

1. Introduction

This paper is an investigation into the dynamics of asset markets with adverse selection à la Akerlof (1970).¹ In particular, I ask whether market failure at some later date might precipitate market failure at an earlier date. The answer I find is there can indeed be “contagious illiquidity” from the future back to the present.

The mechanism works as follows. If the market is expected to break down in the future, then agents holding assets they know to be lemons (assets with low returns) will be forced to hold them for longer – they cannot quickly resell them. As a result, the *effective* difference in payoff between a lemon and a good asset is greater. But we know from the static Akerlof model that the greater the payoff differential between lemons and non-lemons, the more likely is the market to break down. Hence market failure in the future is more likely to lead to market failure today.

Conversely, if the market is *not* anticipated to break down in the future, assets can be readily sold and hence an agent discovering that his or her asset is a lemon can quickly jettison it. In effect, there is little difference in payoff between a lemon and a good asset. The logic of the static Akerlof model then runs the other way: the small payoff differential is unlikely to lead to market breakdown today.

Notice that, although this may a first glance appear like a story of multiple equilibria, at heart it isn't. The conclusion is simply that the nature of today's market – liquid or illiquid – hinges critically on the nature of tomorrow's market, which in turn depends on the next day's, and so on. The tail wags the dog. The core of the paper, in section 3, is a dynamic, *finite horizon* model of an asset market with a *unique* equilibrium, which is discontinuously sensitive to the underlying parameters. The market is either trading normally for the length of the horizon, or is completely broken down.

¹ Other recent papers on this topic include: Bolton, Santos and Scheinkman (2011); Chari, Shourideh and Zetlin-Jones (2010); Daley and Green (2012); Eisfeldt (2004); Eisfeldt and Rampini (2006); Guerrieri and Shimer (2013); Ivashina and Scharfstein (2010); Kurlat (2013); Malherbe (2010); Tirole (2012). For models of dynamic adverse selection in the context of durable goods, see, for example: Hendel and Lizzeri (1999); Hendel, Lizzeri, and Siniscalchi (2005).

That said, in the Appendix of the paper I present an *infinite horizon* model with *two* equilibria, one with trade at all dates, the other with no trade ever.

Before presenting the model, perhaps it will help if I present, in Section 2, a brief recapitulation of a rudimentary example of the static Akerlof model.

2. A rudimentary static Akerlof example

A single item is owned by a seller. There are many buyers. There is a second, divisible, good – which we call “income” and take as the numeraire. Let

$$\begin{aligned}\mu^s &= \text{seller's marginal utility of income} \\ \mu^b &= \text{buyer's marginal utility of income}\end{aligned}$$

where $\mu^s > \mu^b$. (For example, the seller may have an income shortfall.)

The item has 2 possible qualities: utility H or L, where $H > L \geq 0$. Only the seller knows the quality. Buyers know item has quality L with probability λ .

There are two possible equilibria: a high-price equilibrium or a low price equilibrium.

In the *high-price equilibrium*, both qualities are traded. The price \bar{p} satisfies a buyer's indifference condition:

$$\mu^b \bar{p} = (1 - \lambda)H + \lambda L$$

The seller with H-quality must want to trade:

$$\mu^s \bar{p} > H$$

That is, the high-price equilibrium exists iff

$$\lambda \left(1 - \frac{L}{H}\right) < \frac{\mu^s - \mu^b}{\mu^s}$$

If reverse inequality holds:

$$\lambda \left(1 - \frac{L}{H}\right) > \frac{\mu^s - \mu^b}{\mu^s}$$

then only L-quality is traded. In this *low-price equilibrium*, the price, \underline{p} say, satisfies

$$\mu^b \underline{p} = L$$

In limit case $L = 0$, $\underline{p} = 0$: effectively, there is complete market failure, without any trade.

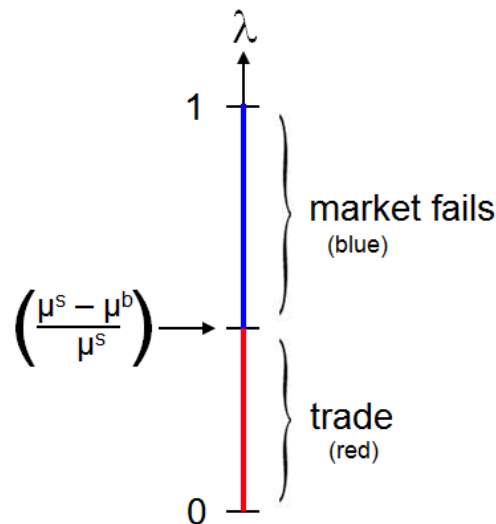
Hence the market “fails” iff

$$\lambda \left(1 - \frac{L}{H}\right) > \frac{\mu^s - \mu^b}{\mu^s}$$

We see that market failure is more likely if there is

- a greater fraction λ of “lemons”
- more percentage difference between H and L
- less percentage difference between the seller’s and the buyers’ marginal utilities of income

When $L = 0$: we can represent our conclusions in this diagram



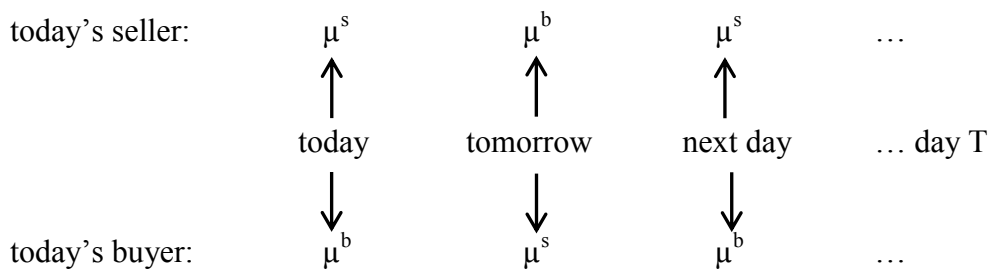
Note that it is tempting to think that in the red region, complete market failure must also be an equilibrium: zero price \Leftrightarrow zero quality. Might this be a theory of illiquidity? No. Although in the red region there exists two “Walrasian” equilibria – the parametric price is either high (\bar{p}) or zero – in fact only the high-price equilibrium is “Nash”, in the sense that

if agents actively make bids/offers, a buyer could profitably deviate from zero by bidding $\epsilon > 0$ below \bar{p} .

3. Contagion Through Time

The economy has discrete time with finite horizon: “days” $t = 1, 2, \dots, T, T+1$. There is a single consumption good, which is divisible and storable. In addition, there are assets, which pay off on day $T+1$.

Agents have a common overnight discount factor $\beta < 1$. Normalise everyone’s marginal utility of day $T+1$ consumption to equal 1. On previous days, agents alternate their marginal utilities ($\mu^s > \mu^b$):



We assume $\mu^b > \beta\mu^s$, implying that in equilibrium there is no storage.

We also assume, crucially, that intertemporal contracts cannot be written. This extreme form of contractual incompleteness implies that the only means agents have for intertemporal redistribution is by trading assets on days $1, 2, \dots, T$.

There are two types of asset, H and L, with a fraction λ of type L. Type H pays an amount $V > 0$ on day $T+1$, but nothing before then. Type L pays zero: a lemon.

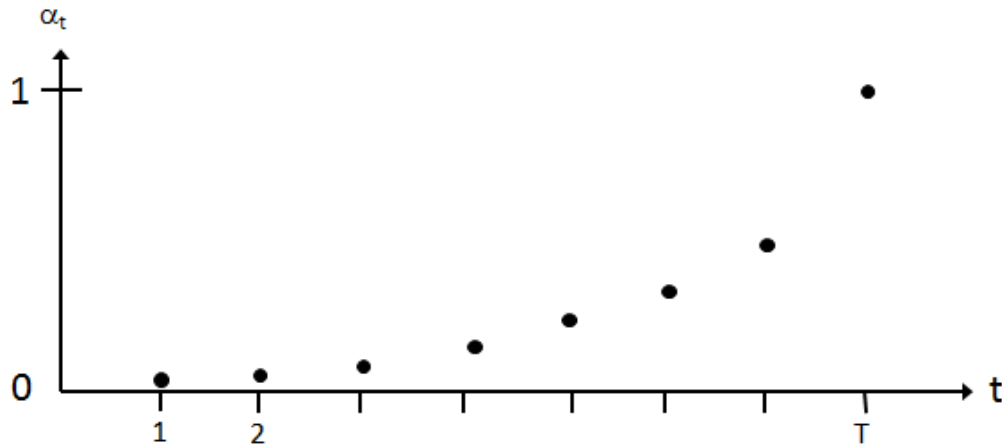
At the end of each day t (after the market closes) current holder of an asset privately learns its type. To greatly simplify the analysis, we assume “anonymity of assets”: namely the trading history of an asset isn’t observed, and no-one can identify an asset he previously sold.

If an asset is a lemon then, again at the end of each day t (after the market closes), with probability α_t there is a public announcement: “This asset is a lemon”. Naturally, on the days after such an announcement, the market price of the asset is zero. If the asset is not a lemon, then there are no such announcements. Note that α_T in effect equals 1, since the type is revealed on day $T+1$.

By way of a *running example*, we might suppose that *all* lemons have the announcement made on one of the days $1, \dots, T$, but the timing is spread uniformly across these days. That is, there is an ex ante probability $(1/T)$ of the announcement occurring on any given day. In this example with “uniformity”,

$$\alpha_t = \frac{1}{T-t+1} \quad \text{for } t = 1, 2, \dots, T$$

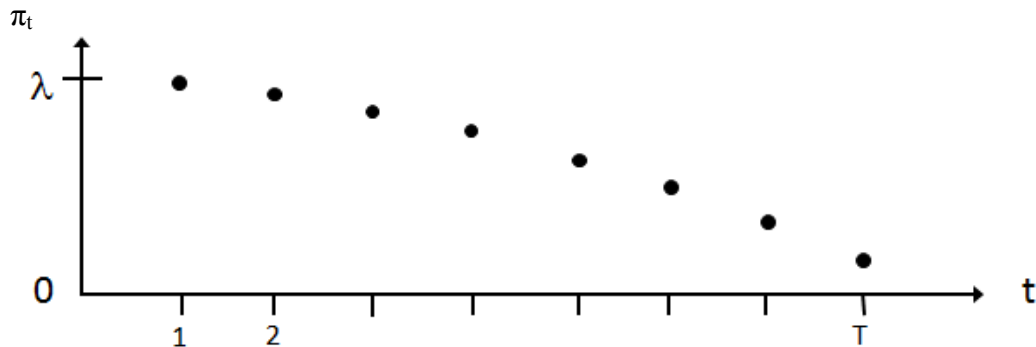
where α_t is the probability of an announcement after the market closes on day t , conditional on the asset being a lemon and there not having been any previous announcement.



From these exogenous parameters $\{\alpha_t\}$, Bayes’ Rule can be used to derive the posterior probabilities, $\{\pi_t\}$ say, that an asset is a lemon. In our running example with “uniformity”,

$$\pi_t = \frac{\lambda(T-t+1)/T}{1-\lambda + \lambda(T-t+1)/T}$$

where π_t is the posterior probability that an asset is a lemon, conditional on there not having been an announcement yet (i.e. no announcement on any of days 1, ..., $t - 1$)



Now the product $\alpha_t \pi_t$ equals the probability that there is an announcement after the market closes on day t , conditional on no announcement yet. We make the assumption about parameters $\{\alpha_t\}$:

Assumption A1: $\alpha_t \pi_t$ is greatest on day $t = T$
 i.e. $\alpha_t \pi_t \leq \alpha_T$ for all t (since $\alpha_T = 1$)

In our running example with “uniformity”,

$$\alpha_t \pi_t = \frac{\lambda/T}{1 - \lambda + \lambda(T - t + 1)/T}$$

is actually *increasing* in t , and so clearly satisfies Assumption A1.

To find the overall equilibrium, start at T (assuming no announcement yet).

There is a high-price \bar{p}_T equilibrium iff

$$\mu^b \bar{p}_T = \beta(1 - \pi_T)V \quad (\text{buyers indifferent})$$

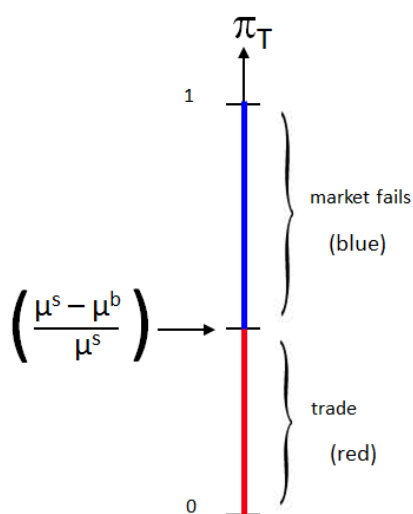
and

$$\mu^s \bar{p}_T > \beta V \quad (\text{holders of H-type wants to sell})$$

That is, there is trade iff

$$\pi_T < \frac{\mu^s - \mu^b}{\mu^s}$$

Summarising day T in a diagram:



Now to the two central results:

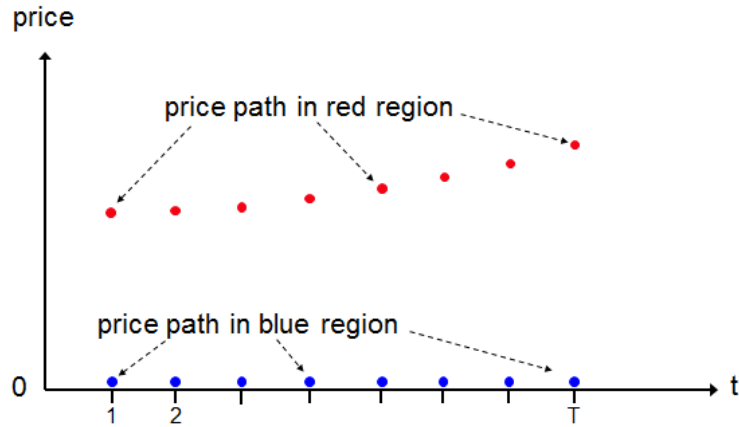
Proposition 1:

If the market fails on day T (blue region), it also fails on the earlier days $t \leq T - 1$.

Proposition 2:

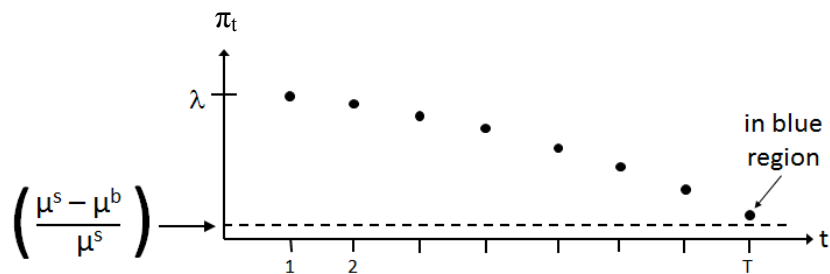
If trade occurs on day T (red region), it also occurs on the earlier days $t \leq T - 1$.
Moreover the price path is increasing, but at a rate no faster than $1/\beta$

To sum up the propositions, the tail wags the dog: the liquidity of the market on day T determines market liquidity on all previous days. See figure:



Proof of Proposition 1 (blue region)

If market fails on days $t+1, \dots, T$, then in effect day t looks like day T except that $\pi_t > \pi_T$:



For all $t \leq T - 1$, $\pi_t > \frac{\mu^s - \mu^b}{\mu^s}$ and hence there is market failure on day t too.

Q.E.D.

Proof of Proposition 2 (red region):

We use backward induction: $t = T-1, T-2, \dots, 1$. Suppose, absent an announcement, there is trade on days $t+1, \dots, T$ at a price path $\bar{p}_{t+1}, \dots, \bar{p}_T$ which is increasing at a rate less than $1/\beta$.

On day t , a high price (\bar{p}_t) equilibrium must satisfy:

$$\mu^b \bar{p}_t = \beta \underbrace{(1 - \alpha_t \pi_t)}_{\substack{\text{probability of no} \\ \text{announcement after} \\ \text{market closes on day } t}} \underbrace{\mu^s \bar{p}_{t+1}}_{\substack{\text{a buyer sells on day } t+1, \\ \text{no matter which type of asset} \\ \text{he learnt that he purchased on day } t}}$$

The price ratio \bar{p}_{t+1}/\bar{p}_t equals

$$\frac{(\mu^b / \mu^s)}{\beta(1 - \alpha_t \pi_t)}$$

which lies strictly between 1 and $1/\beta$, because for $t \leq T-1$,

$$\beta < \frac{\mu^b}{\mu^s} < 1 - \pi_T \leq 1 - \alpha_t \pi_t < 1$$

\uparrow \uparrow
 in red region by Assumption (A1)

Thus the price path is increasing at rate less than $1/\beta$ from day t onwards.

But will seller with asset H want to trade at \bar{p}_t ? Yes, if:

$$\mu^s \bar{p}_{T-1} > \beta^2 V \quad (\text{for } t = T-1)$$

and

$$\mu^s \bar{p}_t > \beta^2 \mu^s \bar{p}_{t+2} \quad (\text{for } t \leq T-2)$$

The final term on the lower right-hand-side is explained by the fact that if the seller didn't sell on day t , she would sell on day $t+2$.

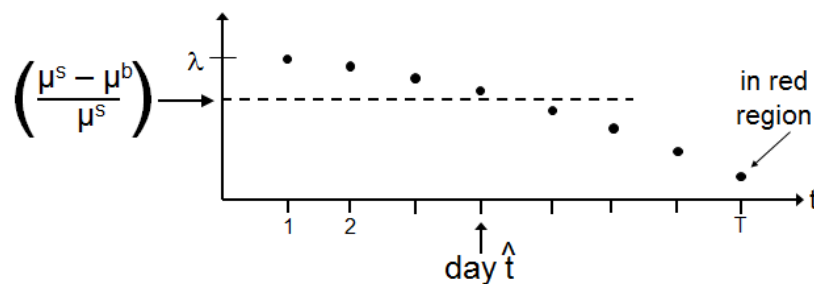
But these two inequalities do hold, given that

$$\frac{\bar{p}_t}{\beta^{T-t}} > \dots > \frac{\bar{p}_{T-1}}{\beta} > \bar{p}_T = \frac{\beta(1 - \pi_T)V}{\mu^b} > \frac{\beta V}{\mu^s}$$

– where the final inequality holds because we are in the red region.

Q.E.D.

Proposition 2 may at first appear surprising. Consider the “backwards” evolution of $\{\pi_t\}$. On day T , π_T is below the critical value $(\mu^s - \mu^b)/\mu^s$; hence the market doesn't break down (we are in the red region). Working back, there is a critical time \hat{t} at which $\pi_{\hat{t}}$ is above $(\mu^s - \mu^b)/\mu^s$ for the first time:



Why doesn't market break down on day \hat{t} ? The answer is because agents have the *option to sell* on day $\hat{t} + 1$.

Recall the intuition from the Introduction. If tomorrow's market is *not* expected to fail, then today's buyer of an unknown asset will sell tomorrow, whether or not he buys a lemon. Thus, the only downside to buying a lemon is the (small) risk of a public announcement after the close of today's market. In other words, the percentage difference in future utility between the good asset and a lemon is small. But, as we saw in Section 2, a small percentage difference means that the market doesn't fail today either.

Conversely, if markets in the future *are* expected to fail (as in the blue region), then today's buyer of an unknown asset will be stuck with it for a long time. Thus, the percentage difference in future utility between the good asset and a lemon is big. But a big percentage difference means that the market fails today too.

It must be emphasised that in this finite horizon model the equilibrium is unique – albeit that there is a discontinuity (from, say, always liquid to permanently illiquid). Which regime the economy is in, red or blue, is dictated by what happens on the last day of potential trading: the tail wags the dog.

In an infinite horizon setting, there is scope for multiple equilibria: liquidity begets liquidity; illiquidity begets illiquidity. In the Appendix, an example of an economy, quite similar to the one discussed here, is presented which can exhibit these two equilibria.

To sum up what we have learnt from the model: the market fails completely iff

$$\pi_T > \frac{\mu^s - \mu^b}{\mu^s}$$

Otherwise, there is trade on days 1, 2, ..., T – unless there is a public announcement.

Given “uniformity”, the market fails iff

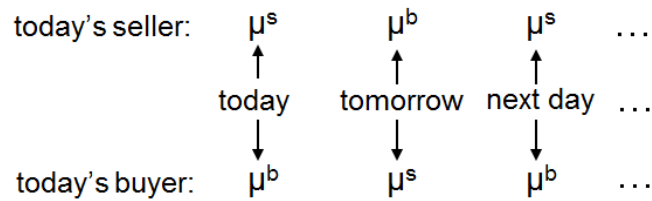
$$\frac{1}{1 + \frac{(1-\lambda)T}{\lambda}} > \frac{\mu^s - \mu^b}{\mu^s}$$

That is, market failure is more likely as λ rises, μ^s/μ^b falls, or the horizon T is shorter. Use is made of these comparative statics results in the second part of Moore (2010).

Appendix

This Appendix presents an example of multiple (two) Nash equilibria in a stationary infinite horizon environment.

There are a countably infinite number of days, and a single consumption good. Agents alternate their marginal utilities:



The daily discount factor $\beta < \mu^b / \mu^s < 1$.

There are two types of asset, with a fraction λ of type L. Asset of type H pays 1 at the start of each day. Asset of type L pays zero. The current owner *privately* receives payment (if any), thus privately learns the type of asset he holds. To greatly simplify, we assume that no-one can identify an asset he previously sold.

First, let us consider the possibility of a stationary high-price equilibrium, a *perpetual trading equilibrium*. Each day, the price \bar{p} is dictated by a typical buyer's indifference condition:

$$\mu^b \bar{p} = \beta \mu^s [(1 - \lambda) + \bar{p}]$$

The term in the square brackets reflects the fact that he sells tomorrow, no matter which type of asset he learns that he has bought today.

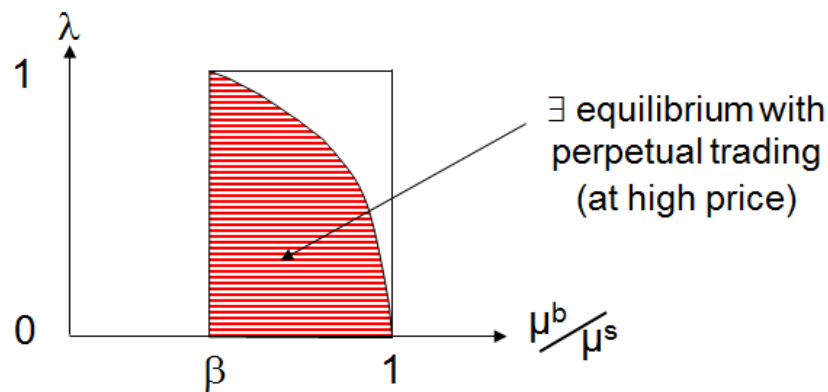
The seller with the H-type asset must want to trade:

$$\begin{aligned}\mu^s \bar{p} &> \beta \mu^b + \beta^2 \mu^s [1 + \bar{p}] \\ &> \beta \mu^b + \beta^2 \mu^s + \beta^3 \mu^b + \beta^4 \mu^s + \dots\end{aligned}$$

That is, a perpetual trading equilibrium exists iff

$$\left(\frac{\mu^b}{\mu^s}\right)^2 < 1 - \lambda(1 - \beta^2)$$

Diagrammatically:



Next, let us consider the possibility of a *perpetual no-trading equilibrium*. Suppose the market is expected to fail from tomorrow onwards. Will it fail today too?

A buyer looking to deviate from a zero-price today, wanting to attract “H-type sellers”, would have to bid a price p satisfying:

$$\mu^s p \geq \beta \mu^b + \beta^2 \mu^s + \beta^3 \mu^b + \beta^4 \mu^s + \dots$$

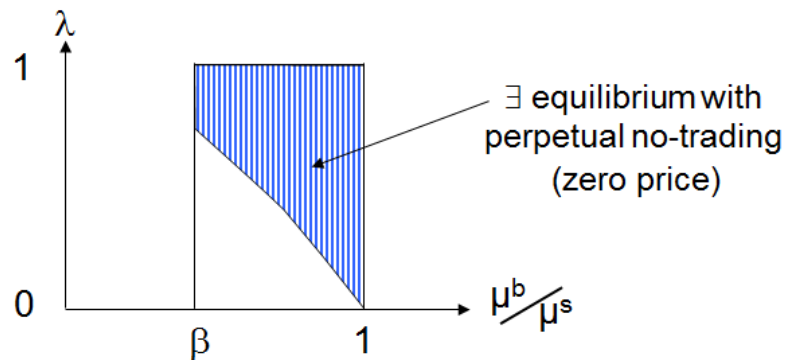
Deviating to this bid p would be profitable for him only if

$$\mu^b p < \beta \mu^s (1 - \lambda) + \beta^2 \mu^b (1 - \lambda) + \beta^3 \mu^s (1 - \lambda) + \beta^4 \mu^b (1 - \lambda) + \dots$$

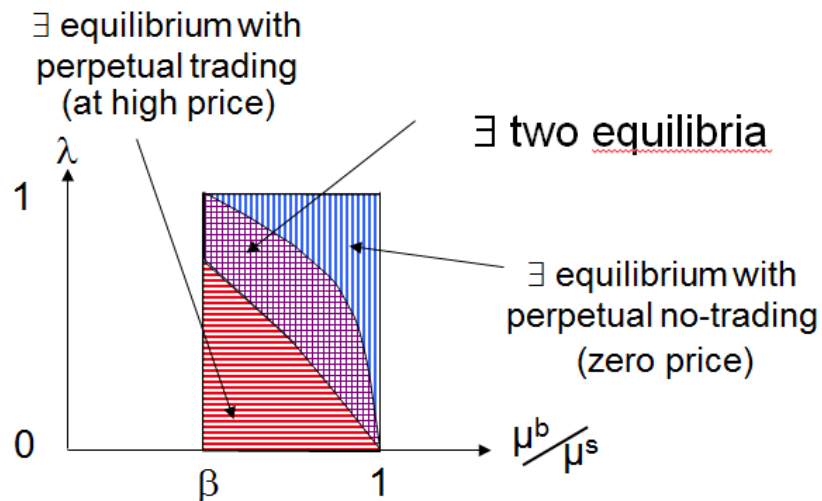
That is, a perpetual no-trading equilibrium exists iff

$$1 - \lambda < \left(\frac{\mu^b}{\mu^s} + \beta \right) / \left(\frac{\mu^s}{\mu^b} + \beta \right)$$

Diagrammatically:



This is interesting, because the two shaded regions overlap:



In the overlap region, there exist *two* stationary equilibria: one equilibrium where both types of asset are always traded and prices are positive; another equilibrium with perpetual market failure (zero prices and no trading). Importantly, both equilibria are “Nash” – that is, robust to agents actively making bids/offers.

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