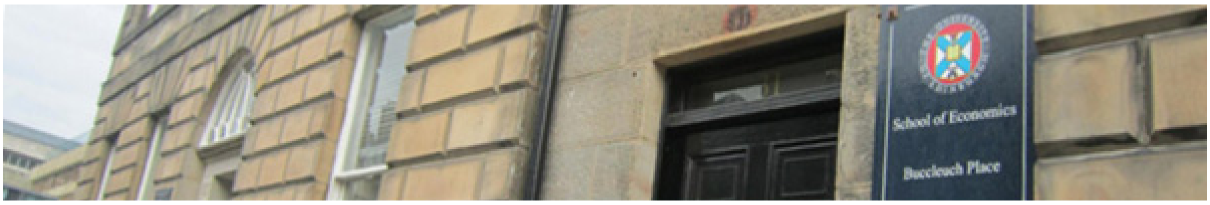

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Bertrand and the long run*

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August 11, 2014

Abstract

We propose a new model of simultaneous price competition, based on firms offering personalized prices to consumers. In a market for a homogeneous good and decreasing returns, the unique equilibrium leads to a uniform price equal to the marginal cost of each firm, at their share of the market clearing quantity. Using this result for the short-run competition, we then investigate the long-run investment decisions of the firms. While there is underinvestment, the overall outcome is more competitive than the Cournot model competition. Moreover, as the number of firms grows we approach the competitive long-run outcome.

Keywords: price competition, personalized prices, marginal cost pricing

JEL numbers: D43, L13

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1 Introduction

In this paper we take a fresh look at markets where the firms compete in prices to attract consumers. This is an elemental topic of industrial organization that has been thoroughly investigated, ever since the original contribution of Cournot (1838).[1] Our excuse for re-opening the case is that we offer a fundamentally new way of modelling price competition, which naturally leads to a unique equilibrium with price equal to (perhaps non constant) marginal cost. The innovation we propose is to allow the firms the option to personalize their prices. Note that we are NOT assuming that they can engage in first-degree price discrimination, as they need not know the valuation of each consumer. Nonetheless, as we explain below, the flexibility allowed by personalized pricing ensures that competition is cut-throat even in situations where attracting too much demand is harmful (because of increasing marginal costs).

In the remainder of this Introduction we give a brief overview of the most relevant literature in three subsections. We then present our model in detail. Section 3 derives the short-run equilibrium, while Section 4 looks at the long-run consequences. We conclude with a brief discussion of our results.

1.1 Deconstructing the Bertrand Paradox

Take the standard model of simultaneous price competition between two producers of a homogeneous good at constant and identical marginal cost, commonly referred to as the Bertrand duopoly. As it is well known, this model has a unique equilibrium, where both firms price at marginal/average cost, thereby earning zero profit. While the model itself seems realistic, the result is clearly not: even though there are only two competitors, they have no market power at all.[2] The literature has dealt with

[1] Yes, Antoine-Augustin Cournot, not Joseph Bertrand. While Cournot (1838) only discussed quantity competion for the more salient case of substitute goods, he did formalize price competition as well, for the case of perfect complements.

[2] As a result, if entry to the market is costly or there are fixed production costs – as we would expect in a duopoly – they have no incentive to enter. This sounds paradoxical: how can we have

this issue enriching the model, by including product differentiation, price-quantity bidding or dynamic competition. While the generalized models are useful in their own right, it is nonetheless conceptually relevant to note that actually nothing is amiss in the basic model.

Recall that if the technology is of increasing returns to scale – that is, average costs are decreasing in output – then we have a natural monopoly: there is room for only one firm in the market. The "paradoxical" situation with constant marginal cost is the limiting case of this, where two (or more) firms can "just" fit. When returns to scale are actually decreasing – that is, average costs are increasing and so marginal costs are above average cost – as we will discuss below in detail, firms do make positive profits in the Bertrand duopoly, despite still pricing at the cost of the marginal unit sold in equilibrium.

The immediate implication is that the seemingly innocuous "simplifying" assumption of constant returns actually leads to a non-generic, knife-edge situation, just between the cases where a duopoly can make profits or losses. Therefore, it should not come as a surprise that constant returns lead to zero profits in oligopoly: there is no paradox. Thus, we need not modify the strategic aspects of the original game to endow the firms with market power: it suffices to observe that marginal costs are likely to be increasing in oligopoly.

1.2 A detour

The traditional approach toward the resolution of the Bertrand Paradox – pioneered by Edgeworth (1897) – has been to allow firms to choose the quantity they are willing to sell at the price they set. In its pure form, this leads to an Edgeworth Cycle, or, in modern parlance, a mixed strategy equilibrium (c.f. Levitan and Shubik, 1972): Even if the equilibrium is unique, the range of prices offered are large[3] and

an operating market where firms cannot recoup their investments?

[3]For example, when demand is $Q = 1 - p$, and cost is quantity squared, with the proportional rationing rule proposed by Edgeworth, prices would oscillate between 1/2 (the competitive price) and 2/3.

the two firms generically set different prices. Allowing firms to set supply functions (complete quantity-price schedules) does not eliminate severe multiplicity either (c.f. Klemperer and Meyer, 1989).

Building on the insights gained from the analysis of Bertrand competition with capacity constraints by Levitan and Shubik (1972), Kreps and Scheinkman (1983) constructed a two-stage model where firms first commit to capacity levels (or simply, produce prior to the realization of demand) and then price competition follows. The remarkable outcome is that in the unique sub-game perfect equilibrium prices and quantities (produced and sold) are the same as those that would result in a one-shot Cournot competition. Unfortunately, Davidson and Deneckere (1986) showed that this result is not robust to the choice of rationing rule: Kreps and Scheinkman used the "efficient" or "surplus-maximizing" rule, where the demand is served starting from the highest valuation buyer. As this rule results in the most pessimistic residual demand curve for a firm with the higher price, for any other rule the outcome is more competitive than the Cournot equilibrium.

Looking at competition from the long-run perspective is indeed insightful and it is the main contribution of Kreps and Scheinkman (1983). However, restricting the "fixed factor" to be a choice of capacity is not only unnecessarily restrictive but it is also somewhat misleading. The latter weakness comes from the undue prominence capacity choice gives to rationing. Allowing for the chosen cost curve to be smooth, avoids rationing altogether as the firms are able to supply – within reasonable limits, see below – the entire demand, even if they wished not to.[4]

1.3 Decreasing returns

Let us re-examine the Bertrand duopoly under diseconomies of scale. As shown by Dastidar (1995), increasing marginal costs are not the panacea either as they lead to multiple equilibria. There exists a range of prices, such that if a firm charges one

[4] Boccard and Wauthy (2000/2004) look at an extension of Kreps and Scheinkman (1983) where the capacity can be *voluntarily* exceeded, at a linear cost.

of them the other's best response is to charge the same price.[5] Denoting demand by $D(.)$ and cost by $C(.)$, the lowest equilibrium price, $\underline{p}$, is where the sellers splitting the demand[6] just break even: $\underline{p}D(\underline{p}) = 2C(D(\underline{p})/2)$. The highest one, $\overline{p}$, is where serving the entire demand gives the same profit as splitting it: $\overline{p}D(\overline{p}) - C(D(\overline{p})) = \overline{p}D(\overline{p})/2 - C(D(\overline{p})/2)$. The reason for this plethora of equilibria lies with the obligation of a deviant firm to serve all comers at the announced price. With constant marginal costs this is not an issue. However, when marginal costs are increasing, selling to the entire market – what happens if a firm undercuts its competitor – may not be an advantageous proposition. Raising the price above the competitor's does not pay either as the residual demand is nil. Thus, with deviations discouraged, equilibria thrive.

In order to regain a unique equilibrium price, Dixon (1992)[7] introduced a modified Bertrand-Edgeworth game, where together with their price the firms also announce a maximum quantity they are willing to sell at it. This "trick" resolves Dastidar's problem that downward deviations are too costly, and by having firms commit to supply – if needed – more than their share in the competitive equilibrium, it removes the incentive for rivals to increase their price above the competitive one (residual demand is zero), thus destroying the Edgeworth Cycle.

In this paper, we wish to enrich Dixon's model by allowing the firms to make a personalized price offer to each consumer. There are a number of reasons for proposing this.

- Firstly, in some applications – especially Internet commerce, where via cookies sellers can price discriminate – the option of posting personalized prices is more realistic.
- Second, from the game-theoretic point of view, our strategies are (nearly) a

[5]The indeterminacy of this result is rather severe. For instance, if demand is $Q = 1 - p$, and cost is quantity squared, the lowest and highest equlibrium prices are $\underline{p} = \frac{1}{3}$ and $\overline{p} = \frac{3}{5}$. The monopoly price would be $\frac{3}{4}$, the Cournot price $\frac{3}{5}$ (it is just a coincidence that it equals $\overline{p}$).

[6]Dastidar assumes equal sharing of the demand for firms charging the same price.

[7]See Allen and Hellwig (1986) as well.

generalization of Dixon's. If we restrict attention to price schedules that take only two values, a sufficiently high one, at which no one buys, and an "interior" price, then such price schedules are equivalent to a single price and a maximum quantity.[8] Thus, Theorem 1 below shows that the larger strategy set does not lead to a different equilibrium price (and neither does it destroy the existence of a deterministic equilibrium price), while it also implies Dixon's result.

- Third, from a conceptual point of view, we feel that despite being a semi-generalization of Dixon's model, ours is closer in spirit to "pure" price competition, as quantities are not explicitly mentioned, and no consumer faces the risk of being rationed.

- Fourth, our model leads to a decentralized implementation, where each consumer decides individually which price to accept in equilibrium, so there is no need to appeal to sharing rules[9] and to an "invisible hand" clearing the market.

- Finally, as our firms do not have finite capacities (self-imposed or otherwise) – for a high enough cost they can satisfy the market demand – our equilibrium is not hostage to an exogenous choice of rationing rule.

2 A model of price competition

Specifically, we assume that there is a set Ω of N producers, indexed by $J = 1, 2, ..., N$, with increasing, strictly convex, twice differentiable costs functions, $C_J(q)$, with $C(0) = 0$, and there is a unit mass of consumers, indexed by $i \in [0, 1]$. Each consumer's valuation, $v \in [0, 1]$, is an i.i.d. draw from the strictly increasing and

[8] Except that the quantity has the names of a subset of consumers on it, which only enriches the set of possible outcomes.

[9] Dixon (1992) assumes equal sharing, though he also assumes that all firms have the same cost function. In fact, it is straightforward to see from the proofs of his Lemmas 1 and 2 that with asymmetric costs and equal sharing, his model generically has no equilibrium. To regain existence the sharing rule must be in proportion of competitive supply, see below.

continuous probability distribution function $F(v)$, resulting in a deterministic aggregate demand function $D(p) = 1 - F(p) : [0,1] \to [0,1]$. We assume that firms do not observe individual buyer valuations (however, see Remark 2 below). Firms simultaneously set a (Lebesgue measurable) price schedule, $P_J(i)$ each, which assigns to each consumer a (personalized) price. Consumers observe their N offers and accept at most one of them. As long as it is feasible, firms are committed to satisfy the demand of each consumer that accepts their offer (see Remark 1 below). Outside options are normalized to zero.

We denote the inverse of firm J's marginal cost function – its price-taking supply function – by $S_J(p)$, and define the competitive price, p^*, as the price that equates aggregate supply with demand : $D(p^*) = \sum_{J=1}^{N} S_J(p^*)$.

The following corollary shows that p^* is a selection from Dastidar's interval.[10] The corollary also highlights a key observation that is useful in comprehending the intuition for our main theorem below: price is below (above) the marginal cost – that would result by selling the competitive share of the demand at that price – if and only if it is less (more) than p^*. That is, there is a pressure to move towards the competitive price from any symmetric hypothetical equilibrium. To simplify matters, we make the following regularity assumption, which basically requires that a firm's residual demand according to the Vives Rule is non-increasing in the price (the second derivatives of the cost functions are not too different):

[10] The exact description of the latter (especially its lower bound), depends on whether there are fixed costs of production (for simplicity we assume not: $C(0) = 0$) and on the rule according to which firms charging the same (lowest) price split demand. For consistency with our endogenously derived result below, we adopt the assumption made by Vives (1999) – see Dastidar (1997) as well – that they split in proportion to their price taking supply ($S_J(p)$): firm J's share as a proportion of the aggregate output if firms in Γ set the lowest price (p) is $\alpha_J(p;\Gamma) = \frac{S_J(p)}{\sum_{K\in\Gamma} S_K(p)}$. With this assumption, the result is straightforward (see Vives' note 7 in Chapter 5), as – by construction – in equilibrium all firms must produce.

An interesting alternative assumption – as it reduces the complexity of the consumers' strategies – could be to assume that the split is according to the competitive supplies at p^* no matter what the price is: $\alpha_J(p;\Gamma) = \frac{S_J(p^*)}{\sum_{K\in\Gamma} S_K(p^*)}$. In this case, as well as with fixed costs, it can happen that in the lower price equilibria some of the firms are excluded from production.

Assumption 1 *For all* J, p *and* Γ, $\frac{d[\alpha_J(p;\Gamma)\cdot D(p)]}{dp} \leq 0$. *Equivalently,* $\frac{d\alpha_J}{dp} \cdot \frac{p}{\alpha_J} \leq -\frac{dD}{dp} \cdot \frac{p}{D(p)}$, *or* $\varepsilon(S_J) - \varepsilon(\Sigma_{K\in\Gamma} S_K) \leq -\varepsilon(D)$.

Corollary 1 *The competitive price is in the Dastidar interval:* $\underline{p} < p^* < \overline{p}$. *Moreover, if Assumption 1 is satisfied, then for every* J, $p >=< p^* \Longleftrightarrow p >=< C'_J(\alpha_J(p,\Omega)\cdot D(p))$.

Proof. The lowest Dastidar equilibrium price is the lowest commonly charged price where all firms make non-negative profits. As at the competitive price they all have the same marginal cost (p^*) which is above average cost (as $C(0) = 0$ and $C'' > 0$) $\underline{p} < p^*$. The highest Dastidar price is the highest commonly charged price at which no firm would prefer to serve all the demand. As at the competitive price they all charge at marginal cost, any additional amount sold would decrease their profits, implying that $p^* < \overline{p}$. To see the second observation, note that by Assumption 1 and the convexity of $C(.)$, $C'_J(\alpha_J(p;\Omega)D(p))$ is a curve non-increasing in p. Thus, it crosses the (strictly increasing) line p from above at p^*. Consequently, when $p >=< p^*$, $p >=< C'(\alpha_J(p;\Omega)D(p))$. ■

3 The short-run equilibrium

The main result of this section is that – assuming that it is feasible for any $N-1$ firms to serve the demand at p^* – our decentralized price setting mechanism leads to the competitive outcome.

Before presenting this, a technical point. When a continuum of agents each ramdomize over a common finite set of actions, there is no guarantee that the set of agents that choose certain action (in this case, accepting trading with seller J) is measurable. If such set is not measurable, then payoffs and best responses cannot be defined. In order to avoid what is but a technical issue, we will consider only equilibria where this indeterminacy is not an issue.[11]

[11] In fact, the concept of "equilibrium" implicitly requires measurability of outcomes.

Thus, let $\mathbf{P} = (P_1, P_2, ..., P_N)$ represent the vector of sellers' strategies. Also, let σ_i represent consumer i's (mixed strategy), where $\sigma_i = (\sigma_i^1, \sigma_i^2, ..., \sigma_i^N)$ and $\sigma_i^J : \mathbb{R}^N \to [0,1]$ with $\sum_J \leq 1$ represent consumer i's probability of accepting J's offer given the price offers $P_J(i)$ for $J = 1, 2, ..., N$. We can finally represent by $\boldsymbol{\sigma}$ the strategies of all consumers, and define

$$\mu_J(i; \mathbf{P}, \boldsymbol{\sigma}) = \sigma_i^J (P_1(i), P_2(i), ..., P_N(i)).$$

as the probability that consumer i accepts seller J's offer when sellers and consumers, respectively, use the strategy profile $(\mathbf{P}, \boldsymbol{\sigma})$. We say that the outcome of $(\mathbf{P}, \boldsymbol{\sigma})$ is *measurable* if for each J, $\mu_J(i; \mathbf{P}, \boldsymbol{\sigma})$ is (Lebesgue) measurable in i. Equivalently, if

$$\int_i \mu_J(i; \mathbf{P}, \boldsymbol{\sigma}) di$$

exists for all J.

Theorem 1 *As long as $C_K \left(\frac{S_K(p^*)D(p^*)}{D(p^*)-S_J(p^*)} \right) < \infty$ for all $K \neq J$,*[12] *the unique measurable equilibrium outcome in pure price schedules is such that all trades are at the competitive price and firms sell in proportion to their competitive supply: firm J's offer of p^* is accepted by a measure $S_J(p^*)$ of consumers.*

The proof is in the Appendix.

Remark 1 *The symmetric equilibrium strategies involve offering the good for the competitive price to all consumers (who then use a mixed strategy of acceptance in proportion to the firms' competitive supply). Thanks to the Law of Large Numbers, this is not any more serious an issue than in Dixon's model, as a unilateral deviation can only move the demand from $S_K(p^*)$ to $\frac{S_K(p^*)D(p^*)}{D(p^*)-S_J(p^*)}$, which by assumption (both his and ours) is still feasible. In practical terms, following a deviation by a competitor a firm would prefer to ration consumers. There must be either sufficient reputational concerns or consumer protection regulation in place to ensure compliance.*[13]

[12] One can replace infinity by any other number that determines the limit of feasibility (like the bankruptcy constraint in Dixon, 1992).

[13] This may involve a substitute good or a "rain check". The crucial assumption is that a consumer who has accepted an offer no longer has unsatisfied demand in the market.

Remark 2 *Note that, if we assume that consumer valuations are observable, our mechanism allows firms to perfectly price discriminate. Theorem 1 would still apply: marginal cost pricing continues to be the equilibrium outcome, so competition drives out price discrimination. Unlike in the case of monopoly, the lack/presence of the ability to price discriminate has no efficiency consequences.*[14]

4 The long run

Now that we have a unique prediction for the outcome of Bertrand competition in the short run, we can turn to the question of the choice of – or investment in – productive technology, which was considered to be fixed in the short run.[15]

We assume that all firms have access to the same meta-technology, described by the differentiable, sub-additive production function $f(K, L)$ which satisfies $f_K, f_L > 0$, $f_{LL}, f_{KK} \leq 0$, and $f_{KL} > 0$. Here K, say capital, priced at r, is considered to be the fixed factor while L, say labor, priced at w, is the short-run decision variable. When K is fixed, the production function results in a cost function $C^{SR} = wL(q; K)$, where $L(q; K)$ is the short-run input demand for L implicitly defined by

$$f(K, L(q; K)) = q. \tag{1}$$

Differentiating both sides with respect to q we obtain $f_L \cdot L'(q) = 1$, implying that $MC^{SR}(q) = wL'(q) = \frac{w}{f_L}$. Differentiating with respect to K, we have that – as required – at any given quantity, marginal cost is reduced by investment:[16]

$$\frac{\partial MC^{SR}(q)}{\partial K} = -\frac{f_{KL} \cdot w}{f_L^2} < 0. \tag{2}$$

[14] This is in contrast to Armstrong and Vickers (1993) but in line with Holmes (1989) and Stole (2007).

[15] Cabon-Dhersin and Drouhin (2014) look at a similar two-stage model, but they use Dastidar's (1995) model of price competition in the second stage, which they then refine by selecting the collusive (highest price) equilibrium.

[16] If we differentiate with respect to q and L, we can also verify that the cost function is indeed convex, as assumed in the previous section.

Thus, firms have an incentive to sink capital into their technology. We model the long-run competition as follows: In a first stage, firms simultaneously choose their "fixed" inputs, K_J, $J = 1, 2, ..., N$, in anticipation that in a second stage this will be followed by the play of the "Bertrand equilibrium" identified in the previous section.

Given the choices of all other firms, $\mathbf{K}_{-J}$, firm J's best response solves

$$\max_{K_J} \left\{ p^*(\mathbf{K}_{-J}, K_J) S_J(p^*(\mathbf{K}_{-J}, K_J)) - C^{SR}(S_J(p^*(\mathbf{K}_{-J}, K_J)); K_J) - rK_J \right\}. \tag{3}$$

The first-order condition for this maximization problem is

$$\frac{\partial p^*(\mathbf{K}_{-J}, K_J)}{\partial K_J} \left[S_J(p^*) + \left(p^* - MC^{SR}\left(S_J(p^*); K_J \right) \right) S_J'(p^*) \right] = r + \frac{\partial C^{SR}\left(S_J(p^*); K_J \right)}{\partial K_J}. \tag{4}$$

As we have discussed in the previous section, $p^* = MC^{SR}(S_J(p^*); K_J)$, so the above condition simplifies to

$$\frac{\partial p^*(\mathbf{K}_{-J}, K_J)}{\partial K_J} \cdot S_J(p^*) = r + \frac{\partial C^{SR}\left(S_J(p^*); K_J \right)}{\partial K_J}. \tag{5}$$

This leads to the following immediate result.

Proposition 1 *In equilibrium all firms will underinvest, not only relative to the first best but even conditional on their equilibrium output.*

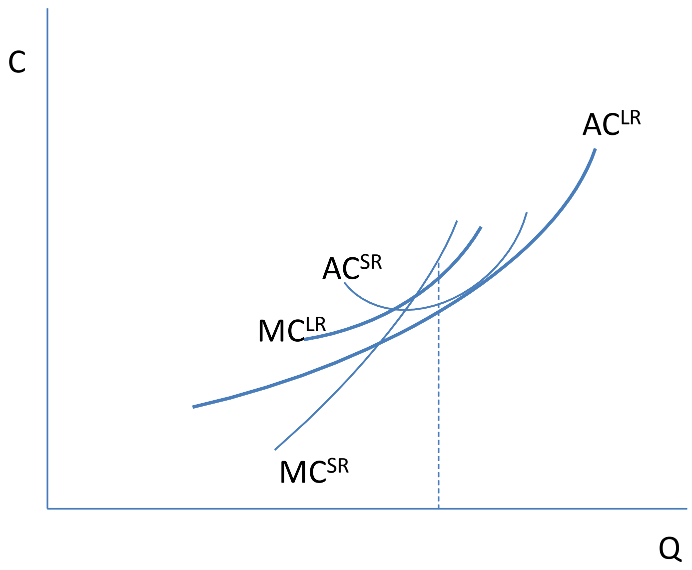
Proof. Note that for a given quantity-price pair, at the cost minimizing mix of K and L, $-\frac{\partial C^{SR}}{\partial K_J} = r$. Thus, at any efficient mix of inputs, the left-hand side of (5) has the same sign as $\frac{\partial p^*}{\partial K_J}$. We will show that $\frac{\partial p^*}{\partial K_J} < 0$, implying that in equilibrium $0 > r + \frac{\partial C^{SR}}{\partial K_J}$. In other words, an extra unit of capital would decrease short-term costs by more than its price, just as claimed in the proposition.

Recall that in short-run equilibrium $\sum_{I \neq J} S_I(p^*) = D(p^*) - S_J(p^*)$. Totally differentiating both sides with respect to K_J we obtain

$$\sum_{I \neq J} \frac{\partial S_I(p^*)}{\partial p} \cdot \frac{dp^*}{dK_J} = D'(p^*) \cdot \frac{dp^*}{dK_J} - \frac{\partial S_J(p^*)}{\partial p} \cdot \frac{dp^*}{dK_J} - \frac{\partial S_J(p^*)}{\partial K_J}. \tag{6}$$

Solving for $\frac{dp^*}{dK_J}$ we have

$$\frac{dp^*}{dK_J} = \frac{\frac{\partial S_J(p^*)}{\partial K_J}}{D'(p^*) - \sum_I \frac{\partial S_I(p^*)}{\partial p}}. \tag{7}$$



By (2) $\frac{\partial S_J(p^*)}{\partial K_J} > 0$. Moreover, we have established already that marginal costs are increasing and thus $\frac{\partial S_I(p^*)}{\partial p} > 0$ for all firms. The fact that demand is downward sloping completes the proof. ■

Note that Proposition 1 points to an effect beyond the hold-up problem. It is not that firms restrict investment because they will not reap its full benefit. Rather, there is a market power effect: taking into account that the final price decreases in their investment, the firms have an additional reason to invest too little in "capital".

Note that this implies that the short-run marginal cost is strictly larger than the long-run marginal cost for the equilibrium level of output. (See graph.)

That is, even though the price equals the short-run marginal cost, the equilibrium is not efficient: the price is larger than the long-run marginal cost (market power effect) and firms do not minimize costs (cost inefficiency).

4.1 Cournot or not, revisited

We can now check what our two-stage model has to say in the discussion of whether the (long-run) Cournot model is a good description of a market where firms first take decisions that affect output, and which they take as given when they set their prices. In the standard Cournot model, when firms choose their inputs, in particular their level of K, they do so taking as given not only their competitors' (conjectured) level of K but also their (conjectured) level of output. We could equivalently formalize this problem in two steps: firms choose their level of K, and then, without observing anything else, they choose their level of L and so their output. This emphasizes that the only difference between the Cournot model and our two-stage model is that, when choosing K, as in (3), in the Cournot model firms would conjecture that – since choices are not observed – their choice of capital does not directly affect their competitors' quantity choices: $\frac{dq_I}{dK_J} = 0$, and so $\frac{\partial Q}{\partial K_J} = \frac{\partial q_J}{\partial K_J}$. This means that in (6) the left-hand side $-\sum_{I \neq J} \frac{dS_I(p^*)}{dK_J}-$ is zero, so that in terms of price responses this is equivalent to conjecturing that $\frac{dp^*}{dK_J} = \frac{\frac{\partial S_J(p^*)}{\partial K_J}}{D'(p^*) - \frac{\partial S_J(p^*)}{\partial K_J}}$ instead of $\frac{\frac{\partial S_J(p^*)}{\partial K_J}}{D'(p^*) - \sum_I \frac{\partial S_I(p^*)}{\partial p}}$, and thereby to "overestimating" the (negative) effect of investment on the market price.

Now, return to our two-step model, and assume for a moment that firm J makes exactly the same conjecture, i.e., that $\frac{dq_I}{dK_J} = 0$, while q_I , $I \neq J$, are fixed at the long-run Cournot output. Obviously, in that case, firm J would also choose the Cournot output *and* the efficient mix of inputs by solving (4). However, compensating for the overestimate of the negative effect of investment, the left-hand side of (4) becomes positive, showing that at that level K_J is too low. Thus, in our equilibrium K_J is larger than in the long-run Cournot solution. As the short-run Cournot production is increasing in K, even a second-stage Cournot competition would lead to higher production. Finally, it is easy to see that the short-run competitive output is always higher than the short-run Cournot one. Thus we have shown that

Proposition 2 *In the two-stage long-run equilibrium with price competition, production is higher than in the one-stage long-run equilibrium with quantity setting.*

Proposition 2 means that the Cournot output is an overestimation of the market power that oligopolistic firms enjoy. This is consistent with Davidson and Deneckere's (1986) critique of Kreps and Scheinkman's (1983) rendering of first long-run quantity, and then short-run price competition. However, our result is not based on the plausibility of one or another rationing rule, but rather on a basic strategic interaction taking into account input substitutability. Once we depart from the (implicit, in Kreps and Scheinkman, 1983, and Davidson and Deneckere, 1986) assumption of a fixed proportion production function, firms take into account how their input decisions affect those of their rivals, and this is what makes them behave more aggressively than predicted by the Cournot model.

4.2 Discussion

Price competition should lead to marginal cost pricing even when firms enjoy market power. Price competition is perhaps the best description of market behavior in the short run, and so we should expect that the price is indeed close to the marginal cost of firms. However, we have been familiar with the distinction between long and short run since the days of our first college studies of Microeconomics. Certain decisions, input decisions in particular, are mostly taken as given, when prices are chosen, as Kreps and Scheinkman (1983) argued. Fixed production factors typically result in decreasing returns even when the technology is constant returns in the long run. Thus, marginal cost pricing and extraordinary profits are compatible. What is important to understand is not so much the difference between price and (short-run) marginal cost, but the incentives for the choice of levels and mix of inputs – and as a result, the level of output – arising from the strategic considerations present when firms do have market power: when firms' decisions affect market output and price.

This is the main message of this paper. We have shown how these strategic considerations typically lead to both an inefficient mix of inputs, with long-run decisions resulting in too low levels of these long-run determined inputs; and result in prices above long-run marginal cost.

We have also shown that, from a long-run point of view, and as argued by Davidson and Deneckere (1986), (short-run) price competition results in more output than predicted by the Cournot model. According to our analysis, the discrepancy comes from the strategic interaction between the long-run decisions of the different firms. When a firm determines its own short-run cost function by investing in the long-run factor of production, it takes into account how these decisions will affect future output decisions of rivals. A lower short-run marginal cost will be answered by rivals with a reduction in their own output. Thus, investments in these production factors have a lower impact on prices than what is predicted by the Cournot model. The result is a stronger incentive on short-run cost reduction and therefore, a larger output.

Despite this stronger incentive to invest in the long-run factor, the equilibrium input mix shows inefficiently low levels of it. As we have shown, this is associated with the effect of the long-run factor on prices, and is a well-understood phenomenon in price competition: at the cost minimizing mix of inputs, a small reduction in the use of the long-run input increases the equilibrium price. When marginal units are sold at marginal (short-run) cost, this effect dominates the second order effect on cost minimization.

Both the departure from the efficient mix of inputs and the departure from long-run marginal cost pricing are, therefore, consequences of market power. Indeed, from (2) if symmetric firms behave symmetrically and there are N active firms in the market, $\left|\frac{\partial p^*}{\partial K_J}\right| < \frac{1}{N-1}$. Thus, as N gets large $\frac{\partial p^*}{\partial K_J}$ approaches zero and, by (5), the input mix approaches efficiency. Moreover, market clearing (and efficient input mix) implies output per firm approaching 0, at which point long-run marginal cost equals short run marginal cost (and then price).[17]

[17] This is not an artifact of our assumption of always increasing average cost, and so marginal cost of 0 at $q = 0$. Indeed, assume more standard, "U-shaped" average cost in the long run, and define the minimum efficient scale

$$q^* = \arg\min_q \{\frac{C(q)}{q}\}.$$

Let $p^* = \frac{C(q^*)}{q^*}$, i.e., the average cost at that level of output. If

$$N^* = \frac{D(p^*)}{q^*}$$

is large, as N approaches N^*, market clearing and efficient input mix implies output per firm approaching q^*, at which point, again, long run marginal cost equals short term marginal cost and so price.

5 Concluding remarks

In this paper we have presented a novel way of modelling price competition, which leads to marginal cost pricing – but positive profits – as the unique equilibrium, without the need to specify rationing (when demand exceeds supply) or sharing (when supply exceeds demand) rules. It should therefore be a useful off-the-shelf workhorse model to embed in more complex scenarios.

We have also developed the most direct implications in a set-up with long-run competition, underlining the consequences of market power as inefficient investments in the fixed factor. This analysis has also shed more light on the literature on two-stage Bertrand-Edgeworth competition.

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6 Appendix

Proof. (of Theorem 1) First, we show that charging p^* to (almost) everyone is indeed an equilibrium. Suppose that consumers use a mixed strategy of acceptance such that if they receive the lowest offer, p, from the firms in Γ, they probabilistically accept them in proportion of the firms' competitive supplies at p: they accept firm J's offer with probability $\alpha_J(p;\Gamma)$.

Assume all firms but J make a price offer p^* to all consumers, and consider the best response of firm J: $P_J(.)$. Let Q_1 be the (Lebesgue) measure of the set $\{i : P_J(i) < p^*\}$ and Q_2 be the measure of the set $\{i : P_J(i) = p^*\}$. Then, the profits of firm J are not larger than $p^*(Q_1+\alpha_J(p^*;\Omega)Q_2) - C_J(Q_1 + \alpha_J(p^*;\Omega)Q_2)$. Indeed, a mass of consumers Q_1 accept J's offer of a price below p^*, and a mass of consumers Q_2 receive N offers of p^*, and proportion[18] $\alpha_J(p^*;\Omega)$ of these accept firm J's offer. The rest of consumers receive better offers than firm J's offer to them so – since, by assumption, the other firms can satisfy all the demand at p^* – they do not buy from it. Now, note that $Q = \alpha_J(p^*;\Omega)D(p^*) = S_J(p^*)$ solves

$$\max_Q p^*Q - C_J(Q),$$

and therefore, $p^*(Q_1+\alpha_J(p^*;\Omega)Q_2) - C_J(Q_1 + \alpha_J(p^*;\Omega)Q_2) \le p^*S_J(p^*) - C_J(S_J(p^*))$. Finally, observe that by using the price schedule $P_J(i) \equiv p^*$, firm J sells exactly $S_J(p^*)$. Therefore, $P_J(i) \equiv p^*$ is indeed a best response. Finally, as the consumers are indifferent, they are clearly happy mixing in the prescribed proportions. Note that the $\mu_J(i;\mathbf{P},\boldsymbol{\sigma})$ is indeed measurable.

We now show that there exists no other measurable equilibrium outcome with pure strategy price schedules. Assume the Law of Large Numbers is satisfied for a continuum -in the index i- of independent random variables -on J- with bounded variance, so that the quantity that firm J sells in the proposed equilibrium is $q_J(\mathbf{P},\boldsymbol{\sigma}) = \int \mu_J(i;\mathbf{P},\boldsymbol{\sigma})di$ almost surely.

Note that for $\mathbf{P}$ to be part of an equilibrium it has to be that $P_J(i) \ge C'_J(q_J(\mathbf{P},\boldsymbol{\sigma}))$ for almost all i such that $\mu_J(i;\mathbf{P},\boldsymbol{\sigma}) > \varepsilon$, for all $\varepsilon > 0$. Indeed, otherwise firm J

[18] By the Law of Large Numbers this proportion is deterministic.

could profit by increasing her offer (up to, say, $P_J(i) = 1$) to a positive measure of these consumers so as not to sell to them. Also, note that $q_J(\mathbf{P}, \boldsymbol{\sigma}) > 0$, if $(\mathbf{P}, \boldsymbol{\sigma})$ is an equilibrium. Indeed, consider a small δ. Since the marginal cost is increasing, there could be no more than $(N-1)\delta$ consumers that receive a price offer below $\min_{J' \neq J} C'_{J'}(\varepsilon) > C'_J(0) = 0$: some producer J' would be selling units below marginal cost and so would profit from withdrawing the corresponding offers. Thus, there are at least $D(\min_{J' \neq J} C'_{J'}(\delta)) - (N-1)\delta$ that are willing to pay $\min_{J' \neq J} C'_{J'}(\delta) > C'_J(0)$ and either don't buy or buy at larger prices. Thus, if $q_J(\mathbf{P}, \boldsymbol{\sigma}) = 0$, J could gain by offering a small measure of those consumes a price $\min_{J' \neq J} C'_{J'}(\delta)$.

Next, observe that $C'_J(q_J(\mathbf{P}, \boldsymbol{\sigma})) = C'_K(q_K(\mathbf{P}, \boldsymbol{\sigma}))$ for all J, K. Otherwise, if $C'_J(q_J(\mathbf{P}, \boldsymbol{\sigma})) > C'_K(q_K(\mathbf{P}, \boldsymbol{\sigma}))$ then firm K could profit by deviating and making a (unique winning) offer $P_J(i) - \delta$ to some arbitrarily small but positive measure υ of consumers i such that $\mu_J(i; \mathbf{P}, \boldsymbol{\sigma}) > \varepsilon$ for some (perhaps very small) $\varepsilon > 0$, for some δ satisfying $(P_J(i) - \delta \geq\ )\ C'_J(q_J(\mathbf{P}, \boldsymbol{\sigma})) - \delta > C'_K(q_K(\mathbf{P}, \boldsymbol{\sigma}) + \upsilon)$.

Next, we show that, for all $\varepsilon > 0$, $P_J(i) = C'_J(q_J(\mathbf{P}, \boldsymbol{\sigma}))$ for all J, and for almost all i such that $\mu_J(i; \mathbf{P}, \boldsymbol{\sigma}) > \varepsilon$. Indeed, if $P_J(i) > C'_J(q_J(\mathbf{P}, \boldsymbol{\sigma})) = C'_K(q_K(\mathbf{P}, \boldsymbol{\sigma}))$ for a positive measure of i such that $\mu_J(i; \mathbf{P}, \boldsymbol{\sigma}) > \varepsilon$, then firm K could profit by reducing her price offer to $P_J(i) - \delta$, to a small but positive measure of these consumers and for a small enough δ. That would increase the sales of firm K by a positive measure, at a price above its marginal cost. It follows that in any equilibrium almost all consumers must buy at the same price in equilibrium. We have left to show that this price must be the competitive price. That is, we need to show that the total sales must be equal to the demand at the price common to all transactions. It cannot be larger, since then some consumers would be buying at a price larger than their willingness to pay. It cannot be smaller either. Indeed, in such a case a positive measure of consumers with willingness to pay higher than $p = C'_J(q_J(\mathbf{P}, \boldsymbol{\sigma}))$ would not buy. Some firm could profit by deviating and offering to a small measure of them a price equal to their willingness to pay (and above its marginal cost). ■