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Confidence and Competence in Communication

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Confidence and Competence in Communication*

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Abstract

This paper studies information transmission between an uninformed decision maker (receiver) and an informed player (sender) who have asymmetric beliefs ("confidence") on the sender's ability ("competence") to observe the state of nature. We find that even when the material payoffs of are perfectly aligned, the sender's over- and underconfidence on his information give rise to information loss in communication, although they do not by themselves completely eliminate information transmission in equilibrium. However, an underconfident sender may prefer no communication to informative communication. We also show that when the sender is biased, overconfidence can lead to more information transmission and welfare improvement.

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1 Introduction

When we learn new information and access its reliability, we often take into account the information provider's competence. However, the information provider and his audience may not always agree on how competent or how well-informed he actually is. This disagreement can be understood as a manifestation of over- and underconfidence in communication. For example, various experts (consultants, securities analysts, lawyers etc.) are often accused of being overconfident about their ability or the quality of information they have. On the other hand, underconfidence (shyness, low self-esteem) is yet another type of obstacle to communication that troubles a number of individuals in society. Such communication apprehension can occur when a sender underestimates his ability or the relevance of his knowledge, and becomes excessively afraid of the audience's reaction to his message. How do over- and underconfidence affect the nature of communication? Do they enhance or diminish information transmission? If so, how? Is more informative communication always better? How do over- and underconfidence interact with the sender's (intrinsic) "bias", which has attracted much attention in the literature on information transmission?

This paper addresses these questions by incorporating asymmetric beliefs ("confidence") on the quality of the sender's information ("competence") into the standard cheap talk model of Crawford and Sobel (1982). We find that overconfidence leads to the sender's incentive to "exaggerate" and send an extreme report, while underconfidence gives rise to incentive to "moderate" his report.[1] When communicating with an overconfident sender, the receiver discounts the quality of the sender's report. That is, in making her decision the receiver puts less weight on the sender's message than the sender wants the receiver to. In order to influence the decision in the face of the discounting, the sender is tempted to send an extreme message, but in equilibrium this results in the feature that extreme messages are less informative than moderate ones. In contrast, the receiver values the information held by an underconfident sender more than the sender himself does. As a

[1] By incentive to "exaggerate", we mean a sender's incentive to misreport in such a way that, if his message refers directly to the value of a signal and is believed literally by the receiver, the sender "overstates" ("understates") the signal when it is high (low) relative to the prior expectation.

Similarly, incentive to "moderate" is a sender's incentive to misreport in such a way that, if his message is believed literally, he "understates" ("overstates") the signal when it is high (low).

In cheap talk games messages used are completely arbitrary and do not have to be taken literally. What matters for the equilibrium outcome is the correspondence between the signal an expert has observed and the receiver's action, so what word (or language) is used to induce a particular action is irrelevant to the equilibrium construction.

result, the sender wants to weaken his influence on the receiver's decision, and hence the sender has incentive to "moderate" his report relative to the actual information he has. In equilibrium, moderate messages become less informative than extreme ones.

At the same time, we show that severe overconfidence and severe underconfidence have a marked similarity as they both may lead to the use of binary communication (e.g. "yes or no", "agree or disagree"), despite the very different nature of the informational distortions they cause. Binary communication is shown to be robust to both over- and underconfidence since it suppresses any incentive to "exaggerate" or "moderate".

We also introduce an intrinsic bias of the sender, which represents the difference between the ideal decision of the sender and that of the receiver (who makes the decision) if they shared the same complete information on the state. It is well known that such bias reduces information transmission when there is no over- or underconfidence. In this paper we demonstrate that, while both overconfidence and underconfidence also reduce the quality of communication in the absence of bias, slight overconfidence on the part of the sender enhances information transmission whenever he is moderately biased. Moreover, overconfidence may increase the prospect of informative communication when he is severely biased.

One striking feature of underconfidence in communication is that, when their expected utilities are computed according to their (asymmetric) beliefs, the sender may be better off with no communication than with informative communication even if both are perfect Bayesian equilibria. The receiver is always better off with more information transmission, so that multiple equilibria may not be Pareto ranked in communication with an underconfident sender. This may provide a strategic basis for "communication apprehension" (or "communication avoidance") by people with underconfidence or low self-esteem. That is, if an underconfident agent can choose whether to participate in a communication game, he may strictly prefer not to do so, since equilibrium communication in the game may hurt himself. This is in contrast to the standard cheap talk models, where the worst that can happen to the sender is "babbling" (no information transmission) so that he weakly prefers playing the communication game to staying out of the game.

Another interesting characteristic of communication with an underconfident sender is that the receiver too may be better off in an equilibrium where a smaller number of messages is used. Specifically, we show that the receiver may prefer the equilibrium with binary messages to an equilibrium with more messages. The intuition is very simple. Consider a situation where three messages are used in equilibrium ("yes, I'm not sure, or no"). When a sender is severely underconfident, he sends a moderate message ("I'm not sure") most of the time, which makes communication hardly informative from the receiver's viewpoint. On the other hand in a binary communication equilibrium the sender has to choose either of the two messages ("yes or no") and the receiver may find this more informative than the three message equilibrium where the agent sends the moderate message ("I'm not sure") with a very high probability. Roughly speaking, when communicating with an underconfident person, one may prefer not to give him a chance to send a moderate message.

Overconfidence has been attracting much attention from psychologists and economists. The literature on judgement under uncertainty has found that people tend to be overconfident about the information they have, in that their subjective probability distributions on relevant events are too tight (Kahneman et al., 1982; Cesarini, Sandewall, and Johannesson, 2004). Overconfidence has been found in various professions such as lawyers (Wagenaar and Karen, 1982), policy experts (Tetlock, 1999), and security analysts (Chen and Jiang, 2006). The implications of overconfidence for economic choices and especially for financial markets have been studied recently by numerous researchers (e.g. Kyle and Wang, 1997; Gervais and Odean 1998; Daniel et al., 1998; Scheinkman and Xiong, 2003). Hvide (2002) offers a theoretical argument for the endogenous emergence of overconfidence.

Despite its prevalence, underconfidence is much less pronounced in the economic literature, and this paper provides a first approach to study both over and underconfidence within a simple framework. Psychologists have analyzed how low self-esteem and shyness, which can be considered as an important aspect of underconfidence, become an obstacle to communication (e.g. Richmond and McCroskey, 1997; Zimbardo, 1990). We are able to offer a game theoretic analysis of the nature of difficulties in communication with underconfident individuals, and also can explain their tendency to avoid communication as we show that an underconfident sender may indeed be better off with no communication than with informative communication, even if engaging in communication itself is completely costless. In experimental settings, Griffin and Tversky (1992) document how overconfidence and underconfidence arise systematically in the context of evidence assessment. Hoelzl and Rustichini (2005) have found that people tend to be underconfident particularly with unfamiliar tasks. Underconfidence can potentially be important even for professional experts. In medical profession, Friedman et al. (2004) have found that physicians are more likely to be underconfident about their diagnoses than to be overconfident.

In order to model the level of confidence, we allow for asymmetric beliefs on the probability that the sender observes the true state of nature. In other words, the receiver and the sender may not share the same "confidence" on the sender's "competence" though they are fully aware of the difference in beliefs. Since we do not have to specify which player a priori has the correct belief, our framework can be applied to communication with "over-reliance" or "under-reliance" on the sender's information, for which a sender is often supposed to hold the correct belief and his audience does not.

Recent papers that involve asymmetric beliefs (non-common priors) include Admati and Pfleiderer (2004), Fang and Moscarini (2005), Van den Steen (2005), and Che and Kartik (2009), among others. Fang and Moscarini (2005) consider the effect of workers' overconfidence in their skills on wage policies, and Van den Steen (2005) examines worker incentives when a worker may disagree with the manager regarding the best course of action. Che and Kartik (2009) develop a verifiable disclosure model between a decision maker and agent who have asymmetric prior beliefs about the state, and study the agent's incentive for information acquisition.

The closest to the present paper is Admati and Pfleiderer (2004) who study an information transmission game where the sender can be overconfident in his ability to observe the true state. They argue that in communication with an overconfident sender extreme messages or ratings are less precise because if the sender reports the truth honestly, from his viewpoint the receiver's reaction is too weak and the sender has incentive to send an extreme report. Admati and Pfleiderer (2004) focus on overconfidence and do not consider underconfidence. Also they assume that the sender is otherwise unbiased: if the sender and the receiver agree on the sender's ability the parties' interests are perfectly aligned. Thus they are unable to address the question how the intrinsic bias of a sender, which has been a focus of attention in the information transmission literature (and an important concern in practice), interacts with his confidence in his competence. In the present paper we analyze underconfidence as well as overconfidence in a systematic way. Moreover we explicitly illustrate the interaction between confidence and the intrinsic bias. Also, while Admati and Pfleiderer (2004) restrict the message space in such a way that the sender chooses one of an exogenously given finite set of messages, we do not impose such a restriction on the message space. Hence we are able to study more naturally how much information a sender can possibly communicate credibly, depending on his confidence and bias. This also enables us to examine interesting issues with multiple equilibria, especially in communication with an underconfident sender.

Although our model is an extension of the canonical model of Crawford and Sobel (1982) we cannot simply adopt their equilibrium characterization because the sender's incentive to misreport may not point in the same direction. Recently Gordon (2010) has provided the general characterization of a class of cheap talk equilibria where the sender's bias can depend on his information. While he is mainly concerned with equilibrium characterization itself, our focus in the present paper is on how parameters regarding confidence and intrinsic bias alter the nature of communication.

We observe that overconfidence and underconfidence lead to different structures of informative equilibria. The informative equilibria with overconfidence are closely related to communication with noise (Krishna and Morgan, 2004; Blume, Board and Kawamura, 2007; Goltsman, Horner, Pavlov, and Squintani, 2009), where the receiver puts less weight on a message than in communication without noise, because due to noise the received message may not necessarily be informative about the state. In the absence of intrinsic bias, weak response to a message leads to the sender's incentive to "exaggerate" his message relative to the prior expected state. A similar intuition has been developed by Kawamura (2011) in a multiple sender setting. This type of informational distortion appears in communication with an overconfident sender too, as we have already mentioned above.

On the other hand, informative equilibria in communication with an underconfident sender shares an important feature with "reputational cheap talk" (Ottaviani and Sørensen, 2006a,b; Gentzkow and Shapiro, 2006), where the sender attempts to look more able through messages when his quality is unknown to the receiver but known to the sender himself. In order to appear to be of high quality, the sender with reputational concerns has incentive to report messages that are closer to the prior, which also arises in communication with an underconfident sender for a different reason.

The rest of this paper is organized as follows. The next section describes the model, and Section 3 characterizes the equilibria. The effects of over- and underconfidence in the absence of bias are studied in Section 4 and Section 5 examines the interplay between confidence and bias. Section 6 concludes.

2 Model

Our analysis builds on the "uniform-quadratic" model of Crawford and Sobel (1982, henceforth CS). There is an uninformed receiver and a sender who observes a private signal on the state of nature. Both parties' payoffs depend on the receiver's action $y \in \mathbb{R}$ and the uniformly distributed state $\theta \in [0, 1]$. The sender's utility function is $U^S = -(y - \theta - b)^2$ and that of the receiver is $U^R = -(y - \theta)^2$, where b represents the sender's bias. We assume

$b \geq 0$ without loss of generality.

Before the receiver chooses her action, the sender observes a private signal $\sigma \in [0, 1]$ on the state θ. After observing the signal, the sender reports a costless, unverifiable message $m \in M$. Following convention we often refer to σ as the sender's "type".

CS assume that the sender observes the true state with probability 1, which is common knowledge. Now we introduce asymmetric beliefs ("confidence") on the quality of the signal (the sender's "competence") as follows: the receiver believes that $\sigma = \theta$ with probability p, and σ has the identical (i.e. uniform on $[0, 1]$) but independent distribution with probability $1 - p$. That is, with probability $1 - p$ the signal is uninformative about the state. On the other hand the sender believes that $\sigma = \theta$ with probability c, and σ has the identical but independent distribution with probability $1 - c$. In other words, the parties may have different beliefs on the probability that the signal observed by the sender coincides with the true state. Moreover they fully recognize the difference in their beliefs. That is, c and p are common knowledge.[2]

In terms of the level of confidence and intrinsic bias, our model is more general than the formulation by Admati and Pfleiderer (2004), who also adopt a similar uniform-quadratic setting, in that we introduce the sender's intrinsic bias b, and we also allow for the possibility that $c < p$ as well as $c \geq p$. In our notation, the widely known uniform-quadratic example of CS assumes $b \geq 0$, $p = c = 1$,[3] and Admati and Pfleiderer (2004) assume $b = 0$, $c = 1$ and $p \leq 1$.[4]

For expositional convenience, throughout this paper we look at the level of the sender's confidence from the receiver's perspective. If $c > p$ the sender is said to be *overconfident* since the sender believes that his signal is more accurate (i.e. he is more competent) than the receiver believes. Likewise if $c < p$ he is *underconfident*. However, since we are interested in how the difference between c and p affects communication, we are not concerned with which party has the correct belief on the sender's ability. It could be

[2] What is important for the intuition behind our results we develop later is the natural and intuitive feature that, the less competent the sender is, the less weight should be put on his signal relative to the prior distribution/expectation. Our formulation of c and p should be thought of as a tractable representation of this feature.

[3] For their general setup CS's equilibrium characterization covers wider parameter values, which we will use later for Proposition 1.

[4] Note also that Admati and Pfleiderer (2004) impose an exogenous restriction on the number of the receiver's actions induced in equilibrium, while it is determined endogenously in our model. Admati and Pfleiderer (2004) allow for general distributions of the state/signal and focus on studying how the distributions affect information transmission.

that neither party does, but this is not important since we calculate the players' expected utilities according to their own subjective beliefs. If we look at the model from the sender's viewpoint and assume that he holds the correct belief, $c > p$ and $c < p$ could be respectively called "under-reliance" and "over-reliance" on a sender. Later we conduct welfare analysis using the expected utilities according to each player's subjective belief.

Let us derive communication equilibrium and examine how the nature of information transmission (i.e. messages communicated in equilibrium) changes depending on the values of b, c, and p. In what follows we look for equilibrium strategies where each party maximizes expected utility according to his/her subjective beliefs on the sender's competence. Given the message the receiver updates her (subjective) belief using Bayes' rule.

From the receiver's viewpoint a message from the sender is informative about the state of nature with probability p. Otherwise the message is completely uninformative. Since neither the receiver nor the sender knows whether the sender has observed the true state of nature, the receiver's maximization problem after she has received the message is given by

$$\begin{aligned}\max_y E[U^R \quad | \quad m, y] &= p\underbrace{E[-(y-\sigma)^2 \mid m]}_{\theta=\sigma} + (1-p)\underbrace{\int_0^1 -(y-\theta)^2 d\theta}_{\theta\sim U[0,1]} \\ &= p\left[-(y - E[\sigma \mid m])^2 - var(\sigma \mid m)\right] + (1-p)\int_0^1 -(y-\theta)^2 d\theta,\end{aligned}$$

where $E\left[\sigma \mid m\right]$ and $var(\sigma \mid m)$ are the expectation and variance of the signal conditional on the message, respectively. Since the utility function is quadratic, what matters for the receiver's choice is the posterior expectation of the signal (and the state). Hence the first order condition for her best response conditional on the message yields

$$y^*(m) = pE\left[\sigma \mid m\right] + (1-p)\frac{1}{2}. \tag{1}$$

$E\left[\sigma \mid m\right]$ is weighted at p because the receiver believes that σ is independent of the true state with probability $1-p$.

Before deriving the sender's best response, let us consider his desired action. Given the signal, from the sender's viewpoint $\theta = \sigma$ with probability c and otherwise θ is uniform on $[0, 1]$. Thus his desired action is given by maximizing expected utility$E[U^S \mid \sigma]$:

$$\max_y E[U^S \mid \sigma, y] = c\underbrace{\left(-(y-\sigma-b)^2\right)}_{\theta=\sigma} + (1-c)\underbrace{\int_0^1 -(y-\theta-b)^2 d\theta}_{\theta\sim U[0,1]}.$$

The first order condition for the sender gives his desired action given σ

$$y^S(\sigma) = c\sigma + (1-c)\frac{1}{2} + b. \tag{2}$$

Hypothetically, suppose that the sender fully reveals his signal. Then, since $E[\sigma \mid m] = \sigma$, (1) implies that the receiver's desired action is

$$y^R(\sigma) = p\sigma + (1-p)\frac{1}{2}. \tag{3}$$

Let us define $\hat{\sigma}$ to be the signal such that the parties' desired actions coincide: $y^S(\hat{\sigma}) = y^R(\hat{\sigma})$. From (2) and (3) we obtain

$$\hat{\sigma} = \frac{1}{2} - \frac{b}{c-p}.$$

We call $\hat{\sigma}$ the *agreement type*. We may have $\hat{\sigma} \in [0,1]$ only when $c \neq p$ and b is not too large. In the CS model ($c = p$) both parties' desired actions never coincide for $b > 0$: we have $y^S(\sigma) > y^R(\sigma)$ for all σ.

An important feature when we have $\hat{\sigma} \in [0,1]$ is that the difference between the parties' desired actions may not be consistently negative or positive. As a result if the sender is overconfident ($c > p$) we have $y^S(\sigma) > y^R(\sigma)$ for $\sigma > \hat{\sigma}$ and $y^S(\sigma) < y^R(\sigma)$ for $\sigma < \hat{\sigma}$: if the receiver takes a reports literally and naively believes it, the sender has incentive to "overstate" his signal for $\sigma > \hat{\sigma}$ and incentive to "understate" it for $\sigma < \hat{\sigma}$.[5] In the previous section we have referred to them as incentive to "exaggerate" since when σ is higher (lower) than $\hat{\sigma}$ the sender wants to convince the receiver that σ is even higher (lower).

In contrast, if the sender is underconfident ($c < p$) we have $y^S(\sigma) < y^R(\sigma)$ for $\sigma > \hat{\sigma}$ and $y^S(\sigma) > y^R(\sigma)$ for $\sigma < \hat{\sigma}$. Now the sender has incentive to "understate" his signal for $\sigma > \hat{\sigma}$ and incentive to "overstate" it for $\sigma < \hat{\sigma}$. We have referred to them together as incentive to send "moderate" messages since when σ is higher (lower) than $\hat{\sigma}$ the sender wants to convince the receiver that σ is lower (higher) and closer to $\hat{\sigma}$ than it actually is.

3 Equilibrium

We have seen that the possibility of $c \neq p$ substantially alters the structure of the sender's incentive to misreport. If $c = p$ and $b > 0$, we have $y^S(\sigma) > y^R(\sigma)$ for all σ. For this case CS have shown that under general assumptions on preferences and distribution every perfect

[5] As we have noted in Footnote 1, in this game the messages are arbitrary and do not necessarily have to refer literally to the values of signals.

Bayesian equilibrium of the cheap talk game is partitional, in that the type (signal) space is divided into a finite number of intervals and all types in an interval induce the same action. The equilibrium characterization is generalized further by Gordon (2010) to cases where, as in our model, the parties' desired actions may coincide for a certain type (the agreement type $\hat{\sigma} \in [0,1]$ in our model). The receiver's best response (1) and the sender's desired action (2) imply that our model constitutes a case of the *admissible problem* formulated and analyzed by Gordon (2010), and we can apply his equilibrium characterization to our model. Therefore following CS and Gordon (2010) we are able to characterize equilibria using an "arbitrage" condition, which says that an equilibrium partition is determined in such a way that any boundary type is indifferent between the actions induced by the types in the interval on left hand side and those in the interval on the right hand side.

Let $\underline{a}$ and $\overline{a}$ be two points in $[0,1]$ such that $\underline{a} < \overline{a}$. Suppose that the sender observes $\sigma \in [\underline{a}, \overline{a})$. Define $\bar{y}(\underline{a}, \overline{a})$ to be the receiver's best response given her belief $\sigma \in [\underline{a}, \overline{a})$. Since σ is uniformly distributed (1) implies

$$\bar{y}(\underline{a}, \overline{a}) = p\frac{\underline{a} + \overline{a}}{2} + (1-p)\frac{1}{2}. \tag{4}$$

In an equilibrium partition with N intervals, each boundary a_i must satisfy an "arbitrage" condition, which says that the sender with $\sigma = a_i$ must be indifferent between inducing $\bar{y}(a_{i-1}, a_i)$ and $\bar{y}(a_i, a_{i+1})$:

$$\begin{aligned} & -c(\bar{y}(a_{i-1}, a_i) - a_i - b)^2 + (1-c)\int_0^1 -(\bar{y}(a_{i-1}, a_i) - \theta - b)^2 d\theta \\ = \; & -c(\bar{y}(a_i, a_{i+1}) - a_i - b)^2 + (1-c)\int_0^1 -(\bar{y}(a_i, a_{i+1}) - \theta - b)^2 d\theta. \end{aligned} \tag{5}$$

By using (4), (5) can be written

$$pa_{i+1} - (4c - 2p)a_i + pa_{i-1} = 4b - 2(c-p). \tag{6}$$

This second-order difference equation describes the (unique) equilibrium partition for given c, p, b, and N where

$$a_0 = 0, a_N = 1 \tag{7}$$

and $a_i < a_{i+1}$ for all $i \in \{0, 1, 2, ..., N-1\}$. In what follows we refer to an equilibrium with at least two non-degenerate intervals as an *informative equilibrium*, where at least two different actions are induced with positive probability.

Before we formally describe equilibria, let us observe their interesting characteristics with respect to the levels of confidence and competence, c and p. (6) can be rewritten as

$$(a_{i+1} - a_i) - (a_i - a_{i-1}) = \frac{4b - 2(c-p)}{p} + \frac{4(c-p)}{p} a_i, \tag{8}$$

which represents the difference in the length of two adjacent intervals. It is easy to check that the right hand side of (8) is 0 when $a_i = \hat{\sigma}$, the agreement type. This implies that i) the size of the intervals becomes *larger* as they are away from the agreement type when the sender is *overconfident*; and ii) the size of the intervals becomes *smaller* as they are away from the agreement type when the sender is *underconfident*. Suppose that $b = 0$ so that $\hat{\sigma} = 1/2$. In this case, (8) indicates that a moderate message (i.e. message closer to the average) is more informative than an extreme message in communication with an overconfident sender (because the size of intervals tends to be larger), while a moderate message is less informative than an extreme message in communication with an underconfident sender. This reflects our earlier observation that an overconfident sender has incentive to "exaggerate" while an underconfident one has incentive to "moderate" messages.

Note that the qualitative characteristics of informative equilibria we have been able to extrapolate from (6) apply to *any* informative equilibria for given parameter values. As we will see later (6) and (7) can generate multiple equilibria with varying degrees of informativeness (in terms of ex ante expected welfare) but whether the size of the intervals become smaller or larger with respect to the distance from the agreement type does not depend on which informative equilibrium we focus on.

We formally describe the set of all equilibria in the following proposition. Let the sender's strategy specify the probability of sending message $m \in M$ conditional on observing signal σ and we denote it by $q(m \mid \sigma)$. The receiver's best response is given by $y^*(m)$ in (1).

Proposition 1

i) Suppose that $\hat{\sigma} \notin [0,1]$ or that $\hat{\sigma} \in [0,1]$ and $c < p$. Then there exists an integer $\bar{N}$ such that, for every integer N with $1 \leq N \leq \bar{N}$, there exists at least one equilibrium $(y^*(m), q(m \mid \sigma))$, where $q(m \mid \sigma)$ is uniform, supported on $[a_i, a_{i+1}]$ if $\sigma \in (a_i, a_{i+1})$; $y^*(m) = \bar{y}(a_i, a_{i+1})$ for all $m \in (a_i, a_{i+1})$; and (7) and (6) hold. Moreover, no equilibrium with $N > \bar{N}$ exists.

ii) Suppose that $c \geq p$ and $\hat{\sigma} \in [0,1]$. Then for every positive integer N there exists at least one equilibrium $(y^*(m), q(m \mid \sigma))$, where $q(m \mid \sigma)$ is uniform, supported on $[a_i, a_{i+1}]$

if $\sigma \in (a_i, a_{i+1})$; $y^*(m) = \bar{y}(a_i, a_{i+1})$ for all $m \in (a_i, a_{i+1})$; and (7) and (6) hold.

All other equilibria are outcome equivalent to the ones identified above.

Proof. See Appendix.

The proposition says that, regardless of the confidence levels c and p, the perfect Bayesian equilibria of this game are partitional. Note in particular that Part ii) of the proposition implies that when the sender is overconfident and the agreement type is in the type space, i.e. $\hat{\sigma} \in [0, 1]$, there always exists an equilibrium with an infinite number of intervals. However, if the sender is underconfident ($c < p$) the number of intervals is always finite. The outcome equivalence means that other equilibria can involve different messages to be communicated, but they all have to feature the same correspondence between the signal σ and action y, for given N.

- **Common Confidence Benchmark ($c = p$)**

If both parties have the same level of confidence about the sender's competence (probability that the sender observes the true state) we have $c = p$. Substituting this into (6) the equilibrium partitions in this case are given by

$$a_{i+1} - 2a_i + a_{i-1} = \frac{4b}{p}. \tag{9}$$

Under common confidence the only difference from CS is that we may have $p < 1$ while they assume $p = 1$. If $b = 0$, we have $y^R = y^S$ for any σ and hence full revelation is possible in equilibrium because both parties' interests are perfectly aligned. Rewriting (9) we have $(a_{i+1} - a_i) = (a_i - a_{i-1}) + \frac{4b}{p}$. In other words, the equilibrium partitions are such that an interval becomes longer as σ becomes larger. A message is less informative about the signal (or state) when the sender observes higher σ. This reflects the assumption that the sender is positively biased ($b > 0$). As in other cheap talk models there are multiple equilibria, including the "babbling" equilibrium where $a_0 = 0$ and $a_1 = 1$ so that no information is transmitted. We will discuss the issue of multiple equilibria later.

Let us define $\bar{b}$ to be the level of intrinsic bias such that an informative equilibrium exists for any $b \in [0, \bar{b})$. This can be obtained by checking at what value of b the equilibrium with two non-degenerate intervals can be supported. Substituting $a_0 = 0$ and $a_2 = 1$ into (9) we obtain $a_1 = \frac{1}{2} - \frac{2b}{p}$. Hence $a_1 > 0$ implies that an informative equilibrium exists if $b < \bar{b}_{c=p} \equiv \frac{p}{4}$.

Remark 1 Suppose confidence and competence coincide ($c = p$). If $b = 0$, then there exists a fully revealing equilibrium exists. For $b > 0$, the less competent the sender is, the less bias is allowed for equilibrium communication to be informative.

When p is lower (4) implies that the message has less impact on the receiver's action. This makes the relative size of intrinsic bias b with respect to the influence of a message (and signal) larger. As a result, given the level of b, lower p leads to more limited information transmission. Later we use $\bar{b}_{c=p}$ as a benchmark to consider how over- and underconfidence interacts with bias in communication.

4 Effects of Over- and Underconfidence

Let us focus on how over- and underconfidence alone affect equilibrium communication, in the absence of bias. We first consider the distortion of communication due to over- and underconfidence, and then examine their welfare implications.

4.1 Information Distortion and Binary Communication

When a sender is overconfident ($c > p$) and $b = 0$, from (2) and (3) we have seen that the sender has incentive to "exaggerate" his message relative to $\hat{\sigma} = 1/2$. Because of this a fully revealing equilibrium does not exist, while the most informative equilibrium always involves an infinite number of intervals where the sender's message becomes more accurate (i.e. the intervals are shorter) as his type becomes closer to 1/2.

Suppose that the sender is underconfident ($c < p$), which implies the sender believes that his competence is lower than the receiver believes. From (2) and (3) we have $y^S(\sigma) > y^R(\sigma)$ for $\sigma \in [0, 1/2)$ and $y^S(\sigma) < y^R(\sigma)$ for $\sigma \in (1/2, 1]$. That is, now the receiver's reaction to a message is stronger than the sender wants it to be. For σ lower than $\hat{\sigma}$ the sender's desired action is higher than that of the receiver and for σ higher than $\hat{\sigma}$ the sender's desired action is lower. Contrary to the case of an overconfident sender, an underconfident sender has incentive to "moderate" his message, which means that in equilibrium there are a finite number of intervals, and that the message becomes less accurate as the sender's type becomes closer to 1/2.

Solving (6) and (7) for $N = 2$ and $b = 0$, we can confirm that there always exists a binary equilibrium where the type space is divided into $[0, 1/2)$ and $[1/2, 1]$. This has a straightforward interpretation: whether the incentive to misreport is to "exaggerate" or "moderate", the sender does not have any incentive to misreport if his is given binary messages (e.g. "yes or no", "agree or disagree"), since with only two messages he can neither exaggerate nor moderate. Our observation can be summarized as follows:

Remark 2 For $c \neq p$, a fully informative equilibrium does not exist even in the absence of bias ($b = 0$). Meanwhile, if there is no bias, then there always exists a binary partition equilibrium, regardless of the level of over- or underconfidence.

The following proposition states that both over- and underconfidence in communication have the remarkable similarity that, as they become severe, binary communication emerges as the *only* way for the sender to transmit his private information.

Proposition 2 If $b = 0$, as the level of overconfidence becomes extreme ($c = 1$ and $p \to 0$) the most informative equilibrium converges to the binary partition. When the degree of underconfidence is large enough, ($c \leq \frac{p}{2}$) the only informative equilibrium is the one with the binary partition.

Proof. See Appendix ■

When the sender's overconfidence becomes extreme, even the types very close to the agreement type have strong incentive to "exaggerate" as his influence on the receiver's action becomes infinitesimally small. As a result the set of sender types (around the agreement type) that are able to send relatively precise information shrinks, which effectively leaves binary communication as the only way to communicate credibly.[6] On the other hand, when the sender is underconfident a message from extreme types is more likely to be precise owing to the incentive to "moderate". When the degree of underconfidence is very high, this incentive becomes larger even for the lowest ($\sigma = 0$) and the highest ($\sigma = 1$) signal. If even those types prefer to pool with the lower half ($\sigma \in (0, 1/2)$) the higher half ($\sigma \in [1/2, 1)$) the only informative equilibrium is binary, which is the case for $c \leq \frac{p}{2}$.

4.2 Underconfidence and Communication Apprehension

As in most cheap talk models, there are multiple equilibria in our model. In Proposition 1 we have seen that if there is an equilibrium with $\bar{N}$ intervals there also exists an equilibrium with N intervals for any $1 \leq N \leq \bar{N}$. In the CS model with quadratic utilities and uniform type distribution, both the receiver's and the sender's expected utilities are higher as the

[6] The probability that the sender's type coincides exactly with the agreement type is zero.

number of intervals is larger. Thus the most efficient (and "informative") equilibrium under given parameter values is the one that has the largest number of intervals.

In our model the calculation of expected utility can be potentially problematic because we have not specified who holds the "correct" belief on the sender's ability (the probability that he observes the correct signal). Rather, neither party's belief (c nor p) may be true for the above equilibrium construction to be valid. In what follows we compute each player's expected utility according to his/her own belief, without making an assumption on the sender's "true" competence, and base our "welfare" statement on the ex ante expected utility.

In what follows we demonstrate through examples that when a sender is *underconfident*, he may be better off (in terms of his expected utility) in an equilibrium with a smaller number of intervals, or in the uninformative equilibrium (i.e. no communication) than in an informative equilibrium. This feature has not been previously pointed out, but may be closely related to communication avoidance by shy or underconfident individuals studied by psychologists (Richmond and McCroskey, 1997; Zimbardo, 1990). As the receiver is shown to be always better off in an informative equilibrium than in the uninformative one, we point to a potential tension between the receiver and underconfident sender as to whether to engage in communication in the first place.

Moreover, we show that the receiver may also be better off in an equilibrium with a smaller number intervals. In particular, she may prefer communication with an even number of answers (such as binary communication, e.g. "yes" or "no") than that with an odd number of answers (e.g. "yes", "I'm not sure", or "no"). This is because an underconfident sender is likely to send a moderate message (such as "I'm not sure") with a high probability, and in an extreme case most types may send it, which makes communication hardly informative. In binary communication (or communication with an even number of intervals) the underconfident sender does not report a moderate message (because such a message is unavailable), and at the very least the receiver can find out which side of the type space ($[0, 1/2), [1/2, 1]$) the underconfident sender belongs to. Since the binary communication equilibrium exists for any level of underconfidence, asking a binary question could be a powerful "tool" for the receiver when the sender is severely underconfident.

On the other hand, as we suggested earlier this property does not hold in communication with an underconfident sender ($c < p$). Let us first consider an example where the receiver is worse off in an equilibrium in more intervals. Suppose that $c = 0.55$, $p = 1$ and $b = 0$. Then we have $\bar{N} = 3$: the largest number of intervals supported in equilibrium is three, and the corresponding partition is given by $\{[0, 1/12), [1/12, 11/12), [11/12, 1]\}$ where $EU^R = -0.0483$ and $EU^S = -0.0444$. On the other hand in the equilibrium where the partition has two intervals $\{[0, 1/2), [1/2, 1]\}$, we obtain $EU^R = -0.0208$ and $EU^S = -0.0771$ (Table 1). Thus the receiver is better off in the equilibrium with two intervals, while the underconfident sender is better off in the equilibrium with three intervals.

	$N = 2$	$N = 3$
EU^R	-0.0208	-0.0483
EU^S	-0.0771	-0.0444

Table 1: $c = 0.55, p = 1$

	$N = 1$	$N = 2$
EU^R	-0.0833	-0.0208
EU^S	-0.0833	-0.0958

Table 2: $c = 0.40, p = 1$

Note that the length of an interval reflects the precision of the information conveyed by messages. The partition $\{[0, 1/12), [1/12, 11/12), [11/12, 1]\}$ implies that a message is very imprecise (the receiver's posterior is such that $\sigma \in [1/12, 11/12))$ with large probability (0.8333), while only with small probability (0.1777) it is relatively precise (i.e. the posterior is either $\sigma \in [0, 1/12)$ or $\sigma \in [11/12, 1]$). In contrast, when the equilibrium partition is $\{[0, 1/2), [1/2, 1]\}$ the message is always moderately precise (the posterior is either $\sigma \in [0, 1/2)$ or $\sigma \in [1/2, 1]$) compared with the other equilibrium. Overall the receiver, who wants to minimize the expected variance of σ, prefers to be always moderately informed in this example. The underconfident sender prefers the equilibrium with $N = 3$ because due to underconfidence he prefers an action around the one based on the prior $y^S(1/2) = 1/2$, and indeed the action is likely to be (with probability 0.8333) $y = 1/2$ in this equilibrium.

Suppose also that the sender is even more underconfident $c = 0.4$, $p = 1$ and $b = 0$. We then have $\bar{N} = 2$: one equilibrium is uninformative and the other has two intervals $\{[0, 1/2), [1/2, 1]\}$. In the uninformative equilibrium $EU^R = EU^S = -0.0833$. In the equilibrium with two intervals we have $EU^R = -0.0208$ and $EU^S = -0.0958$ (Table 2). Therefore while the receiver prefers the informative equilibrium with two intervals, the sender prefers the uninformative equilibrium. The receiver prefers $N = 2$ to $N = 1$ because in the former equilibrium she can update her belief on the true state, which never hurts her. On the other hand from the underconfident sender's viewpoint the receiver's action varies "too much" according to his message since the receiver puts more weight on the message than the sender wants her to. This reduces the sender's utility because the ex ante difference between the receiver's action and the sender's ideal action is larger for $N = 1$ than $N = 2$.

The findings from the examples above can be summarized as follows:

Remark 3 The expected utility of an underconfident sender may not be monotonic in the number of intervals supported in equilibrium. In particular, a severely underconfident sender (with a large difference between p and c) may be better off staying out of communication than engaging in an informative equilibrium communication.

On the other hand it is easy to see that the receiver always prefers informative communication to no communication. The receiver chooses her action to maximize her expected utility according to her posterior belief. An equilibrium with any non-degenerate intervals enables the receiver to update her belief on the signal and gives higher expected utility than the uninformative equilibrium where the decision depends only on the prior.

Remark 4 The receiver's expected utility is higher in an equilibrium with two or more intervals than in the uninformative equilibrium. Thus an underconfident sender and a receiver may disagree on whether to engage in communication.

The receiver can put an appropriate weight on the message from the sender according to p in order to maximize her expected utility. Therefore when she updates her belief on σ she is better off than in the uninformative equilibrium. However, since his message has a stronger influence on the receiver's action than optimal, the sender may prefer the uninformative equilibrium to an informative one especially when he is severely underconfident, as we have seen in Table 2. This might provide a game theoretic insight into "communication avoidance" or "shyness" discussed in the Introduction. Those concepts typically entail the presumption that one may avoid communication when he should not. Indeed, if we assume that the receiver has the "correct" assessment of the sender's competence p, the expected utilities of the two parties are identical, which implies that a severely underconfident sender *should* engage in communication while he may choose to avoid it.

4.3 Overconfidence and Multiple Equilibria

Let us conclude this section with the following proposition, which confirms that when the players share the same confidence ($c = p$) or the sender is overconfident ($c > p$) both parties are better off in an equilibrium with more intervals. Naturally this implies that, in contrast to the case with an underconfident sender, whenever there exists an informative equilibrium both parties prefer to engage in communication. The proposition is given more generally for any bias $b \geq 0$, which we focus on in the next section.

Proposition 3 For given $c \geq p$ and $b \geq 0$, both the receiver's and the sender's expected utilities are higher in an equilibrium with a larger number of intervals.

Proof. See Appendix ■

5 Confidence and Bias

In the previous section, for the most part we assumed $b = 0$ so that we can clearly separate effects of over- and underconfidence from bias. This section examines how they interact with the sender's bias ($b > 0$) in equilibrium communication. Specifically, we will see that overconfidence increases the scope for information transmission with respect to bias, while underconfidence reduces it. Recall that when the receiver and the sender share the same confidence ($c = p$) there exists an informative equilibrium for all $b \in \left[0, \frac{p}{4}\right)$ and thus an informative equilibrium exists if $b < \bar{b}_{c=p} \equiv \frac{p}{4}$.

5.1 Overconfidence

In communication with a biased overconfident sender ($c > p$), substituting $a_0 = 0$ and $a_2 = 1$ into (9) the equilibrium with two intervals is given by

$$\left\{ \left[0, \frac{1}{2} - \frac{2b}{2c-p}\right), \left[\frac{1}{2} - \frac{2b}{2c-p}, 1\right] \right\}.$$

Hence this equilibrium is informative for

$$a_1 = \frac{1}{2} - \frac{2b}{2c-p} > 0,$$

which can be written as

$$b < \frac{2c-p}{4} = \bar{b}_{c>p},$$

and we have $\bar{b}_{c>p} > \bar{b}_{c=p}$. Thus we have the following remark:

Remark 5 For any given the level of p, more underconfidence allows more bias for informative equilibrium.

The intuition behind this is as follows. Comparing (1) and (2) we can see that when $c > p$ the sender's desired action is more weighted towards the signal σ and away from the prior 1/2. Suppose that the sender's type is low. He has two opposing incentives. One is to "overstate" his type due to bias b, and the other is to "understate" due to overconfidence. These two kinds of informational distortion partly offset each other, so that he may have more incentive to reveal (partially). Therefore overconfidence may mitigate the level of conflict and encourage information transmission. We will see later that slight overconfidence improves communication also when b is positive but small.

In fact, we are able to show that at least slight overconfidence is strictly beneficial also for any $b \in (0, 1/4]$:

Proposition 4 Suppose that there is an informative equilibrium for given levels of $b > 0$, and $p = c$. Then for $c = p + \epsilon$ with some small $\epsilon > 0$, there exists an equilibrium where the receiver's expected utility is higher than the case with $c = p$.

Proof. See Appendix ■

Given the same levels of competence p and intrinsic bias b, the receiver is better off by communicating with a slightly overconfident sender than a sender with common confidence $c = p$. In order to obtain the intuition, consider the simple case where the equilibrium has two intervals $\{[0, a_1), [a_1, 1]\}$ such that $a_1 = \frac{1}{2} - \frac{2b}{2c-p}$. The most desirable binary partition for the receiver is $a_1 = 1/2$ for which the expected ex post variance of the signal is lowest. Note that this is the binary equilibrium partition for $b = 0$. For any $b > 0$ we have $a_1 \neq 1/2$ but since a_1 is increasing in c, compared to the common confidence case $c = p$, as c becomes larger a_1 is closer to the "best" binary partition $a_1 = 1/2$. The receiver clearly favours this change. In other words, overconfidence has the same effect as lower b. While excessive overconfidence reduces informativeness when b is small, slight overconfidence always helps as long as b is positive but not too large.[7]

5.2 Underconfidence

Let us consider communication with a biased underconfident sender. We can compute the upper bound of b such that an informative equilibrium exists, by focusing on the equilibrium partition with two intervals. Solving (6) with $N = 2$, we obtain $a_0 = 0$, $a_1 = \frac{1}{2} - \frac{2b}{2c-p}, a_2 = 1$. First, suppose that $\frac{1}{2}p < c < p$, then we have $a_1 < \frac{1}{2}$ and a_1 is decreasing in b. The informative equilibrium partition is supported in equilibrium if $a_1 > 0$ or[8]

$$b < \frac{2c-p}{4}.$$

[7] Blume, Board and Kawamura (2007) show that a small noise weakly increases the expected utilities of both the sender and the receiver by allowing more information transmission to occur, as in this model.

[8] The probability that the sender's type coincides exactly with the agreement type is zero.

Second, if $0 \leq c < \frac{1}{2}p$, then $a_1 > \frac{1}{2}$ and a_1 is increasing in b. Hence we must have $a_1 < 1$, which implies

$$b < \frac{p - 2c}{4}.$$

Third, if $c \to 1/2p$ then $a_1 \to \infty$ for any $b > 0$ so that the partition cannot be supported in equilibrium for $b > 0$. To summarize, an informative equilibrium exists for

$$b < \bar{b}_{c<p} \equiv \begin{cases} \frac{2c-p}{4} & \text{if } \frac{1}{2}p < c < p \\ \frac{p-2c}{4} & \text{if } 0 < c < \frac{1}{2}p \end{cases} \text{ and} \quad (10)$$

$$b = 0 \text{ if } c = \frac{1}{2}p. \quad (11)$$

It is easy to see that $\bar{b}_{c<p} < \bar{b}_{c=p} = \frac{p}{4}$.

Remark 6 For any given level of p, underconfidence allows less bias for informative equilibrium.

The effect of underconfidence on communication is in stark contrast to that of overconfidence, with which *more* bias is allowed for informative equilibrium. Note that the effect of changes in c and p on the upper bound of the bias b for an informative equilibrium varies according to the values of c and p. To see the intuition for (10) and (11), consider the common confidence case $c = p$ and $b > 0$. In this case lower types can transmit more precise information in equilibrium, as they have less incentive to overstate. Relative to this benchmark, underconfidence ($c < p$) weakens the incentive for the lower types to reveal because it makes desired actions of lower types higher through the incentive to "moderate" towards the prior $1/2$ (i.e. incentive for lower types to "overstate"), which creates the same type of informational distortion as $b > 0$ does. What happens to higher types' incentive to reveal? Underconfidence might lead to incentive to separate through the incentive to "moderate" towards the prior (i.e. incentive for higher types to "understate") but when the sender is only moderately underconfident ($\frac{p}{2} < c < p$) the incentive to "understate" is not strong enough for higher types to overcome the incentive to "overstate" caused by b. In other words, both opposing incentives do not offset each other. Therefore, for this range, underconfidence discourages lower types to reveal and does *not* give enough incentive for higher types to reveal more.

On the other hand, when the sender is extremely underconfident ($0 < c < \frac{1}{2}p$) higher types may partially separate because the incentive to "moderate" for those types (i.e. incentive to "understate") may now offset the positive bias due to b. Consequently, for this range of c the possibility of information transmission increases as the sender becomes more underconfident.

6 Conclusion

This paper offers a first systematic analysis of confidence in communication, including both over- and underconfidence. We have studied information transmission between a receiver (decision maker) and a sender (informed party) who have asymmetric beliefs ("confidence") on the sender's ability ("competence") to observe the state of nature. Overconfidence in communication is characterized by the sender's incentive to "exaggerate", and underconfidence entails incentive to "moderate" his report. Consequently, signals closer to the prior expectation are more accurately reported in communication with an overconfident sender, whereas extreme singles are more accurately reported in communication with an underconfident sender.

Those features generate some interesting insights, especially into communication with underconfident individuals which has not been studied in the economics literature. We have shown that an underconfident sender may prefer to stay out of informative communication, even if there exists an informative equilibrium. This is the case even if underconfidence itself does not involve any physical or mental cost in engaging in information transmission. In other words, this paper offers an alternative (game theoretic) explanation for communication avoidance caused by underconfidence. We have also shown that in communication with an underconfident sender, the receiver may be better off in an equilibrium which effectively involves an even number of messages (actions). In particular, the receiver may prefer binary communication ("yes or no") to ternary communication ("yes, I'm not sure, or "no"), because in the ternary communication equilibrium, a severely underconfident sender would almost always send a moderate message ("I'm not sure"), in which case communication is seldom informative.

We have also shown that, when the sender is intrinsically biased in a particular direction, overconfidence on the sender's side may mitigate the intrinsic bias, while underconfidence exacerbates it. This might explain why overconfidence seems prevalent in various types of expertise, even though underconfidence itself (in the absence of bias) harms communication. Overconfident experts do survive despite potential incentive to "exaggerate" reports: since they may reveal more information when bias is inevitable, those seeking expert advice might in fact welcome a certain degree of overconfidence.

7 Appendix: Proofs

7.1 Proposition 1

Proof.

Part i) This part of the proposition for the case where $\hat{\sigma} \notin [0,1]$ and $c \geq p$ is implied by Theorem 1 of CS, since $y^S(\sigma) > y^R(\sigma)$ for all σ.

For $c < p$, consider a partition satisfying (6) and (7). The roots are complex and any equilibrium partition must be an increasing sequence $0 = a_0 < a_1 < ... < a_N = 1$ given by

$$a_i = \hat{\sigma} + A_N \cos(i\rho + \omega_N), \tag{12}$$

where $\cos\rho = (2c - p)/p$ and $\hat{\sigma}$ is the particular solution. The constants A_N and ω_N are determined by (7), that is,

$$\hat{\sigma} + A_N \cos(\omega_N) = 0 \tag{13}$$

$$\hat{\sigma} + A_N \cos(\omega_N + N\rho) = 1. \tag{14}$$

Combining (13), (14) we have

$$\frac{\cos(\omega_N)}{\cos(\omega_N + N\rho)} = -\frac{\hat{\sigma}}{1 - \hat{\sigma}}. \tag{15}$$

In order for (12) to be increasing in i we must have

$$\omega_N \in (\pi - \rho/2, 2\pi + \rho/2 - N\rho) \tag{16}$$

since, as the cosine function is strictly increasing between π and 2π and symmetric around the extrema, for $a_0 < a_1$ we need $\pi < \rho/2 + \omega_N$; and for $a_{N-1} < a_N$ we need $2\pi > (N-1)\rho + \rho/2 + \omega_N$. Any equilibrium must satisfy (15) and (16). Since (16) is empty for large N, the number of intervals that can be supported in equilibrium is finite.

The range of the left hand side of (15) that also satisfies (16), when (16) is nonempty, is

$$\frac{\cos(\omega_N)}{\cos(\omega_N + N\rho)} \in \left(\frac{\cos(\pi - \rho/2)}{\cos(\pi - \rho/2 + N\rho)}, \frac{\cos(2\pi + \rho/2 - N\rho)}{\cos(2\pi + \rho/2)} \right).$$

The infimum of the range is negative and nondecreasing in N, and the supremum is non-increasing in N. Thus if there exists ω_N that satisfies (15) and (16) for N, then there also exists ω_{N-1} that satisfies (15) and (16) for $N-1$, which implies the equilibrium with $N-1$ intervals.

Part ii) From (2) and (3), for $c \geq p$ and $\hat{\sigma} \in [0,1]$, the sender has an *outward bias* in Gordon's (2010) terms, which means the sender's ideal action given the signal is higher than that of the receiver if for $\sigma > \hat{\sigma}$ and lower for $\sigma < \hat{\sigma}$. (2) and (3) also imply the sender's preference is continuous in y, and the receiver's ideal policy is continuous in σ. Thus we can directly apply Theorem 4 of Gordon (2010) to establish that there exists an informative equilibrium with an infinite number of intervals. Theorem 2 of Gordon (2010) proves that, more generally, if there is an infinite equilibrium then there exists an equilibrium with N intervals for every positive integer N. Hence Part ii) of Proposition 1 holds. ■

7.2 Proposition 2

Proof. Let us first show that the most informative equilibrium with an extremely over-confident sender converges to the binary partition. Solving (6) explicitly for $c > p$ with $a_0 = 0$ and $a_N = 1$ we obtain

$$a_i = \hat{\sigma} + \frac{1 - \hat{\sigma}(1 - B^N)}{A^N - B^N} A^i + \frac{-1 + \hat{\sigma}(1 - A^N)}{A^N - B^N} B^i \tag{17}$$

and equivalently with $a'_0 = 1$ and $a'_N = 0$

$$a'_i = \hat{\sigma} + \frac{-B^N - \hat{\sigma}(1 - B^N)}{A^N - B^N} A^i + \frac{A^N + \hat{\sigma}(1 - A^N)}{A^N - B^N} B^i,$$

where A and B are two distinct roots

$$A = \frac{2c - p + 2\sqrt{c(c-p)}}{p} \text{ and } B = \frac{2c - p - 2\sqrt{c(c-p)}}{p}. \tag{18}$$

Solving (17) for $N \to \infty$ we obtain two sequences

$$\begin{cases} a_i = \hat{\sigma} - \hat{\sigma} B^i & \text{for } a_i \in [0, \hat{\sigma}) \\ a'_i = \hat{\sigma} + (1 - \hat{\sigma}) B^i & \text{for } a'_i \in (\hat{\sigma}, 1] \text{ such that } a'_0 = 1 \end{cases}.$$

Since $0 < B < 1$, both sequences converge to $\hat{\sigma}$. If we fix $c = 1$ and $p \to 0$, then $B \to 0$. Hence for $b = 0$ both a_1 and a'_1 converge to $\hat{\sigma} = 1/2$, which implies that the infinite partition equilibrium converges to the binary equilibrium.

For $c < p$, solving (6) with $b = 0$, $a_0 = 0$ and $a_3 = 1$ (equilibrium with three intervals) we obtain $a_1 = \frac{2c-p}{4c-p}$ and $a_2 = \frac{2c}{4c-p}$. We have $a_1 \leq 0$ or equivalently $a_2 \geq 1$ if $c \leq \frac{p}{2}$, which implies the equilibrium with three intervals does not exist and from Proposition 1 no equilibrium with three or more intervals exists. Thus we conclude that the binary equilibrium with partition $\{[0, 1/2), [1/2, 1]\}$ is the only informative equilibrium for $c \leq \frac{p}{2}$.

■

7.3 Proposition 3

Before we prove the proposition, we provide some useful lemmas and outline how we construct the main proof. Let us call a sequence $(a_0, a_1, ..., a_N)$ that satisfies the arbitrage condition (6) a "*solution*" to (6). The monotonicity condition (M) in CS requires that, for given b, c, and p, if we have two solutions a^+ and a^{++} with $a_0^+ = a_0^{++}$ and $a_1^+ > a_1^{++}$, then $a_i^+ > a_i^{++}$ for all $i = 2, 3, ...$ In other words, (M) says that starting from a_0, all solutions to (6) must move up or down together. Also, solving (6) in terms of a_1

$$a_i = \hat{\sigma} + \frac{a_1 - \hat{\sigma}(1-B)}{A-B}A^i - \frac{a_1 - \hat{\sigma}(1-A)}{A-B}B^i,$$

where A and B are the distinct roots in (18). Since $da_i/da_1 > 0$ for all $i = 2, 3, ..., N$ we can see that the equilibrium partition of our model satisfies (M) for $c \geq p$.

In order to show that the expected utilities of both players are higher in an equilibrium with more intervals, CS deform the partition with N intervals to that with $N+1$ intervals, continuously increasing the player's expected utility throughout the deformation. We follow this method, but we need to proceed by two step deformation, rather than one, because when $\hat{\sigma} \in (0, 1]$ the deformation takes place towards the opposite directions for the right-hand and left-hand sides of $\hat{\sigma}$. Intuitively, as the number of interval increases, each boundary type on the left hand side of $\hat{\sigma}$ move to the left (except for $a_0 = 0$) while each boundary type of the right hand side of $\hat{\sigma}$ move to the right (except for $a_N = 1$). We need to perform a different comparative statics for each case.

Let $a(N)$ be the equilibrium partition of size N. We show that $a(N)$ can be deformed to $a(N+1)$ by two steps, continuously increasing the players' expected utility in each step. Here we consider the case where $\hat{\sigma} \in (0, 1]$. We omit the case where $\hat{\sigma} \notin (0, 1]$ because the Proposition for this case can be proven similarly, by using the first step only.

Let the sub-partition of $a(N)$ equal or below $\hat{\sigma}$ be $\underline{a}(N) \equiv (a_0(N), a_2(N), ..., a_K(N))$ where $a_0(N) = 0$. Also, suppose that $a_K(N)$ is closer to $\hat{\sigma}$ than $a_{K+1}(N)$ is, in other words, $\hat{\sigma} - a_K(N) < a_{K+1}(N) - \hat{\sigma}$. In the following we proceed in two steps:[9]

1. We fix $a_K(N)$ and make the sub-partition $(a_K(N), a_{K+1}(N), ..., a_N(N))$ deform continuously to $(a_K(N), a_{K+1}(N+1), a_{K+2}(N+1), ..., a_{N+1}(N+1))$, increasing the expected utility.

2. We make the sub-partition $(a_0(N), a_1(N)..., a_K(N))$ deform continuously to $(a_0(N+1), a_2(N+1), ..., a_K(N+1))$, increasing the expected utility.

[9] If $\hat{\sigma} - a_K(N) \geq a_{K+1}(N) - \hat{\sigma}$ then the two deformation steps must be reversed. See below.

- If $\hat{\sigma}-a_K(N) \geq a_{K+1}(N)-\hat{\sigma}$ then the first step deforms $(a_0(N), a_1(N), ..., a_K(N), a_{K+1}(N))$ to $(a_0(N+1), a_1(N+1), ..., a_{K+1}(N+1), a_{K+1}(N))$ while fixing $a_{K+1}(N)$, and the second step deforms $(a_{K+1}(N), a_{K+2}(N), ..., a_N(N))$ to $(a_{K+2}(N+1), a_{K+3}(N+1), ..., a_{N+1}(N+1))$. Except for this, the same method and result as the case where $\hat{\sigma} - a_K(N) < a_{K+1}(N) - \hat{\sigma}$ apply.

Lemma 1 If $a(N)$ and $a(N+1)$ are two equilibrium partitions for the same values of b, c and p, then $a_{i-1}(N) < a_i(N+1) < a_i(N)$.

Proof. See Lemma 3 (p.1446) in CS. The proof follows directly from (M). ■

The first step of deformation is carried out as follows. Let $(a_K^x, a_{K+1}^x, ..., a_i^x, ..., a_{N+1}^x)$ be the sub-partition that satisfies (6) for all $i = K+1, K+2, ..., N$ with $a_K^x = a_K(N)$, $a_N^x = x$ and $a_{N+1}^x = 1$. If $x = a_{N-1}(N)$ then $a_{K+1}^x = a_K^x = a_K(N)$. If $x = a_N(N+1)$ then we have $(a_K(N), a_{K+1}(N+1), ..., a_N(N+1))$, where (6) is satisfied for all $i = K+2, K+3, ..., N$. We are going show that, if $x \in [a_{N-1}(N), a_N(N+1)]$, which is again a non-degenerate interval by Lemma 1, then the sender's expected utility is strictly increasing in x.

In the second step, let $(a_0^z,\ a_1^z, ..., a_i^z, ..., a_K^z)$ be the sub-partition that satisfies (6) for $i = 1, 2, ..., K-1$, with $a_0^z = 0$ and $a_K^z = z$. If $z = a_K(N)$ then $a_i^z = a_i(N)$ for all $i = 0, 1, ..., K$. If $z = a_K(N+1)$ then $a_i^z = a_i(N+1)$ for all $i = 0, 1, ..., K$. We will show that when $z \in [a_K(N+1), a_K(N)]$, which is again a non-degenerate interval by Lemma 1, the sender's expected utility is strictly decreasing in z.

Lemma 2 Suppose that $(a_0, a_1, ..., a_i, ..., a_N)$ is a solution to (6). Then for all $i = 1, 2, ..., N-1$ if $a_i > (<)\hat{\sigma}$ then $a_i - a_{i-1} < a_{i+1} - a_i$ $(a_i - a_{i-1} > a_{i+1} - a_i)$. If $a_i = \hat{\sigma}$ then $a_i - a_{i-1} = a_{i+1} - a_i$.

Proof. Rearranging (6) we have

$$(a_{i+1} - a_i) - (a_i - a_{i-1}) = \frac{4ca_i + 4b - 2c}{p} - 4a_i + 2. \tag{19}$$

The left hand side $(a_{i+1} - a_i) - (a_i - a_{i-1}) = 0$ if

$$\begin{aligned} \frac{4ca_i + 4b - 2c}{p} - 4a_i + 2 &= 0 \Rightarrow \\ 4a_i(c-p) &= -4b + 2c - 2p \Rightarrow \\ a_i &= \frac{1}{2} - \frac{b}{c-p} \equiv \hat{\sigma}. \end{aligned}$$

Since the right hand side of (19) is increasing in a_i for $c > p$. Thus if $a_i > \hat{\sigma}$ then $(a_{i+1} - a_i) - (a_i - a_{i-1}) > 0$, and if $a_i < \hat{\sigma}$ then $(a_{i+1} - a_i) - (a_i - a_{i-1}) < 0$. ■

The above lemma says that an interval $[a_i, a_{i+1})$ is longer (shorter) than the previous interval $[a_{i-1}, a_i)$ when $a_i > (<)\hat{\sigma}$. The following Lemma is similar but cannot be implied by Lemma 2. Since by definition a^x_K and a^z_{K+1} are fixed throughout the respective deformation, (6) is not satisfied at $a_i = a^x_{K+1}$ for $x \in (a_{N-1}(N), a_N(N+1))$ or $a_i = a^z_K$ for $z \in (a_K(N+1), a_K(N))$.

Lemma 3 $a^x_{K+1} - a^x_K < a^x_{K+2} - a^x_{K+1}$ and $a^z_K - a^z_{K-1} > a^z_{K+1} - a^z_K$.

Proof. Suppose $a^x_{K+1} \leq \hat{\sigma}$. By construction $\hat{\sigma} - a_K(N) < a_{K+1}(N) - \hat{\sigma}$. Since $a^x_K = a_K(N)$ and $a^x_{K+2} > a_{K+1}(N)$, we have $\hat{\sigma} - a^x_K < a^x_{K+2} - \hat{\sigma}$ and hence $a^x_{K+1} - a^x_K < a^x_{K+2} - a^x_{K+1}$ as stated. Suppose $a^x_{K+1} > \hat{\sigma}$. From Lemma 2 we have $a^x_{K+1} - \tilde{a}_K < a^x_{K+2} - a^x_{K+1}$ where $\tilde{a}_K$ is defined such that $\{a_{i-1} = \tilde{a}_K,\ a_i = a^x_{K+1}, a_{i+1} = a^x_{K+2}\}$ satisfies (6). Since $a_K(N+1) < \tilde{a}_K < a_K(N) = a^x_K$ from Lemma 1, we have $a^x_{K+1} - a^x_K < a^x_{K+2} - a^x_{K+1}$. This proves the first part of the Lemma.

Similarly we have $a^z_K - a^z_{K-1} \geq \check{a}_{K+1} - a^z_K$ where $\check{a}_{K+1}$ is defined such that $\{a_{i-1} = a^z_{K-1}, a_i = a^z_K, a_{i+1} = \check{a}_{K+1}\}$ satisfies (6). Lemma 1 implies $a^z_{K+1} = a_{K+1}(N+1) < \check{a}_{K+1} < a_{K+1}(N)$. Hence we have $a^z_K - a^z_{K-1} > a^z_{K+1} - a^z_K$. ■

Proof of Proposition 3. The sender's expected utility for the first part of deformation is given by

$$\begin{aligned}
EU^S \equiv\ & -c\left[\sum_{i=1}^{K}\int_{a_{i-1}}^{a_i}\left(p\frac{a_{i-1}+a_i}{2}+\frac{1-p}{2}-b-\theta\right)^2 d\theta\right.\\
&\left.+\sum_{i=K+1}^{N+1}\int_{a^x_{i-1}}^{a^x_i}\left(p\frac{a^x_{i-1}+a^x_i}{2}+\frac{1-p}{2}-b-\theta\right)^2 d\theta\right]\\
&-(1-c)\left[\sum_{i=1}^{K}(a_i-a_{i-1})\int_0^1\left(p\frac{a_{i-1}+a_i}{2}+\frac{1-p}{2}-b-\theta\right)^2 d\theta\right.\\
&\left.+\sum_{i=K+1}^{N+1}(a^x_i-a^x_{i-1})\int_0^1\left(p\frac{a^x_{i-1}+a^x_i}{2}+\frac{1-p}{2}-b-\theta\right)^2 d\theta\right].
\end{aligned}$$

It follows that

$$\frac{dEU^S}{dx} \equiv \sum_{i=K+1}^{N+1} \frac{da_i^x}{dx} \times$$

$$\Bigg\{ -c \Bigg[\left(p\frac{a_{i-1}^x + a_i^x}{2} + \frac{1-p}{2} - b - a_i^x \right)^2 - \left(p\frac{a_i^x + a_{i+1}^x}{2} + \frac{1-p}{2} - b - a_i^x \right)^2$$

$$+p\int_{a_{i-1}^x}^{a_i^x} \left(p\frac{a_{i-1}^x + a_i^x}{2} + \frac{1-p}{2} - b - \theta \right) d\theta + p\int_{a_i^x}^{a_{i+1}^x} \left(p\frac{a_i^x + a_{i+1}^x}{2} + \frac{1-p}{2} - b - \theta \right) d\theta \Bigg]$$

$$-(1-c)\Bigg[\int_0^1 \left(p\frac{a_{i-1}^x + a_i^x}{2} + \frac{1-p}{2} - b - \theta \right)^2 d\theta - \int_0^1 \left(p\frac{a_i^x + a_{i+1}^x}{2} + \frac{1-p}{2} - b - \theta \right)^2 d\theta$$

$$+p(a_i^x - a_{i-1}^x) \int_0^1 \left(p\frac{a_{i-1}^x + a_i^x}{2} + \frac{1-p}{2} - b - \theta \right) d\theta$$

$$+p(a_{i+1}^x - a_i^x) \int_0^1 \left(p\frac{a_i^x + a_{i+1}^x}{2} + \frac{1-p}{2} - b - \theta \right) d\theta \Bigg] \Bigg\}$$

$$= \sum_{i=K+1}^{N-1} \frac{da_i^x}{dx} \frac{(a_{i+1}^x - a_i^x)(2c-p)p}{4} (a_{i-1}^x - 2a_i^x + a_{i+1}^x) > 0 \qquad (20)$$

The inequality follows since $\frac{da_i^x}{dx} > 0$ and from Lemmas 2 and 3 we have $a_i^x - a_{i-1}^x < a_{i+1}^x - a_i^x \Rightarrow a_{i-1}^x - 2a_i^x - a_{i+1}^x > 0$ for $i = 1, 2, ..., N-1$.

Let us look at the second part of deformation.

$$\frac{dEU^S}{dz} \equiv \sum_{i=1}^{K} \frac{da_i^z}{dz} \times$$

$$\Bigg\{ -c \Bigg[\left(p\frac{a_{i-1}^z + a_i^z}{2} + \frac{1-p}{2} - b - a_i^z \right)^2 - \left(p\frac{a_i^z + a_{i+1}^z}{2} + \frac{1-p}{2} - b - a_i^z \right)^2$$

$$+p\int_{a_{i-1}^z}^{a_i^z} \left(p\frac{a_{i-1}^z + a_i^z}{2} + \frac{1-p}{2} - b - \theta \right) d\theta + p\int_{a_i^z}^{a_{i+1}^z} \left(p\frac{a_i^z + a_{i+1}^z}{2} + \frac{1-p}{2} - b - \theta \right) d\theta \Bigg]$$

$$-(1-c)\Bigg[\int_0^1 \left(p\frac{a_{i-1}^z + a_i^z}{2} + \frac{1-p}{2} - b - \theta \right)^2 d\theta - \int_0^1 \left(p\frac{a_i^z + a_{i+1}^z}{2} + \frac{1-p}{2} - b - \theta \right)^2 d\theta$$

$$+p(a_i^z - a_{i-1}^z) \int_0^1 \left(p\frac{a_{i-1}^z + a_i^z}{2} + \frac{1-p}{2} - b - \theta \right) d\theta$$

$$+p(a_{i+1}^z - a_i^z) \int_0^1 \left(p\frac{a_i^z + a_{i+1}^z}{2} + \frac{1-p}{2} - b - \theta \right) d\theta \Bigg] \Bigg\}$$

$$= \sum_{i=1}^{K} \frac{da_i^z}{dz} \frac{(a_{i+1}^z - a_i^z)(2c-p)p}{4} (a_{i-1}^z - 2a_i^z + a_{i+1}^z) < 0 \qquad (21)$$

The inequality follows since we have $\frac{da_i^z}{dz} > 0$ and Lemmas 2 and 3 imply $a_i^z - a_{i-1}^z > a_{i+1}^z - a_i^z \Rightarrow a_{i-1}^z - 2a_i^z + a_{i+1}^z < 0$. Therefore, EU^S is increasing throughout the second part of deformation for which z decreases from $a_K(N)$ to $a_K(N+1)$.

Since we have completed the deformation from $a(N)$ to $a(N+1)$ by two steps while increasing the expected utility, we conclude that the sender's expected utility is higher in an equilibrium with more intervals.

Following the above two-step deformation, the receiver's expected utility for the first part of deformation is given by

$$\begin{aligned} EU^R \equiv & -p\left[\sum_{i=1}^{K}\int_{a_{i-1}}^{a_i}\left(p\frac{a_{i-1}+a_i}{2}+\frac{1-p}{2}-\theta\right)^2 d\theta \right. \\ & \left. +\sum_{i=K+1}^{N+1}\int_{a_{i-1}^x}^{a_i^x}\left(p\frac{a_{i-1}^x+a_i^x}{2}+\frac{1-p}{2}-\theta\right)^2 d\theta\right] \\ & -(1-p)\left[\sum_{i=1}^{K}(a_i-a_{i-1})\int_0^1\left(p\frac{a_{i-1}+a_i}{2}+\frac{1-p}{2}-\theta\right)^2 d\theta \right. \\ & \left. +\sum_{i=K+1}^{N+1}(a_i^x-a_{i-1}^x)\int_0^1\left(p\frac{a_{i-1}^x+a_i^x}{2}+\frac{1-p}{2}-\theta\right)^2 d\theta\right]. \end{aligned}$$

Note that the expected utility is identical to that of the sender, except that $b=0$ and $c=p$. Therefore, in order to show that the receiver's expected utility is higher in an equilibrium with more intervals, we can directly apply the the argument we have used for the sender's expected utility. ■

7.4 Proposition 4

Proof. Differentiating (17) with respect to $\hat{\sigma}$ we have

$$\lim_{c\to p}\frac{da_i}{dc}=\frac{\overbrace{i(i-N)}^{-}[-2bN(1-i^2+iN+N^2)+(2i-N)p]}{\underbrace{3Np^2}_{+}}>0 \text{ for } i=1,2,...,N-1. \quad (22)$$

For the inequality to follow, the term in the square brackets must be negative

$$\underbrace{-2bN(1-i^2+iN+N^2)}_{-}+(2i-N)p<0.$$

If this inequality holds for $i=N-1$ (so that the second term on the left hand side is the largest), it is also satisfied for all $i=1,2,...,N-1$. Thus substituting $i=N-1$ we obtain

$$b>\frac{(N-2)p}{2N^2(N+1)}. \quad (23)$$

The assumption that the equilibrium with N intervals is the most informative equilibrium implies

$$\underbrace{\frac{p}{2N(N-1)}}_{\text{Sup of bias for eqm with } N \text{ intervals.}} > b > \underbrace{\frac{p}{2N(N+1)}}_{\text{Sup of bias for eqm with } N+1 \text{ intervals}} \tag{24}$$

and it is easy to check that

$$\frac{p}{2N(N+1)} > \frac{(N-2)p}{2N^2(N+1)}.$$

This implies that in the most informative equilibrium (23) and hence (22) hold.

The receiver's expected utility is given by

$$\begin{aligned} EU^R &= -p\sum_{i=1}^{N}\int_{a_{i-1}}^{a_i}\left(p\frac{a_{i-1}+a_i}{2}+\frac{1-p}{2}-\theta\right)^2 d\theta \\ &\quad -(1-p)\sum_{i=1}^{N}(a_i-a_{i-1})\int_0^1\left(p\frac{a_{i-1}+a_i}{2}+\frac{1-p}{2}-\theta\right)^2 d\theta \end{aligned}$$

Fix the level of p and let $a(N,\bar{c})$ be the partition with N non-degenerate intervals and $p = c = \bar{c}$. Since $b > 0$ any informative equilibrium has a finite number of intervals (Proposition 1). By continuity we can construct the equilibrium partition with N intervals with $c = \bar{c}+\epsilon$ for small enough ϵ, which we denote by $a(N,\bar{c}+\epsilon)$.

Now we can deform $a(N,\bar{c})$ into $a(N,\bar{c}+\epsilon)$, increasing the receiver's expected utility throughout the deformation, as we have done in (20). ■

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