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Unemployment Risk and Wage Differentials*

ROBERTO PINHEIRO[†] AND LUDO VISSCHERS[‡]

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Abstract

Workers in less secure jobs are often paid less than identical-looking workers in more secure jobs. We show that this lack of compensating differentials for unemployment risk can arise in equilibrium when all workers are identical and firms differ only in job security (i.e. the probability that the worker is not sent into unemployment). In a setting where workers search for new positions both on and off the job, the worker's marginal willingness to pay for job security is endogenous: it depends on the behavior of *all* firms in the labor market and increases with the rent the employing firm leaves to the worker. We solve for the labor market equilibrium, finding that wages *increase* with job security for at least all firms in the risky tail of the distribution of firm-level unemployment risk. Meanwhile, unemployment becomes persistent for low-wage and unemployed workers, a seeming pattern of 'unemployment scarring' created entirely by firm heterogeneity. Higher in the wage distribution, workers can take wage cuts to move to more stable employment.

Keywords: Layoff Rates, Unemployment risk, Wage Differentials, Unemployment Scarring

JEL Codes: J31, J63

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1 Introduction

When a transition to unemployment inflicts a loss of income or utility, workers are willing to give up a part of their wages in exchange for more job security. In this set up, the risk of becoming unemployed could be seen as just another job attribute that workers dislike, e.g. the need to spend a lot of additional time commuting or traveling or a requirement to put in higher levels of effort. However, job security, i.e. the absence of unemployment risk, is quite different from typical amenities.

For one, job security is naturally complementary to the desirability of the job along all other dimensions, including, prominently, the wage. For workers, it is one thing to lose a job that is marginally better than unemployment; it is quite another thing to lose the very best possible job out there. In a frictional labor market, a job loss incorporates not only the immediate drop in income when becoming unemployed but also the subsequent labor market outcomes in employment. It is much more difficult and time-consuming to recover to the best job than it is to find a marginally attractive job. We find that a given incremental increase in job security is valued more in high-wage employment than low-wage employment, *ceteris paribus*. Moreover, the rate at which this valuation of job security increases with the wage is higher, the higher the wage is. Put differently, the marginal willingness to pay for job security is an increasing and convex function of the firm's wages. In contrast, in most hedonic wage models, the marginal willingness to pay is taken to be exogenously given and *constant* across wage levels. (See e.g. Bonhomme and Jolivet (2009) and Hwang, Mortensen, and Reed (1998), who have studied the inclusion of amenities in a frictional labor market – and also have this feature.¹)

A higher unemployment risk can affect firms because it lowers the value of employment for the firm's workers, and therefore increases the risk of losing workers to other firms, for a given wage. But it also may affect the profit maximization of the firm directly, because safer firms have longer employment relationships, *ceteris paribus*, which change the firms' recruitment and retention incentives when they consider which wage to set. This additional effect cannot be captured in some general utility specification on the workers' side; to capture fully the impact of job security and job loss, one rather needs to bring both sides of the market together and solve for the equilibrium.

Overall, the cost of a job loss depends on market characteristics, prominently the extent of frictions and the firms' optimal wage setting responses (which, in turn, take into account the endogenous valuation of job security on the workers' side). Given that it takes time for a laid-off worker to climb the job ladder to his

¹In order to a hedonic wage model deliver this complementarity, we would need to assume a complementarity between wages and job amenities *within* the utility function. However, this leads one to easily ignore that anything that influences firms' behavior, labor market flows, or labor market conditions in general, will affect this utility relation.

previous position, there is a clear loss of earned wages during the process. However, in a market where different firms offer jobs with distinct levels of job security, a job loss may also imply an increase in future layoff risk since the displaced worker may take a job at a firm with lower job security. This potential increase in probability of future unemployment spells, is another component of the cost of job loss².

In this paper, we propose an equilibrium theory of frictional labor markets that incorporates the different, endogenous, nature of the valuation of job security. In our model, we follow Burdett and Mortensen (1998), henceforth referred to as BM, and introduce search frictions and on-the-job search into an otherwise competitive setting. Our sole deviation from the BM setup is that firms differ in the job security they provide³. Behind these differences in unemployment risk across firms could, for example, be differences in managerial practices and their quality (see Syverson, 2004). Bloom and Bloom and Reenen (2007) and Huselid (1995) show that different managerial practices are at the core of differences in firms' economic performance. To further provide an intuitive micro-foundation for firms' differences in unemployment risk, one can take a page from the 'knowledge hierarchy' literature⁴ and see the firm's production in terms of a problem solving technology. In a firm with better managerial practices and more organizational knowledge, there is a higher chance that the worker is able to satisfactorily accomplish the new task, while relying on knowledge embodied in codes and procedures. Therefore, the transfer of workers from old to new tasks is more likely to be successful in better managed firms, reducing worker turnover.

In our model, job security enters directly both in the firm's optimization and also in the worker's valuation in an endogenous, nonlinear way. However, we are able to characterize the equilibrium almost as easily as the standard BM model. In our model's equilibrium, riskier firms need to increase their wages more than safer firms when offering higher values, while they do not gain as much in firm size, relatively. As a result, riskier firms will find it optimal to offer lower values to workers, and workers move, job by job, towards increasingly safe firms. Wages will increase with job security at the bottom of the wage/job security distribution, causing the firms with the lowest job security to be the ones with the lowest wages, contrary to competitive compensating wage theory. While on the workers' side the marginal willingness to pay for job security is increasing and convex in wages, on the firms' side competition will keep pushing up wages with job security throughout the

²The empirical literature finds that a layoff raises the prospect of shortened employment spells in the future, increasing the probability of future job losses (e.g. Stevens 1997, Kletzer 1998). This reduction in subsequent employment durations is responsible for a significant part of the cost of a transition into unemployment (Eliason and Storrie 2006, Boheim and Taylor 2002, Arulampalam et al. 2000). Moreover, for displaced workers of a given quality, commonly, new jobs come with lower wages, and simultaneously with a higher risk of renewed unemployment (Cappellari and Jenkins 2008, Uhlendorf 2006, Stewart 2007).

³In order to isolate the impact of the job security channel, firms will not differ in e.g. the instantaneous productivity of jobs.

⁴See Garicano, 2000 and Garicano and Rossi-Hansberg, 2006

firm distribution, as long as the density of the firm-level job security distribution is increasing, constant, or not decreasing too sharply. Only when the density of the firm-level job security distribution decreases sharply enough could wages drop with job security, in which case some workers would be observed to take wage cuts. Whether wage cuts occur depend also on the rate at which workers receive alternative offers on the job; the higher the incidence of on-the-job search, the less job security is valued, and the less likely we are to observe a negative relationship between job security and wages. In fact, if that incidence is high enough, wages can be strictly increasing in job security.

Overall, our equilibrium is consistent with, first, the lack of compensating wage differentials, and a positive correlation between wages and job security, as has been documented e.g. in Bonhomme and Jolivet (2009) and Mayo and Murray (1991), and second, with a pattern of ‘unemployment scarring’ through both repeated unemployment spells and lower wages, as found e.g. by Stevens (1997) and Kletzer (1998), while the heterogeneity lies entirely on the firm’s side of the market.⁵

Finally, we note that, if firm heterogeneity is a leading factor behind the equilibrium patterns described, this has further implications for policy. After a worker falls off the job ladder into unemployment, the first rungs of the wage ladder will be more slippery than the higher rungs. Therefore bad labor market outcomes can persist, and time and luck is needed before a recently unemployed worker can find traction on the job ladder. Until then, unemployment begets low wages and unemployment. Overall, bad outcomes will be correlated over time, as will be good outcomes, increasing the distance between best-case and worst-case outcomes in the labor market in terms of life-time discounted utility (hence increasing overall risk, given the same aggregate flows). The resulting increase in inferred risk might have further policy implications⁶. Consider the case of policies which increase the earnings capacity of unemployed workers; these are often thought to decrease the probability that these workers will subsequently slide back into unemployment. If, however, firm heterogeneity is what matters and the workers in question keep taking jobs in the same risky firms, the foreseen reduction in repeat-unemployment might not fully come to pass.

⁵Additionally, the model produces a correlation between firm size and job security through the same channel as Burdett and Mortensen (1998), once the ranking of values provided to workers is taken into account.

⁶This has further implications for consumption and saving choices of workers who have just become unemployed, and, ex ante, for employed workers who face differing risks of becoming unemployed, with more dire consequences if they do fall off the job ladder into unemployment. See Lise (2011) for a model linking the climbs of the wage ladder through on-the-job search, and the drops from the wage ladder, to consumption and savings decisions. In his model, following BM, the risk of dropping from the ladder is constant across workers and jobs.

2 Model

A measure l of risk-neutral firms and a measure m of risk-neutral workers live forever in continuous time, discounting the future at rate r . When not matched with a firm, a worker receives unemployment benefits b . When matched with a firm, the worker produces output p , which is the same in any firm. Firms, however, differ in the probability δ with which they send workers back into unemployment. We index firms by this probability, and will refer to a high- δ firm as a “risky firm” and a low- δ firm as a “safe firm.” One can think that the heterogeneity in δ as standing in for the quality of the firm’s management, manifested, for example, when the tasks that workers’ do occasionally become obsolete and they need to be given new tasks. A safe firm with good managers, then, is more likely to provide good guidance for the new task, such that the worker is likely to keep his productivity level p . In a risky firm, managers are less likely to be able to support the worker’s transition to the new task and hence, they will have to dismiss the worker more often.⁷

The distribution function of firm types is $H(\delta)$; the distribution may (but does not have to) contain mass points. Apart from the differences in firms’ layoff risk δ , the remaining setup follows Burdett and Mortensen (1998). This means that search frictions in the labor market prevent workers from instantaneously matching with the best firm in the market. Instead, from time to time workers meet randomly with one of the firms in the market. Concretely, unemployed workers receive a single job offer at a time at Poisson arrival rate λ_0 , while employed workers do so at rate λ_1 . An offer is a wage-layoff risk combination (w, δ) which specifies the wage w that the firm, which has layoff rate δ , commits to pay as long as the match lasts (it is the same as the wage of workers already in the firm). The job offer must be accepted or rejected on the spot, and when rejected it cannot later be recalled. Firms are able to hire everyone who accepts their wage, and they choose this wage to maximize their steady state profits. In setting profit maximizing wages, firms take into account both the distribution of wages posted by the other firms in the market and also how workers compare wage offers from firms with different layoff risks. We first turn to *workers’* decisions in the face of differentially risky firms posting different wages.

⁷In this interpretation, unproductive tasks by construction have a productivity low enough that no firm wants to keep the worker in it, just in case any future change in the task portfolio render the worker productive again. Additionally, all productive tasks have productivity p for simplicity. We take elements here from the ‘knowledge hierarchy’ literature, following Garicano (2000) and Garicano and Rossi-Hansberg (2006). See also appendix C in which we have spelled out these microfoundations more elaborately. The environment as we sketch it can also be, loosely, considered to be a normalization with respect to trend, so becoming ‘unproductive’ means becoming unproductive relative to trend.

2.1 Workers' Job Offer Acceptance Decisions

Obstructed by the frictions in the labor market, a worker receives a new offer of employment only occasionally (at Poisson arrival rate λ_0 when unemployed, or λ_1 when employed). The worker is offered a wage w offered by a firm with layoff risk δ , and he will accept any offer that improves his life-time expected discounted income. Because a new offer could come with higher job security but also with a lower wage (or vice versa), the worker needs to fully rank these two-dimensional job offers by their life-time expected discounted income.

Call V_0 the life-time expected discounted income of a worker who is currently unemployed, and $V(w, \delta)$ the value for a worker currently employed at a firm that pays wage w and has a layoff risk δ . To establish notation, consider that firms with layoff risk δ symmetrically post according to a possibly pure strategy with cdf $\hat{F}(w|\delta)$. We can express the value functions of workers as follows: for an unemployed worker, the flow Bellman equation is

$$rV_0 = b + \lambda_0 \int \int \max\{V(w, \delta) - V_0, 0\} d\hat{F}(w|\delta) dH(\delta). \quad (1)$$

The opportunity cost of searching for a job when unemployed (which equals the interest rate on its asset value, rV_0) equals the benefit flow b and the expected capital gain of job search. The latter, in the right-most term in equation (1), results from receiving, at rate λ_0 , a job offer (w, δ) randomly drawn from $\int \int d\hat{F}(w|\delta) dH(\delta)$. An accepted job offer (w, δ) will improve the worker's lifetime expected value by $V(w, \delta) - V_0$ and therefore an offer will be accepted if and only if this term is strictly positive.⁸ Similarly, for employed workers we find

$$rV(w, \delta) = w + \lambda_1 \int \int \max\{V(w', \delta') - V(w, \delta), 0\} d\hat{F}(w'|\delta') dH(\delta') + \delta(V_0 - V(w, \delta)), \quad (2)$$

where, in addition to the wage and expected capital gains from moving up to better jobs in the middle term on the right-hand side, the worker also faces an expected capital *loss* from becoming unemployed, $\delta(V_0 - V(w, \delta) - V_0)$.

Suppose that offers arrive from firms with layoff risks identical to the current firm. In this case, the reservation property in wages follows naturally from equation (2); values $V(w, \delta)$ increase with w , and hence wages above the current wage are acceptable, those below are not. To compare across types of firms, we can start by looking at *indifferences* in wages and layoff risks. Consider a worker's current employment in a firm with (w_1, δ_1) , and consider an alternative job offer from a firm with (w_2, δ_2) : the worker is indifferent when $V(w_2, \delta_2) = V(w_1, \delta_1)$; by equation (2) this occurs precisely when

$$w_2 = w_1 + (\delta_2 - \delta_1)(V(w_1, \delta_1) - V_0). \quad (3)$$

⁸In case of ties, the worker will remain in his current job.

Then, if the worker, currently with $V(w_1, \delta_1)$, is offered a larger wage than w_2 in a new firm with layoff risk δ_2 , he will accept the new job, rejecting it otherwise. Thus, to move to a more risky firm, the worker needs ‘compensation’ in the form of a wage increase; this increase must cover the additional amount of risk taken on, multiplied by the cost of the layoff ($V(w, \delta) - V_0$). We will return to discuss the ‘value’ of job security in more detail, but let us first use equation (3) to completely characterize the value $V(w, \delta)$ associated with employment in any (w, δ) -firm. The reservation property and the established indifference condition above, lead to

$$rV(w, \delta) = w + \lambda_1 \int \left(\int_{w + (\delta' - \delta)(V(w, \delta) - V_0)} (V(w', \delta') - V(w, \delta)) d\hat{F}(w'|\delta') \right) dH(\delta') + \delta(V_0 - V(w, \delta)). \quad (4)$$

This implies that (given the indifference at the optimal acceptance choices),⁹

$$\frac{\partial V(w, \delta)}{\partial w} = \frac{1}{r + \delta + \lambda_1 \int \left(1 - \hat{F}(w + (\delta' - \delta)(V(w, \delta) - V_0) | \delta') \right) dH(\delta')}. \quad (5)$$

To characterize $V(w, \delta)$ along the δ -dimension, we can similarly find

$$\frac{\partial V(w, \delta)}{\partial \delta} = - \frac{V(w, \delta) - V_0}{r + \delta + \lambda_1 \int \left(1 - \hat{F}(w + (\delta' - \delta)(V(w, \delta) - V_0) | \delta') \right) dH(\delta')}. \quad (6)$$

Together with the relevant initial conditions, equations (5) and (6) form a partial differential equation that we can solve to fully characterize $V(w, \delta)$. This leads to the results stated in lemma 1, where, for notational ease in its last line (and for use in the subsequent analysis of the firms’ decisions), we define $w(V, \delta)$ as the wage in a firm with layoff risk δ that implies a life-time expected value V to the worker. In other words, $w(V, \delta)$ is the inverse of $V(w, \delta)$ in w , given δ .

Lemma 1. (a) *Given the reservation wage out of unemployment R_0 and wage distributions $\hat{F}(w|\delta)$, the value function of employed workers $V(w, \delta)$ is the solution to the partial differential equation, defined by (5) and (6) with initial conditions for every δ ,*

$$w(V_0, \delta) = R_0, \text{ and } V_0 = \frac{\lambda_0 R_0 - \lambda_1 b}{r(\lambda_0 - \lambda_1)} \quad (7)$$

⁹Equation (5) has a familiar appearance to the differential equation that captures the relation between values and wages in the standard Burdett and Mortensen setup (where there is only one value of δ for all firms); there it is given by $dV(w)/dw = 1/(r + \delta + \lambda_1(1 - F(w)))$. Notice, however, that in equation (5) the derivative $dV(w)/dw$ is a function of *both* w and $V(w, \delta)$, instead of only the former, while $V(w, \delta)$ is precisely the endogenous object that we are after. Even with this additional dimension, equation (5), as a differential equation, remains standard and tractable.

(b) In equilibrium, R_0 satisfies

$$R_0 = b + (\lambda_0 - \lambda_1) \int_{V_0} (V - V_0) dF(V) \quad (8)$$

with

$$F(V) = \int \hat{F}(w(V, \delta) | \delta) dH(\delta). \quad (9)$$

We have relegated all proofs to the appendix. Lemma 1 shows that, given the reservation wage out of unemployment, R_0 , it is possible to solve directly (in one iteration) for all values as a function of the associated wage and the firm-level layoff risk $V(w, \delta)$. In the second part of the lemma, R_0 can be found as the solution of the fixed point problem in (8), given any set of distributions $\hat{F}(w | \delta) \forall \delta$, the posting strategies per firm-type – for all types, and distribution $H(\delta)$ of firm types. It will turn out that we can proceed to the firms' maximization and resulting distributions, leaving R_0 implicit for the time being. Then, we will find the *equilibrium* R_0 as the fixed point of a mapping with value distribution $F(V)$ that incorporates all other equilibrium relations.

In the process of deriving lemma (1), we have quantified the wage increase that is needed to offset a discrete increase in unemployment risk, $w_2 - w_1 = (\delta_2 - \delta_1)(V(w_1, \delta_1) - V_0)$, from equation (3). This difference is directly related to the concept of the *marginal willingness to pay* (MWP), employed in 'hedonic' estimations of the value of job amenities, including job security. The latter refers to the derivative of the workers' indifference curve in (w, δ) -space, which here is the derivative of equation (3) with respect to δ

$$MWP = \left. \frac{dw}{d\delta} \right|_{V \text{ constant}} = - \frac{\frac{\partial V(w, \delta)}{\partial \delta}}{\frac{\partial V(w, \delta)}{\partial w}} = V(w, \delta) - V_0. \quad (10)$$

Looking more closely at equation (10), note that at the reservation wage out of unemployment, the marginal willingness to pay for job security is zero.¹⁰ Intuitively, if one is indifferent between being in state A or B, whether one transits from one to the other, and thus also how frequently one transits, is irrelevant for well-being. Thus, at the indifference with unemployment, no compensating wages are required to hire out of unemployment even when $R_0 > b$, and, as a result, the reservation wage out of unemployment is identical for all firm types. This is why the initial condition in the partial differential equation, R_0 , in lemma 1, is invariant with δ in equation (7). At employment values strictly above the value of unemployment, a worker is willing to give up some of his wage in exchange for an increase in job security. The amount of wage the worker is willing to give up for the same difference in job security increases as the worker's value in his current job increases. Thus, there is an intuitive, but critical complementary between the attractiveness of a job, i.e. the

¹⁰See also Burdett and Mortensen 1980.

rent a worker receives in a job, and how he values job security. In terms of wages: since the value of the job increases with the wage paid, and more strongly so the higher the wage level is, the MWP can be easily shown to be increasing and convex in the wage paid, *ceteris paribus*.¹¹

Second, the worker's marginal willingness to pay for job security depends not only on the wage of the firm in question, but also on the extent of frictions in the labor market and on the choices of the firms, captured in the joint distribution of firms' job security and the wages these firms post. These dimensions are all subsumed in the rent of the worker's employment in his current firm, $V(w, \delta) - V_0$, and can be made visible, e.g. by writing this difference out, using equations (1) and (2),

$$MWP = \frac{1}{r} \left((w - b) - \lambda_0 \left((1 - F(V(w, \delta))) (V(w, \delta) - V_0) \right) - \lambda_0 \int_{V_0}^{V(w, \delta)} (V - V_0) dF(V) \right. \\ \left. - (\lambda_1 - \lambda_0) \left(\int_{V(w, \delta)} (V - V(w, \delta)) dF(V) \right) - \delta (V(w, \delta) - V_0) \right). \quad (11)$$

In equation (11), the first term, $w - b$, captures that the value of job security (i.e. MWP) in a job indeed strictly increases with its wage, compared to the benefit flow b in unemployment. When a worker loses his job and becomes unemployed, the adverse shock to life-time income is mitigated when he can re-enter employment quickly, particularly when the new job is at least as good as the old job. The probability that the displaced worker finds such employment depends on both the arrival rate λ_0 of offers while unemployed and the measure of firms that provide a value at least as good as the old job, i.e. $V \geq V(w, \delta)$. This probability is captured in the second term in equation (11). A laid-off worker might also find employment inferior in value to his previous job; this allows him to at least partially recover the lost income (in the third term of equation (11)); and also decreases his willingness to pay for job security. The next term, on the second line, accounts for the possibility that an employed worker occasionally receives offers allowing a move to an even better firm. The rate of meeting these 'desirable' firms might increase or decrease upon being laid off, depending on whether the arrival rate of offers in unemployment, λ_0 , is larger or smaller, respectively, than the arrival rate of offers when employed, λ_1 . This will increase or decrease, respectively, the value of job security, while also depending on the measure of firms offering such a (further) improvement of conditions relative to $V(w, \delta)$. Finally, as shown in the last term of equation (11), the willingness to trade off wages for an incremental change in job security also depends on the baseline level of unemployment risk in the worker's existing job; the same (absolute) change in job security in a very safe job has a more profound effect on the worker's value than is a job which very likely will end soon anyway.

¹¹The derivative of the marginal willingness to pay with respect to w equals $\partial V(w, \delta) / \partial w > 0$ in equation (5). The second derivative $\partial^2 V(w, \delta) / (\partial w \partial w)$ is also positive.

To put this in a broader perspective, the valuation of job security we derive above differs from the way it is usually considered, in which standard methods for inferring the value of a generic amenity are applied to job security as well. The assumptions underlying these methods are (implicitly) thought to apply equally to the ‘amenity’ job security. However, we can draw some interesting contrasts between the assumptions and implications of these methods, and those of job security in our setting. To do so, we will take a step back and first consider the valuation of firm-level job security when using conventional methods, before returning to highlight differences with our setting.

The perhaps most-used method of valuing amenities in general is based on hedonic wage regressions, in which the wage as dependent variable is regressed on amenity measures and controls, e.g. for worker quality. In its most straightforward interpretation, the estimated relationship is thought to trace out identical workers’ indifference curve between wages and amenities. To link the regression coefficient of an amenity measure to the marginal willingness to pay for it, a static utility function is posited that is linear and *additively separable* in wages and amenities.¹² However, the coefficients in these hedonic wage regressions, estimated in the cross section, are problematic: they are often statistically insignificant, and in many case have an unexpected sign after controlling for observable and unobservable worker heterogeneity. (See e.g. Bonhomme and Jolivet (2009), who discuss this in detail, both with regard to their data findings and the literature.)

Hwang et al. (1998) show that when search frictions matter in the labor market, the empirical cross-sectional relationship between wages and amenities can structurally deviate from the underlying marginal willingness to pay of workers. For example, firms which are better at producing amenities, *ceteris paribus*, will choose to provide higher overall utility to workers. If all else is indeed equal, the ‘marginal willingness to pay’ as found in hedonic wage regressions is biased downward. While Hwang et al. confine themselves to simulation exercises, Bonhomme and Jolivet (2009) estimate a partial-equilibrium version of their model on a set of amenities that includes subjectively reported job security, and find that the model implies a high marginal willingness to pay, even though the cross-sectional relationship is very different, often wrongly-signed.

In Hwang et al. (1998) and Bonhomme and Jolivet (2009) all job amenities enter the workers’ static (or instantaneous) utility function in a non-pecuniary way. In their setup, the valuation of an amenity is, in a sense, taken to be ‘fundamental’, an invariant aspect of the environment. In addition, the utility function of wages

¹²To make this explicit, let the static utility be $v(w, a)$, where w is wages and a is the vector of amenities. To be able to construct a ‘hedonic wage function’ which rationalizes the regression under the assumption that all firms offer the same utility, i.e. $v(w, a) = \bar{v}$, we must be able to isolate w on one side of equation, which means that w enters this utility additively separable. Moreover, given these assumptions, observing combinations of wages and amenities (w, a) and (w', a') allows us to calculate the average willingness to pay; to let this pin down the *marginal* willingness to pay as well, one assumes that the marginal willingness is constant, hence the utility resulting utility function is $v(w, a) = w + \gamma a$.

and amenities in these papers keeps the same additive and linear form as was behind the standard hedonic wage functions. In contrast, in our setting job security enters differently in the worker's utility. It is, rather, 'instrumental': workers care about increasing their job security, not because it is instantaneously enjoyable, but because it implies that the realized stream of future incomes –which is what they fundamentally care about– will likely be larger.¹³

Then, the valuation of job security is not an unknown that needs to be estimated from the wages and/or quit behavior in workers' current jobs as a function of those jobs' professed layoff risk. Instead it can be inferred by comparing streams of income realizations when the worker would keep his job, to those that would result after losing his job.¹⁴ In other words, incorporating labor market dynamics more fully, as we have done above in a setting with on-the-job search and firm heterogeneity in layoff risk, could inform us about the valuation of job security itself and more broadly, how worker overall well-being varies within the labor market. The theory thus pushes us to use additional dimensions of labor market data to measure the value of job security and its impact on the labor market.

Furthermore, this approach draws attention to how the shape of the valuation of job security is molded by labor market conditions. We have already discussed the intuitive complementarity between a job's wages (or even better, the worker's rent) and the job's security when taking the 'instrumental' approach towards job security. This stands in contrast to a valuation of job security that is independent of the job's wage, as is required in standard hedonic wage analyses (and analyses building on these), to be able to isolate wages on one side of the equation and amenities on the other side. In other words, it assumes that job security is equally valued across the wage distribution.

Fundamentally, a worker values job security more when he has a lot to lose from becoming unemployed, which occurs precisely when he is in high-wage, otherwise relatively secure job, which cannot be found easily when looking for jobs in the labor market. The latter part of this sentence captures (as did equation (11) beforehand) that frictions enter the valuation of job security. In contrast, in previous analyses that incorporated

¹³Other amenities, such as a company car, or sport or child care facilities, are clearly well-captured by a contemporaneous 'fundamental' utility flow. Job security differs because it affects future outcomes, not today's consumption. It would interact with these amenities in the same way as it interacts with income in our setting. In general, the worker would compare the expected stream of *utility* and how it is affected by job security.

¹⁴Confining oneself to wage and turnover data within a job spell to estimate amenity valuation is more difficult in the case of job security, as observed transitions into unemployment imply that quits will not be observed for those spells. Bonhomme and Jolivet avoid this by the additional use of self-reported data on the worker's *satisfaction level* with respect to job security". However, the mapping from an answer to this question to objective job security measurement is not without difficulties. Bonhomme and Jolivet (2009) deal with these by considering the survey answer to map to a binary (threshold) job security measure and incorporating controls for the unobservable individual-level heterogeneity in answering the survey question.)

frictions, they help to explain the wedge between the coefficient in the hedonic wage regression and the actual marginal willingness to pay, but do not enter the marginal willingness to pay itself. This leads to different results when the degree of frictions is varied. Hwang et al. (1998) and Bonhomme and Jolivet (2009) discuss at length how a reduction in frictions can bring the estimated coefficient of the cross-sectional hedonic regression, which is affected by frictions, closer to the marginal willingness to pay, which is ‘deep’ and invariant to frictions.¹⁵ While this applies to the generic amenity that is consumed on the spot, job security is different: a change in frictions also changes the ease with which an laid-off worker can make up his losses in new employment. Now there are two counteracting effects when frictions decrease: firms will be forced to offer more similar utility; simultaneously, workers will consider job security a less important dimension of jobs, and will have a reduced marginal willingness to pay for it. We will study the overall impact that changes in frictions have on equilibrium outcomes in section 3.

Finally, the fact that in some jobs the worker values job security more than in other jobs, is not only an important feature behind workers’ choices, it also reverberates through the choices which firms make in competition, as we shall see in the remainder of section 2. In the next sections, we will see that this comes up in a number of ways. Because wages and job security complement interact in the worker’s lifetime utility, job security may (or may not) bring a lot of advantage in recruitment. However, job security also affects the firms’ profit maximization in separate ways. The reduced outflow rate into unemployment in a more secure firm prolongs the expected time its workers contribute to the firm’s life-time profit, which affects the wages the firm will optimally choose (with an eye on recruitment and retention). As some firms send more of their workers back to unemployment, these firms have less workers for other firms to poach. All these issues affect the firms’ decision making and all would be missed if job security would be placed in the utility only.

Overall, the workers’ heterogeneous valuation of job security, which depends on the distribution of wages posted by firms, affects individual firms’ behavior in turn, making the valuation of job security a true equilibrium object. Then, in general, any change in policy variables which are thought to affect wages or flows in the labor market will affect the valuation of job security as well. In section 2.2, we will proceed to consider the firms’ profit maximization, taking into account the endogenous valuation of job security. In section 2.3, we show how this affects the way workers are distributed over firms. Finally, we will put the two sides of the market together, to fully characterize the equilibrium in section 2.4.

¹⁵Although interestingly, Bonhomme and Jolivet (2009) find that local changes around the estimated extent of frictions will not radically alter the inferred difference between the marginal willingness to pay and the estimated coefficient of the hedonic wage regression, which remains substantial.

2.2 The Firm's Problem and Labor Market Equilibrium

Firms face the problem to pick the wage that maximizes their steady state profits. If a firm pays a higher wage, workers will value being employed in this firm more relative to other firms. As a result, workers employed in other firms are more likely to move into the firm whenever they receive the firm's wage offer. Similarly, when the firm's own workers receive alternative offers, they are more likely to reject those and stay with the firm. Therefore, with higher wages facilitating recruitment and retention, the firm employs more workers in the steady state. While this force pushes overall profit upwards, it comes at the cost of earning less profit per worker, $p-w$, which pushes profits downwards. At the optimal wage, the firm balances those two counteracting dimensions, maximizing *total* steady-state profits. As we will see below, this balance works out differently for firms with different degrees of job security.

To explicitly find the profit-maximizing wage for each firm, we first look at how steady state firm sizes vary by the value provided to the worker and the firm's layoff rate. The latter is directly relevant for firm size: even if the value they offer to worker is the same, risky firms lose workers at a faster rate and will be smaller as a result. When firms are in steady state, their inflows must be completely balanced by their outflows. Since we are dealing with measures of firms and workers, we first look at the inflow and outflow measures of workers in a positive measure of firms, then take the appropriate limit to isolate the firm size, $l(V, \delta)$.

Using $V(w, \delta)$, we can define $\tilde{F}(V, \delta)$ as the joint distribution of firm-level unemployment risk δ and firm-offered *value* V . Here we rather use values V than wages, because worker mobility, and therefore worker flows, are directly dictated by these values. Distribution $\tilde{F}(V, \delta)$ is constructed with the equilibrium wage choices of firms of each type δ , $\hat{F}(w|\delta)$, the translation of these wages into values $V(w, \delta)$ (and this function's inverse $w(V, \delta)$), and the distribution of the firms' layoff risk δ , in $H(\delta)$. Formally, $\tilde{F}(V, \delta)$ equals $\int_{\delta' \leq \delta} \hat{F}(w(V, \delta')) dH(\delta')$.

With standard random matching (see Podczeck and Puzello 2011), and in steady state, the measure of workers who are employed at values weakly below V , in firms with a layoff risk rate weakly below δ , $\tilde{G}(V, \delta)$, must satisfy¹⁶

$$\int_{\delta' \leq \delta, V_0 \leq V' \leq V} \left(\delta' + \lambda_1 \int_{\tilde{V} > V} d\tilde{F}(\tilde{V}, \tilde{\delta}) + \lambda_1 \int_{\substack{\tilde{\delta} > \delta, \\ V \geq \tilde{V} > V'}} d\tilde{F}(\tilde{V}, \tilde{\delta}) \right) d\tilde{G}(V', \delta')(m - u) = \int_{\delta' \leq \delta, V_0 \leq V' \leq V} \left(\lambda_0 u + \lambda_1 \int_{\tilde{\delta} > \delta, \tilde{V} < V'} d\tilde{G}(\tilde{V}, \tilde{\delta})(m - u) \right) d\tilde{F}(V', \delta'). \quad (12)$$

The left-hand side captures the outflow, consisting of (in order of appearance) the outflow to unemployment (δ'), to firms with a higher value $\tilde{V} > V$, and to firms with higher $\delta' > \delta$ which offer values better than the

¹⁶Note that, since mass can be concentrated at a single (V, δ) , we are explicit whether the boundaries are included.

worker's value V' in his current firm, but weakly lower than V . The right-hand side accounts for the inflows, first from unemployment, and second, from firms with $\delta' > \delta$ with values \tilde{V} below the new firm's V' .

Together with the joint firm-type offer distribution $\tilde{F}(V, \delta)$, the steady-state equality of inflows and outflows in (12) pins down the distribution of employees across firms, $\tilde{G}(V, \delta)$. It follows that $\tilde{G}(V, \delta)$ is absolutely continuous with respect to $\tilde{F}(V, \delta)$: if a subset $A \in \mathbb{R}^2$ has probability $\int_A d\tilde{F}(V, \delta)$ equal to zero, then the LHS of (12), adapted to integrate only over the set A , equals zero; since $\delta > 0, \forall \delta$, it must be that $\int_A d\tilde{G}(V, \delta) = 0$ as well. Then, by the Radon-Nikodym theorem, a function $l(V, \delta)$ exists such that $(m - u)\tilde{G}(V, \delta) = \int_{V_0}^V \int_{\underline{\delta}}^{\delta} l(V', \delta') d\tilde{F}(V', \delta')$; Roughly, $l(V, \delta)$ corresponds to the measure of employed workers divided by the measure of firms, as both get very small, and we take this as the firm's size, in lemma 2.

Lemma 2. *The size of a firm posting a wage of which the equivalent wage is V , only depends on aggregate offer distributions of values, $F(V)$, the aggregate distribution of workers over values, $G(V)$, and on the firm's own δ ,*

$$l(V, \delta) = \frac{\lambda_0 u + \lambda_1 G^-(V)(m - u)}{\lambda_1(1 - F^+(V)) + \delta}, \quad (13)$$

where $G^-(V) = \int_{V' < V} d\tilde{G}(V', \delta')$, $F^+(V) = \int_{V' \leq V} d\tilde{F}(V', \delta')$. (Note, these are integrated over the entire set of δ).

Likewise for unemployment,

$$\lambda_0 u \int_{V \geq V_0} d\tilde{F}(V, \delta) = (m - u) \int \delta d\tilde{G}(V, \delta) \quad (14)$$

The fact that the size of a firm will be affected by both the value (or wage) offered to the worker and its own unemployment risk stands in contrast to BM, and to Bontemps et al., who allow many sources of heterogeneity on the firm and worker side, but keep the property that the firm size only depends on the wage. As a direct consequence of the dependence of firm size on both value V and unemployment risk δ , the distribution function of workers, $G(V) = (m - u)^{-1} \int_{V' \leq V} l(V', \delta') d\tilde{F}(V', \delta')$, depends also on the type of the firms that offer each value V . Consider a subset of values which are offered predominantly by riskier firms: these firms send workers into unemployment at a faster rate. If these firms offer high values, they will attract many workers, who all will be subjected to a high unemployment risk, which generates a large inflow into unemployment; if they offer low values, only a smaller subset of employed workers will be subject to this increased risk. Hence, the average unemployment risk in the labor market depends on the joint distribution of firm types and value. Likewise, a firm's recruitment inflow from other firms depends not only on the number of firms that offer less

attractive employment, but also on the unemployment risk at these firms. A higher risk at these firms, *ceteris paribus*, translates into less workers that can be poached from other firms with more workers in unemployment. This affects the competition for workers and therefore the firms' wages and values posted. Because the inflow from unemployment is not responsive to wages (as long as the wage remains above R_0), and smaller firms imply less gains of offering higher wages to poach workers, different wages will be posted. In equilibrium, we have to take these dependencies into account.

Formally, a firm with layoff rate δ chooses w to maximize $(p - w)l(V(w, \delta), \delta)$. Combining lemma 1 and lemma 2, we can derive that safer firms will offer better values, and the rank of the firm in the value distribution corresponds to the ranking with respect to job security.

Proposition 1 (Ranking Property). *Suppose in equilibrium a riskier firm (with layoff risk δ_h) and a safer firm (with layoff risk δ_l , with $\delta_l < \delta_h$) offer wages of resp. w_s and w_r . Then, we must have $V(w_s, \delta_l) \geq V(w_r, \delta_h)$.*

Below we prove proposition 1 without reference to the shape of $H(\delta)$, and therefore holds also whether it is a discrete, continuous, or a mixture distribution. To establish the proposition, we look at both the relative gains in the steady state number of workers in firm and the relative losses in profit-per-worker, when offering a higher value. First, consider how much larger a firm will be in steady state when offering a larger V' , as opposed to V

$$\frac{l(V', \delta)}{l(V, \delta)} = \frac{\lambda_0 u + \lambda_1 G^-(V')(m-u)}{\lambda_0 u + \lambda_1 G^-(V)(m-u)} \times \frac{\lambda_1(1-F^+(V)) + \delta}{\lambda_1(1-F^+(V')) + \delta}. \quad (15)$$

Now, consider how for an increase of offered value from V_l to V_h , firm size increases by a relatively larger amount when considering a safer δ_l -firm (with $\delta_l < \delta_h$.) As the *inflows* into a firm only depend on the value offered, here V_h or V_l , the first term on the right-hand side is the same for any δ -type of firm. *Outflows* on the other hand, depend on the unemployment risk. The flow of workers lost to other firms, is a relatively more important component of overall outflows in safer firms, $\frac{\lambda_1(1-F^+(V_l))+\delta_l}{\lambda_1(1-F^+(V_h))+\delta_l} \geq \frac{\lambda_1(1-F^+(V_l))+\delta_h}{\lambda_1(1-F^+(V_h))+\delta_h}$, for $\delta_l < \delta_h$, and hence a change in this component has a larger effect on firm size for safer firms. As a result, safer firms gain relatively more in firm size from offering a higher value,

$$V_h \geq V_l \implies \frac{l(V_h, \delta_l)}{l(V_l, \delta_l)} \geq \frac{l(V_h, \delta_h)}{l(V_l, \delta_h)}, \quad (16)$$

and the converse holds in strict inequalities. Put differently, for a given increase in value V , the accompanying relative increase in expected job duration for the firm's workers is higher in a safer firm: as a result, safer firms care more about worker retention than riskier firms. Thus, job security does not only enter the valuation of workers, it also enters the firm's objective function directly.

Second, consider the profit flow per worker, $p - w(V, \delta)$ for a δ -type firm providing a value V to its workers. The *relative* loss of profit when offering a higher value V_h , instead of V_l , to the worker, is given by $\frac{w(V_h, \delta) - w(V_l, \delta)}{p - w(V_l, \delta)}$. From the worker's indifference in 'compensating wage' equation (3), we know that, when two firms need to provide the same worker's value $V > V_0$, the safer firm can do so while paying a lower wage, i.e. $w(V, \delta_l) < w(V, \delta_h)$, making a higher profit per worker at value V . Moreover, equation (5) has told us that a change of value in riskier firms requires a larger change in wage than in safer firms; for $V_h > V_l$, we have

$$\forall V, \left. \frac{dV(w, \delta_h)}{dw} \right|_{V(w, \delta_h)=V} < \left. \frac{dV(w, \delta_l)}{dw} \right|_{V(w, \delta_l)=V} \implies w(V_h, \delta_h) - w(V_l, \delta_h) > w(V_h, \delta_l) - w(V_l, \delta_l). \quad (17)$$

Then, the relative loss of profit of offering larger values is larger for risky firms for two reasons: because they need to raise wages more (in the numerator), and because, to begin with, their profits at baseline value $V_l > V_0$ (in the denominator) are lower, so even if the required change in wages would be the same for the safe and risky firm, it would still lead to a larger relative profit loss still for the risky firm,

$$V_h > V_l \geq V_0 \iff \frac{w(V_h, \delta_h) - w(V_l, \delta_h)}{p - w(V_l, \delta_h)} > \frac{w(V_h, \delta_l) - w(V_l, \delta_l)}{p - w(V_l, \delta_l)}. \quad (18)$$

$$\iff \frac{p - w(V_h, \delta_h)}{p - w(V_l, \delta_h)} < \frac{p - w(V_h, \delta_l)}{p - w(V_l, \delta_l)}. \quad (19)$$

Now consider the optimal value V_r for a riskier firm, and the optimal value V_s for a safer firm. The riskier firm makes more profits at V_r than at V_s , while this is reversed for the safer firm. This means that

$$\frac{(p - w(V_s, \delta_l))l(V_s, \delta_l)}{(p - w(V_r, \delta_l))l(V_r, \delta_l)} \geq \frac{(p - w(V_s, \delta_h))l(V_s, \delta_h)}{(p - w(V_r, \delta_h))l(V_r, \delta)}. \quad (20)$$

But by equation (19) and the converse of equation (15), it follows from the optimality conditions for δ_h and δ_l -firms in inequality (20) that it must be that $V_s \geq V_r$, proving proposition 1.

Overall, intuitively, a safer firm gains relatively more in firm size from offering a higher value, while giving up strictly less (in relative terms) in profit per worker. Combined, a safer firm will gain strictly more in overall profit (or lose less) than the riskier firm when offering a higher value. Then, if V_r is optimal for the riskier firm, the safer firm will strictly gain when instead of offering $V < V_r$, it offers V_r itself (as the risky firm gains weakly from doing this). As a result, the safer firm will indeed optimally offer V_s or higher.

In the case that amenities would enter additively in the utility, and some firms could provide higher levels of the amenity more cheaply, these firms would also offer higher values in equilibrium, *ceteris paribus* (as is shown in Hwang et al. 1998). However, this effect would go entirely through the *denominator* of equation (18): when raising their value to workers, these firms would incur the same additional wage cost as any other firm copying the raise in worker value, they would also reap the same benefit in firm size as any other firm, but since their level of profit per worker is higher to begin with, they would lose less profit per worker in relative

terms. Compared to other firms, this makes it more profitable for them to offer higher values. In our case, there are two additional forces that, crucially, all work in the same direction: for the safer firm, the relative gain in firm size is larger when raising the value, in equation (15), and they need to raise wages less (in absolute terms) to provide the additional value, in equation (17).

While this establish that safer firms offer better values, we cannot make inferences yet about the *actual wages* posted – the higher job security of safer firms by itself might (more than) deliver the higher value required in equilibrium. Below will be able to explicitly link firm types to wages, in a tractable way. Towards this, let us first define the state state equilibrium properly, and then take several steps to characterize this equilibrium.

Definition 1. *The **steady state equilibrium** in this labor market consists of distributions $\hat{F}(w|\delta)$, $\tilde{F}(V, \delta)$, $\tilde{G}(V, \delta)$, $F(V)$, $G(V)$; an unemployment rate u ; a value function $V(w, \delta)$ for employed workers, and a value V_0 and reservation wage R_0 for unemployed workers, such that*

1. *workers' utility maximization: optimal mobility decisions result in a value function $V(w, \delta)$ for employed workers, and a reservation wage R_0 , with associated value V , according to equations (5)-(9), given $\hat{F}(w|\delta)$ and $H(\delta)$.*
2. *Firms' profit maximization: given $F(V)$, $G(V)$ and $V(w, \delta)$, for each δ , $\exists \pi$ such that $\forall w \in \text{supp } \hat{F}(w|\delta)$, it holds that $\pi = (p-w)l(V(w, \delta), \delta)$ and $\forall w \notin \text{supp } \hat{F}(w|\delta)$, $\pi \geq (p-w)l(V(w, \delta), \delta)$, where $l(V, \delta)$ is given by (13).*
3. *steady state distributions follow from individual decisions aggregated up. For firms: $\tilde{F}(V, \delta)$ is derived from $\hat{F}(w|\delta)$ and $H(\delta)$ using $V(w, \delta)$. For workers: $\tilde{G}(V, \delta)$, and u follow from the steady state labor market flow accounting in (12)-(14). 'Aggregate' value distributions $F(V)$ and $G(V)$ follow from $\int_{V' \leq V} d\tilde{F}(V, \delta)$, and $\int_{V' \leq V} d\tilde{G}(V, \delta)$.*

Adapting the proofs in BM and Bontemps et al. to incorporate the heterogeneity in δ , we can show that $F(V)$ is a continuous, strictly increasing distribution function, and so is $G(V)$. The intuition for this also follows the aforementioned papers: mass points in the distribution of offered wages or intermediate intervals where no firms offer wages, allow discrete gains in firm size or profit per worker, while the costs of such deviation can be made arbitrarily small.

Proposition 2. *In equilibrium, we can derive the following about derived distribution $F(V)$: (i) The support of the distribution of values offered in equilibrium is a connected set, (ii) there are no mass points in $F(V)$, (iii) the lowest value offered is V_0 , i.e. $F(V_0) = 0$. Properties (i)-(iii) likewise hold for $G(V)$ derived from $F(V)$ and (12).*

This implies that $F(V)$ and $G(V)$ are strictly increasing, continuous functions between V_0 (with $F(V_0) = G(V_0) = 0$) and some \bar{V} (with $F(\bar{V}) = G(\bar{V}) = 1$). Combining proposition 1 and proposition 2, the conditional distribution function $\hat{F}^{-1}(\delta|V)$ has all probability mass concentrated at a unique δ . Conversely, if $H(\delta)$ has a continuous probability density, it also follows that each δ posts a unique value. This does not imply that an *actual* wage w is offered by at most one δ -type of firm: overlaps in the actual wage distribution (with concomitant wage cuts in transitions) are possible, as we show below.

One of the strengths of the results is that, although now workers and firms are affected by two dimensions, wages and job security, and the valuation of the latter is endogenous, solving for the firm size distribution and subsequently the equilibrium wage distributions can be done (almost) as easily and as explicitly as in BM. We turn to this now.

2.3 Equilibrium Firm Sizes

Already employed workers move to new jobs in different firms when these offer a higher value of employment. Thus, for worker mobility, the level of worker's value itself is not relevant, it only matters how it is ranked relative to the other firms' values (and to the value of unemployment V_0). Now, in addition, the 'ranking property' in proposition 1 tells us that the firm with rank z in the value distribution $F(V)$ must be the firm with unemployment risk that has rank $1 - z$ in firm-type distribution $H(\delta)$ in equilibrium (since values offered are strictly decreasing in layoff risk). It follows that the firm's rank z captures both the workers' job-to-job and the job-to-unemployment mobility and, as a result, we are able to solve for equilibrium firm sizes as a function of rank z only, without any further reference to equilibrium wages paid or value levels.

Formally, define firm rank z as $z = F(V)$; by proposition 2, we have the $F(V)$ is continuous and strictly increasing, so $V(z) = F^{-1}(z)$ exists, and is unique. We can also define the firm's layoff risk as a function of *equilibrium* firm rank in the value distribution, $\delta(z)$; in equilibrium this is the layoff risk associated with z^{th} firm, starting from the most *risky* firm.¹⁷ Similarly, define $G^z(z) = G(V(z))$ as the proportion of employed workers at firms ranked z or lower. Moreover, using that $\hat{F}^{-1}(\delta|V)$ concentrates mass at a unique δ , and the absolute continuity of $F(V)$, which both follow from proposition 2, we can write down an intuitive

¹⁷To formally deal with mass points in $H(\delta)$, define $\bar{H}(\delta)$ as the closed graph of $1 - H(\delta)$, then let $\delta(z) \stackrel{def}{=} \max\{\delta | conv(\bar{H}(\delta)) = z\}$. Taking the maximum here is without loss of generality for our results, since alternative assumptions at points where the convex closure of $H(\delta)$ is an interval, would change δ only for a zero measure of firms.

correspondence between firm size and rank as follows, given $z = F(V)$,

$$\begin{aligned} G^z(z)(m-u) &= G(V)(m-u) = \int_{V' \leq V} l(V, \delta) dF(V, \delta) \\ &= \int_0^z \frac{\lambda_0 u + \lambda_1 G^z(z')(m-u)}{\lambda_1(1-z') + \delta(z')} dz' \end{aligned} \quad (21)$$

Though the necessary substitutions above involve a surplus of notation, the intuition of the firm size is straightforward: the steady state firm size is given by the ratio of worker inflows to the rate of outflows, now functions of ranking z only.¹⁸ Rank z determines layoff risk through equilibrium mapping $\delta(z)$, while proportion of employed workers below rank z , $G^z(z)$, is implicitly defined by equation (21) itself. From the last line in (21), it follows that the individual firm size $l(z)$ is given by

$$l(z) = \frac{dG^z(z)}{dz}(m-u) = \frac{\lambda_0 u + \lambda_1 G^z(z)(m-u)}{\lambda_1(1-z) + \delta(z)}. \quad (22)$$

Comparing this to firm size in (13), $l(V, \delta) = (\lambda_0 u + \lambda_1 G(V)(m-u))/(\lambda_1(1-F(V)) + \delta)$, using the continuity of $F(V)$ and $G(V)$, we see that a reformulation of firm size in terms of firm rank z is as intuitive as before. Inflows in the numerator of (22) consist of a flow from unemployment $\lambda_0 u$, and a flow from the measure of workers who are employed by lower-ranked firms, $(m-u)G^z(z)$, who would move to the firm whenever they get the chance, which depends on offer arrival rate λ_1 . The outflow *rate*, in the denominator, is determined by the amount of higher-ranked firms $(1-z)$, the rate at which workers meet these firms, λ_1 , and the firm's own layoff rate, $\delta(z)$. Proposition 2 tells us that $F(V)$ is strictly increasing and continuous on a connected support of values V , and hence the mapping $z = F(V)$ is invertible, continuous and strictly increasing as well, allowing us to change the variable to z in employment distribution function $G^z(z)$ in (21), and in firm size $l(z)$ in (22) (using the steps in footnote 18, where proposition 1 allows us to construct the mapping $\delta(z)$).

¹⁸To see the substitutions needed to yield the result in equation (21), consider

$$\begin{aligned} \int_{V' \leq V} l(V, \delta) dF(V, \delta) &= \int_{V_0}^V \int_{\delta} \frac{\lambda_0 u + \lambda_1 G(V')(m-u)}{\lambda_1(1-F(V')) + \delta} d\hat{F}(\delta|V') dF(V') \\ &= \int_{V_0}^V \frac{\lambda_0 u + \lambda_1 G(V')(m-u)}{\lambda_1(1-F(V')) + \delta(F^{-1}(V'))} dF(V') \\ &= \int_0^z \frac{\lambda_0 u + \lambda_1 G^z(z')(m-u)}{\lambda_1(1-z') + \delta(z')} dz' = \int_0^z l(z') dz' \end{aligned}$$

where in the first step, we split up $dF(V, \delta)$ into $dF(\delta|V)dF(V)$, after which proposition 2 tells us that $F(\delta|V)$ all mass concentrated at one and only one δ . In the next line, we substitute this into the function $\delta(z)$ (which uses the type-ranking of proposition 1 and the strict monotonicity of $F(V)$ of proposition 2), leaving us with an integration over V' . Finally, the last line incorporates the change of variable to $z = F(V)$.

The implicit relationship between z and $G^z(z)$ in equation (21) is the solution to differential equation

$$\frac{dG^z(z)}{dz} = \frac{\frac{\lambda_0 u}{m-u} + \lambda_1 G^z(z)}{\lambda(1-z) + \delta(z)} \quad (23)$$

with initial condition $G^z(0) = 0$, which follows directly from (22). Note that there is no reference to another equilibrium object in this formulation of the distribution but $G^z(z)$ itself. Given this, we can now solve *explicitly* for firm size $l(z)$ and distribution $G^z(z)$, which is done in the next lemma.

Lemma 3. *The cumulative density function $G^z(z)$, and equilibrium firm size $l(z)$ and measure of unemployed, are given by: $u = \frac{m \int_0^1 \delta(z)g(z)dz}{\lambda_0 + \int_0^1 \delta(z)g(z)dz} = (\lambda_0)^{-1} \int_0^1 \delta(z)l(z)dz$, and*

$$G^z(z) = \frac{\lambda_0 u}{\lambda_1(m-u)} \left(e^{\int_0^z \frac{\lambda_1}{\lambda_1(1-z') + \delta(z')} dz'} - 1 \right), \quad (24)$$

$$l(z) = \frac{\lambda_0 u}{\lambda_1(1-z) + \delta(z)} e^{\int_0^z \frac{\lambda_1}{\lambda_1(1-z') + \delta(z')} dz'}. \quad (25)$$

The dependence of the size of the z th-ranked firm on the unemployment risks of all lower ranked firms is explicit in the integral term in the exponent. Solving firm size as a function of the firm rank does not only yield a clean and simple expression for firm size, because it does rely on a minimal set of relevant parameters, it also allows for general probability distributions of firm types through $\delta(z)$. Concretely, this means that the formulation in lemma 2 can deal with discrete distributions as easily as it can deal continuous distributions, or any mixture of these. We expect that this approach can be applied more generally when a firm-specific factor affects the firm size separately from the rank that the firm occupies in the workers' preference. This would work as long as one is able to establish a mapping between this factor and the equilibrium *rank* in the firm distribution of values offered to workers, as proposition 1 does for unemployment risk.¹⁹

¹⁹We can be more explicit in the case of a pure discrete and a purely continuous distribution. For a continuous probability density $h(\delta)$ with $H'(\delta) = h(\delta) > 0$, $\delta(z)$ is differentiable everywhere with $\delta'(z) > 0$, and with the appropriate change of variable, this results in

$$l(\delta) = \frac{\lambda_0 u}{\lambda_1 + \bar{\delta}} e^{\int_{\bar{\delta}}^{\delta} - \left(\frac{2\lambda_1 h(\delta) + 1}{\lambda_1(1-H(\delta)) + \bar{\delta}} \right) d\bar{\delta}}.$$

In case of a discrete distribution $h(\delta_j), j = 1, \dots, J$, with $\sum_j h(\delta_j) = 1$ and $\bar{\delta} = \delta_1 > \dots > \underline{\delta} = \delta_J$, lemma 3 tells us that the mass of workers in δ_i firms, $v(\delta_i)$ can be derived from (24), using $e^{\int_a^b \frac{\lambda_1}{\lambda_1(1-z) + \delta} dz} = \frac{\lambda_1(1-z_a) + \delta}{\lambda_1(1-z_b) + \delta}$. Suppose that $\sum_{i=1}^{j-1} h(\delta_i) < z < 1 - \sum_{i=j+1}^J h(\delta_i)$ for some j . Then from (24),

$$G(z) = \frac{\lambda_0 u}{\lambda_1(m-u)} \left(\frac{\lambda_1(1 - \sum_{i=j}^J h(\delta_i)) + \delta_j}{\lambda_1(1-z) + \delta_j} \prod_{i=1}^{j-1} \frac{\lambda_1(1 - \sum_{h=i}^J h(\delta_i)) + \delta_i}{\lambda_1(1 - \sum_{h=i+1}^J h(\delta_i)) + \delta_i} - 1 \right)$$

2.4 Equilibrium Wage Distributions

In the previous section, we have derived equilibrium firm size as a function of firm rank in the value distribution. In this section, we link these firm sizes to the wage setting of firms. Instead of formulating the firm's problem as choosing the optimal wage (which in equilibrium implies a certain rank $z = F(V(w, \delta))$), we formulate it equivalently as the firm choosing a rank z (which in equilibrium implies a wage w and an associated value V). Firm optimization in equilibrium requires that a firm that is ranked z by job security does not want to deviate to another rank z' in the value distribution. Therefore, a first order condition that needs to hold for each firm in equilibrium pins down how locally values change with firm rank. This condition, however, depends on the wage w and on the value V at a given firm rank. Hence, we need to keep track of both wages and values as a function of firm rank; this introduces a second condition that needs to be satisfied in equilibrium. It makes our setting different from the standard model, where one equation is sufficient to characterize equilibrium wages (given initial condition R_0). Here, we show that the resulting system of two ordinary differential equations helps to pin down all elements of the equilibrium. It also helps to establish the existence and uniqueness of the labor market equilibrium.

Concretely, we can set up the firm's profit maximization problem such that the firm chooses the rank z it wants to occupy in the firm distribution, given the equilibrium objects $V(z) = F^{-1}(z)$, $G^z(z)$, $l(z)$ and $w(V, \delta)$ (the inverse of $V(w, \delta)$).²⁰ In particular, firm optimization in equilibrium requires that no firm of rank z , with layoff risk $\delta(z)$, strictly prefers to offer a value $V(z')$ associated with a different rank z' in firm distribution $F(V)$, compared to the value, $V(z)$, associated with rank z . Formally, this means that in equilibrium, the following must be satisfied:

$$\max_{z'} (p - w(V(z'), \delta(z))) l^d(z', \delta(z)), \quad (26)$$

where $w(V(z'), \delta(z))$ is the wage the z -th-most risky firm, with its immutable unemployment risk $\delta(z)$, must pay to provide value $V(z')$. The term $l^d(z', \delta(z))$ is the steady state firm size of a deviating z -type firm which offers its workers a value $V(z')$ instead. Then the rank- z firm will have exactly the same measure of worker inflows as the rank- z' firm enjoys, and additionally also rank- z' firm's *outflow rate to other jobs*, but (again) the rank- z firm cannot do anything to change its own outflow rate *to unemployment*, $\delta(z)$. This means that

²⁰Using these functions, the resulting optimization problem is equivalent to firm optimization with respect to value V or wage w , taking as given the corresponding functions $F(V)$, $G(V)$, $l(V, \delta)$. There is one additional aspect that needs to be taken care of: deviations to values that are not part of the support of $F(V)$, which by proposition 2 means only values below V_0 , or above $V(1)$. This is easily verified: $V < V(0)$ implies zero profit, since no workers will ever find it optimal to accept employment; $V > V(1)$ implies less profit per worker than $V(1)$ with a firm size equal to the firm size when offering $V(1)$. Hence if a firm does not want to deviate to $V(1)$, it does not want to deviate to $V > V(1)$.

outflow rate of a z -firm deviating to $V(z')$, relative to the outflow rate of the z' -firm itself, is larger by a factor $\frac{\lambda_1(1-z')+\delta(z)}{\lambda_1(1-z')+\delta(z')}$, which implies that the size of the deviating firm is the size of the z' th-most risky firm divided by this factor:

$$l^d(z', \delta(z)) = \frac{\lambda_1(1-z') + \delta(z)}{\lambda_1(1-z') + \delta(z')} l(z') = \frac{\lambda_0 u}{\lambda_1(1-z') + \delta(z)} e^{\int_0^{z'} \frac{\lambda_1}{\lambda_1(1-\bar{z}) + \delta(\bar{z})} d\bar{z}}. \quad (27)$$

The firm size upon deviating in equation (27) can be formally derived using the firm size as a function of values in equation (13) in conjunction with lemmas 2 and 3. From equation (27), we see that $l^d(z', \delta(z))$ is differentiable in z' . The first order condition of profit in equation (26) with respect to z' , evaluated at the equilibrium choice, $z' = z$ is

$$(p - w(V(z), \delta(z))) \left. \frac{\partial l^d(z', \delta(z))}{\partial z'} \right|_{z'=z} - \left(\frac{\partial w(V(z), \delta(z))}{\partial V(z)} \frac{dV(z)}{dz} \right) l^d(z, \delta(z)) = 0 \quad (28)$$

(Second-order conditions that establish that this indeed maximizes profit are established in theorem 1 below.)

This can be rewritten as

$$\frac{\frac{\partial w(V(z), \delta(z))}{\partial V(z)} \frac{dV(z)}{dz}}{(p - w(V(z), \delta(z)))} = \frac{\left. \frac{\partial l^d(z', \delta(z))}{\partial z'} \right|_{z'=z}}{l(z, \delta(z))}. \quad (29)$$

On the right-hand side is the relative gain in firm size when incrementally increasing the ‘mimicked’ rank z' . It is profit-maximizing for the z th-most risky firm to be ranked z th in the firm value distribution $F(V)$ when the relative gain in firm size from marginally increasing rank at rank z is offset exactly by the relative profit loss per worker on the left-hand side. Rearranging first-order condition (29), we find that this is satisfied when

$$\begin{aligned} \frac{dV(z)}{dz} &= (p - w(V(z), \delta(z))) \frac{\left. \frac{\partial l^d(z', \delta(z))}{\partial z'} \right|_{z'=z}}{l(z, \delta(z))} \frac{\partial w(V(z), \delta(z))}{\partial V(z)} \iff \\ \frac{dV(z)}{dz} &= (p - w(V(z), \delta(z))) \frac{2\lambda_1}{\lambda_1(1-z) + \delta(z)} \frac{1}{r + \delta(z) + \lambda_1(1-z)}. \end{aligned} \quad (30)$$

Thus, equation (30) is a differential equation that captures how much the value needs to change with firm rank z to guarantee that rank- z firms indeed find it profit-maximizing to offer $V(z)$. It is interesting to briefly highlight the difference between firms at the bottom of the job security distribution and those higher up. Safer firms have a higher term $2\lambda_1/(\lambda_1(1-z) + \delta(z))$ in equation (30), which can be seen from the derivative of this term with respect to layoff risk δ , which is $-\frac{2\lambda_1}{(\lambda_1(1-z')+\delta)^2} < 0$.²¹ Safer firms thus compete more aggressively, *ceteris paribus*; this raises $dV(z)/dz$ on the left-hand side of equation (30). A higher $dV(z)/dz$ will be a force

²¹This term is the cross-derivative $\frac{\partial^2 \ln l(z', \delta)}{\partial z' \partial \delta} = -\frac{2\lambda_1}{(\lambda_1(1-z')+\delta)^2}$, which is the differential analogue (now in terms of rank z) to condition (15).

that also pushes up wages as a function of firm rank z . It is, however, not the entire story for wages, as the change in unemployment risk $\delta(z)$ also to be taken into account.

Note that equation (30) incorporates wage function $w(V(z), \delta(z))$. This mapping depends on *distribution* of values offered by firms and the distribution of unemployment risk across firms, as can be seen in section 2.1 on the workers' optimization, and hence is not an object that can be replaced by an explicit solution at this stage. However, the *change in wage* as a function firm rank z can be established more easily, leading to a second first-order differential equation, which combined with (30) gives rise to a system of two differential equations that are crucial to characterize the equilibrium.

The pattern of wages offered by differently ranked firm indeed takes into account not only how values change with firm rank in $V(z)$, but also how simultaneously these higher values can be delivered in part, or in whole, by the increased job security that a higher-ranked firm provides. As one moves through the ranks of firms the value offered by firms changes simultaneously with the job security these firms provide, while at any z , the wage must be given by $w(V(z), \delta(z))$. This means that at any $\delta(z)$ that is differentiable, we have ²²

$$\frac{dw(V(z), \delta(z))}{dz} = \frac{\partial w(V(z), \delta(z))}{\partial V(z)} \frac{dV(z)}{dz} + \frac{\partial w(V(z), \delta(z))}{\partial \delta(z)} \delta'(z). \quad (31)$$

Then, we can decompose the equilibrium wage change with firm rank in into two parts: the competition component coming from the firm's wage setting in the firm's first-order condition (29), $\frac{\partial w(V(z), \delta(z))}{\partial V(z)} \frac{dV(z)}{dz}$, and the effect through the composition of firms on the labor market; with $\delta'(z)$ derived from $H(\delta)$, capturing how fast job security increases with firm rank. Substituting (29) into the last expression, yields

$$\frac{dw(z)}{dz} = (p - w(z)) \frac{2\lambda_1}{\lambda_1(1 - z) + \delta(z)} + \delta'(z)(V(w(z), \delta(z)) - V_0). \quad (32)$$

While the firm's optimization pins down $\frac{dV(z)}{dz}$, increased job security of the higher ranked firm itself could deliver part of the increased value V . If workers value job security a lot, i.e. $V(w(z), \delta(z)) - V_0$ is high, or the higher-ranked firms' job security is much higher, i.e. $\delta'(z)$ is far below zero, this force can even lead to wage cuts.

The strength of these forces varies with the rank of the firm in the firm-level job-security distribution. For firms with high job security, the value of employment is significantly different from the value of unemployment; this means that the term $V(w(z), \delta(z)) - V_0$ is relatively large. However, simultaneously, for these safer firms, the gains of holding on to their workers are larger, and hence these firms will compete more fiercely,

²²Since it is derived from the cumulative density function $H(\delta)$ the function $\delta(z)$ is differentiable a.e. With abuse of notation, we use $\delta'(z)$ as a function that is defined everywhere, and consistent with the derivative of $\delta(z)$ almost everywhere. Moreover, note that $V(z)$ is continuous everywhere; if there is a discontinuity in $\delta(z)$ at z , 'compensating wage' indifference (3) will tell us the size of the discrete drop in wages at this z (as stated more explicitly in theorem 1).

as argued below equation (29). It is therefore still not a foregone conclusion whether wages paid will rise or fall with firm-level job security, nor is it immediate that a greater valuation of job security on the worker side will indeed lead to wage cuts, as it simultaneously also raises the value of retaining a worker to the firm. One can see that $\delta'(z)$ can potentially play an important role here, scaling $V(w(z), \delta(z)) - V_0$, and therefore the strength of the forces. In the next section, we study the occurrence or absence of wage cuts in exchange for job security: both can occur depending in part on the distribution of firm types, which is an important determinant of the extent of competition among firms. The workers' marginal rate of substitution between wages and job security (derived in section 2.1) by itself is only half of the story; firms' imperfect competition is the other half.

Let us finish this section by putting all pieces together: we can find $(w(z), V(z))$ as the solution a system of *two* differential equations, one from using (28) which tells us $\frac{\partial V(z)}{\partial z}$, and (32) combined with (3), which tells us $\frac{\partial w(z)}{\partial z}$, both as functions of parameters, distributions and wages $w^s(z), w(z)$ themselves. The solution to this system of differential equations $\{V(z), w(z)\}$, in combination with the appropriate initial conditions fully characterizes the equilibrium. Moreover, we are able to establish the existence and uniqueness of this solution, and therefore of the labor market equilibrium, as spelled out in definition 1

Theorem 1 (Existence, Uniqueness, Characterization). *Consider functions $\{w(z), V(z)\}$, and $R_0 \in \mathbb{R}$ (and the associated $V_0 = \frac{\lambda_0 R_0 - \lambda_1 b}{r(\lambda_0 - \lambda_1)}$), such that $w(z), V(z)$ are a solution to the system of two ODEs, with, for all z at which $\delta(z)$ is continuous,*

$$\frac{dV(z)}{dz} = (p - w(z)) \frac{2\lambda_1}{\lambda_1(1-z) + \delta(z)} \frac{1}{r + \delta(z) + \lambda_1(1-z)} \quad (33)$$

$$\frac{dw(z)}{dz} = (p - w(z)) \frac{2\lambda_1}{\lambda_1(1-z) + \delta(z)} + \delta'(z)(V(z) - V_0), \quad (34)$$

and a jump discontinuity at every \hat{z} such that $\lim_{z \uparrow \hat{z}} \delta(z) > \delta(\hat{z})$, $w(z)$ will jump down according to

$$w(z) = \lim_{z \uparrow \hat{z}} w(\hat{z}) - \left(\delta(z) - \lim_{z \uparrow \hat{z}} \delta(\hat{z}) \right) (V(z) - V_0), \quad (35)$$

$$V(z) = \lim_{z \uparrow \hat{z}} V(\hat{z}) \quad (36)$$

under initial conditions $w(0) = R_0$, and $V(0) = V_0$, where R_0 additionally satisfies

$$R_0 = b + (\lambda_0 - \lambda_1) \int_0^1 (V(z) - V_0) dz = b + (\lambda_0 - \lambda_1) \int_0^1 (1-z) \frac{dV(z)}{dz} dz \quad (37)$$

Denote the inverse of $V(z)$ as $F(V)$. This distribution, and $G(V(z)) = G^z(z)$, value functions $V(w, \delta)$, and $u, \hat{F}(w|\delta), G(V, \delta), F(V, \delta)$, all constructed from $\{w(z), \tilde{V}(z), R_0\}$ are the functions associated with the steady state equilibrium in the environment; this steady state exists and is unique.

In this setting it has been necessary to follow a path different from BM and Bontemps et al. towards characterizing the equilibrium wage distribution: neither wages or values alone are sufficient to characterize the equilibrium. How powerful competition is driving up the values offered to workers depends on wages through the instantaneous profit flows $p - w$, and it depends on the job security of the firm in question. On the other hand, how wages co-move with job security depends on the valuation of job security, which consist of the job value lost when losing a job, and how likely this transition is. However, not all elements of equilibrium depend on both wages and values: firm size only depends on the rank of the firm and its associated job security. Exploiting this, we are able to solve for equilibrium firm sizes first, and then simultaneously find the wages and value distributions that have to arise with these firm size.

Given the characterization in theorem 1, it is perhaps insightful to compare the case with heterogeneous unemployment risk to the standard case in BM and Bontemps et al. without this heterogeneity ($\delta(z) = \tilde{\delta} \forall z$). In the absence of heterogeneity in $\delta(z)$, the two equations (33) pre-multiplied by dw/dV and (34) are identical to each other: the differential equation $w'(z) = (p - w(z))(2\lambda_1/(\lambda_1(1 - z) + \delta))$ with initial condition $w(0) = R_0$ has solution

$$\frac{p - w(z)}{p - R_0} = \left(\frac{\lambda_1(1 - z) + \delta}{\lambda_1 + \delta} \right)^2 \implies F(w) = \frac{\delta + \lambda_1}{\lambda_1} \left(1 - \left(\frac{p - w}{p - R_0} \right)^{0.5} \right), \quad (38)$$

which, on the LHS, is precisely the wage distribution in BM.²³

Note that one way of showing existence and uniqueness in Burdett-Mortensen model would combine the left expression in (38) with (37), to show that $T(R_0)$ is linear in R_0 , while $T(p) = b$, and $T(r) < r$, for r small enough. The proof of theorem 1 relies on the same method: we can rescale the differential equation (34) by $p - w$, and show that the term $\frac{V(z) - V_0}{p - w(z)}$, which captures closely how the overall rent is divided up between firm and worker, is independent of R_0 , and therefore just a function of parameters, unemployment risk distribution $H(\delta)$ and firm rank z . Then, the strategy of the proof is to show that – even though the problem has two dimensions– there still exists a (now more involved) term $A(z)$ which again depends only on parameters, $H(\delta)$, and firm rank, such that $p - w(z) = (p - R_0)A(z)$. From this the continuity of $T(R_0)$ in R_0 , needed for the existence proof, can be established, and so can the uniqueness.

²³Exploiting the ranking property inherent in BM-type models allows one to incorporate more heterogeneity in standard models. Here, we deal with firm heterogeneity that cannot be incorporated straightforwardly in the standard model, where e.g. there is a simple, unique one-dimensional mapping between wages and worker's values. Moscarini and Postel-Vinay (2010) exploit a similar ranking property to deal with time-varying aggregate productivity, an otherwise notoriously difficult problem.

3 Wages and Transition Hazards

In the previous section, we derived equations which characterized how wages, worker's values, and firm qualities are linked in equilibrium. In this section, we look more concretely at the labor market outcomes implied by the characterization.

First, since safer jobs are more attractive jobs, workers in safe jobs are much less likely to separate from these jobs, whether to unemployment or to another job. This implies the following (where we have, once again, relegated all proofs to the appendix),

Result 1. *The transition rate into unemployment as a function of tenure is decreasing.*

To condense language, we will refer to this particular transition rate as the unemployment hazard. Thus, the standard BM model, augmented with firm heterogeneity in unemployment risk, is able to reproduce an unemployment hazard that in the aggregate declines with firm tenure (as well as with the overall duration of the incomplete employment spell), as it does in the data.²⁴

Next, we consider the relationship between the unemployment risk a worker faces, and the wage he receives. If wages are increasing in the job security that the firm offers, there is in some sense a *strong failure* of compensating wage differentials: not only do riskier firms offer lower employment values (established in proposition 1), but in fact they offer values so much lower that in addition to a higher unemployment risk they actually pay lower wages. For those jobs at the bottom of the wage distribution, we can derive the following, without restrictions on parameters or the firm distribution.

Result 2. *The lowest wage, R_0 , is paid by the firm with the highest unemployment risk. There exists a nontrivial interval of wages $[R_0, \hat{w}]$ where job security increases with wages*

Under typical conditions, spelled out next, this interval can span a large part of the wage distribution, while on the other hand, wage cuts can also occur higher up in the wage distribution. For analytic simplicity and to be consistent with steady state profit maximization, we let $r \rightarrow 0$, and consider mainly the case of a distribution of firm unemployment risk with a differentiable p.d.f. $h(\delta)$.

Result 3. *In equilibrium the relation between wages and job security depends on the firm distribution of unemployment risk in the following way:*

1. *Wages increase with job security if $h'(\delta) \leq 0$ (i.e. $\frac{dw}{dz} > 0$ at \tilde{z} , where $\tilde{z} = 1 - H(\delta)$.)*

²⁴See e.g. Menzio et al. 2012.

2. *Wage cuts for increased job security will occur if*

$$\frac{h(\delta)}{\delta + \lambda(1 - H(\delta))} < \int_{\delta}^{\bar{\delta}} \frac{h(\tilde{\delta})}{(\tilde{\delta} + \lambda(1 - H(\tilde{\delta})))^2} d\tilde{\delta} \quad (39)$$

The first point highlights that wage cuts will not occur for all firms at or above the mode of the distribution (or the right-most local maximum in a multimodal distribution), and will not occur at any point during the distribution if the density is (weakly) decreasing in unemployment risk at any δ . Some wage cuts can occur, e.g., when densities $h(\delta)$ have a thin left-tail with sufficient kurtosis, where $\frac{h(\delta)}{\delta}$ rises sufficiently fast in the left tail. In particular, for any distribution with $\frac{h(\delta)}{\delta} < \int_{\delta}^{\bar{\delta}} \frac{1}{\delta} \frac{h(\tilde{\delta})}{\tilde{\delta}} d\tilde{\delta}$, wage cuts will occur when λ_1 is small enough.²⁵

In figure 1, we have drawn, in the left panel, the density of log-normal distributions over unemployment risk. In the right panel, wages are drawn as a function of underlying unemployment risk, for a particular set of parameters.²⁶ On the x-axis in the right-hand panel is unemployment risk, and therefore job mobility occurs from right to left. Hence, wage cuts can occur at those unemployment risks δ in the right panel where the graph is upward-sloping, when the worker is previously employed at a firm with a not too distinct unemployment risk. Wages increase with unemployment risk for sure wherever the graph is decreasing. Note that indeed the firm distribution with almost completely decreasing density does not generate any wage cuts, but for the other two distributions, with the clear left tails, some wage cuts occur when moving from relatively safe firms to the very safest firms. At the lowest wages, which come with significantly higher unemployment risk, no wage cuts are observed. At these wages, who are taken in relatively large proportion by the unemployed, we see a complete absence of compensating wage differentials. In general, the complementarity between wages and job security works both on the firm's and worker's sides, and can push actual wages either up or down, depending on the distribution. We think that this is a nice illustration of the value of studying unemployment risk and wage setting in a full-fledged labor market equilibrium setting, as the willingness to take wage cuts for safety for workers could be offset by the increased competition by firms for the, *ceteris paribus*, longer-employed and thus more more valuable workers.

The degree to which firms are in competition with each other is linked to parameter λ_1 : an increase in this parameter makes it easier for higher ranked firms to poach workers from lower-ranked firms, and thus raises firm competition. One could think that an increase in competition among firms will lead firms to offer less

²⁵In case of a discrete distribution, the existence of wage cuts will follow directly from condition (35) in theorem 1. We discussed this type of wage cuts extensively in a previous version of the paper. This case however is intuitively close to the case where $h(\delta)$ is very close to zero on an interval, in this case (39) implies that for $h(\delta)$ small enough over an interval, wage cuts will also occur.

²⁶Note that the proof of theorem 1 establishes that the shape of the function that links wages to unemployment risk does not depend on R_0, b, λ_0 .

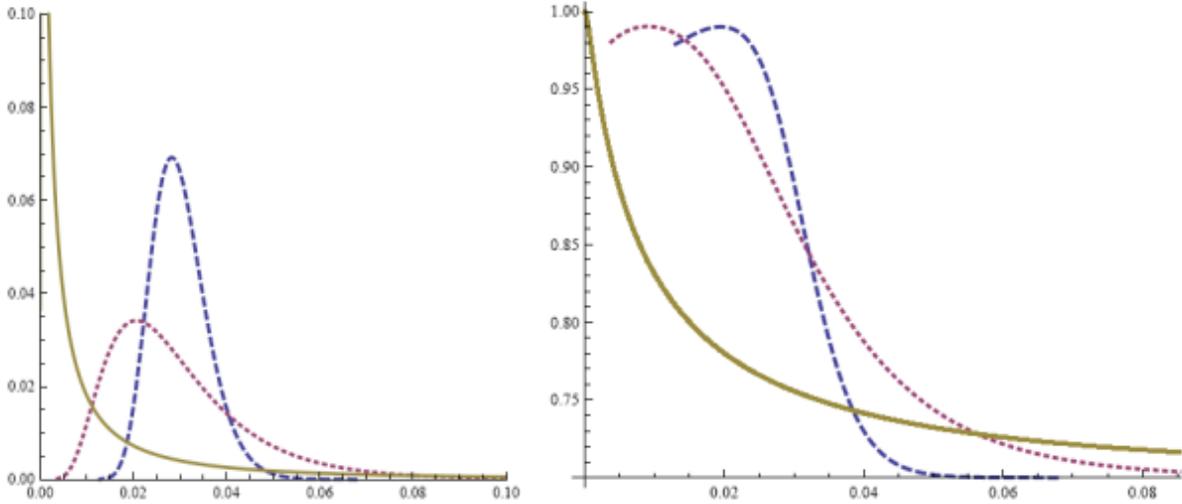


Figure 1: Left panel: log-normal p.d.f.s of δ , with standard deviation 0.03 (dashed), 0.2 (dotted), 2.0 (safe). Right panel: wage as a function of unemployment risk (x-axis) ($\lambda_1 = 0.23, r = 0.0025, R_0 = 0.7, p = 1$)

dispersed employment values in equilibrium and hence trace out more closely the workers' marginal rate of substitution between job security and wages. This turns out not to occur; instead we can find a lower bound on $\tilde{\lambda}_1$ for a given distribution $H(\delta)$ (with $h'(\delta)/h(\delta)^2$ bounded from above), such that above this $\tilde{\lambda}_1$, for any λ_0 and b , wages will be increasing in job security throughout.

Result 4. *If $\lambda_1 > h'(\delta)/(h(\delta))^2$, wages will be increasing in job security at δ ($dw(z)/dz > 0$ at $z = 1 - H(\delta)$), for any λ_0, b . If $h'(\delta)/(h(\delta))^2$ is bounded from above, there exists $\tilde{\lambda}_1$ such that for all $\lambda_1 > \tilde{\lambda}_1$ wages are increasing in job security for all δ , for any b, λ_0 .*

Thus, as the labor market gets more competitive, the scope for wage cuts disappears. In this result, we keep b and λ_0 constant²⁷; while it becomes progressively easier for employed workers to move from job to job, unemployed workers keep leaving unemployment at the same rate. This keeps the cost of losing one's job bounded away from zero, even as λ_1 becomes very large. (In the limit: $V_1(\underline{\delta}) - V_0 = \frac{p-b}{r+\lambda+\underline{\delta}}$.) Thus when λ_1 becomes large enough, the increased competition between firms will drive up wages with job security throughout the *entire* wage distribution, even though workers keep experiencing a loss of lifetime utility when becoming unemployed. Increased competition among firms does not lead to the payment of compensating wage differentials, it does, quite surprisingly, lead to the opposite, as it strengthens the motive of the low- δ firms to compete with similar firms. To prove this, we heavily rely on the ranking property, which holds for

²⁷Result 4 is actually stronger, it says that this bound on λ_1 will hold, entirely independent of λ_0 and b .

every λ_1 , and thus the firm ranking is preserved throughout any limit taking with respect to λ_1 (and λ_0). Thus, we can calculate firm sizes easily as a function of the rank of the firm as we approach the limit without having to recalculate the wage distribution. In turn, firm profit maximizing decisions are then still easily characterized, following theorem 1, even as we move towards the limit, $\lambda_1 \rightarrow \infty$.

We can also study the case where we let search frictions for both unemployed and employed workers disappear in the limit.

Result 5. *Let $\lambda_0 > \lambda_1$, $\lambda_1 \rightarrow \infty$, $\lambda_0 \rightarrow \infty$, while keeping $\frac{\lambda_1}{\lambda_0} = \alpha < 1$ constant. Then $w(z) \rightarrow p$ for all z .*

If we let the frictions for the unemployed disappear, we converge to the same limit as in the standard BM model without heterogeneity in δ , which equals competitive outcome $w(z) = p \forall z$, again without any compensation for unemployment risk. To see this, note that for reservation wage out of unemployment, R_0 , the following holds

$$\frac{R_0 - b}{p - R_0} = (\alpha - 1) \int_0^1 \frac{\lambda_1(1-z)}{\bar{\delta} + \lambda(1-z)} \frac{\bar{\delta} + \lambda_1(1-z)}{\delta(z) + \lambda_1(1-z)} \frac{2\lambda_1}{\delta(z) + \lambda_1(1-z)} \frac{p - w(z)}{p - R_0} dz \quad (40)$$

$$\geq (\alpha - 1) \int_0^1 \frac{2\lambda_1^2(1-z)}{(\bar{\delta} + \lambda(1-z))^2} dz = -2 \frac{\lambda_1}{\lambda_1 + \bar{\delta}} + 2 \log \frac{\bar{\delta} + \lambda_1}{\bar{\delta}} \quad (41)$$

As we let λ_1 (and $\lambda_0 = \lambda_1/\alpha$ with it) go to infinity in (40), it follows that $R_0 \rightarrow p$, as the RHS goes to infinity. Since $p > w(z) \geq R_0$, it follows that all wages go to p . Since the bound in result 4 is uniform in λ_0 , we also know that for λ_1 large enough, wages will become increasing in job security for all δ in the process.

As we approach the competitive limit no compensating wages are paid. In the case where search frictions also disappear in the limit for the unemployed, job security will cease to be a payoff relevant dimension for workers. This is intuitive because, apart from the loss of ‘search capital’, there is no additional cost to unemployment.²⁸ Decreasing λ_1, λ_0 means that search frictions become more important, which implies that job security becomes more important, and competition between firms becomes more limited, which raises the potential for wage cuts. Thus somewhat ironically, wage cuts, which seem to relate closely to the notion of compensating wages paid in competitive settings, are in the environment we study actually associated with a low degree of competition among firms. Though ironic, the result is intuitive: a low λ_1 means that climbing up the job ladder is a slow process in which gains are lost when becoming unemployed; therefore, at a lower λ_1 , workers will value job security more, ceteris paribus. Likewise, a lower λ_1 lowers the competition among firms, i.e. the relative gains of being higher in the value ranking are lower when λ_1 is low, hence higher ranked firms will not increase the values that they offer workers as much. This, however, does not mean that

²⁸If a transition into unemployment comes with an explicit cost instead or in addition to a search cost, then in the limiting economy only the low turnover firms would survive.

result 4 immediately follows from the intuition: we need to use, explicitly, the equilibrium characterization, because the lower values offered by the firms due to the lower competition reduce the valuation of job security, potentially more than offsetting the direct (ceteris paribus) effect of the decrease in λ_1 on the workers' valuation of job security. However, result 4 implies that this is not the case, and with a lower λ_1 more cases can occur where firms with higher job security promise a lower value increase than is delivered by their increased job security alone, and as a result will offer lower wages, thus leading to wage cuts in equilibrium.

4 Discussion

Above, we have established that for a substantial portion of (standard) wage distributions, wages will be increasing in job security, thus providing a foundation for the observed lack of compensating wage differentials for job security. The theory emphasizes that, in particular in the lower part of the wage distribution, the forces that push towards the positive correlation between wages and job security are strong. A further implication is that the unemployed are the predominant takers of the riskiest jobs which also have a low pay, implying that unemployed workers are particularly vulnerable for 'no-pay/low-pay'-cycles in their subsequent labor market outcomes. When treating job security as any other amenity, entering additively separate in the utility function, the locus of wage cuts appears less clear: wage cuts could occur at any part of the distribution, while, on the flip-side, an overall positive correlation could also be generated in any part of the wage offer distribution. In contrast, the endogenous complementarity in our setting closely ties the low-pay to the no-pay in workers' histories.

Behind this, we want to emphasize, on-the-job search plays an important role: by itself, it creates heterogeneity in worker's rent and in the expected match termination rate due to employer-to-employer flows. Workers in different firms take jobs in other firms at different rates, and this crucially interacts with heterogeneity in the rate at which firms send workers into unemployment. Workers care more about job security, ceteris paribus, when the expected match duration is longer (due to lower outflow rate to other jobs) and firms care relatively more about retention when the expected match duration is longer (due to a lower separation rate into unemployment). This complementarity is missed in models that only consider unemployment-to-employment flows and vice versa.

The randomness of the search technology, on the other hand, does not appear to be essential for our results. In Pinheiro and Visschers (2013), we show in a *directed* search setting with job-to-job mobility adapted from Delacroix and Shi (2006), that safer firms offer higher values than riskier firms. At work are the same forces as before, including the aforementioned complementarity between outflows to other jobs and outflows

to unemployment. Because of the job-to-job mobility, workers are again heterogeneous in the lifetime value they have in their current employment. In higher-value jobs they optimally direct their search towards jobs with even better conditions – but these come with a lower matching probability. As before, since safer firms care more about worker retention, they have an additional incentive to offer the high values that attract employed workers. Unemployed workers care the least of all workers about the unemployment risk of the jobs, and target employment in risky firms. It then follows that mobility patterns would look similar to our setting, to the extent that unemployed workers end up in the most risky firms, and long-term employed workers will only switch to safe firms.

To put our results in a wider perspective: it would be of great interest to further distinguish generally and quantitatively among the sources of patterns of repeated unemployment and persistence in future low pay of the unemployed.²⁹ This matters because, each underlying cause has different implications how risk is distributed in the labor market. If workers are heterogeneous in the propensity to become unemployed, i.e. if some workers have more trouble than others to hold a job, a more ‘stable’ worker who nevertheless has become unemployed by a stroke of bad luck will have better prospects than the typical unemployed. The danger of repeated falls of the job ladder into unemployment would not apply to the same extent to him.³⁰

The concentration of actual inflows into unemployment across workers and firms (that send workers into unemployment) is likely to tell us more about the importance of each of these dimensions, but the picture is made murkier by the force of selection on unobservables on both the workers’ and firms’ side of the market, and also on unobservable match quality. Repeated unemployment spells and low-wage employment spells for a subset of individuals can arise not only due to worker heterogeneity, but can also occur because workers who have become unemployed will accept different jobs, in different firms, than long-employed workers. In particular, the jobs that unemployed workers take, will include jobs with lower wages and a higher risk of becoming unemployed again.

It is possible to make inferences about the extent of selection by looking in more detail e.g. at the labor

²⁹Perhaps in part because of a strong intuition about the need for compensating differentials to arise when the only heterogeneity is on the firm side, firm heterogeneity in unemployment risk does not appear to us to have been considered on the same level as other explanations in the more applied literature focussed (narrowly) on the effect of becoming unemployed on future labor market outcomes.

³⁰A correlation between wages and job security could occur because low-ability workers are also unstable workers: they *prefer* not to stay with the same employer for long (Salop and Salop 1976). Alternatively, their skills are less job specific, making them more mobile (Neal 1998), or they are repeatedly screened out during a lower-wage probationary period (Wang and Weiss 1998). Sorting could also be behind the worker-specific unemployment risk, with low-ability workers could also sort into risky firms (Evans and Leighton 1989, which could be modeled in a frictional labor market e.g. by extending Albrecht and Vroman (2002) with on-the-job search and firm heterogeneity in layoff rates). See also Kaas and Carrillo-Tudela (2011)

market histories for firms and workers. Using workers' labor market histories, if unobservable worker heterogeneity is important for unemployment patterns, then the entire labor market history of a worker before a current unemployment spell is informative for future labor market outcomes. In contrast, if only firm and match quality matter, past matches with firms become irrelevant upon when a worker becomes unemployed, and hence previous labor market history should not predict future labor market outcomes for the unemployed. In general, after attempting to control for observable and unobservable worker heterogeneity, the empirical literature that investigates the causal effects of being unemployed on subsequent employment outcomes, typically find these to be substantial (see e.g. Arulampalam et al. (2000) and Böheim and Taylor (2002)). This is consistent with a substantial role for firm or job match heterogeneity in combination with search frictions: these imply that it is often not easy or quick to find a similarly good employment relationship, or relationship with a good firm, after an unemployment spell.

Learning about match quality gives rise to an unemployment hazard that could potentially first increase, but will subsequently decrease with tenure *within a firm*. (To observe this, one needs data that keeps track of individual workers over time while matched with a particular firm, while also observing when/whether this match is broken up.³¹) Ex ante known differences among firms in unemployment risk, as captured in our model, shift up or down the firm-specific unemployment hazard *uniformly* at all tenures *across firms*. The role of firm-level heterogeneity in unemployment risk is supported by the empirical finding that observable firm characteristics indeed can correlate with the unemployment hazard after controlling for tenure effects (e.g. Winter-Ebmer 2001). Hence firm-differences in unemployment risk can be deduced, at least in part, ex ante. In the face of firm-level heterogeneity, this paper furthermore emphasizes the equilibrium wage choices: while there is room for these differences in firm quality to be absorbed in compensating wage differences, these differences also affect how firms compete, leading them to post wages that typically more than offset any compensating wage considerations.³²

In general, it would be interesting to combine, more formally, ex ante known heterogeneity among firms with ex-post learning, to draw inferences e.g. about precisely how much a worker knows ex ante about his outcomes in a potential new employment, how much uncertainty is resolved later, during the match, and what

³¹With regard to this, longitudinal matched-employer-employee data could simultaneously allow for estimation of worker, firm, and tenure-within-firm effects.

³²Ex ante known firm-level differences in job security, compared to uncertain match quality that is revealed slowly over the course of the employment relationship can have different implications for worker mobility in the case of workers who have taken a wage cut. In our model, when workers move to firm taking a wage cut, this firm will have a lower unemployment risk, and a lower total separation risk (incl. separations to other firms). In the case of uncertain match quality, workers take a wage cut to move to a match with *more* variance. After such a move, separation and even unemployment risk can increase.

this means for the distribution of risk over the labor market, which will feed back to many economic decisions of workers.³³

5 Conclusion

In this paper, we have presented a model with homogeneous workers and search frictions in which, in equilibrium, wages do not compensate for differences in unemployment risk. Therefore, workers move, whenever they have the chance, from risky companies to more stable firms, which are also larger. We are able to characterize the joint distribution of equilibrium wages and job security in a very tractable way, making this model amenable to further extensions and estimation. Theoretically, we find that wages increase with job security for the lowest wages; this pattern can extend over a significant part of the wage distribution. While safer firms can offer lower wages while still attracting more workers, the increased job duration makes a worker more valuable to the firm, and hence raises their incentive to prevent worker mobility to other firms, which puts an upwards force on wages. The second force dominates at low wages, and also for large parts of standard firm distributions, though higher in the wage distribution, depending on the distribution of the heterogeneous firms, wage cuts could occur.

In our model, a willingness to pay for job security did not arise from some deep discomfort in the utility function, but rather from the loss of life-time discounted income. As a result, workers at different positions in the job ladder value job security differently, and a complementarity between the job's rent and job security arose endogenously. Taking also into account the firms' incentives in their wage setting, a potentially large discrepancy can result between the workers' willingness to pay for job security (which is different for workers in different firms), and the overall cross-sectional correlation between wages and job security, which can very well be positive.

The model also generates an unemployment hazard rate that is declining with tenure, as in the data, while in the standard Burdett and Mortensen (1998) model it is counterfactually constant. We thus show that unemployment scarring in terms of wages *and* risk of repeated job losses, arises in equilibrium, resulting neither

³³In firm panel data or matched-employer-employee data one could link the firm-average outflow rate into unemployment with the patterns that occur as function of worker's tenure. For example, one could attempt to link the how firm-specific *slope* of the unemployment outflow rate as a function of job tenure relates to the firm-average *level* of unemployment risk. To our knowledge, this is not an dimension of the data that has received substantial attention, and neither do we have clear theoretical predictions. Learning models set in frictional labor markets most often abstract from the notion of multi-worker firms which differ in an ex-ante known quality component. In richer models, strategic wage setting is also often abstracted from, by tying wages one-for-one to productivity by assumption (a notable exception to the latter is Moscarini (2005)).

from an decline in (perceived) productivity of workers when they become unemployed, nor as a manifestation of a selection effect on workers, but because of heterogeneity on the firm side.

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APPENDIX

A Proofs

Proof of lemma 1 What remains to be done is to fill in the few gaps that were not taken care of in the main text. First, note that $V(w, \delta)$ exists as the fixed point of the functional mapping $T : \mathcal{C} \rightarrow \mathcal{C}$

$$TV(w, \delta) = \frac{1}{r + \delta + \lambda_1} \left(w + \lambda_1 \int \int \max\{V(w', \delta'), V(w, \delta), V_0\} d\hat{F}(w|\delta) dH(\delta) + \delta V_0 \right). \quad (42)$$

It further follows straightforwardly from the above equation that $V(w, \delta)$ is continuous, increasing in w , and decreasing in δ when $V(w, \delta) \geq V_0$. Given that the support of $H(\delta)$ and $\hat{F}(w|\delta)$ is bounded by assumption, $V(w, \delta)$ is bounded as well. Then, since $V(w, \delta)$ is monotone, continuous and bounded, it is also a.e. differentiable with respect to w (Kolmogorov and Fomin (1975), 31.2, th. 6); similarly, it is a.e. differentiable with respect to δ . At those points, using the right-hand side of equation (2) we find $\partial V(w, \delta)/\partial w$ in (5). Again, similarly, we find $\partial V(w, \delta)/\partial \delta$ in (6). From equations (2) or (42), in particular the integration on the right-hand side of these equations, it follows that $V(w, \delta)$ is in fact absolutely continuous (Kolmogorov and Fomin (1975), 33.2 Th. 5), and therefore, the derivatives in (5) and (6), together with the initial conditions characterize $V(w, \delta)$. (cf. Kolmogorov and Fomin, 33.2 Th. 6). At a zero measure set of points $V(w, \delta)$ is not differentiable; in our formulation, we use (5)-(6) at those points, without affecting the solution $V(w, \delta)$.

The results for R_0 and V_0 in (7) follow from the compensating differential equation (3), which now implies a reservation wage from unemployment R_0 that is unaffected by δ at value $V(w, \delta) = V_0$. Moreover, substituting out the double integral term in (1), using $V(R_0, \delta) = V_0$ in equation (2), yields V_0 as a function of R_0 (or vice versa). As $V(w, \delta)$ is strictly increasing in w above V_0 , we can invert it (keeping δ fixed); hence $w(V, \delta)$ exists, is continuous, strictly increasing, and a.e. differentiable. Changing the variable of integration yields (8).

Proof of lemma 2 In this proof, we show that the appropriate ratio of limits of a sequence of sets agrees with (13) a.e. (with respect to $F(V, \delta)$). (We do not have to worry about firm sizes at a set of measure zero of firms for overall outcomes: anything that happens on a set of firms of measure zero will not affect the choices or utility and profit attained by workers and other firms.) First, we can define

$$I(\delta, V) \stackrel{def}{=} \int_{\delta' \leq \delta, V_0 \leq V' \leq V} \left(\delta' + \lambda_1 \int_{\tilde{V} > V} dF(\tilde{V}, \tilde{\delta}) + \lambda_1 \int_{V \geq \tilde{V} > V'}^{\tilde{\delta} > \delta} dF(\tilde{V}, \tilde{\delta}) \right) dG(V', \delta') (m - u) \\ - \int_{\delta' \leq \delta, V_0 \leq V' \leq V} \left(\lambda_0 u + \lambda_1 \int_{\tilde{\delta} > \delta, \tilde{V} < V'} dG(\tilde{V}, \tilde{\delta}) (m - u) \right) dF(V', \delta') \quad (43)$$

Then, for $\delta'' > \delta'$ and $V'' > V'$, we have $I(\delta'', V'') - I(\delta', V'') - I(\delta'', V') + I(\delta', V') = 0$, because in steady state $I(\delta, V) = 0$ for every (δ, V) . After some tedious algebra, in which we drop the flow-terms that cancel each other out, add up the remaining flows where possible, but split the integral such that in one set the upper bound is not included and in the other set the value to integrate over is a singleton $\{V''\}$; this results in

$$\int_{V' < \tilde{V} < V''}^{\delta' < \delta \leq \delta''} \left(\delta + \lambda_1 \int_{\tilde{V} > V''} dF(\tilde{V}, \tilde{\delta}) + \lambda_1 \int_{V < \tilde{V} \leq V''}^{\tilde{\delta} \notin (\delta', \delta'']} dF(\tilde{V}, \tilde{\delta}) \right) dG(V, \delta) \\ + \int_{V = \tilde{V}''}^{\delta' < \delta \leq \delta''} \left(\delta + \lambda_1 \int_{\tilde{V} > V''} dF(\tilde{V}, \tilde{\delta}) \right) dG(V, \delta) \\ = \int_{V' < \tilde{V} < V''}^{\delta' < \delta \leq \delta''} \left(\lambda_0 u + \lambda_1 \int_{\tilde{V} \leq V'} dG(\tilde{V}, \tilde{\delta}) + \lambda_1 \int_{V' \leq \tilde{V} < V}^{\tilde{\delta} \notin (\delta', \delta'']} dG(\tilde{V}, \tilde{\delta}) \right) dF(V, \delta) \\ + \int_{V = \tilde{V}''}^{\delta' < \delta \leq \delta''} \left(\lambda_0 u + \lambda_1 \int_{\tilde{V} \leq V'} dG(\tilde{V}, \tilde{\delta}) \right) dF(V, \delta) \quad (44)$$

Now, we can take the limit as $\delta' \rightarrow \delta''$ and $V' \rightarrow V''$. There are two cases: (i) $\int_{V=V'', \text{all } \delta} dF(V, \delta) = 0$, and (ii) $\int_{V=V'', \text{all } \delta} dF(V, \delta) > 0$. In case (i), the terms on the second and fourth line equal zero, while the rightmost terms in the integral on the first and third line are equal in value to an integral that has a strict upper or lower bound on values, i.e. $\lambda_1 \int_{V < \tilde{V} \leq V''}^{\tilde{\delta} \notin (\delta', \delta'']} dF(\tilde{V}, \tilde{\delta}) = \lambda_1 \int_{V < \tilde{V} < V''}^{\tilde{\delta} \notin (\delta', \delta'']} dF(\tilde{V}, \tilde{\delta}) \stackrel{def}{=} T(V, \delta')$. Moreover, T is continuous at (δ'', V'') , with $T(\delta'', V'') = 0$, so we have

$$(\delta + \lambda_1(1 - F(V''))) \frac{\lim_{V' \rightarrow V''} \int_{V' < \tilde{V} \leq V''}^{\delta' < \delta < \delta''} dG(V, \delta)}{\lim_{V' \rightarrow V''} \int_{V' < \tilde{V} \leq V''}^{\delta' < \delta < \delta''} dF(V, \delta)} = (\delta + \lambda_1(1 - F(V''))) l(V'', \delta) = \lambda_0 u + \lambda G(V''),$$

using that $\frac{dG(V'', \delta)}{dF(V'', \delta)} = l(V'', \delta)$, and that $G(V'')$, $F(V'')$ are continuous at V'' . Rearranging yields (13).

For case (ii), we can first take the limit $V' \rightarrow V''$ on both sides of the equation. The terms on the first and third line go to zero. If the second and fourth line are zero as well, we are dealing a set $\{(V, \delta) | V = V'', \delta \in (\delta', \delta'']\}$ that is of measure zero in F , which wlog for the aggregate patterns, we can ignore. Suppose therefore that $B(\delta', \delta'') \stackrel{def}{=} \{(V, \delta) | V = V'', \delta \in (\delta', \delta'']\}$ is of positive measure. Then in the limit as $V' \rightarrow V''$ (44)

reduces to

$$\int_{\substack{\delta' < \delta \leq \delta'' \\ V = \tilde{V}''}} \left(\delta + \lambda_1 \int_{\tilde{V} > V''} dF(\tilde{V}, \tilde{\delta}) \right) dG(V, \delta) = \int_{\substack{\delta' < \delta \leq \delta'' \\ V = \tilde{V}''}} \left(\lambda_0 u + \lambda_1 \int_{\tilde{V} < V''} dG(\tilde{V}, \tilde{\delta}) \right) dF(V, \delta) \quad (45)$$

Consider now the limit as $\delta' \rightarrow \delta''$ while $B(\delta', \delta'')$ stays of positive measure (if it becomes of zero measure, we can ignore it, wlog). The terms between brackets inside the integrals stay constant, and hence can be taken outside the integrals. Dividing both sides by $\int_{\substack{\delta' < \delta \leq \delta'' \\ V = \tilde{V}''}} dF(V, \delta)$, and taking the limit wrt δ , we have

$$(\delta + \lambda_1(1 - F^+(V''))) \frac{\lim_{\delta' \rightarrow \delta''} \int_{\substack{\delta' < \delta \leq \delta'' \\ V' = \tilde{V}''}} dG(V, \delta)}{\lim_{\delta' \rightarrow \delta''} \int_{\substack{\delta' < \delta \leq \delta'' \\ V' = \tilde{V}''}} dF(V, \delta)} = (\delta + \lambda_1(1 - F^+(V''))) l(V'', \delta) = \lambda_0 u + \lambda G^-(V''),$$

where $(1 - F^+(V'')) = \int_{\tilde{V} > V''} dF(\tilde{V}, \tilde{\delta})$ and $G^-(V) = \int_{\tilde{V} < V''} dG(\tilde{V}, \tilde{\delta})$. \square

Proof of proposition 1 To spell out the last step in the proof, consider

$$\frac{(p - w(V_s, \delta_l))l(V_s, \delta_l)}{(p - w(V_r, \delta_l))l(V_r, \delta_l)} \geq \frac{(p - w(V_s, \delta_h))l(V_s, \delta_h)}{(p - w(V_r, \delta_h))l(V_r, \delta)} \quad (46)$$

This implies that either

$$\frac{(p - w(V_s, \delta_l))}{(p - w(V_r, \delta_l))} \geq \frac{(p - w(V_s, \delta_h))}{(p - w(V_r, \delta_h))}, \quad (47)$$

in which case it must be that $V_s \geq V_r$ by equation (19); and/or

$$\frac{l(V_s, \delta_l)}{l(V_r, \delta_l)} \geq \frac{l(V_s, \delta_h)}{l(V_r, \delta)}, \quad (48)$$

in which case it again must be that $V_s \geq V_r$ by the converse to equation (15). Since these two cases cover all possibilities (using that all terms are positive), it must be that $V_s \geq V_r$. \square

Proof of proposition 2. First, the same argument that established that $\tilde{G}(V, \delta)$ is absolutely continuous with respect to $\tilde{F}(V, \delta)$ can be made to establish that $\tilde{F}(V, \delta)$ is absolutely continuous with respect to $\tilde{G}(V, \delta)$, which then necessarily implies that each property (i)-(iii) applies to $F(V)$, if and only if it applies to $G(V)$.

To establish property (ii), towards a contradiction, consider a V which is offered by a strictly positive mass of firms, with δ_1 as the infimum of those types offering this value. The expected profit gain for this type when offering a wage that implies a value $V + \varepsilon$ is greater than

$$\frac{d\pi}{d\varepsilon} > -(r + \delta + \lambda_1) \varepsilon l(V, \delta_1) + (p - w(V, \delta)) \frac{\lambda_1 G^-(V + \varepsilon) - \lambda_1 G^-(V)}{\lambda_1(1 - F^+(V)) + \delta_1}, \quad (49)$$

where the first term follows from the fact that $dw(V, \delta)/dV < r + \delta + \lambda(1 - F^+(V))$ almost everywhere. The change in firm size (from offering a value ε higher than V) is larger than the left-most term. Note that a mass

point at V implies that, there exists a $\eta > 0$, such that for all $\varepsilon > 0$, $G^-(V + \varepsilon) - G^-(V) \geq \frac{\eta(\lambda_1 + \bar{\delta})}{\lambda_1} > 0$. Then, as $\varepsilon \rightarrow 0$, $\frac{d\pi}{d\varepsilon} \geq (p - w(V, \delta))\eta > 0$ and hence a strictly profitable deviation exists to offering for firms who offer V , to offer $V + \varepsilon$ with ε small enough. Moreover, this applies to a non-zero measure of agents, since it holds *all* firms offering V : the argument above does not depend on the size of δ .

Next, (towards contradiction) consider the case where the support of $F(V)$ is not connected. Let \underline{V} be the minimum value in the support of $F(V)$, and \bar{w}^s the corresponding maximum value. Then, there exist $\underline{V} < V_1 < V_2 < \bar{V}$ such that $F(V_1) = F(V_2)$, and $G(V_1) = G(V_2)$ by the continuity of $F(V)$, $G(V)$. Consider a firm posting at V_2 , this firm can keep the same firm size but make strictly more profit when deviating to V_1 . If $\underline{V} > V_0$, the same argument applies: the firm offering \underline{V} can deviate to V_0 , which does not affect his firm size, but strictly raises the profit per worker. \square

Proof of theorem 1 There are three steps in this proof. First, one can show that the equilibrium objects $F(V), G(V), V(w, \delta), \hat{F}(w|\delta), R_0$ constructed from $V(z), w(z)$ satisfy workers' and firms' optimization. This is straightforward, with this and the derivation in the paper, we have established that a steady state equilibrium corresponds to $\{V(z), w(z), V_0\}$ and vice versa.³⁴ Second, we establish that the second-order conditions are also satisfied whenever the first-order conditions hold. Finally, we show that the existence of the equilibrium is guaranteed, and its uniqueness.

Pseudoconcavity of the firm's problem Secondly, we have to check that the first-order conditions indeed pick the maximum in the firm's problem, at any point where $\delta(z)$ is continuous and differentiable. We can verify that the problem is pseudo-concave, using $dV(z)/dz$ in equation (34) and the firms's first-order condition (28) by showing the derivative of the first-order condition below is negative when (29) holds.

$$\frac{d}{dz'} \left((p - w(V(z'), \delta(z))) \frac{\partial l^d(z', \delta(z))}{\partial z'} - \left(\frac{\partial w(V(z'), \delta(z))}{\partial V(z')} \frac{dV(z')}{dz'} \right) l^d(z', \delta(z)) \right) \quad (50)$$

Evaluated at a point where the first order condition is equal to zero ($z = z'$), this has the same sign as

$$\frac{\partial}{\partial z'} \left((p - w(V(z'), \delta(z))) \frac{2\lambda_1}{\lambda_1(1 - z') + \delta(z)} \right) - \frac{\partial}{\partial z'} \left(\frac{\partial w(V(z'), \delta(z))}{\partial V(z')} \frac{\partial V(z')}{\partial z'} \right) \quad (51)$$

Evaluating the above term at $z = z'$, the second term equals $\frac{\partial}{\partial z'} \left((p - w(V(z'), \delta(z))) \frac{2\lambda_1}{\lambda_1(1 - z') + \delta(z)} \right)$, which differs from the first, left-most, term only by $\delta(z)$ vs. $\delta(z')$ in the denominator. Therefore, the only term that does not cancel out between the first and second term of second-order condition (51) is

$$(p - w(V(z), \delta(z))) \frac{\delta'(z)}{(\lambda(1 - z) + \delta(z))^2} < 0, \quad (52)$$

which is negative, since $\delta'(z) < 0$.

³⁴For completeness, this in appendix B.

Existence and uniqueness of the fixed point R_0 . Finally, we have to show existence and uniqueness of the reservation wage $R_0 = w(0)$, and associated $V_0 = \frac{\lambda_0 R_0 - \lambda_1 b}{r(\lambda_0 - \lambda_1)} = V(0)$, satisfying (33), (34), (37). From this, the existence and uniqueness of the steady state equilibrium then follows. Index $\frac{dV(z; R_0)}{dz}$, $\frac{dw(z; R_0)}{dz}$ by initial condition R_0 ; then (37) is the solution to the following fixed point

$$R_0 = T(R_0), \text{ where } T(R_0) = b + (\lambda_0 - \lambda_1) \int_0^1 (1-z) \frac{dV(z; R_0)}{dz} dz \quad (53)$$

Note that $\frac{dV(z)}{dz}$ depends implicitly on the reservation only through $(p - w(z))$.

Manipulating (33) and (34), we can define $x(z) = \frac{V(z) - V_0}{p - w(z)}$ and find that the system of two equations $dV(z)/dz$ and $dw(z)/dz$ can equivalently be written as two equations of which one differential equation only takes itself as argument,

$$\frac{d(p - w(z))}{dz} = -(p - w(z)) \left(\frac{2\lambda_1}{\lambda_1(1-z) + \delta(z)} + \delta'(z)x(z) \right) \quad (54)$$

$$\frac{dx(z)}{dz} = \frac{2\lambda_1}{\lambda_1(1-z) + \delta(z)} \frac{1}{r + \lambda_1(1-z) + \delta(z)} + \left(\frac{2\lambda_1}{\delta(z) + \lambda_1(1-z)} \right) x(z) + \delta'(z)x(z)^2. \quad (55)$$

Note that $p - w(0) = p - R_0$, and $x(0) = 0$. Note that $\frac{d(p-w(z; R_0))}{dR_0}$, by standard FODE theory, is continuous in R_0 , and we will see this derived below as well. Consider first the interval $[0, \tilde{z}]$ on which $\delta(z)$ is continuous.

On this interval, $x(z)$ does not depend on R_0 . We can rewrite (54) to get

$$\frac{\frac{d(p-w(z))}{dz}}{(p-w(z))} = - \left(\frac{2\lambda_1}{\delta(z) + \lambda_1(1-z)} + \delta'(z)x(z) \right); \quad (56)$$

Integrating over z yields

$$p - w(z) = e^{-\int_0^z \left(\frac{2\lambda_1}{\delta(z) + \lambda_1(1-z)} + \delta'(z)x(z) \right) dz} (p - R_0), \quad (57)$$

where the exponential term does not depend on R_0 . It follows immediately that

$$\frac{d(p-w(z))}{dR_0} = -e^{-\int_0^z \left(\frac{2\lambda_1}{\delta(z) + \lambda_1(1-z)} + \delta'(z)x(z) \right) dz} < 0.$$

To generalize this to general distributions $H(\delta)$, consider next a point where $\delta(z)$ is discontinuous: this a point where $\delta(z)$ drops discretely. We want to show that the properties of $\frac{d(p-w(z))}{dR_0}$, $\frac{dx(z)}{dR_0}$ are preserved. Consider first $x(z)$, from (35), which in turn comes from the worker's indifference curve in equation (3),

$$(p - w(\tilde{z})) = \lim_{z \uparrow \tilde{z}} (p - w(z)) - (\delta(\tilde{z}) - \lim_{z \uparrow \tilde{z}} \delta(z))(V(z) - V_0) \quad (58)$$

To shorten notation, let, for a generic function $y(z)$, the limit $\lim_{z \uparrow \tilde{z}} y(z)$ be denoted by $y_L(z)$. Then we can rewrite the above equation (58) as

$$(x(z))^{-1} = (x_L(z))^{-1} - (\delta(z) - \delta_L(z)) \iff \quad (59)$$

$$x(z) = \frac{x_L(z)}{x_L(z) - (\delta(z) - \delta_L(z))} < x_L(z). \quad (60)$$

Hence, if $dx_L(z; R_0)/dR_0 = 0$, it follows that $dx(z; R_0)/dR_0 = 0$.

Thus, the irresponsiveness of $\frac{dx(z)}{dR_0}$ is also preserved whenever $\delta(z)$ drops discretely. Let $Z = \{\zeta_i\}$ be the countable set of ranks z at which $\delta(z)$ drops discretely; define additionally $\zeta_0 = 0$. Then, letting $\bar{\zeta}(z) = \sup\{\zeta \in Z | \zeta < z\}$

$$p - w(z) = (p - R_0) \left(\prod_{\{i | \zeta_i \in Z, \zeta_i < z\}} e^{-\int_{\zeta_{i-1}}^{\zeta_i} \left(\frac{2\lambda_1}{\delta(z') + \lambda_1(1-z')} + \delta'(z')x(z') \right) dz'} \left(\frac{x_L(\zeta_i)}{x_L(\zeta_i) - (\delta(\zeta_i) - \delta_L(\zeta_i))} \right) \cdot e^{-\int_{\bar{\zeta}(z)}^z \left(\frac{2\lambda_1}{\delta(z') + \lambda_1(1-z')} + \delta'(z')x(z') \right) dz'} \right) \iff \quad (61)$$

$$p - w(z) = (p - R_0)\mathbf{A}(z), \quad (62)$$

summarizing the entire bracketed term, which only depends on firm rank z and fundamentals but not on R_0 , in equation (61) into term $\mathbf{A}(z)$ in (62).³⁵ From this it immediately follows that $\frac{d(p-w(z; R_0))}{dR_0} = -\mathbf{A}(z) < 0$. Moreover the $T(R_0)$ mapping becomes

$$T(R_0) = b + (p - R_0)(\lambda_0 - \lambda_1) \int_0^1 \frac{2\lambda_1(1-z')}{\lambda_1(1-z') + \delta(z')} \frac{1}{r + \delta(z') + \lambda_1(1-z')} \mathbf{A}(z') dz' \quad (63)$$

Denoting the term post-multiplying $(p - R_0)$ by \mathbf{B} , we find

$$R_0 = \frac{(b/p + \mathbf{B})}{1 + \mathbf{B}} p, \quad (64)$$

which for any $b \leq p$ gives the reservation wage R_0 . This establishes the existence, and the uniqueness of the equilibrium reservation wage of the unemployed, and given the existence and uniqueness of the firm posting and workers' value function given the reservation wage R_0 , it establishes the overall existence and uniqueness of the equilibrium. \square

Proof of Result 1 The inflow into employment $\lambda_0 u + \lambda G(z) = l(z)(\delta(z) + \lambda(1-z))$, the probability that an inflow at time t survives until $t + \tau$ is $e^{(\delta(z) + \lambda(1-z))\tau}$; thus the number of workers in the z^{th} firm who have been around τ periods $t_{eu}(z, \tau) \stackrel{def}{=} l(z)(\delta(z) + \lambda(1-z))e^{-(\delta(z) + \lambda(1-z))\tau}$. Then, the derivative of the

³⁵A similar result holds true in the standard Burdett and Mortensen model, where

$$p - w(z) = (p - R_0) \left(\frac{\lambda_1(1-z) + \delta}{\lambda_1 + \delta} \right)^2,$$

however, here we have take care of the heterogeneity in δ , and the resulting influence on the wages (with wage cuts etc.). Notice that if we set $\delta'(z) = 0$ and hence $\Delta(z) = 0$ and $x(z) = 0 \forall z$, the Burdett and Mortensen result in fact follows. The observation that a similar property is preserved in our more complicated setting is encouraging for the wider applicability of the BM-type wage posting framework, e.g. when incorporating further heterogeneity.

empirical hazard rate with respect to tenure is

$$d \ln \left(\frac{\int_0^1 \delta(z) t_{eu}(z, \tau) dz}{\int_0^1 t_{eu}(z, \tau) dz} \right) / d\tau = - \int_0^1 \frac{(\delta(z) - \delta^{ave}) t_{eu}(z, \tau) (\delta(z) + \lambda_1(1-z)) dz}{\int \delta^{ave} t_{eu}(z', \tau) dz'} < 0 \quad (65)$$

The derivative $dt_{eu}(z, \tau)/d\tau = -t_{eu}(z, \tau)(\delta(z) + \lambda_1(1-z))$. Define $\delta^{ave} \int t_{eu}(z', \tau) dz' = \int \delta(z') t_{eu}(z', \tau) dz'$. Then $\int_0^1 (\delta(z) - \delta^{ave}) t_{eu}(z, \tau) dz$ equals zero; since $\delta(z) - \delta^{ave}$ and $(\delta + \lambda_1(1-z))$ are both decreasing, the latter one strictly, the integral term in (65) is positive, establishing the result.

Proof of Result 2 We have to make sure not only that $w(z)$ is increasing on an interval $[0, \tilde{z}]$ itself, but also that there exists an interval $[0, \tilde{z}]$ where additionally for no further $z > \tilde{z}$ we have that $w(z) < w(\tilde{z})$. By theorem 1, locally, for z close 0, we have $w(z)$ strictly increasing, while by proposition 2 $V(z)$ strictly increasing everywhere. Then immediately there exists $\tilde{z} > 0$ such that $w(z)$ is strictly increasing for all $0 < z < \tilde{z}$. Now, towards a contradiction, suppose that there does not exist a $\tilde{z} > 0$ such that for all $z > \tilde{z}$, it holds that $w(z) > w(\tilde{z})$. Then, there must exist a sequence $\{z_n\}$ with $z_n > \tilde{z}$, such that $w(z_n) \rightarrow R_0$. By (3), then also $V(z_n) \rightarrow V_0$. But then, there exists an n such that $V(z_n) < V(\tilde{z})$, contradicting the ranking property/the strict monotonicity of $V(z)$.

Proof of Result 3 Note that since $dw(0)/dz > 0$, at the the \tilde{z} from which onwards an *interval* of wage cuts occurs, both $dw(\tilde{z})/dz = 0$ and $d^2w^*(\tilde{z})/(dzdz) < 0$, i.e. $dw^*(z)/dz$ cuts 0 from above, at \tilde{z} . Note that a point at which first and second derivative are zero will not translate in any wage cuts, or strict decrease of wage in job security. The second derivative at \tilde{z} equals

$$\begin{aligned} \frac{d^2w^*}{dzdz} &= - \frac{dw^*}{dz} \frac{2\lambda_1}{\lambda_1(1-z) + \delta(z)} + (p - w^*(z)) \frac{2\lambda_1(\lambda_1 - \delta'(z))}{(\lambda_1(1-z) + \delta(z))^2} + \delta''(V(w^s(z), \underline{\delta}) - V_0) \\ &\quad + \delta'(z) \frac{dV(z)}{dz}. \end{aligned} \quad (66)$$

Note that after substituting in $dV(z)/dz$ from (33), the terms with $\delta'(z)$ cancel out. Evaluated at a point where $dw/dz = 0$, and letting $r \rightarrow 0$, this turns into

$$\left. \frac{d^2w^*}{dzdz} \right|_{\frac{dw^*}{dz}=0} = (p - w^*(z)) \frac{2\lambda_1^2}{(\lambda_1(1-z) + \delta(z))^2} + \delta''(V(w^s(z), \underline{\delta}) - V_0) \quad (67)$$

This can be smaller than zero only if $\delta''(z) < 0$, which in turn occurs if and only if $h'(\delta) > 0$, since $\delta'(z) = -1/h(\delta)$ and $\delta''(z) = \frac{h'(\delta)}{h(\delta)^2} \delta'(z)$.

The second point follows from (34) being negative. Substituting in $V(z) - V_0 = \int_0^z dV(z)/dzdz$ into $dw(z)/dz$ in equation (34), a change of the integrating variable ($dz = -h(\delta)d\delta$), this can be written equiva-

lently as

$$(p - w(1 - H(\delta))) \frac{2\lambda_1}{\delta + \lambda_1(1 - H(\delta))} < \frac{1}{h(\delta)} \int_{\delta}^{\tilde{\delta}} \frac{2\lambda_1}{(\tilde{\delta} + \lambda_1(1 - H(\tilde{\delta})))^2} (p - w(1 - H(\tilde{\delta}))) h(\tilde{\delta}) d\tilde{\delta}$$

Since $p - w(1 - H(\tilde{\delta})) > p - w(1 - H(\delta))$ for $\tilde{\delta} > \delta$ if no other wage cuts with increased job security have occurred, the above equation will be negative whenever (39) holds. (In the other case, if wage cuts have occurred lower in the value distribution (at lower z), then point 2. holds trivially.)

Proof of Result 4 We can show this by establishing that $\frac{d^2 w(\tilde{z})}{dw d\tilde{z}} < 0$, and $\frac{dw(\tilde{z})}{d\tilde{z}} = 0$ cannot occur for $\lambda_1 > \frac{h'(\delta)}{(h(\delta))^2}$. Note that the existence of wage cuts implies a z such that $dw(z)/dz = 0$, $\frac{d^2 w(z)}{dz dz} \leq 0$. This implies that at that z , from (34) and (67),

$$(p - w(z)) \frac{2(\lambda_1)^2}{\delta(z) + \lambda_1(1 - z)} < -\delta''(z) \int_0^z \frac{dV(z')}{dz'} dz \quad (68)$$

$$(p - w(z)) \frac{2\lambda_1}{\delta(z) + \lambda_1(1 - z)} = -\delta'(z) \int_0^z \frac{dV(z')}{dz'} dz \quad (69)$$

Dividing the RHS of (68) by the RHS of (69), and similarly for the LHS, this yields

$$\lambda_1 < \delta''(z)/\delta'(z) = \frac{h'(\delta)}{(h(\delta))^2}$$

as a necessary condition. Hence if $\lambda_1 > \frac{h'(\delta)}{(h(\delta))^2}$, we do not satisfy the necessary condition, and therefore can rule out $dw(z)/dz < 0$ at $z = 1 - H(\delta)$.

Proof of Result 5 is in the main text

ADDITIONAL MATERIAL (NOT FOR PUBLICATION)

B Theorem 1, verification

Theorem 1: Direct Verification in terms of Original Equilibrium Objects In the paper, we derived $\{V(z), w(z), R_0\}$ satisfying (33), (34), (37) from the equilibrium conditions. Here, we outline the reverse argument: any set $\{V(z), w(z), R_0\}$ satisfying (33), (34) and (37) corresponds to a steady state equilibrium with the equilibrium objects listed in definition 1.

The reverse construction of equilibrium objects, such as $\hat{F}(w|\delta)$ etc., is very straightforward, and we will mention it here for the sake of completeness. We construct (i) the aggregate conditions and distributions, then show that these are consistent with the original (ii) firm's optimization, and (iii) the original worker's optimization. First invert $V(z)$, to construct $F(V)$, $G(V) = G^z(F^{-1}(V))$, and $V(w, \delta)$ implicitly from $V(z), w(z), \delta(z)$ for those wages that are actually posted on the equilibrium path. Then we can construct $\hat{F}(w|\delta)$ from $\delta(z), w(z)$, using that if $\delta(z)$ is strictly decreasing at z , then $F(w(z)|\delta(z)) = 1$, while if $\delta(z)$ is constant on a compact interval, then we isolate this interval, rescale it to $[0,1]$ and invert the re-scaled $w(z)$ to find $\hat{F}(w|\delta)$. $\tilde{F}(V, \delta), \tilde{G}(V, \delta)$ can be constructed straightforwardly from $F(\hat{w}|\delta)$ and $H(\delta)$, or from $V(z)$ and $\delta(z)$ directly by the monotonicity of the latter functions. Similarly for $\tilde{G}(V, \delta)$.

Now we have to show that these distribution are consistent with optimal decision making, (ii)-(iii). Given $F(V), V(w, \delta)$, where for off-equilibrium (w, δ) we use the same partial derivatives as in lemma 1, to derive the entire function $V(w, \delta)$. For worker optimization and firm optimization, consider that firms choose w in the profit maximization, which leads to the following first order condition

$$(p - w) \frac{\partial l(V(w, \delta), \delta)}{\partial w} - l(V(w, \delta), \delta) = 0, \quad (70)$$

which can be rewritten as

$$1 = (p - w) \frac{\frac{\partial l(V(w, \delta), \delta)}{\partial V}}{l(w^s, \delta)} \frac{dV(w, \delta)}{dw} = 0, \iff \frac{\frac{\partial w}{\partial V(w, \delta)}}{p - w(V(z), \delta(z))} = \frac{2\lambda_1 F'(V)}{\delta + \lambda(1 - F(V))} \quad (71)$$

Using $F'(V) = (dV(z)/dz)^{-1}$, we can verify that (28) and (31) combined with (33) and (34) imply the equivalence of (71) and (28). In other words, given the constructed distributions, firms indeed maximize their profit when taking the original optimization problem with respect to posted wage w to post. Moreover, if $\{V(z), w(z), R_0\}$ does not satisfy (33), then the first order condition of wages (71) is likewise violated.

Finally, consider the worker's optimization. This is fully captured in $V(w, \delta)$ and R_0 . Value function $V(w, \delta)$ in turn is characterized by partial derivatives (5) and (6). Function list $\{V(z), w(z), \delta(z)\}$ defines points of $V(w, \delta)$ that occur in equilibrium as a parametric equation of z . For a generic 'parametric equation'

it has to hold that $dV/dz = \partial V/\partial w \cdot dw/dz + \partial V/\partial \delta \cdot d\delta/dz$. The equivalence of this and (34) after the appropriate substitutions for distributions using dw/dz from (33) follows. Finally, the equivalence of R_0 in (37), and R_0 defined in (8), follows directly from a change of the variable of integration.

Hence the existence and uniqueness of the equilibrium $\{V(z), w(z), R_0\}$ maps into the existence and uniqueness of the equilibrium as laid out in definition 1.

C Putting a foundation under the heterogeneous unemployment risk

Model

Following the knowledge hierarchy literature (see Garicano (2000) and Garicano and Rossi-Hansberg (2006), for example), we assume a firm is an organization that handles a flow of production tasks. These tasks can be scaled, so that any worker can be fully occupied with her own flow. Each task requires specialized knowledge to be completed. If a task is successfully completed, it results in a flow output of p . If the worker fails to complete a task, the flow output is 0 (or otherwise low enough to make it worthless to keep a worker). We assume that there is a change in tasks faced by a given worker at a Poisson rate $\zeta \in (0, \infty)$. The new task is drawn³⁶ from a distribution $H(\cdot)$ with support $[\underline{z}, \bar{z}]$ and density $h'(\cdot) < 0$. Therefore, based on the distribution's characteristics, problems are ranked from the most to the least common ones, and the most common problems require the least amount of knowledge.

We consider that there is no team production, in the sense that each worker in the firm either solves a task by herself or not at all. We also assume that workers cannot be relocated to a different task. Suppose that such a worker acquires the knowledge necessary to solve the tasks in a set $A \subset [\underline{z}, \bar{z}]$. Since $h'(\cdot) < 0$, if there is any cost to acquire knowledge, it is optimal that A is a subset starting from \underline{z} . Therefore, if $A = [\underline{z}, z_0]$, the probability that a worker solves a given task is $H(z_0)$. Imagine that workers decide how much knowledge to acquire before they enter the labor market. Since workers are homogeneous, there is an optimal amount $A^* = [\underline{z}, z^*]$ that workers will acquire³⁷. Notice that, if the worker cannot solve a problem, given task persistence and a bad enough productivity in this case, the optimal decision by the firm as well as the worker it is to destroy the job match. Therefore, a worker is dismissed with probability $\zeta [1 - H(z_0)]$.

Following Cremer (1993) and Cohen and Bacdayan (1994), we assume that a firm is a set of knowledge and management practices that are shared by workers in an organization, boosting these workers' ability to solve problems. In this sense, we consider that a firm is able to provide the worker the knowledge to solve problems in the range $[z_0, z_1]$, for example. Then, the probability a worker is dismissed becomes $\zeta [1 - H(z_1)]$. We assume that this additional knowledge is tacit and cannot be transferred across firms, being part of the firm's total factor productivity³⁸.

³⁶Therefore, as in Mortensen and Pissarides (1994), we have persistence in task-specific shocks.

³⁷We assume a convex cost of education with $\lim_{z \rightarrow \bar{z}} c(z) = \infty$ such that it is not optimal for workers to cover the entire support.

³⁸There is clear evidence in the literature that managerial practices survive changes in top managerial team (Bloom and Van Reenen, 2007) as well as major firm restructures, as spin offs - Cronqvist, Low, and Nilsson (2009) find that spin-off firms have practices closer to their parent companies than to its industry peers even after over long periods of time as well as after changes in the managerial team.

For example, let's assume that there are two types of firms, that are identical on every aspect but their levels of institutional knowledge, i.e., knowledge embodied in their management practices and procedures. Therefore, let's consider type s firms that provide workers with knowledge over the interval $[z_0, z_s]$ and type d firms that provide workers with knowledge over the interval $[z_0, z_d]$ with $z_d < z_s$. While workers from firm s will be unable to solve a problem and be dismissed with probability $\zeta [1 - H(z_s)]$, workers in a type d firm will be dismissed with probability $\zeta [1 - H(z_d)]$. Therefore, workers in d firms are more likely to be laid off - They face an additional $\zeta [H(z_s) - H(z_d)]$ probability of being dismissed. Therefore, these differences in managerial practices and or codified knowledge that can be access by workers generate differences in job destruction rates among firms.

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