2.2 UK Central Government Spending

The secondary dataset for this research is UK central government spending which is split between current expenditure and capital expenditure. For the 2013-2014 fiscal year capital expenditures contributed 5.5% of the spending total.

The spending moments for the final month of each fiscal year from 1997-1998 to 2013-2014 are presented in figure 2. It can be seen that in every fiscal year capital spending was significantly heightened in the last month of the fiscal year. This figure also displays when the rollover systems were changed. The first change was from no rollover to EYF system for the 1998-1999 fiscal year. The second change is the replacement of EYF by BES for the 2011-2012 fiscal year. It can be seen that neither the implementation of the EYF scheme or its abolition and replacement with a weaker scheme had any sizable effect of the size of end of year spending spikes\textsuperscript{4}. This is an

\textsuperscript{3}The size of capital spending spikes relative to current expenditure spending spikes is consistent with Liebman & Mahoney’s (2013) finding (using USA data) of greater spending spikes for “maintenance and repair of buildings, furnishings and office equipment, and I.T. services and equipment” all of which are typically capital expenditure items.

\textsuperscript{4}The caveat to this analysis is the lack of variation in the dataseries. There is only data for one year previous to the implementation of EYF and 3 years since its abolition and the establishment of BES. There does exist a longer series of governmental cash outlays (ONS’s RUUP series) that can be seen as a proxy variable for governmental spending. This series is of longer duration and was examined for the period 1988-1989 to 2008-2009 by Crawford et al. (2009, Figure 3.2) to see the effect of EYF on end of year spending. They find a similar conclusion to this paper with high expenditures towards the end of the fiscal year being relatively unaffected by the implementation of EYF.
important testable prediction and is discussed in section 4.2.

2.3 The quality of end of fiscal year spending

Both the UK central government and the Northern Ireland government exhibit heightened spending at the end of fiscal years. An important consideration is the quality of this heightened spending.

Whilst no research is available based on British data a rigorous examination of the quality of end of year spending is provided by Liebman & Mahoney (2013) using data from the United States. They use data on US government IT expenditures to examine the quality of year-end spending. As they note IT spending is ubiquitous across government agencies and the quality of IT spending may be a valid proxy for spending more generally. They have access to some ordered categorical data series for the quality, timeliness and cost of IT project commissioned at various points of the fiscal year. They find that IT projects commissioned at the end of a fiscal year are between 2.2 and 5.6 times more likely to rated as low quality. An earlier paper was that of McPherson (2007) who interviewed 20 US Department of Defence managers in various organizations with budgets ranging from $200 000 to $20 billion. 95% of these managers agreed that end of year spending was a problem with the managers estimating that 24% of spending from the last two months of the fiscal year goes to low priority projects and 8% is “at least partially wasted (McPherson 2007, page 40)”. Aside from this quantitative evidence both papers feature a number of anecdotes regarding wasteful end
of year spending.

This paper does not contribute additional evidence for the low quality of end of year spending but accepts the consensus of this previous research. As was the case in Liebman & Mahoney (2013) this paper models production as having diminishing returns to spending\(^5\).

### 3 Procrastination

There are two observations that motivate the model proposed by this paper. The first is that it is well known there are effort costs to departments from efficiently spending money. There often exists an extensive procedure for spending public funds, implemented to ensure probity, transparency and value for money. Spending money in a way that satisfies all such requirements is likely to be a strenuous activity. Outsourcing this activity to professional project managers typically costs around 5% of the value acquired which gives an idea of the effort costs involved. The second observation is that performance evaluation is typically organized around financial years. This is certainly the case with the performance management systems utilised by the UK government in the last two decades. The performance pay bonus system for public servants distributes bonuses in October for performance in the preceding fiscal year (Hope 2012). These bonuses can be significant relative to public sector wages with the average worker in the Serious Fraud Office taking £674 in the 2008 – 2009 fiscal year and £534 in the 2009 – 2010 fiscal year (Serious Fraud Office 2010). The average SCS grade (top level management) worker took £3900 in each year. Aside from direct performance pay there are career incentives for achieving good performance evaluations\(^6\).

This paper will argue that the measurement of performance at discrete intervals can lead to a spike of activity towards the end of the interval. In the same way students often exert heightened effort in the weeks before their exams, departments spur to action in the lead up to the end of the fiscal year when performance is evaluated. This burst in activity explains end of fiscal year spending sprees.

This mechanism will be termed “procrastination” in this paper. It is important to note however that the model presented does not depend on any time inconsistency for the public service. Delayed measurement of performance is sufficient to cause spending spikes even in a rational framework.

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\(^5\)Further discussion of the diminishing returns assumption is available in appendix C.3.

\(^6\)More discussion of the UK’s performance monitoring system is available in appendix C.2
3.1 A deterministic model of procrastination

Here a simple model will be developed in order to illustrate how procrastination can lead to end of fiscal year spending spikes. This is done in a deterministic setting and without loss of generality departments are not allowed to rollover funds\textsuperscript{7}. Aside from the tractability advantages of a deterministic setting, this highlights the differences between this model and the precautionary savings model (to be outlined in section 4.1) where stochastic elements are entirely responsible for heightened end of fiscal year spending. The mechanism shown here is robust to stochastic elements however which is shown numerically in section 3.2.

There are two time periods and two agents, a representative department and the parliament.

The representative department chooses when to spend in the fiscal year subject to an annual budget of $B$. Spending in period $t$ is denoted $x_t$. A department receives positive utility from its performance, a monotonically increasing function of its expenditure denoted $v(x_t)$ - however departmental performance is only realised at the end of the fiscal year\textsuperscript{8}. A department realizes disutility from effort which is realised immediately. Effort in a period is a continuously differentiable increasing function of spending and is denoted $e(x_t)$. Discounting applies between periods via a parameter $D < 1$.

Subject to an annual budget of $B$, a department chooses an expenditure level each period. A department has the choice variables of $x_1$ and $x_2$ to maximise:

\[
\max_{x_1, x_2 \geq 0} D [v(x_1) + v(x_2)] - e(x_1) - De(x_2)
\]

s.t. $x_1 + x_2 \leq B$ (1)

Here it can be seen that performance in the first two periods $v(x_1)$ and $v(x_2)$ is realised in the second period. Effort is always realised in the period in which spending occurs however.

Production is denoted $g(x_t)$. The parliament gains utility from production realized instantaneously. The parliament also realizes disutility (instantaneously) from the social cost of the funds denoted $\lambda x_t$. For simplicity the

\textsuperscript{7}Note there is no loss of generality in not allowing rollover in a model lacking stochastic elements and with concave utility functions. Discounting and the lack of precautionary savings ensure departments would never want to save even if they were allowed.

\textsuperscript{8}One example may be that departments get utility from their production, but this is only recognised at the end of the fiscal year. In this case we could have $v(x_t) = g(x_t)$. Another case is that departments are assessed solely on how much they spend by the end of the fiscal year (as may occur when production is unobservable).
government discounts within the fiscal year at the same rate as the department. Subject to the autonomy of a department in selecting spending levels in each period, a parliament has the choice variable of the budget, $B$, to maximise:

$$\max_B g(x_1) - \lambda x_1 + D [g(x_2) - \lambda x_2]$$

The following conditions are placed on the performance, production and effort functions:

$$g(0) = 0 \quad v(0) = 0 \quad e(0) = 0$$
$$g'(0) > 0 \forall x \quad v'(x) > 0 \forall x \quad e'(x) > 0 \forall x$$
$$v''(x) < e''(x) \forall x$$
$$Dv'(0) > e'(0)$$

The first two conditions (3, 4) are elementary and simply state that production, performance and effort are zero when spending is zero and rise as spending increases. The third condition (5) ensures that the marginal utility for performance for a department approaches the marginal effort. This assumption is necessary to avoid corner solutions where all funds are expended in the last period\(^{10}\). The fourth condition (6) is necessary so that the government prefers an even spending pattern (Proposition 2). The fifth condition (7) represents a weakened inada condition for $v(x)$.

First order conditions yield the following results (proofs in appendix B)

**Proposition 1.** For any given budget departments will choose strictly higher spending in the second period relative to the first.

The intuition for this result is that the department expects the same reward from spending in either the first or second period. In planning their year however they face a lower effort cost (due to discounting) from spending in the second period. As a result the total utility from spending increases with time and departments spend more in the second period.

\(^9\)This model expands upon the model of Liebman & Mahoney (2013) and reduces to the same model if $e(x_1) = e(x_2) = 0 \forall x$, performance is recognised immediately and if there are stochastic shocks in each period. As will become apparent however the mechanism behind this procrastination model is very different to the precautionary savings that drive the Liebman & Mahoney (2013) model and both models lead to different testable predictions.

\(^{10}\)For instance if both functions are linear, but $v'(x) > e'(x)$, all spending will occur in the last period.
Proposition 2. Constant spending in each period is optimal for the parliament at some level which will be denoted $x_{Par}$.

The parliament realises the social cost of spending and the utility from production instantaneously. There do not exist any time distortions and hence the same level of spending that is optimal in every period.

Proposition 3. It is not possible for the parliament to induce its optimal spending through budgeting.

This is immediate from propositions 1 and 2.

3.2 A calibrated procrastination model

The previous discussion in a deterministic environment was useful in presenting the central mechanism through which procrastination can lead to delayed spending. In this section we calibrate a procrastination based model to the data to show that the procrastination mechanism is robust to the presence of shocks and is capable of describing the full size of spending spikes seen in the data.

The model is calibrated to the Northern Ireland departmental capital spending dataset over the years 2009-2010, 2010-2011 and 2011-2012. As this data is monthly the model is expanded to encompass 12 periods per year. It is necessary to provide functional forms of various functions. The form of the performance function $v(x_m)$ is given by $x^\delta_m$ where $\delta < 1$. The form of the effort function $e(x_m)$ is linear and is given by $\Omega x_m$.

Rollover is allowed in the model as was the case for Northern Ireland departments over this period. In each period the problem of the department is:

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11Whilst spending data is available for 2008-2009 and 2012-2013, data is not available for how much money was saved from these years. The data is adjusted for the length of each month. Departments with a budget of less than £1 million are dropped as well as months where spending is negative (A small number of negative spending values are in the dataset as a result of low capital expenditure along with some revenues from sale of capital items) or very close to zero (This is necessary in order to identify the model implied residuals as detailed later). The annual budget for each department is estimated based on the amount spent in each month as well as the amount saved into the next fiscal year. Monthly spending is then divided by the annual budget so that annual spending and net rollover accrued in a year add to one.

12Imposing linearity on the effort function is not a quantitatively important restriction as flexibility is allowed in $v(x_m) = x^\delta_m$ and it is the net of this curvature against the curvature of the effort function which dictates how departmental utility from spending changes with time and hence the pattern of spending.
\[ V_m(B_{m,y}, \alpha_{m,y}) = \max_{B_{m,y} \geq x_{m,y} \geq 0} D_{12-m} \alpha_{m,y} x_m^\delta - \Omega x_{m,y} + D_1 E_{m,y} [V_{m'}(B_{m',y'}, \alpha_{m',y'})] \]  

(8)

Where:

\[ B_{m',y'} = B_{m,y} - x_{m,y} + \kappa_m \]  

(9)

\[ B_{m,y} \geq 0 \forall m, y \]  

(10)

Where \( m, y \) give the current month and year and \( m', y' \) give the month and year in one month's time. \( D_n \) is the discounting \( n \) periods into the future. Note that the performance function is discounted between the current month and the end of the fiscal year (at time 12). \( \kappa_m \) is the receipt of the annual budget. This is taken to be 1 if \( m = 1 \) and 0 otherwise\(^{13}\).

The \( \alpha_{m,y} \) shocks are drawn from a normalised log normal distribution:

\[ \alpha_{m,y} \sim \frac{LN(0, \sigma^2)}{E[LN(0, \sigma^2)]} \]  

(11)

Two calibrations are performed, one for exponential discounting and one for hyperbolic discounting. Exponential discounting is done with the form:

\[ D_n = \beta^n \]  

(12)

with \( \beta \) taking the value of 0.996 (5% annual discounting\(^{14}\)). The second model has quasi-hyperbolic discounting (Laibson 1997) of the form:

\[ D_n = k_n \beta^n \]  

(13)

Where \( k \) is a constant if \( n \geq 1 \) and 1 if \( n = 0 \). This model is calibrated to the values \( k = 0.975, \beta = 0.998 \) (5% annual discounting).

These two models are calibrated in the following way. The value functions are first found numerically for each of the twelve months. The first order condition of (8) with respect to \( x_t \) is then taken and rearranged to give:

\[ \alpha_{m,y} = \frac{\Omega - D_1 E_{m,y} [V_{m'}(B_{m',y'})]}{D_{12-m} \delta x_m^{\delta - 1}} \]  

(14)

\(^13\)In choosing a constant budget there is an implicit simplifying assumption here that the parliament does not choose a budget conditional on departmental savings but instead choose a stationary budget that maximises long run expected utility. Discussion of this assumption is included in appendix H.

\(^14\)There have been a number of papers who have found evidence of discounting rates in excess of 5%, including Warner & Pleeter (2001) and Harrison, Lau & Williams (2002). Whilst here we impose a discounting rate of 5% it is shown in appendix E that a similar model fit is available with a discounting rate of 10%.
\( x_t, B_t \) and \( t \) can be input to this equation to give the \( \alpha_{m,y} \) shock that would have caused the spending decision, \( x_{m,y} \). The lognormal pdf is then used with these \( \alpha_{m,y} \) shocks to calibrate the model with maximum likelihood.

<table>
<thead>
<tr>
<th></th>
<th>Exponential discounting model</th>
<th>Quasi-hyperbolic discounting model</th>
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<tr>
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<td>( \delta )</td>
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<td>( \Omega )</td>
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<td><strong>Discounting (Imposed parameters)</strong></td>
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<tr>
<td>( \beta )</td>
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<td>0.998</td>
</tr>
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<td>( k )</td>
<td>1.000</td>
<td>0.975</td>
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<tr>
<td><strong>Moments</strong></td>
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<td>Shocks Variance</td>
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<td>Log Likelihood</td>
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Here the shocks mean and variance refer to the first 2 moments of the implied shocks after they are transformed to the normal distribution. They should be distributed \((0, \sigma)\) to match the shock process anticipated by departments in the model. The RSS refers sum of squared differences between the predicted monthly spending values (from the long term average of simulations) and the average spending from the data over the fiscal year. This is also summarised in figure 3.

Table 1: Calibrated Parameters

The calibrations can be seen in figure 3\(^\text{15}\) as well as in table 1. The nature of exponential discounting means that spending increases are constrained to be increasing regularly throughout the year. Thus while this model performs reasonably well for most months it cannot describe the large increase at the end of the fiscal year. The hyperbolic model performs better in this regard.

When the implied residuals from the model\(^\text{16}\) are examined it can be seen that both models predict residuals from the Northern Ireland data that have almost the same distribution as the assumed shock process. In particular the implied residuals have almost the same variance as the model variance and the mean of the residuals is close to 0. These observations are important for verifying model consistency so that the shocks that departments actually encounter are drawn from the same distribution as the shocks incorporated into their value function.

In both calibrations the curvature of the performance function is close to the (linear) curvature of the effort function. This indicates that in order

\(^{15}\)Note that the data differs slightly from table 1 due to only using data from 2009 and 2012 and the removal of outliers. This is as discussed in footnote 11 on page 11

\(^{16}\)Found using equation 14
for modest amounts of discounting to explain the size of observed spending spikes, departments need to have a weak preference for evenly spending money throughout the fiscal year. There also exists a tradeoff between the curvature of the performance function and discounting. It is shown in appendix E that it is possible to calibrate 10% (total) discounting versions of the exponential and hyperbolic discounting models that achieve almost exactly the same model fit as the 5% discounting models. These calibrations require more curvature of the performance function ($\delta$) to compensate for the departments greater desire to delay spending. If the same performance function was used with this greater discounting rate departments would delay too much spending leading to excessively large spending spikes. There also exists a link between the curvature of the performance function and the variance of the shock process. If the performance function is more concave the departments have a stronger preference towards smoothing spending and a greater volatility shock process is necessary to explain deviations from the average spending pattern over the year.

One interesting feature of simulations that were performed was the relationship between shock volatility and the size of spending spikes (holding other parameters constant). The procrastination model predicts that more volatility would induce more even expenditure and smaller end of year spending spikes. The intuition for this is simple: when the shock variance is greater the incentive for departments is to time spending to take advantage of beneficial shocks rather than timing towards the end of the year to delay the
expenditure of effort. As shocks are i.i.d. across each month this is a force which evens out spending over the year. This is a key testable prediction of the procrastination model which is examined in section 4.2.2.

4 Discussion

A natural question is how the procrastination model compares to alternate models of government spending. Currently the most developed model in the literature is the precautionary savings model of Liebman & Mahoney (2013). This section will firstly briefing recount the precautionary savings model before moving to a discussion of its performance against the procrastination model.

Whilst alternate explanations of spending spikes have been proposed outside the economics literature these generally fail to stand up to close scrutiny and will not be discussed further in this section. A brief discussion of these alternate explanations for spending spikes is included in appendix C.4.

4.1 The Precautionary Savings Mechanism

Liebman & Mahoney (2013) present a model to explain end of year spending spikes based on precautionary savings. In their model the value of spending is stochastic and as a result departments build up precautionary savings throughout the year to spend in the case of positive productivity shocks. At the end of the year these savings stocks are then expended as no rollover of unspent funds is allowed.

Government departments want to maximize the value of their production without regard for the social cost of this spending. Their objective function can be written as:

\[ V_m(B_{y,m} | \alpha_{y,m}) = \max_{B_{y,m} \geq x_{y,m} \geq 0} \alpha_{y,m} g(x_{y,m}) + \beta E_{y,m} [V_{m'}(B_{y',m'} | \alpha_{y',m'})] \] (15)

The remaining budget is given by

\[ B_{y',m'} = \begin{cases} B_{y,m} - x_{y,m} & \text{if } m < M \\ B_{y+1} + \chi(B_{y,M} - x_{y,M}) & \text{if } m = M \end{cases} \] (16)

Where \( y, m \) are the current year and month and \( y', m' \) are the year and month in one month’s time. \( x_{y,m} \) refers to monthly spending, \( g(x_{y,m}) \) is a monotonically increasing concave production function, \( B_{y+1} \) is the next year’s budget, \( B_{y,m} \) is the available funds at the beginning of period \( y, m \),
$V_m(A_{y,m})$ is the value function for the agency and $\alpha_{y,m}$ is a monthly stochastic parameter without persistence. $\chi$ is a parameter describing the amount of rollover into the next fiscal year, where $\chi = 1$ describes full rollover and when $\chi = 0$ implies that no rollover is allowed. $M$ is the number of months in the fiscal year.\footnote{Note that the key differences between this model and the procrastination are detailed in footnote 9 (page 10)}

The mechanism for end of year spending spikes here is precautionary savings driven by the $\alpha_{y,m}$ parameter. At the start of the year agencies accrue a rainy day fund to allow sufficient funds to spend in the event of a high future $\alpha_{y,m}$. At the end of a year this cannot be carried over and so agencies expend this money. The concavity of the production means that this concentration of funds in the end of the year represents inefficient spending.

The clear policy implication is to allow agencies to save unspent funds and spend these savings in future years. This would eliminate the spending spike and avoid low value spending.

While this mechanism sounds plausible and may provide a partial explanation in some cases, there are reasons for believing that the majority of the effect is left unexplained by this mechanism.

4.2 Model comparison

4.2.1 The U.K.’s changes between budgetary rollover mechanisms

The UK traditionally had expiring budgets where unspent money was returned to treasury at the end of every fiscal year. A full rollover system (the EYF system) was then implemented for the 1998-1999 fiscal year before it was repealed and replaced with a more restrictive rollover system (the BES system) for the 2011-2012 fiscal year. The prediction of the precautionary savings model in this case is straightforward. The size of spending spike should have abated while the EYF scheme was in effect. The new BES scheme should have resulted in spending spikes greater than existed under the EYF scheme.\footnote{Note that while a particularly high spending spike in 2010-2011 might be expected due to the planned abolition of stocks, measures were taken to prevent departments from accessing these funds (The National Assembly for Wales 2010).}

These predictions are not backed up by the data however. The size of the final month spending moment can be seen in figure 2. It appears that neither the implementation of the EYF scheme or its abolition had any sizeable effect of the size of end of year spending spikes. Whilst inconsistent with the precautionary savings model, the lack of an impact for rollover is completely
consistent with the procrastination model where rollover or the lack thereof does not play the central role.

4.2.2 Uncertainty and Spike Magnitude

The precautionary savings model holds that spending spikes that are a result of uncertainty in the production process for public goods. Therefore an implication of the model would be that higher levels of uncertainty would lead to greater spending spikes.

This is the opposite prediction from the procrastination model. In the procrastination model the shock process is a force which evens spending through the fiscal year. This is because departments must balance the utility benefit (through discounting) of procrastination with the exploitation of beneficial shocks.

This can be tested with data from Northern Ireland for the five financial years from 2008–2009 to 2012–2013. Figure 4 shows on the y axis the $R^2$ for government spending regressed against dummies for month and fiscal year which acts as a basic measure of stochasticity of spending for each department. A higher $R^2$ can be interpreted as indicating a more predictable spending environment. On the x axis is the size of the spending moment for the last month.

As can be seen in figure 4, the size of spending spikes does not increase with more uncertainty. When a regression is performed on the observations in this figure there is a positive relationship between certainty in spending and the size of the spending spike at the 5% significance level. The procrastination model is more consistent with this data than the precautionary savings model.

4.2.3 The uncertainty required to generate the data

When Liebman & Mahoney’s (2013) calibration of their baseline CRRA precautionary savings model is examined it is immediately clear that it is very stochastic. The stochastic parameter, $\alpha_{g,m}$ is calibrated to have a log normal distribution. Each department’s $R^2$ was calculated with the five fiscal years from 2008–2009 to 2012–2013. Some departments (Most notably the Department of Justice which was established in 2010 as a result of an expansion of powers delegated from the UK central government) were omitted as data was not available for the whole period. In this case the $R^2$ would be too large and hence not comparable with other departments.

The output for this regression is shown in appendix D. The number of residual degrees of freedom available limits the use of many controls in this regression. Nonetheless the presence of unobserved heterogeneity is limited by the fact that each department served the same population, over the same period and with the same budgetary framework.
This calibration is done in order to hit a final month spending moment of 2.18, meaning that final month spending is 2.18 times the annual average. Changes in the distribution of the $\alpha_{y,m}$ parameter would have two effects however, the first is in uncertainty and hence the amount of precautionary savings generated, the second is the extent to which spending changes month to month. A more comprehensive calibration strategy would take this into account calibrating $\alpha_{y,m}$ to fit both aspects of the data.

It should be noted that variation in the effectiveness of government spending is not precisely what is needed for precautionary savings - what is need is unpredictability. In this way the variance of the $\alpha_{y,m}$ could alternatively be thought of as uncertainty about future production rather than variation in future production. This interpretation allows the distribution of $\alpha_{y,m}$ to be checked against real world measures of governmental uncertainty.

In the real world it is likely that a large part of such variations could be forecast. As one introductory public sector management textbook states "the great bulk of spending in any one year represents commitments from the year before" (Flynn 2007, p. 56). To make more precise the statement that Liebman & Mahoney’s (2013) calibration requires too much uncertainty it is

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**21** Due to the multiplicative way $\alpha_{y,m}$ enters equation (15) this distribution would indicate that the top end of the middle 66% (The shock greater than 83.3% of the distribution and less than 16.7% of the distribution) would result in 28 times more output than the bottom end of the middle 66%. The top end of the middle 95% of this distribution would result in 834 times more output than the bottom end of the middle 95%.

---
useful to examine the average forecast errors of macroeconomic variables like
unemployment which are significant sources of uncertainty for government
departments.

Stark (2010, Tables 1-4) has published estimates of the errors of the survey
of professional forecasters over the period 1985 Q1 to 2007 Q4 in predicting
real output growth, inflation, the unemployment rate as well as the interest
rate on 10-year Treasury bonds. The one quarter ahead Root Mean Square
Errors (RMSEs) of these 4 variables are 1.65, 0.95, 0.25 and 0.53 respectively.
By contrast on a month to month basis the implied RMSE of the calibration
shocks is 19.6\textsuperscript{22}. Whilst it is possible that departmental uncertainty is higher
than the uncertainty in predicting these macroeconomic variables it is hard
to believe it is so much greater. This reliance on an incredible amount of
uncertainty is a shortcoming of the precautionary savings model that is not
shared by the procrastination model where shocks do not play the crucial
role\textsuperscript{23}.

5 Policy Responses to heightened end of year
spending

From Proposition 3 the government is not able to induce its desired spending
levels in every period. This section examines what measures are available to
even spending across the fiscal year. Throughout this section it is assumed
that it is possible for the desired level of spending to be achievable in any
singular period through offering a sufficiently high budget. This implies that
the marginal utility from spending at the parliamentary optimal level has
positive marginal utility for the department in every period. Were this not
the case the parliament would need to increase $v(x_t)$ (for instance through
increasing performance pay) or decrease $e(x_t)$ (for instance through hiring
more public servants) in order to induce the parliament optimal level of
spending. By making this restriction we focus attention on the problem of
inducing even expenditure (the shape of spending) over the year rather than

\textsuperscript{22}Calculated by taking $E[\alpha_{y,m}]$ as the forecast and a sample of 10 million random draws
of the distribution as the realized values.

\textsuperscript{23}One solution to this criticism might be to interpret the uncertainty as being iid
amongst low level departmental managers. In this case excessively large month to month
changes in spending would not be seen in aggregate data. This assumption would be
very tenuous however as shocks to different departmental managers are likely to be highly
correlated as they service the same population and buy goods and services in the same
economy at the same time. It would also raise the question why managers in a department
could not collectively insure each other against these shocks.
adjusting the absolute level of spending over the year.

It is useful to first discuss what features are attractive in a practical policy solution that could actually be implemented. The first property is that any policy measure should be of low cost. Aside from rollover, auditing is the most relied upon technique for preventing low value spending. A former Australian finance minister has suggested the only way to prevent excessive spending is to apply external scrutiny (Mannheim 2012). A key disadvantage of auditing is that it can be expensive which limits the potential for its widespread use.

Due to the strong reticence for elected representatives to give up control of spending (McPherson 2007, page 28), a policy would be more likely to be implemented if it does not result in a loss of power for elected officials. The loss of parliamentary control associated with rollover budgeting is the leading reason the UK’s rollover system was weakened in 2010.

A successful measure should also be simple to understand and implement. Furthermore it should be robust to interference by different levels of the bureaucracy. For instance if rollover were considered, it is possible that rollover strategies would face the risk of not being transmitted throughout the bureaucracy due to senior managers in an organization raiding the rolled-over savings of more junior managers. Where junior managers expect senior managers to raid their savings there is no disincentive to spend money at the end of a fiscal year.

5.1 Auditing

In this section we examine the use of auditing to achieve even spending using the simple deterministic model introduced in section 3.1. We first assume there exists a penalty that is incurred by a department in the event of them being found guilty of poor spending. All spending in the fiscal year is audited with a continuously differentiable increasing convex function $g(x)$ which gives the expected utility loss from audits. This can be thought of as the penalty for being caught spending poorly multiplied by a function that gives the probability of being caught as a function of spending. The convexity of this $g(x)$ function indicates that high spending levels (which will be at low marginal value) increase the chance of being penalised (or the penalty itself conditional on being caught) in an audit.

24Specifically the conditions:

$$g(0) = 0 \quad g'(0) = 0 \quad g''(x) > 0 \quad \forall x$$
Thus the problem of the department becomes:

\[ U = [Dv(x_1) - e(x_1) - Dq(x_1)] + [Dv(x_2) - De(x_2) - Dq(x_2)] \]  \hspace{1cm} (17)

We can now get the following result:

**Proposition 4.** Auditing with imperfect monitoring and constant penalties is insufficient to induce even expenditure.

The reason is clear - a constant level of auditing does not change the fact that late spending is relatively more attractive than early spending. To fix this a time variant strategy would be needed to target late spending and make spending in each period equally attractive.

Now consider the case where auditing frequency increases or penalties increase throughout the year. That is there are two \( q(x) \) functions where the second period function \( q_2(x) \) is greater than \( q_1(x) \) for all \( x \). This means that for any given spending level the expected utility loss from auditing is higher in the second period than the first. This relationship between \( q_1(x) \) and \( q_2(x) \) will be formalised as:

\[ q_2(x) = \eta q_1(x) \]  \hspace{1cm} (18)

for \( \eta > 1 \). With this relationship we get the following proposition.

**Proposition 5.** There exists \( \eta > 1 \) that is sufficient to induce even spending.

The \( \eta \) that induces optimal spending is shown in the proof (see appendix B) to be given by:

\[ \eta = 1 + \frac{(1 - D)e'(x_{\text{Par}})}{Dq_1'(x_{\text{Par}})} \]  \hspace{1cm} (19)

By implementing harsher auditing throughout the year spending in later periods can be made less attractive than it was. A sufficiently harsh second period audit is enough to make spending in either period equally attractive which neutralises the incentive for departments to defer spending and leads to a constant level of spending throughout the fiscal year. Note that the parliament could induce time-variant auditing either through increasing the probability of being caught or through increasing the penalty from getting caught.

While a time variant auditing strategy can both induce even expenditure such a strategy is not the recommendation of this paper. Variable penalties for the same behaviour throughout the year are unlikely to be tenable to most governments. For two departments that make the same infraction (albeit
in different periods) it would be inequitable to have them receive different penalties. Increasing auditing frequency for end of year spending is another option but not an attractive one on the basis of cost. It is likely that the next proposed solution would be close to costless.

5.2 Time-variant budgetary tax

In this section we propose a new technique of time variant budgetary taxes to achieve even spending using the simple deterministic model introduced in section 3.1. With a time-variant budgetary tax (hereafter simply referred to as a tax), spending in later months of the year is more expensive relative to earlier years of the fiscal year. These taxes are not taxes in the normal sense but represent reductions on a department’s appropriated budget at a higher level than what the department actually spent. The implementation of these taxes would not necessarily mean lower overall spending as it is anticipated that the department’s budget could be increased to compensate for these taxes being levied on purchases made later in the year.

Let $\theta$ denote a tax rate by which expenditures in the second period are more expensive. The budget constraint faced by a department in equation 1 is replaced by:

$$x_1 + \theta x_2 \leq B$$

(20)

otherwise the department’s problem is unchanged. Note here that both $\theta$ and $B$ are choice variables for the parliaments and hence taxes give parliaments more power to control spending in the fiscal year.

**Proposition 6.** Time-variant budgetary taxes are sufficient to induce parliament optimal expenditure levels in each period.

The tax that induces optimal spending is shown in the proof (see appendix B) to be given by:

$$\theta = \frac{Dv'(x_{Par}) - De'(x_{par})}{Dv'(x_{Par}) - e'(x_{par})}$$

(21)

This is the ratio of marginal utility from spending in the second period divided by marginal utility of spending in the first period with both evaluated at the parliamentary optimal spending level. This optimal $\theta$ may differ between every department leading to the parliament taxing each department at different rates. If for any reason a parliament was constrained to implement a common $\theta$ for several different departments it is easy to show that choosing the minimum $\theta$ rate across all departmental specific $\theta$ values would
bring every department closer to parliamentary optimal spending relative to the no taxation case \(^{25}\).

### 5.3 Calibrated time variant budgetary tax

With the calibrated parameters of section 3.2, the long term spending behaviour of departments is well defined. In order to assess time variant taxes in a stochastic setting we must assume a production function \(^{26}\). We assume a function of:

\[
\text{Production}(\alpha_{y,m}, x_{y,m}) = \alpha_{y,m}x_{y,m}^{\gamma}
\]

Where \(\alpha_{y,m}\) are the same shocks that enter the department’s utility function and \(\gamma\) is a curvature parameter to be discussed. The assumption of a shared shock process is reasonable in the principal-agent context of the parliament and department relationship. If shocks were uncorrelated with parliamentary utility then parliaments would seek to contract departments in such a way as to align shocks.

With the assumption of a production function and the calibration of departmental behaviour with a budget of 1 we can numerically estimate the social cost that would have lead to an optimal budget of 1 using the equation (derivation in appendix H):

\[
\lambda = \left. \frac{\partial \text{Expected Annual Production}(B)}{\partial B} \cdot \frac{\partial B}{\partial \text{Expected Annual Expenditure}(B)} \right|_{B=1}
\]

The functional form taxes will take over the year \(^{27}\) needs to be chosen. For the exponential discounting model, a deterministic version of the calibrated model has an analytical solution for the tax rates that will yield even spending.

\[
\theta_t = \frac{\delta D_{t1}x_{\text{par}}^{\delta-1} - \Omega D_{t-1}}{\delta D_{t1}x_{\text{par}}^{\delta-1} - \Omega}
\]

\(^{25}\)This may not be the constrained optimal \(\theta\) for the parliament however. To find an optimal common \(\theta\) level the inefficiency of excessive early spending (from a too high \(\theta\)) would have to be balanced with the inefficiency of excessive late spending (from a too low \(\theta\)).

\(^{26}\)One option would be to assume the production function is congruent to the calibrated performance function. This is a different way of saying that performance reviews simply sum up all production done in the year. This is not done in this paper as the performance function is calibrated relative to the imposed linear effort function.

\(^{27}\)Whilst more effective taxes could be found by optimising each different tax rates this is computationally not feasible as it encounters the curse of dimensionality.
Where $\theta_t$ gives the tax in month $t$. The annual budget is then set so that the department can afford to spend $x_{\text{par}}$ every period while paying the tax:

$$B = x_{\text{par}} \left[ \sum_{t=1}^{12} \theta_t \right]$$  \hspace{1cm} (25)

Where $x_{\text{par}}$ is a deterministic approximation of parliamentary optimal spending found by taking a first order condition for parliamentary utility resulting in the expression:

$$x_{\text{par}} = \left[ \frac{\lambda}{\gamma} \right] \frac{1}{1 + \gamma}$$  \hspace{1cm} (26)

For the quasi-hyperbolic discounting model the taxation function (24) performed poorly in evening spending and so tuning parameters, $\nabla$ and $\Upsilon$ were added to give a form of $\theta_t = \frac{\nabla D_{11} x_{\text{par}}^{\gamma - 1} - \Omega D_{t-1}}{\nabla D_{11} x_{\text{par}}^{\gamma - 1} - \Omega}$ for $t \leq 11$ and $\theta_{12} = \frac{\nabla D_{11} x_{\text{par}}^{\gamma - 1} - \Omega D_{t-1}}{\nabla D_{11} x_{\text{par}}^{\gamma - 1} - \Omega} + \Upsilon$ for the final month.\(^{28}\)

An assumption also needs to made for either the value of $\gamma$ or the value of $\lambda$, the remaining parameter being able to be found from equation (23). A conservative approach is to choose a $\gamma$ that will tend to underestimate the gains from the imposition of a tax. A near linear production function may be thought to be a conservative choice in that it will have less inefficiency from spending spikes and will thus underestimate gains from smoothing spending. The complicating factor is that a near linear production will also have less inefficiency from shock driven spending away from the parliamentary optimum, a problem that will be referred to as the shock over-exploitation problem. If the imposition of a tax (and the larger budgets needed to compensate for it) lead to greater departures from the parliamentary optimum in the early months taxes could actually decrease welfare.

With the prior belief that a near linear production function would be a conservative choice it was chosen to impose a curvature value of $\gamma = 0.95$, implying a modest level of diminishing returns. Robustness checks were then

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\(^{28}\)Exponential discounting is used for $D_{t-1}$ in the functional form equation (24)The quasi-hyperbolic form was not used after a calibration with this form performed worse than $D_{t-1}$ with an exponential form. These tuning parameters were optimised by maximising parliamentary expected utility for both parameters with budgets being set according to equation 25.

\(^{29}\)The closeness of the calibrated performance functions to the effort function indicates that it is more likely departments will overreact to shocks than underreact. This is a result of departments needing to have a weak preference for spending smoothing in order for a 5% discount rate to explain end of year spending spikes
carried out with the somewhat surprising finding that the choice of $\gamma$ does not make a large difference to the estimated percentage gains from imposing a tax\textsuperscript{30}. This robustness check is detailed in appendix F.

For each of the two models a perfect agency simulation is done with money being spent exactly in accordance with the parliament’s utility function. This is done to give the maximum possible gains from implementing some policy mechanism in this stochastic environment\textsuperscript{31}. The output of simulations for both models along with simulations for the tax augmented models and perfect agency models are presented in table 2.

Even with this simple tax formula it can be seen that quite large gains are possible. The exponential and quasi-hyperbolic models have available utility benefits of 16.1% and 12.6% respectively. These gains are realized in three ways. The first is that heightened spending in the final months of the fiscal year is less likely to encounter inefficiency from diminishing returns. The second is that the parliament can offer larger budgets and induce greater spending in earlier months of the year when previously there were an unexploited surplus available. The third is that more even expenditure better utilises shocks which are iid throughout the fiscal year\textsuperscript{32}.

This third mechanism suggests that in the presence of shocks it is possible for there to be inefficiencies caused by spending spikes even when there is a production function exhibiting constant returns. A tax to even spending therefore can lead to more efficient exploitation of shocks and higher welfare even when there is a linear production function.

\textsuperscript{30}Although the production function imposed does change the absolute level of parliamentary utility both before and after the imposition of a tax.

\textsuperscript{31}Note that in a deterministic environment the parliament can get their ideal spending level in every month through budgeting and taxation and hence the tax model would be congruent to the perfect agency model. This is not necessarily the case in a stochastic environment however.

\textsuperscript{32}The intuition for this third mechanism is as follows. In order for the department to have been given a finite budget it must be true that the shock process leads to the marginal value of spending fluctuating above and below social cost. As spending in later months is more attractive for departments they will tend to overspend in these months and underutilise beneficial shocks earlier in the year.
<table>
<thead>
<tr>
<th></th>
<th>5% Exponential discounting model</th>
<th>5% Quasi-hyperbolic discounting model</th>
</tr>
</thead>
</table>
| \( \gamma \) and \( \lambda \) | \begin{align*} \gamma &= 0.95 \\
|                              | \lambda &= 1.046                | \begin{align*} \gamma &= 0.95 \\
|                              | \lambda &= 1.047                |                                       |
| **No Tax Model**             |                                  |                                       |
| \( B \)                     | 1                               | 1                                    |
| Average \( x_1 \)           | 1.4\%                          | 3.18\%                               |
| Average \( x_{12} \)        | 21.5\%                         | 29.6\%                               |
| Average Annual Production    | 1.101                           | 1.102                                |
| Average Annual Spending      | 1                               | 1                                    |
| Parliament Utility           | 0.055                           | 0.055                                |
| **Tax Model**                |                                  |                                       |
| \( B \)                     | 1.788                           | 1.752                                |
| Tuning Parameters \((\nabla, \Upsilon)\) | NA                             | 0.491, 0.022                         |
| Final Month Tax \((\theta_{12})\) | 1.049                         | 1.048                                |
| Average \( x_1 \)           | 8.98\%                         | 9.09\%                               |
| Average \( x_{12} \)        | 7.52\%                         | 9.55\%                               |
| Average Annual Production    | 1.881                           | 1.869                                |
| Average Annual Spending      | 1.747                           | 1.726                                |
| Parliament Utility           | 0.064                           | 0.062                                |
| **Perfect Agency**           |                                  |                                       |
| Average Annual Production    | 1.961                           | 1.953                                |
| Average Annual Spending      | 1.781                           | 1.773                                |
| Parliament Utility           | 0.098                           | 0.098                                |
| Gains from tax               | 16.1\%                         | 12.6\%                               |
| Maximum possible gain        | 78.8\%                         | 76.7\%                               |

Note that start month and end month spending amounts are not listed for the direct parliament case as monthly spending will be even spending in all cases at 8.3\%. Utility gains from the tax and from the perfect agency models are relative to the no tax model. The tax equation formula can be found in equation 24 for the exponential model, the taxation formula for the quasi-hyperbolic model is as described on page 24.

Table 2: Calibrated Time Variant Budgetary Tax
These gains are robust to what $\gamma$ value is imposed. As presented in appendix F, the gains from a tax were evaluated for the 5% exponential discounting model for 9 different values of $\gamma$ in the interval $[0.5, 1]$. The gains were all at least 10.3% which offers some robustness to the finding that taxes are utility improving\(^{33}\).

Moving back to examining the $\gamma = 0.95$ cases in table 2, it can be seen that the assumed functional forms were moderately successful in smoothing spending. The exponential has long run spending decreasing from a moment of 1.08 in the first month to 0.90 in the last month. The quasi-hyperbolic model has its smallest average spending moment of 0.91 in the 7th month with its greatest spending moment at 1.15 in the last month of the fiscal year. The tax formula used was simple however and a time variant budgetary tax with more parameters could improve upon the results reported in table 2.

The maximum gains from the perfect agency simulations are larger than the benefits possible with a tax at 78.8% and 76.7%. In both cases there is little difference between annual average spending in the tax model and the perfect agency simulations. In conjunction with the even spending in the tax model this tends to suggest that the reason the tax model underperforms the perfect agency simulations is the shock over-exploitation problem\(^{34}\).

Two calibrations with 10% discounting were also performed (presented in appendix E) which illustrate this point. As the higher discounting rates indicate a stronger preference for delayed spending these 10% calibrations offset this tendency by having a more concave performance function which encourages even spending\(^{35}\). When the tax is calibrated there are utility benefits of 45.3% and 25.1% possible for the exponential and quasi-hyperbolic versions which compare to the perfect agency benefits of 64.3% and 62.9%. Intuitively the reason the tax gets closer to the perfect agency optimum in the 10% calibrations is because the more concave performance function reduces

\(^{33}\)In general the imposition of $\gamma$ does not have a large impact when considers the relative utility gains from taxes. Whilst the absolute level of long term average parliamentary utility depends heavily on the imposed $\lambda$ and $\gamma$ values, the change in long term parliamentary utility with and without a tax is not significantly affected by the choices of $\gamma$ and $\lambda$.

\(^{34}\)This shock over-exploitation problem may be able to be reduced by having taxes depend on the amount of spending as well as the time of the year such that the cost of spending to a department is convex in the amount spent (thus taxes are amount variant as well as time variant). At the cost of adding to the complexity of the budgeting system, these amount & time variant taxes may be successful in preventing departments from spending too much when there is a beneficial shock. The effectiveness of such a policy is beyond the scope of this paper but may merit future research.

\(^{35}\)These calibrations also lead to a greater variance in the shock process. This would tend to exacerbate the problem of shock over-exploitation problem however and so does not undermine the argument here.
the problem of shock over-exploitation. These calibrations indicate that the benefits from taxes have some level of robustness to the discounting rates and time consistency applied. In general every calibration performed in this research achieved a taxation benefit greater than 10% over the no taxation case.

6 Conclusion

This paper has detailed the governmental end of fiscal year spending spike phenomena with a focus on the United Kingdom. The key contributions of this paper are threefold. The first contribution is the development of a model for heightened end of year spending based on procrastination. The second is the model comparison of the procrastination model against the precautionary savings model which demonstrates the poor performance of the precautionary savings model in the UK experience. The third contribution is in the invention of time variant budgetary taxes as a policy tool to even spending and facilitate more efficient spending from government departments.

Time variant budgetary taxes are the recommendation of this paper as they are low cost, easy to implement and understand, do not take power away from elected politicians and are shown to be capable of smoothing spending throughout the fiscal year. Based on calibration results this paper estimates that the surplus that departments deliver for the parliament (and taxpayers) could be increased by more than 10% with the imposition of a time variant budgetary tax.

References


Hall, H. (1896), The Red Book of the Exchequer, Printed for Her Majesty’s Stationary Office by Eyre and Spottiswoode.

36 All calibrations are detailed in table 2, appendix E or appendix F


Serious Fraud Office (2010), ‘How much was paid out in bonuses to your departments staff?’, http://www.sfo.gov.uk/media/165070/bonuses.pdf.


Appendices

A Data

Budgets are further split by the planning horizon of the funds in two categories: Annual Managed Expenditure (AMEs) and Departmental Expenditure Limits (DELs). AMEs are set annually and are intended for demand driven expenses including welfare payments, hospital admissions and public sector pension scheme payments. DELs are set in the spending review process and are intended for expenses which can be planned for in advance including wages and rent costs. Spending reviews occur every three years (Crawford et al. 2009) and each time set DELs for the coming three years. The purpose of this system is for departments to know their budgets for the coming fiscal years and be able to plan accordingly with AME funds being available on a shorter horizon to cover demand driven expenses that are more difficult to plan for.

A.1 Northern Ireland Data

The Northern Ireland data was accessed from a Freedom of Information request to the Northern Ireland government. This data is based on accrual accounting and only includes DEL data. The restriction to DEL data is due to the division of responsibilities between the UK and Northern Ireland government (who supplied the data). Although AMEs constitute about 40% of spending in Northern Ireland (Northern Ireland Department of Finance and Personnel 2012, author calculations) this restriction is not particularly important. The first reason is that DELs are spending items that are driven by departmental decisions rather than being demand driven and outside of departmental hands. A restriction to DEL expenditures therefore focuses attention to those areas of government spending that are discretionary and thus where the underlying principal agent relationship of interest occurs. This is in contrast to AMEs which are demand driven and largely include transfer payments such as public sector pension schemes. Secondly the data used for calibration purposes later on is the capital spending data. Almost all Northern Irish capital spending is DEL spending and as such is represented in our data. The Department for Employment and Learning is the only exception receiving about 80% of its capital budget from AMEs with all other departments receiving less than 5% of their budget from AMEs. The qualitative conclusions of the calibrations are robust to the omission this department.

The data is split by current and capital expenditure for every year except for the fiscal years 2011-2012 and 2012-2013 when ”Non Ringfenced Resource Expenditure” and ”Ringfenced Resource Expenditure” were summed up to get current expenditure. All raw spending figures have been adjusted to reflect the differing number of days in each month. All monthly spending figures are normalised to their proportion of the annual total spending (after taking account for rollover). A regression of current and capital spending on month dummies can be seen in
The observations in this case include monthly departmental spending for each department for each of the 5 years.

Table A.1: Regressions on Northern Ireland Spending

<table>
<thead>
<tr>
<th></th>
<th>Capital Expenditure</th>
<th>Current Expenditure</th>
</tr>
</thead>
<tbody>
<tr>
<td>April</td>
<td>0.013 (0.016)</td>
<td>0.071*** (0.003)</td>
</tr>
<tr>
<td>May</td>
<td>0.043** (0.016)</td>
<td>0.076*** (0.003)</td>
</tr>
<tr>
<td>June</td>
<td>0.041** (0.016)</td>
<td>0.079*** (0.003)</td>
</tr>
<tr>
<td>July</td>
<td>0.045*** (0.016)</td>
<td>0.077*** (0.003)</td>
</tr>
<tr>
<td>August</td>
<td>0.057*** (0.016)</td>
<td>0.082*** (0.003)</td>
</tr>
<tr>
<td>September</td>
<td>0.063*** (0.016)</td>
<td>0.073*** (0.003)</td>
</tr>
<tr>
<td>October</td>
<td>0.066*** (0.016)</td>
<td>0.082*** (0.003)</td>
</tr>
<tr>
<td>November</td>
<td>0.056*** (0.016)</td>
<td>0.084*** (0.003)</td>
</tr>
<tr>
<td>December</td>
<td>0.084*** (0.016)</td>
<td>0.081*** (0.003)</td>
</tr>
<tr>
<td>January</td>
<td>0.064*** (0.016)</td>
<td>0.082*** (0.003)</td>
</tr>
<tr>
<td>February</td>
<td>0.120*** (0.016)</td>
<td>0.081*** (0.003)</td>
</tr>
<tr>
<td>March</td>
<td>0.350*** (0.016)</td>
<td>0.132*** (0.003)</td>
</tr>
<tr>
<td>Observations</td>
<td>1,008</td>
<td>960</td>
</tr>
<tr>
<td>R²</td>
<td>0.391</td>
<td>0.918</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.383</td>
<td>0.917</td>
</tr>
<tr>
<td>Residual Std. Error</td>
<td>0.149 (df = 996)</td>
<td>0.025 (df = 948)</td>
</tr>
<tr>
<td>F Statistic</td>
<td>53.192*** (df = 12; 996)</td>
<td>886.638*** (df = 12; 948)</td>
</tr>
</tbody>
</table>

Notes:
***Significant at the 1 percent level.
**Significant at the 5 percent level.
Northern Ireland’s NIAUR (Energy Regulator) current expenditure observations were omitted as outliers.
Note no constant term was included.

Devolved administrations have certain privileges that Whitehall departments do not. The extent of these depends on the budgetary system and therefore changed after the 2010-2011 financial year. An important privilege is that devolved administrations do not need to notify treasury in advance of any planned underspends (and associated rollover) (Hutt 2011). Taken in total the differences in budgetary restrictions are relatively minor however.

A.2 The UK Government

The UK Central Government Spending data was taken from the Office of National Statistics as the series: MF6S - Central Government: Total current expenditure: (£m) and MF6T - Central Government: Total capital expenditure: (£m). These series are accrual based meaning that expenditure is recorded in the month the
underlying economic transaction took place rather than the month in which cash was exchanged. All raw spending figures have been adjusted to reflect the differing number of days in each month. These series do not divide between DEL and AME expenditure but represent aggregate spending.

B Proofs

The important feature of the procrastination model is that departmental utility from spending is time dependent. For ease and clarity of the proofs we define and use a concept of departmental time dependent utility.

A time dependent departmental utility function \( u_t(x_t) \) in which in each period satisfies:

\[
\frac{\partial u_t(x_t)}{\partial x_t} > 0 \quad \forall t \text{ when } x_t = 0 \quad (B.1)
\]

\[
\frac{\partial^2 u_t(x_t)}{\partial x_t^2} < 0 \quad \forall t, x \quad (B.2)
\]

\[
u_t(x) > u_T(x) \quad \forall x \text{ for } t > T \quad (B.3)
\]

\[
\frac{\partial u_t(x_t)}{\partial x_t} > \frac{\partial u_T(x_T)}{\partial x_T} \quad \forall x \text{ for } t > T \quad (B.4)
\]

Clearly the procrastination model above is an example of such a model with \( u_t(x_t) = Dv(x_t) - D_t e(x_t) \) where \( D_1 = 1 \) and \( D_2 = D < 1 \).

As discussed in the first paragraph of section 5 we assume that the marginal utility of spending is positive at the parliament optimal level of spending in any period of the year. This assumption translates to the following expression:

\[
\frac{\partial u_t(x_{\text{par}})}{\partial x} > 0 \quad \text{for } t = t_{\text{min}} \quad (B.5)
\]

Where \( t_{\text{min}} \) is the start of the fiscal year.

37This is imperfect however with several large capital receipts relating to the sales of 3G licenses in 2000 and the decommissioning of British Nuclear Fuels in 2005 being ascribed to particular months. For this reason some months of the 2000-2001 fiscal year have been omitted and the remaining months spending moments renormalised.

38If this was violated the parliament would need to adjust \( v_t(x_t) \) in order to induce their desired spending level.
Proof of Proposition 1

We can write a Lagrangian for the department:

\[ V = u_1(x_1) + u_2(x_2) + \Lambda[B - x_1 - x_2] \]

\[
\frac{\partial V}{\partial x_1} = \frac{\partial u_1(x_1)}{\partial x_1} - \Lambda \quad \text{(B.6)}
\]

\[
\frac{\partial V}{\partial x_2} = \frac{\partial u_2(x_2)}{\partial x_2} - \Lambda \quad \text{(B.7)}
\]

Note that if the budget constraint doesn’t bind \( \Lambda = 0 \), if it does \( \Lambda \) is positive.

In either case the department will spend in each period until the marginal value of spending is equal to the shadow price \( \Lambda \). From condition (B.4) when the marginal value at time 1 is \( \Lambda \), the marginal value at time 2 is higher. Hence the department will spend more in the second period.

Proof of Proposition 2

Suppose the parliament could choose spending in each period. We write the government objective function as:

\[ G = g(x_1) - \lambda x_1 + D[g(x_2) - \lambda x_2] \]

\[
\frac{\partial G}{\partial x_1} = g'(x_1) - \lambda \\
\frac{\partial G}{\partial x_2} = g'(x_2) - \lambda
\]

Condition 6 implies a unique solution for both FOCs at \( x_1 = x_2 \).

Proof of Proposition 3

From Proposition 1 we have the strict inequality \( x_2 > x_1 \). Therefore it is not possible to get \( x_2 = x_1 = x_{\text{Par}} \).

Proof of Proposition 4

We can note that the periodic utility is:

\[ u_t(x_t) = Dv(x_t) - D_t e(x_t) - D_t q(x_t) \quad \text{(B.8)} \]

where \( D_1 = 1 \) and \( D_2 = D < 1 \).

This satisfies the conditions (B.1), (B.2), (B.3) and (B.4) such that the proof of Proposition 1 is applicable.
Proof of Proposition 5

We have the objective function:

\[ V = u_1(x_1) - Dq_1(x_1) + u_2(x_2) - Dq_2(x_2) \]  (B.9)

We assume \( u_1(x_1) - Dq_1(x_1) \) and \( u_2(x_2) - Dq_2(x_2) \) are such that equation B.5 holds.

We get the following first order conditions

\[ \frac{\partial V}{\partial x_1} = u_1'(x_1) - Dq_1'(x_1) \]  (B.10)
\[ \frac{\partial V}{\partial x_2} = u_2'(x_2) - Dq_2'(x_2) \]  (B.11)

These are equal at the optimal point:

\[ u_1'(x_1) - Dq_1'(x_1) = u_2'(x_2) - Dq_2'(x_2) \]  (B.12)

Substituting in \( q_2'(x) = \eta q_1'(x) \) we want to find \( \eta \) such that both sides are equalised when \( x_1 = x_2 = x_{Par} \). This solves to get:

\[ \eta = 1 + \frac{u_2'(x_{Par}) - u_1'(x_{Par})}{Dq_1'(x_{Par})} \]  (B.13)

Proof of Proposition 6

We assume the parliament will set a budget such that it will be fully expended\(^{39}\):

\[ V = u_1(x_1) + u_2(B - \theta x_2) \]  (B.14)

This gives the first order conditions

\[ \frac{\partial V}{\partial x_1} = \frac{\partial u_1(x_1)}{\partial x_1} - \theta^{-1} \frac{\partial u_2(B - \theta x_1)}{\partial x_2} \]  (B.15)

For this to be 0 when \( x_1 = x_2 = x_{Par} \)

\[ \theta = \frac{\frac{\partial u_2(x_{Par})}{\partial x_2}}{\frac{\partial u_1(x_{Par})}{\partial x_1}} \]  (B.16)

The assumption in equation (B.5) guarantees that this result will exist. The corresponding budget choice for the parliament is:

\[ B = (1 + \theta)x_{Par} \]  (B.17)

\(^{39}\)This will be true of any parliament optimally setting a budget and tax. Time variant budgetary taxes will be ineffective if the budget constraint never binds.
C Further discussion

C.1 Assumption of disutility from effort in spending money

A crucial assumption for the procrastination model is that there is some level of disutility in spending. The project management industry gives some basic evidence for this. It is also possible however to motivate this assumption by starting with a constant returns performance function incorporating effort and money and with a cost of effort function involving only effort.

Denoting \( P(e_t, x_t) \) as the performance in a period from spending \( x_t \) and using effort \( e_t \) and denoting \( k(e_t) \) as the disutility from effort level \( e_t \) the problem for a department is:

\[
\max_{x_1, x_2, e_1, e_2} D [P(e_1, x_1) + P(e_2, x_2)] - k(e_1) - Dk(e_2) \quad (C.1)
\]

\[\text{s.t.} \quad x_1 + x_2 \leq B\]

In any period we can use first order conditions to get an equation for optimal effort as a function of spending in the current period. As there is no intertemporal constraint on effort only the current period’s spending will enter this function.

\[e_t = f(x_t) \quad (C.2)\]

We can substitute this into the performance and functions as:

\[P(e_t, x_t) = P(f(x_t), x_t) \quad k(e_t) = k(f(x_t)) \quad (C.3)\]

\[= v(x_t) \quad = e(x_t) \quad (C.4)\]

Substituting back into equation C.1 yields equation 1.

C.2 Assumption of performance monitoring on a fiscal year basis

A key assumption for the procrastination model is that performance is monitored on a fiscal year basis. This is a reasonable assumption when the UK is considered. While the performance frameworks of devolved administrations (such as Northern Ireland) will not be explicitly discussed they can be considered (in the timing dimension) to mirror the central government framework.

Since at least the time of the red book of the exchequer was written around the year 1230 (Hall 1896, page 855) fiscal reporting has been organised around annual increments. This annual increment was maintained as performance-related pay was beginning to be introduced in 1991 by the Major government (Major 1991). An early implementation was in local government where a number of performance
metrics were introduced to encourage greater public accountability (Panchamia & Thomas 2010). A broader based framework was introduced with the incoming Labour government in 1998 with the Public Service Agreement System where departments were given measurable performance targets (termed PSAs) along with completion dates. Despite the presence of in year completion dates and bimonthly Prime Ministerial meetings with department heads these performance targets were inexorably linked to the fiscal year. With the passage of the 2000 Government Resource Accounting Act the public spending committee was tasked with monitoring progress towards PSAs by requiring departments to provide performance reporting alongside spending figures which were published on a fiscal year basis. Furthermore the negotiation of PSAs was “explicitly tied to the spending review process, which increased the potential for the treasury to incentivise departments to deliver on government’s overall objectives in return for appropriate funding. (Panchamia & Thomas 2010, page 3)”.

There exists further evidence of performance and spending being scrutinised together in practice from the reports of the House of Commons Scrutiny Unit (2009, page 4). In 2010 the incoming coalition government replaced the PSA system with the Departmental Business Plans system (Stephen, Bouchal & Bull 2013). These documents were updated on an annual basis and included regular goalsetting and performance measurement in a similar fashion to the PSA system. One new feature was the public release of individual performance targets for senior public servants on an annual (fiscal year) basis (UK Cabinet Office 2013).

Regular performance appraisal is also common in other countries. The United States Office of Personnel Management describes the performance award framework for the US government with awards for performance awarded on a period basis similarly to the UK (US Office of Personnel Management 2015). In a recent case in Russia the governor of Saint Petersburg, Georgy Poltavchenko, stopped the awarding of bonuses to public sector budget managers as they collectively only spent 92.4% of the budget allocated to them in the 2014 fiscal year (In Russia the calendar and fiscal years coincide), an amount he thought insufficient (NTV News 2015).

There are good reasons for examining performance and spending together. The first is that performance is often measured in cost per output terms and therefore cannot be examined independently of expenditures. There are economies of scale in examining performance at the same intervals as the budgeting interval. The second is that performance based budgeting (OECD 2007, page 94) can be used to give more funding to well performing departments as well as using funding as a carrot to incentivize performance. These reasons suggests that delayed utility from expenditure is likely a feature of other budgetary systems where spending spikes are observed.
C.3 Assumption of diminishing returns from spending

In this paper the low quality of spending at the end of fiscal years in interpreted as being a result of diminishing returns to spending. Diminishing returns from spending emerges as a credible way to model spending when one considers what mechanisms could explain the link between heightened spending and low quality. One way this could occur is that managers have less time to spend on each spending project and projects could blow out costs due to mistakes being made. Whilst money expended (as a factor of production) can be scaled up, manager’s time is another factor of production that cannot be scaled up as easily. Another may be frivolous purchases that occur while senior managers cannot check on all of the purchases made by more junior budget holders. In this way heightened spending may lead to a higher proportion of funds being spent on items which may increase personal utility for these budget holders but do not deliver value to the organisation (for instance new office furniture or high-end computer monitors). Crowding out of suppliers may be a further mechanism - due to higher demand at the end of the fiscal year suppliers raise their prices. A further mechanism is that shorter timeframes for contracting as well as more contracts available for suppliers to quote on result in fewer quotes per project (a mechanism which finds direct support in Liebman & Mahoney’s (2013, page 24) IT project analysis). In addition there exists the possibility that departments simply run out of profitable spending opportunities and need to fall back on less valuable spending opportunities.

There may be alternative ways of modeling why fiscal-year ends have spending of low quality. The end of year is not an inherently special time however and so it is difficult to devise a feasible mechanism for this low quality that does not depend upon the heightened level of spending in a way that is isomorphic to diminishing returns.

Note that this assumption of diminishing returns from spending is not important for showing how heightened end of year spending can be caused by procrastination. It is important in a deterministic setting however for establishing that there are welfare gains from eliminating these spending spikes however. As is discussed in section 5.3 in a stochastic setting end of year spending spikes can lead to inefficiencies even with a constant returns production function.

C.4 Alternate explanations of end of fiscal year spending spikes

Two explanations frequently came up at early seminars of this paper and in discussions with public servants. The first is that if money is not spent by the end of the fiscal year it is lost by a government department. This has not been the case in the UK since 1998, nonetheless even if rollover were available this is still only an argument for spending an entire budget within a fiscal year and is not an argument for spending the funds in the final months of a fiscal year. A further feature is needed in any model to explain why spending is delayed until late in the year. The second explanation offered is that if funds are not spent in a fiscal year
then the budget for the next fiscal year is reduced by an amount corresponding to the underspend. This is subject to the same criticism as the first explanation in that it doesn’t explain why spending is delayed until the end of the fiscal year. In addition it is likely that spending spikes at the end of the fiscal year would be a bad signal to the parliament which sets the budget. This mechanism may therefore actually work to push spending away from the end of the fiscal year towards more even spending. Furthermore Danny Alexander⁴⁰, a former UK secretary of the treasury, indicated that he was aware of this argument but did “not believe it is entirely the case”. He cites that even after allowing for the rolling over of funds, “since 2010 departments have on average underspent by an average of £5 billion a year”.

Another potential mechanism is the release of some funds (Departmental Unallocated Provisions (DUPs)) that are typically authorised and available for departments in January. There are a few reasons to believe that this effect would have a minor impact on spending spikes. The first is that as these funds are awarded by January it would be expected that the effect of these funds would be realised in February and March. As can be seen in figure 1 however March spending is much greater than February. The second is that these funds are approved as DUPs at the start of the fiscal year and there exists an expectation that they would be approved for spending if they are required. In the absence of ambiguity about whether these funds would be approved these funds would be anticipated and funding more evenly dispersed. The third is that heightened spending is found in every Northern Irish department not all of which receive additional funding provision within the fiscal year.

Two further putative explanations can be dismissed as an explanation for end of year spending spikes. One is a general growth in demand and cost for government services as a result of population growth and inflation. The size of the spending spikes observed is far too large to be accounted for in this way. The second is measurement error where spikes are exaggerated by imperfect accrual accounting which tends to record a number of expenditure items in the last month of the fiscal year. Whilst it is credible that this could occur to some degree the survey data collected by Liebman & Mahoney (2013), Teich (2013) and McPherson (2007) support the existence of a spending spike independently of any evidence of substantial measurement error. Furthermore even if the last month is excluded it can be seen that spending is increasing over the first 11 months of the fiscal year for both Northern Ireland and the UK central government.

⁴⁰Source: Communications with the author
D  Regression for Spending Spike Vs R-Squared

Table D.1: Regression for Spending Spike Vs $R^2$

<table>
<thead>
<tr>
<th></th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spike</td>
<td>0.094**</td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
</tr>
<tr>
<td>Current Budget dummy</td>
<td>0.161</td>
</tr>
<tr>
<td>(Capital Budget is baseline)</td>
<td>0.100</td>
</tr>
<tr>
<td>Constant</td>
<td>0.224</td>
</tr>
<tr>
<td></td>
<td>(0.156)</td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Observations</td>
<td>22</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.231</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.150</td>
</tr>
<tr>
<td>Residual Std. Error</td>
<td>0.132 (df = 19)</td>
</tr>
<tr>
<td>F Statistic</td>
<td>2.858* (df = 2, 19)</td>
</tr>
</tbody>
</table>

Note: *p<0.1; **p<0.05; ***p<0.01

E  5% and 10% discounting calibrations
### Table E.1: All 5% and 10% discounting calibrations

<table>
<thead>
<tr>
<th></th>
<th>5% Exponential</th>
<th>10% Exponential</th>
<th>5% Quasihyperbolic</th>
<th>10% Quasihyperbolic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Calibrated Parameters</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.993</td>
<td>0.986</td>
<td>0.992</td>
<td>0.984</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>0.536</td>
<td>0.523</td>
<td>0.257</td>
<td>0.251</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.010</td>
<td>0.020</td>
<td>0.011</td>
<td>0.022</td>
</tr>
<tr>
<td><strong>Discounting</strong></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.996</td>
<td>0.992</td>
<td>0.998</td>
<td>0.996</td>
</tr>
<tr>
<td>$k$</td>
<td>1.000</td>
<td>1.000</td>
<td>0.975</td>
<td>0.950</td>
</tr>
<tr>
<td><strong>Moments</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shocks Mean</td>
<td>0.001</td>
<td>0.002</td>
<td>0.001</td>
<td>0.002</td>
</tr>
<tr>
<td>Shocks Variance</td>
<td>0.011</td>
<td>0.021</td>
<td>0.012</td>
<td>0.023</td>
</tr>
<tr>
<td>Final Month Spending Moment</td>
<td>2.584</td>
<td>2.555</td>
<td>3.554</td>
<td>3.601</td>
</tr>
<tr>
<td>RSS</td>
<td>3.139</td>
<td>3.119</td>
<td>0.478</td>
<td>0.457</td>
</tr>
<tr>
<td>Log Likelihood</td>
<td>1310.602</td>
<td>1018.531</td>
<td>1281.764</td>
<td>997.181</td>
</tr>
<tr>
<td>$\gamma$ and $\lambda$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>1.046</td>
<td>1.054</td>
<td>1.047</td>
<td>1.055</td>
</tr>
<tr>
<td><strong>No Tax Model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Average $x_1$</td>
<td>1.4%</td>
<td>1.5%</td>
<td>3.18%</td>
<td>3.2%</td>
</tr>
<tr>
<td>Average $x_{12}$</td>
<td>21.5%</td>
<td>21.3%</td>
<td>29.6%</td>
<td>30.0%</td>
</tr>
<tr>
<td>Average Annual Production</td>
<td>1.101</td>
<td>1.101</td>
<td>1.102</td>
<td>1.111</td>
</tr>
<tr>
<td>Average Annual Spending</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Parliament Utility</td>
<td>0.055</td>
<td>0.055</td>
<td>0.055</td>
<td>0.056</td>
</tr>
<tr>
<td><strong>Tax Model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B$</td>
<td>1.788</td>
<td>1.597</td>
<td>1.752</td>
<td>1.532</td>
</tr>
<tr>
<td>Tuning Parameters ($\nabla$, $\Upsilon$)</td>
<td>NA</td>
<td>NA</td>
<td>0.491, 0.022</td>
<td>0.470, 0.059</td>
</tr>
<tr>
<td>Final Month Tax ($\theta_{12}$)</td>
<td>1.049</td>
<td>1.112</td>
<td>1.048</td>
<td>1.112</td>
</tr>
<tr>
<td>Average $x_1$</td>
<td>8.98%</td>
<td>9.14%</td>
<td>9.09%</td>
<td>9.45%</td>
</tr>
<tr>
<td>Average $x_{12}$</td>
<td>7.52%</td>
<td>7.45%</td>
<td>9.55%</td>
<td>7.61%</td>
</tr>
<tr>
<td>Average Annual Production</td>
<td>1.881</td>
<td>1.665</td>
<td>1.869</td>
<td>1.638</td>
</tr>
<tr>
<td>Average Annual Spending</td>
<td>1.747</td>
<td>1.515</td>
<td>1.726</td>
<td>1.487</td>
</tr>
<tr>
<td>Parliament Utility</td>
<td>0.064</td>
<td>0.069</td>
<td>0.062</td>
<td>0.070</td>
</tr>
<tr>
<td><strong>Perfect Agency</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Annual Production</td>
<td>1.961</td>
<td>1.823</td>
<td>1.953</td>
<td>1.824</td>
</tr>
<tr>
<td>Average Annual Spending</td>
<td>1.781</td>
<td>1.643</td>
<td>1.773</td>
<td>1.643</td>
</tr>
<tr>
<td>Parliament Utility</td>
<td>0.098</td>
<td>0.091</td>
<td>0.098</td>
<td>0.091</td>
</tr>
<tr>
<td>Gains from tax</td>
<td>16.1%</td>
<td>24.3%</td>
<td>12.6%</td>
<td>25.1%</td>
</tr>
<tr>
<td>Maximum possible gain</td>
<td>78.8%</td>
<td>64.3%</td>
<td>76.7%</td>
<td>62.9%</td>
</tr>
</tbody>
</table>

Here the shocks mean and variance refer to the first 2 moments of the implied shocks after they are transformed to the normal distribution. They should be distributed $(0, \sigma)$ to match the shock process anticipated by departments in the model. The RSS refers sum of squared differences between the predicted monthly spending values (from the long term average of simulations) and the average spending from the data over the fiscal year. This is also summarised in figure 3. Note that start month and end month spending amounts are not listed for the direct parliament case as monthly spending will be even spending in all cases at 8.3%. Utility gains from the tax and from the perfect agency models are relative to the no tax model. The tax equation formula can be found in equation 24 for the exponential models, the taxation formula for the quasi-hyperbolic models are as described on page 24.
F Curvature of production function robustness checks

Table 2 and the discussion of section 5.3 concentrates on the case of $\gamma = 0.95$. Here a robustness check was carried out to assess the extent to which the benefits of a tax change when a different $\gamma$ value is assumed. This was done for the 5% exponential discounting model. For a range of $\gamma$ values from 1 to 0.5 time implied social cost ($\lambda$) was found from equation 23. $x_{Par}$ was then found using equation 26. For the purposes of the $\gamma = 1$ case a $\gamma$ value of 0.95 was used for the purposes of equation 26.

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\lambda$</th>
<th>$x_{Par}$</th>
<th>Benefits From Tax</th>
<th>Benefits from Perfect Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.007</td>
<td>0.31</td>
<td>68.629</td>
<td>$\infty$</td>
</tr>
<tr>
<td>0.95</td>
<td>1.046</td>
<td>0.145</td>
<td>1.161</td>
<td>1.79</td>
</tr>
<tr>
<td>0.9</td>
<td>1.086</td>
<td>0.152</td>
<td>1.109</td>
<td>1.847</td>
</tr>
<tr>
<td>0.85</td>
<td>1.128</td>
<td>0.152</td>
<td>1.103</td>
<td>1.838</td>
</tr>
<tr>
<td>0.8</td>
<td>1.171</td>
<td>0.149</td>
<td>1.107</td>
<td>1.809</td>
</tr>
<tr>
<td>0.733</td>
<td>1.229</td>
<td>0.144</td>
<td>1.116</td>
<td>1.756</td>
</tr>
<tr>
<td>0.666</td>
<td>1.288</td>
<td>0.139</td>
<td>1.123</td>
<td>1.696</td>
</tr>
<tr>
<td>0.583</td>
<td>1.36</td>
<td>0.131</td>
<td>1.127</td>
<td>1.614</td>
</tr>
<tr>
<td>0.5</td>
<td>1.426</td>
<td>0.123</td>
<td>1.125</td>
<td>1.529</td>
</tr>
</tbody>
</table>

All of these benefits are for the 5% exponential calibration. For the $\gamma = 1$ estimation for the purposes of equation 26, a $\gamma$ value of 0.95 was chosen in finding the parliament desired spending level. Furthermore in this case the the utility improvement proportion should be interpreted with caution as a result of this assumption as well as the fact that utility in the no tax case was close to zero and simulation errors can have a large relative impact when calculating utility gains.

Table F.1: Robustness checks for the gains from imposition of a tax for the 5% exponential discounting case.

G Derivation of optimal tax for calibrated model in deterministic setting

As analytical results are not possible a deterministic approximation was used to give a functional form for a time variant budgetary tax. A parameter was then inserted that could be calibrated to better reflect the stochastic environment.

The departments have a problem represented by the lagrangian:

$$L = \sum_{t=1}^{12} D_{11} \left[ x_{t}^{\delta} - \Omega D_{t-1} x_{t} \right] + \Lambda \left[ B - \sum_{t=1}^{12} \theta_{t} x_{t} \right]$$

(G.1)

Where $\Lambda$ is the shadow price and $D_{i}$ is the discounting $i$ periods into the future. Taking the first order condition and setting it equal to zero we get:

$$\Lambda = \frac{\delta D_{11} x_{t}^{\delta-1} - \Omega D_{t-1}}{\theta_{t}}$$

(G.2)
Combining this equation for two different months $x_t$ and $x_T$ we get:

$$\frac{\theta_T}{\theta_t} = \frac{\delta D_{11} x_T^{\delta - 1} - \Omega D_{t-1}}{\delta D_{11} x_t^{\delta - 1} - \Omega D_t} \quad (G.3)$$

Setting $\theta_1 = 1$ and choosing the other tax rates such that $x_t = x_T = x_{\text{par}} \forall t, T$ we get the expression:

$$\theta_t = \frac{\delta D_{11} x_{\text{par}}^{\delta - 1} - \Omega D_{t-1}}{\delta D_{11} x_{\text{par}}^{\delta - 1} - \Omega} \quad (G.4)$$

The annual budget is then set so that the department can afford to spend $x_{\text{par}}$ every period while paying the tax:

$$B = x_{\text{par}} \left[ \sum_{t=1}^{12} \theta_t \right] \quad (G.5)$$

### H Social Cost and Optimal Budgeting

We find the social cost ($\lambda$) that implies an optimal budget choice of 1. This has the implicit assumption that the parliament does not choose a budget conditional on departmental savings but instead choose a stationary budget that maximises long run expected utility.

This is justified by the fact that this data refers to departmental expenditure limits which are decided upon in four year cycles (although departments still receive annual budgets. See appendix A for a general discussion of this feature of governmental budgeting as well as Northern Ireland Executive (2008) and Northern Ireland Executive (2011)). In this framework it is not possible for parliaments to budget conditional on departmental savings accrued within a spending review period. Parliaments could only budget conditional on long term department savings (accrued between spending review periods). Thus this is a mild assumption. Aside from being close to the reality this assumption also offers a large computational advantage as the behaviour of the department can be determined independently of the parliament as they are not optimising against the actions of the other party.

We assume that budgets are chosen to maximise the expected annual surplus without taking current departmental savings into account. The expected production is given by:

$$\text{Expected Production}(B) = \int_S E[P(B, S_y)]d\mu(S_y) \quad (H.1)$$
Where $B$ is the budget, $S$ is the state space of departmental savings, $\mu(S_y)$ is the measure of each savings level and $E[P(B, S_y)]$ is the expected production for a department with budget $B$ and savings $S_y$. Expected expenditure will be equal to:

$$\text{Expected Expenditure}(B) = \int_S E[X(B, S_y)]d\mu(S_y) \quad (H.2)$$

Where $E[X(B, S_y)]$ is expected expenditure for a department with budget $B$ and savings $S_y$.

When the parliament is optimising we have:

$$\frac{\partial\text{Expected Production}(B)}{\partial B} = \lambda \frac{\partial\text{Expected Expenditure}(B)}{\partial B} \quad (H.3)$$

$$\lambda = \frac{\frac{\partial\text{Expected Production}(B)}{\partial B}}{\frac{\partial\text{Expected Expenditure}(B)}{\partial B}} \quad (H.4)$$

It is possible to evaluate this equation using numerical estimates of the gradients $\frac{\partial\text{Expected Production}(B)}{\partial B}$ and $\frac{\partial\text{Expected Expenditure}(B)}{\partial B}$. 

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