The probability of this event equals

$$\sum_{i=2}^{n} \frac{n!}{i!\,(n-i)!} \left(F\left(x^{a}\right)-F\left(a\right)\right)^{i} F\left(a\right)^{n-i},$$

which simplifies to

$$F\left(x^{a}\right)^{n}-n\left(F\left(x^{a}\right)-F\left(a\right)\right)F\left(a\right)^{n-1}-F\left(a\right)^{n}.$$

Taking the expectation over n yields the desired expression. ■

Proof of Lemma 5

For the planner's problem, the proof proceeds by induction, much like the proof of lemma 1. Suppose that n buyers visit a seller, the first $n-1$ buyers learn their valuation, and no trade has taken place because $\widehat{x}_{n-1} \equiv \max\{y, x_1, \ldots, x_{n-1}\} < x^*$. In this case, the planner has two options: either let buyer n incur the inspection cost k and base the ensuing trading decision on $\widehat{x}_n$, or avoid the inspection cost by instructing the seller to trade with buyer n without knowing his valuation.[39]

In the former case, expected surplus generated by the match is $Z_{n-1}\left(\widehat{x}_{n-1}\right)-y$, where

$$Z_{n-1}(\widehat{x}) = -k + \widehat{x}F\left(\widehat{x}\right) + \int_{\widehat{x}}^{\overline{x}} xf\left(x\right)dx,$$

while the latter case yields an expected surplus equal to $\int_{\underline{x}}^{\overline{x}} xf\left(x\right)dx - y$. Clearly, inspection is preferred if and only if $Z_{n-1}\left(\widehat{x}_{n-1}\right)-y > \int_{\underline{x}}^{\overline{x}} xf\left(x\right)dx - y$, or equivalently

$$k < \int_{\underline{x}}^{\widehat{x}_{n-1}} \left(\widehat{x}_{n-1}-x\right)f\left(x\right)dx.$$

This condition needs to hold for any feasible value of $\widehat{x}_{n-1}$ in order to guarantee inspection by the last buyer. Since the right-hand side is strictly increasing in $\widehat{x}_{n-1}$, (22) is a necessary and sufficient condition. The final step is then to show that this condition implies that inspection is also optimal after meeting with $n-j-1$ buyers for $j \in \{1,\ldots,n-1\}$. This follows immediately from $Z_{n-j-1}\left(\widehat{x}\right) \geq Z_{n-1}\left(\widehat{x}\right)$ for all $\widehat{x}$ and j, as shown in the proof of lemma 1.

Next, we analyze the market equilibrium described in section 4. Consider a deviating buyer who does not inspect the good and therefore does not know his valuation. This deviant has three options: 1) submit a bid below y, which will be rejected; 2) submit a bid between y and a; or

[39] The planner can of course also instruct the seller to immediately trade with the agent with valuation $\widehat{x}_{n-1}$, but, as shown in lemma 1, this is dominated by learning the valuation of the last buyer since $\widehat{x}_{n-1} < x^*$.

3) bid the asking price and trade immediately. The choice between these options is equivalent to choosing a type of buyer $x' \in [\underline{x}, \overline{x}]$ to mimic.

We first establish that it is optimal for the deviant to behave like a buyer who has a valuation x' equal to the unconditional expected value of x, which we denote by $x^e = E_F[x] \equiv \int_{\underline{x}}^{\overline{x}} xf(x)\,dx$. To see this, consider the expected payoffs under each of the three options. First, a deviant who acts like a buyer with valuation $x' \in [\underline{x}, y]$ receives a payoff of 0. Second, if the deviant instead imitates a buyer of type $x' \in (y, x^a)$ and bids $\hat{b}(x')$, his payoff conditional on meeting with the seller is

$$u(x'|x^e) \;=\; \frac{\lambda(1-F(x^a))\,q_0(x')}{1-q_0(x^a)}\left[x^e - \hat{b}(x')\right].$$

Since $\hat{b}(x')$ is optimal in equilibrium, it follows that the deviant should choose $x' = x^e$ and bid $\hat{b}(x^e)$. Evaluating the expected payoff from this strategy yields

$$u(x^e) = \frac{\lambda(1-F(x^*))}{1-q_0(x^*)}\int_y^{x^e} q_0(\tilde{x})\,d\tilde{x}, \tag{33}$$

where $u(x^e)$ is increasing and convex in its argument. Finally, if the deviant mimics a buyer of type $x' \in [x^*, \overline{x}]$ and bids the asking price, he obtains a payoff

$$x^e - a^* = x^e - x^* + u(x^*). \tag{34}$$

Comparing the three payoffs reveals that the deviant maximizes his payoff by behaving as a buyer with valuation x^e. That is, he should submit a bid below y if $x^e < y$ and should bid $\hat{b}(x^e)$ if $x^e \in [y, x^*)$. Note that the remaining case, $x^e \in [x^*, \overline{x}]$, cannot occur under (22), since it implies

$$\begin{aligned} x^e \;&=\; -\int_{\underline{x}}^{x^*}(x^*-x)f(x)\,dx + k + x^* \\ &< -\int_{\underline{x}}^{y}(y-x)f(x)\,dx + k + x^* < x^*. \end{aligned}$$

To see whether the deviant benefits from not inspecting the good, define an auxiliary distribution $\widetilde{F}(x)$ that resembles $F(x)$, except that the mass below y and above x^* is concentrated as mass points at y and x^*, respectively. That is,

$$\widetilde{F}(x) = \begin{cases} 0 & \text{if } x < y \\ F(x) & \text{if } y \le x \le x^* \\ 1 & \text{if } x^* < x. \end{cases}$$

Let $\widetilde{x}^e = E_{\widetilde{F}}[x]$ denote the expectation of x under this modified distribution. Under (22), it then follows that $\widetilde{x}^e > x^e$, since

$$\begin{aligned}\widetilde{x}^e - x^e &= \int_{\underline{x}}^{y} (y - x)\, dF(x) - \int_{x^*}^{\overline{x}} (x - x^*)\, dF(x) \\ &= \int_{\underline{x}}^{y} (y - x)\, dF(x) - k > 0.\end{aligned}$$

The fact that $u(x)$ is an increasing function then implies that $u(x^e) < u(\widetilde{x}^e)$, while the convexity of $u(x)$ implies that $u(\widetilde{x}^e) < E_{\widetilde{F}}[u(x)]$ by Jensen's inequality. Note, however, that $E_{\widetilde{F}}[u(x)]$ exactly equals the payoff from inspection, since

$$\begin{aligned}E_{\widetilde{F}}[u(x)] &= u(y)\,\widetilde{F}(y) + \int_{y}^{x^*} u(x)\, d\widetilde{F}(x) + \left(1 - \widetilde{F}(x^*)\right) u(x^*) \\ &= \int_{y}^{x^*} u(x)\, dF(x) + (1 - F(x^*))\, u(x^*) \\ &= \frac{\lambda(1 - F(x^a))}{1 - q_0(x^a)} \left[\int_{y}^{x^*} \int_{y}^{x} q_0(\tilde{x})\, d\tilde{x} dF(x) + (1 - F(x^*)) \int_{y}^{x^*} q_0(\tilde{x})\, d\tilde{x}\right]\end{aligned}$$

Hence, the payoff from inspection is strictly higher than the payoff from not inspecting.

To show that equation (22) is also necessary, consider the limit $\Lambda \to 0$, such that a buyer who meets with a seller knows that, with probability 1, he does not face competition from other buyers. The optimal bid in that case is y and it follows immediately that inspection is better only if equation (22) holds. ■

Proof of Lemma 6

Compared to the homogeneous model, the environment gives rise to one extra decision, which concerns the allocation of buyers across the two goods. Suppose that a fraction ψ_i of the buyers is allocated to sellers of good i, with $\psi_1 + \psi_2 = 1$. Conditional on ψ_i, the game within each submarket is essentially the same as in the homogeneous model. Hence, it follows directly from proposition 1 that a planner assigns each seller of good i a queue $\lambda_i(\psi_i) = \frac{\psi_i}{\phi_i}\Lambda$ and instructs him to implement a cutoff x_i^*, which is the solution to $k = \int_{x_i^*}^{\overline{x}} (x - x_i^*)\, f_i(x)\, dx$. The surplus created by the seller then equals

$$\mathbf{S}_i^*(\lambda(\psi_i)) = x_i^* - y_i - \int_{y_i}^{x_i^*} e^{-\lambda_i(\psi_i)(1 - F_i(x))} dx.$$

Conditional on sellers using asking price mechanisms, it follows from proposition 2 that—for a given ψ_i—all sellers of good i post an asking price equal to

$$a_i(\psi_i) \equiv x_i^* - \frac{\lambda_i(\psi_i)(1 - F_i(x_i^*))}{1 - e^{-\lambda_i(\psi_i)\left(1 - F_i\left(x_i^*\right)\right)}} \int_{y_i}^{x_i^*} e^{-\lambda_i(\psi_i)(1-F_i(x))} dx.$$

A buyer who visits a seller of good i obtains an expected payoff

$$\overline{U}_i(\psi_i) = \int_{y_i}^{x_i^*} (1 - F_i(x)) e^{-\lambda_i(\psi_i)(1-F_i(x))} dx,$$

which is exactly his marginal contribution to surplus, $\frac{\partial}{\partial\lambda}\mathbf{S}_i^*(\lambda(\psi_i))$. Hence, by proposition 3, asking price mechanisms are optimal even when sellers have access to a larger mechanism space.

Consider now the allocation of buyers to the two different goods. In the decentralized market, each buyer chooses the submarket that maximizes his expected payoff, while a planner wants to allocate buyers to the submarket in which their marginal contribution of surplus is highest. Since $\overline{U}_i(\psi_i) = \frac{\partial}{\partial\lambda}\mathbf{S}_i^*(\lambda(\psi_i))$, these choices will coincide. There are three possible cases:

1. If $\overline{U}_1(0) \leq \overline{U}_2(1)$, then $\psi_1^* = 0$ and $\psi_2^* = 1$. That is, all buyers visit sellers of good 2.
2. If $\overline{U}_1(1) \geq \overline{U}_2(0)$, then $\psi_1^* = 1$ and $\psi_2^* = 0$. That is, all buyers visit sellers of good 1.
3. If $\overline{U}_1(0) > \overline{U}_2(1)$ and $\overline{U}_1(1) < \overline{U}_2(0)$, then—by the Intermediate Value Theorem—there exists a $\psi_1^* \in (0,1)$ such that $\overline{U}_1(\psi_1^*) = \overline{U}_2(1 - \psi_1^*)$. That is, buyers randomize between the sellers.

■

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Figure 1: Optimal Mechanisms

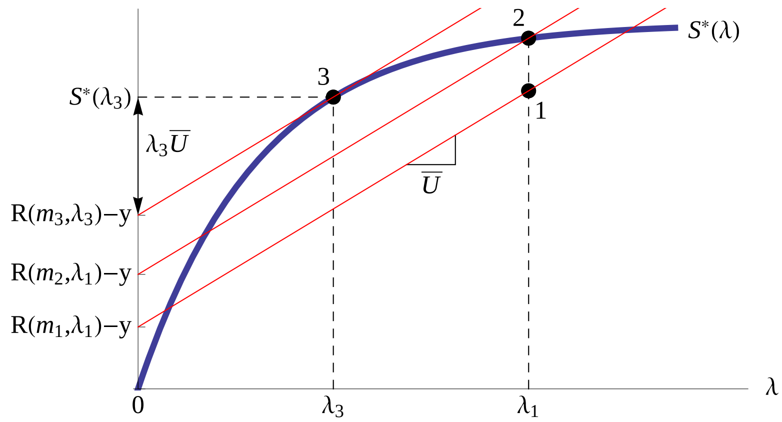


Figure 2: Transaction Price CDF

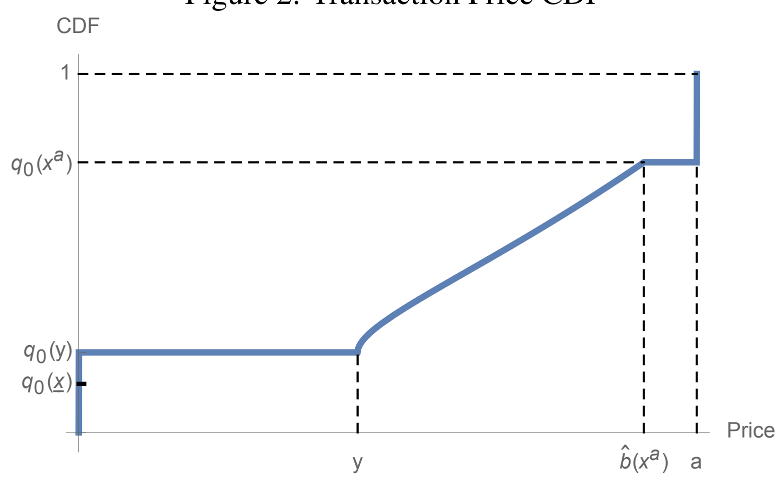
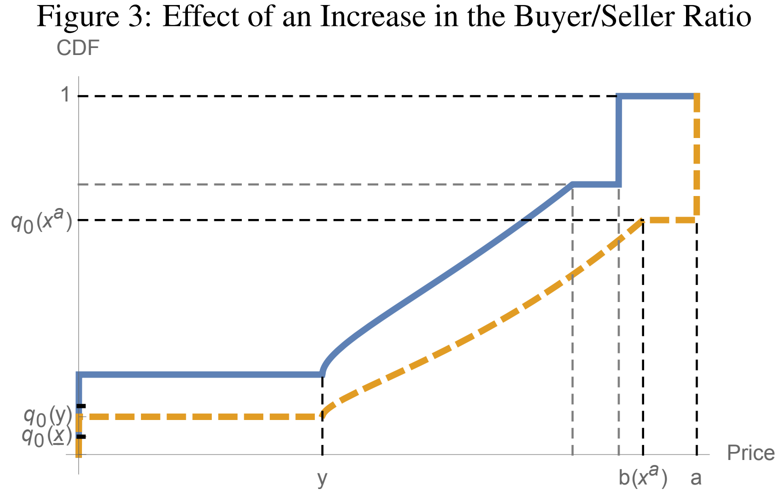
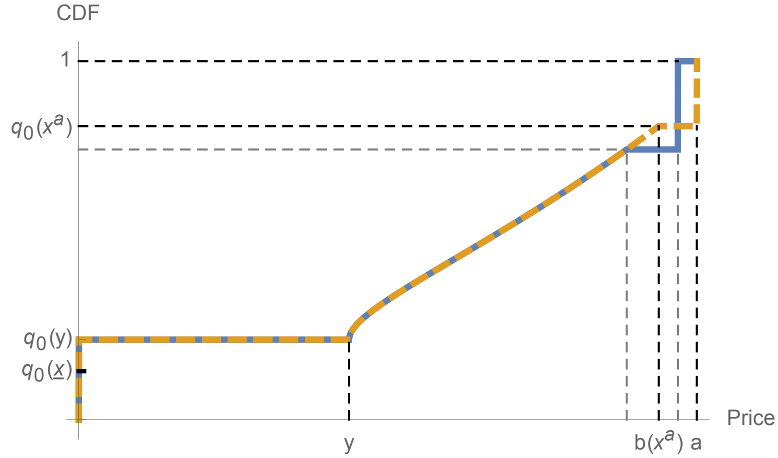


Figure 3: Effect of an Increase in the Buyer/Seller Ratio



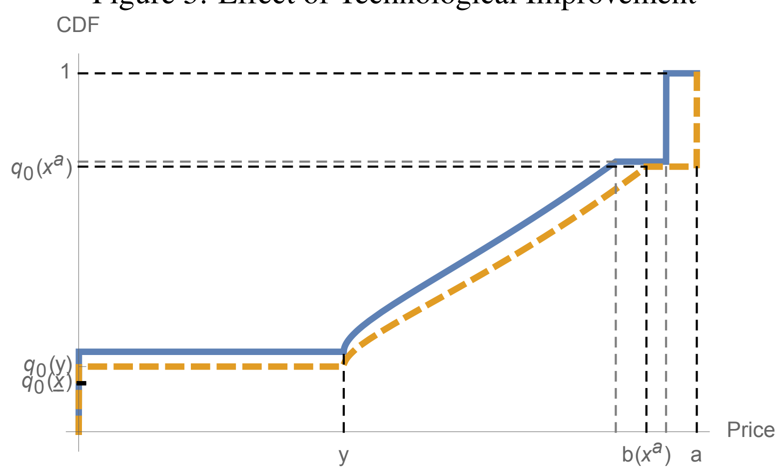
solid blue line = original equilibrium; dashed orange line = equilibrium with higher Λ.

Figure 4: Effect of a Decrease in the Inspection Cost



solid blue line = original equilibrium; dashed orange line = equilibrium with lower k.

Figure 5: Effect of Technological Improvement



solid blue line = original equilibrium; dashed orange line = equilibrium with more information.

Figure 6: Transaction Prices Above the Asking Price