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Rational Expectations Dynamics: A Methodological Critique

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Abstract

This paper analyses RE macromodels from the methodological perspective. It proposes a particular property, robustness, which should be considered a necessary feature of scientifically valid models in economics, but which is absent from many RE macromodels. To restore this property many macroeconomists resort to detailed and implausible assumptions, which take their models a long way from simple Rational Expectations. The paper draws attention to the problems inherent in the technique of local linearisation and concludes by proposing the use of nonlinear models, analysed globally.

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1 Introduction

Since the 1970s, inspired by Muth's seminal paper (Muth, 1961 [35]), economists have adopted the Rational Expectations (RE) Hypothesis as a cornerstone of macroeconomic modelling. The hypothesis asserts that rational economic agents can be assumed to learn from their experience, and that it would be irrational to hold beliefs which were systematically refuted by experience. Agents with rational expectations would behave as if they knew the "true" model of the macroeconomy (as known to the economist) and based their expectations on that model. They might make errors in the formation of their expectations, but not systematic errors. The RE Hypothesis is best thought of as an equilibrium condition of a learning process (much as the zero-profit condition is an equilibrium condition of the process of free entry under perfect competition). Macroeconomic learning is a complicated business, and it is useful for modelling purposes to have a simple equilibrium condition of the learning process, which can be incorporated into macroeconomic models. The RE Hypothesis is consistent with a wide variety of different macroeconomic models, including New Classical models with detailed microfoundations and full market clearing, (e.g. Lucas and Sargent, 1981 [32] and Turnovsky, 1995 [44]), as well as New Keynesian models with price or wage stickiness (e.g. Buiter and Miller, 1981 [10] and Chiarella and Flaschel, 2000 [16])

The RE Hypothesis clearly provides a simple way to represent the complicated process of expectations formation, and as such it is hard to object to. In practice however it rarely does much work on its own. To derive useful predictions requires that the RE hypothesis be embedded in some kind of dynamic macroeconomic model, which is typically a linear (or linearised) model with saddlepoint dynamics. Picking the year 1991 at random, the following papers contain linear saddlepoint models: Chadha, 1991 [11]; Froot and Obstfeld, 1991 [22]; Manase, 1991 [33]; Montiel and Haque, 1991[34]; Nielsen and Sorensen, 1991 [37]; Turnovsky and Sen, 1991, [46]; Sussman, 1991, [43]; Van der Ploeg, 1991 [48]. Earlier objections to saddlepoint dynamics were apparently banished, or even turned to the macroeconomists's advantage, according to Begg, 1982 [4]:

> This (a saddlepoint solution) used to trouble macroeconomists: only by a fluke would the economy happen to begin at a point on the unique convergent path. The comforting belief in the underlying stability of the economic system seemed to have been challenged. The literature on Rational Expectations stands this argument on its head. It is now argued that, when the steady state is a saddlepoint, the economy will succeed in locating the unique convergent path. (Begg,1982 [4])

An advantage of adopting a linear model is that it permits the macroeconomist to invoke the Certainty Equivalence Principle. This asserts that the solution of a stochastic model differs from its deterministic counterpart only in the sense that actual values of future variables are replaced with current expectations of those future variables. Certainty Equivalence allows any truly stochastic elements to be washed out of the system, so that stochastic perturbations will have no effect on the deterministic elements of the model. Most RE models invoke Certainty Equivalence in order to circumvent the statistical distribution problems inherent in the Muth definition of Rational Expectations. It is important to note that, in a nonlinear model Certainty Equivalence cannot be invoked.

This class of macroeconomic models, involving Rational Expectations plus linear (or linearised) saddlepoint dynamics will be referred to as the "Macrodynamic Orthodoxy" throughout this paper. A main object of the paper is to analyse these models from the methodological perspective. We start by proposing a property we call "robustness", which, we argue, should be required of any scientifically valid model. We then show that some Macrodynamic Orthodoxy models do not have this property. Other models in this class do have the robustness property, but to ensure this their authors have had to invoke many detailed and usually implausible assumptions, which take their models a long way from their origins in the Rational Expectations Hypothesis. Finally we argue that the linear model was only ever a local approximation, which can easily mislead the economist: it is time therefore for macroeconomists to abandon it and turn instead to global, nonlinear dynamic modelling.

2 Scientific Method

Lucas and Sargent, 1979 [32] and 1981 [31] attack their Keynesian predecessors on the grounds that their approach was "non-scientific" and should be replaced with the scientifically more demanding methods of the New Classical Macroeconomics. They look forward to the evolution of macroeconomics into a quantitative scientific discipline. It is widely accepted that scientific assertions, as distinct from say theological ones, should refer to entities which are, in principle observable. If this were not the case theoretical assertions would be immune to empirical testing. It may be that the underlying assumptions of a theory are not themselves open to empirical test but that testable implications can be drawn from them. However, there immediately arises the problem of *verisimilitude.* The underlying assumptions of any theory (particularly in macroeconomics) are unlikely to be exactly true descriptions of the real world but, one hopes, are close approximations to it. Under such circumstances it is important that the implications of a scientific theory are robust with respect to small variations in the underlying assumptions. Such variations should only produce small variations in the theory's implications, not wild and dramatic ones. Without this property empirical testing of theories becomes impossible, because of random environmental perturbations in the conditions under which observations are made. Consider, for example, a chemical theory which predicts the outcome of a particular chemical reaction under conditions of constant ambient temperature. Whatever care the experimental chemist may take, she will not be able to hold the ambient temperature *exactly* constant, it is bound to fluctuate slightly during the course of the experiment. Suppose the outcome of the experiment is substantially different from what the theory predicted. Is the theory refuted? The theorist can always reply that the ambient temperature was not exactly constant, as his theory requires and that the experiment does not, therefore constitute a refutation. This would not be the case if the robustness property, discussed above, had been required of the theory *ab initio.* Had the theory satisfied this property, the experimenter could be sure that, according to the theory, small fluctuations in the ambient temperature could only generate small fluctuations in the outcome of the reaction. An experimental outcome substantially different from the theory's predictions would then constitute a genuine refutation of the theory. Non-robust theoretical predictions are, in practice, *non-observable,* and therefore of no scientific interest.

This kind of problem clearly arises in economics as well as chemistry. Economists rarely obtain their data from experiments, so that testing of theories is usually undertaken by statistical and econometric means. The theory under test is typically expressed as a model involving some parameters which are assumed to be constant. The marginal propensity to consume or the interest elasticity of the demand for money might fall into this category. Of course no-one actually believes that parameters such as these are exactly constant over time: they are bound to vary slightly, just as the ambient temperature would in the chemical example discussed above. It is clear then that the robustness property should be required as a necessary (though not sufficient) property of any economic theory, if that theory is to be regarded as scientifically valid. This point was made by Baumol, 1958 [2] in connection with linear difference equation models of the trade cycle. In such models persistent, regular cycles occur only for certain exact parameter values. Arbitrarily small perturbations in these parameters induce a transmutation to either damped or explosive cycle. Baumol's (1958 [2]) argument is as follows:

> But our statistics are never fine enough to distiguish between a unit root (of the characteristic equation of a linear difference equation) and one which takes values so close to it....it is usually possible to show that a slight amendment in one of the simplifying assumptions will eliminate the unit roots and so have profound qualitative effects on the system. As Solow has pointed out, since our premises are necesssarily false, *good theorizing* consists to a large extent in avoiding assumptions like these, where a small change in what is posited will seriously affect the conclusions. (Baumol, 1958 [2], emphasis added)

To make the robustness property operational it is necessary to define it more rigorously We adopt the following definition:

Definition 1 *Any property of a model will be called* robust *if the set of parameter values for which it occurs is of strictly positive Lebesgue measure.*

This definition ensures that small random perturbations of parameters will not cause the given property to disappear. A non-robust property is one which occurs for a set of parameter values of measure zero, and thus can be thought of as having a zero probability of occurring. Of course it is a well known conundrum of probability theory that, although an event which cannot occur has a probability of zero, the converse does not hold. An event with zero probability could occur, though we think it appropriate to label such events as *unobservable.* Note that the definition has been framed in such a way as to ensure that the randomness of perturbations is appropriately captured. Suppose that a certain property P occurs for given parameter values. There may be parameter values arbitrarily near the given values, which cause the property P to disappear, but that does not necessarily mean that P is a non-robust property. For example the property of "having a chaotic trajectory" (to which we return in section 4 below) can easily be robust even though, in models with a chaotic attractor, there often exists a set of periodic points which is dense in that attractor. In this case, arbitrarily close to an initial state of a chaotic trajectory there are unstable periodic points. In fact dense sets may easily have measure zero. For example the set of rational numbers is dense in the set of reals, but is countable and therefore certainly of measure zero.

3 The Macrodynamic Orthodoxy

As in section 1, we will use the term "Macrodynamic Orthodoxy" to refer to the class of macroeconomic models with Rational Expectations embedded in a model with linear (or linearised: we return to this point in section 4) saddlepoint dynamics. Such models have the reduced form:

$$\dot{\mathbf{y}} = A\mathbf{y} - \mathbf{b} \tag{1}$$

where $\mathbf{y}$ is a variable n-vector, $\mathbf{b}$ is a constant n-vector and A is an nxn matrix with a strictly negative determinant and n distinct eigenvalues. The elements of the vector $\mathbf{y}$ may be the natural logs of economic variables which cannot, by their nature, be negative. An *equilibrium* of (1) is simply a vector $\mathbf{y}^*$ such that

$$A\mathbf{y}^* - \mathbf{b} = \mathbf{0} \tag{2}$$

Since A has a strictly negative determinant, it must be invertible, hence equation 2 yields:

$$\mathbf{y}^* = A^{-1}\mathbf{b} \tag{3}$$

and there is a unique equilibrium. By the change of variables:

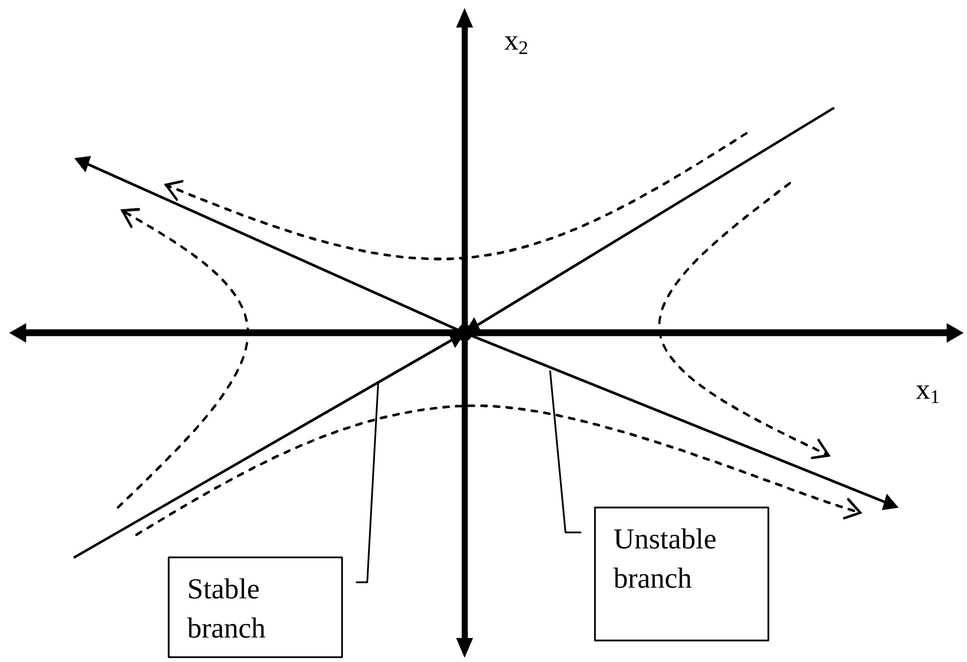
$$\mathbf{x} = \mathbf{y} - \mathbf{y}^* \tag{4}$$

equation 1 can be reduced, without loss of generality, to the homogeneous case:

$$\dot{\mathbf{x}} = A\mathbf{x} \tag{5}$$

to which attention is now turned. Clearly the only equilibrium of (5) is at the origin.

The set of solutions to equation (5) depends on the eigenvalues of the matrix A. Because A has a strictly negative determinant and no repeated



eigenvalues, some of its eigenvalues must have positive real parts and some must have negative real parts. Suppose there are k eigenvalues with negative real parts and m with positive real parts, so that k + m = n. Then $\Re^n$ can be split into two subspaces, intersecting only at the origin, of dimension k and m respectively. The first subspace is spanned by the eigenvectors associated with the eigenvalues with negative real parts ($\mathbf{e}_1.....\mathbf{e}_k$), and the second is spanned by the remaining eigenvectors (i.e. those associated with eigenvalues having positive real parts, $\mathbf{e}_{k+1}.....\mathbf{e}_n$). The first subspace is called the *stable manifold* and the second is called the *unstable manifold.* Equation 5 has a family of solutions which can be represented in a phase portrait in $\Re^n$. The case n = 2, k = 1, m = 1 is depicted in figure 1. In this case the stable and unstable manifolds are referred to as unstable and stable *branches* respectively.

To select the solution to (5) from the family of solutions depicted in its phase portrait requires n independent boundary conditions which may, for example take the form of initial conditions, say $\mathbf{x}(0) = \overline{\mathbf{x}}$. A solution path is therefore a function with $n^2 + n$ parameters:

$$\mathbf{x} = \mathbf{x}(t; A, \overline{\mathbf{x}}) \tag{6}$$

Note that only solution paths with initial conditions lying in the stable manifold converge to the equilibrium. Their parameter values must satisfy:

$$\overline{\mathbf{x}} = \sum_{1}^{k} \lambda_i.\mathbf{e}_i \tag{7}$$

for some scalars λ_i. The parameter set defined by equation (7) is of Lebesgue measure zero, so the property of convergence must be non-robust in this class of models: arbitrarily small random perturbations in any parameter will cause a convergent path to transmute into a divergent one. By a similar argument, divergence is robust in this class of model.

Consider three examples of macrodynamic orthodoxy: Buiter and Miller, 1981 [10]; Eastwod and Venables, 1982 [19] and Neary and Purvis, 1982 [36]. All three are RE models having the reduced form of equation (5) (in suitably transformed coordinates). In the first two models n = 2, k = 1 and m = 1, while in the third n = 3, k = 2 and m = 1. In the Eastwood/Venables model the two endogenous variables are the domestic price level, p and the exchange rate, e. The phase portrait of the model is a two-dimensional linear saddlepoint (similar to figure 1) in which *every* solution path is consistent with Rational Expectations concerning the exchange rate. Note the following remarks by Eastwood and Venables:

> The stable branch plays an important role in the analysis to follow, since we rule out *by assumption* all paths which do not converge to a steady state......The uniqueness of the path (actually followed by the economy) evidently depends on *the assumption that* rational agents anticipate convergence to a steady state. (Eastwood and Venables, 1982 [19] emphasis added)

It is clear that, in addition to the Rational Expectations hypothesis, an extra, *ad hoc*, assumption is required in the Eastwood/Venables model to force the desired result, namely that the economy converges to its steady state. For well-rehearsed reasons, *ad hoc* assumptions violate the standards of good scientific methodology. The same problem besets the Neary/Purvis model:

> This gives rise to a typical saddlepoint structure: the single positive root contributes a direction of instability, but exchange rate speculators are *assumed* to choose an initial value of e (exchange rate) and hence of π (real exchange rate) which ensures that the model converges towards a long-run equilibrium. (Neary and Purvis, emphasis added)

Blanchard, 1981 [6], is not convinced by the standard approach:

> Following standard, if not entirely convincing practice, I will assume that q always adjusts so as to leave the economy on the stable path equilibrium. (Blanchard, 1981 [6])

Buiter and Miller, 1981 [10] try to escape from this problem by invoking a transversality condition. Their model also has the standard linear saddlepoint structure with the standard problem: convergence is non-robust. In the Buiter/Miller model, a factory burning down unexpectedly could lead to ever expanding liquidity and ever falling competitiveness. Thoroughly undesirable no doubt, but perfectly consistent with Rational Expectations. Buiter and Miller rule out outcomes such as this because:

> *The assumption of........*the transversality condition that rational agents will not choose an unstable solution means that the jump variable (e or c) will always assume the value required to place the system on the unique convergent solution trajectory. (Buiter and Miller, 1981 [10] emphasis added)

Jump variables also make an appearance in Christiano and Harrison, 1999 [17]; King and Watson, 1998 [30] and Weder, 2005 [49].

Two new elements enter the story here, (a) the transversality condition and (b) jump variables. The transversality condition arises from a maximising problem. Assume that there exists some agent who maximises:

$$\int_0^\infty U(c,m)e^{-\delta t}dt \tag{8}$$

where c represents consumption, m represents real money balances and the discount rate δ is strictly positive.

Definition 2 *The* transversality condition of (8) *is the condition that* $\mu(t).m(t) \to 0$ *as* $t \to \infty$*, where* μ *is the costate variable associated with the state variable* m*. (*μ *can be interpreted as the discounted shadow value of the money stock).*

The transversality condition can be interpreted as a *terminal condition* of the model and it is easy to show that the transversality condition is only satisfied along convergent paths. Imposing the transversality condition therefore reduces the phase portrait of the model to the stable manifold alone, thus dealing with the non-robustness problem. But what would justify imposing this condition? It is often asserted to be a necessay condition for the maximisation of (8), (see e.g. Obstfeld and Rogoff, 1983 [38]) but this is not the case in general (see Halkin, 1971 [28] for counter examples). Even supposing the class of models can be reduced further to those for which transversality is a necessary condition, two further problems remain. Firstly, who is the agent supposedly maximising (8)? Presumably not the Central Planning Board of a socialist economy, so perhaps a representative household. Accepting the second interpretation, it is clear that the assumption of particular (infinite horizon) maximising behaviour by the household is doing far more work than the Rational Expectations Hypothesis: the maximising assumption selects, from the class of all RE solution paths, a subclass of measure zero.

The second problem concerns the behaviour of the economy when it is away from the stable manifold (perhaps because some policy parameter has changed, causing that manifold to shift). At this stage, the jump variables come into play. These are variables which are assumed to vary discontinuously: they are usually interpreted as the most flexible variables in the model (in the Buiter/Miller model the exchange rate is a jump variable, while the domestic price level, dependent on long-term wage contracts, is a non-jump, or "backward-looking" variable) and are assumed to jump in such a way as to ensure that the economy is in the stable manifold and thus converging to its steady state. While this helpful jump occurs the underlying dynamic of the model is instantaneously suspended, only to be restored again when the economy is back in its stable manifold. Blanchard and Khan, 1980 [7] have shown that, in a linear saddlepoint model, it is generically necessay and sufficient for a unique jump that the number of jump variables is equal to the codimension of the unstable manifold. More jump variables would mean many possible jumps, while fewer would mean no suitable jump at all. In the Buiter/Miller model there is one jump variable (the exchange rate) and an unstable manifold of codimension 1. By an agreeable coincidence, all macrodynamic orthodoxy models have this happy equality of the number of jump variables and codimension of the unstable manifold. However this does nothing for the methodological standing of the macrodynamic orthodoxy. The representative household must now somehow engineer the the appropriate jump if for some reason (such as a shift in policy parameters) the economy should become displaced from its stable manifold: yet another assumption is added to the model to circumvent the non-robustness problem. Buiter, 1984 [9] remarks:

> The problem lies in the economic motivation of the boundary conditions. In *ad hoc* macromodels this motivation can never be fully satisfactory. (Buiter, 1984 [9])

Turnovsky and Nguyen, 1980 [45] note the following:

> In analysing how the system responds to an exogenous policy disturbance we shall impose the assumption that the sytem reamins stable....In this literature (optimal monetary growth) an important role is played by transversality conditions. In most cases the effect of imposing these conditions is to ensure that the optimal (in the sense of an infinite horizon individual utility maximisation problem) path remains stable (i.e. convergent). And while *it may not necessarily be feasible to derive behavioural relationships in macroeconomic models from a full dynamic optimisation,* it is generally desirable for descriptive models (such as this) to be generally consistent with corresponding optimising models, insofar as their stability properties are concerned.....However, models based on perfect myopic foresight are typically inherently unstable.......In this situation the introduction of the transversality condition imposes one initial jump on the sytem allowing it to move instantaneously to some stable adjustment path therefore eliminating all unstable roots....Moreover the nature of the jump depends on the nature of the roots. For example, if all roots are unstable then the system must jump instantaneously to the steady state. In other cases the jump may be onto the stable arm of the saddlepoint. (Turnovsky and Nguyen, 1980 [45] emphasis added)

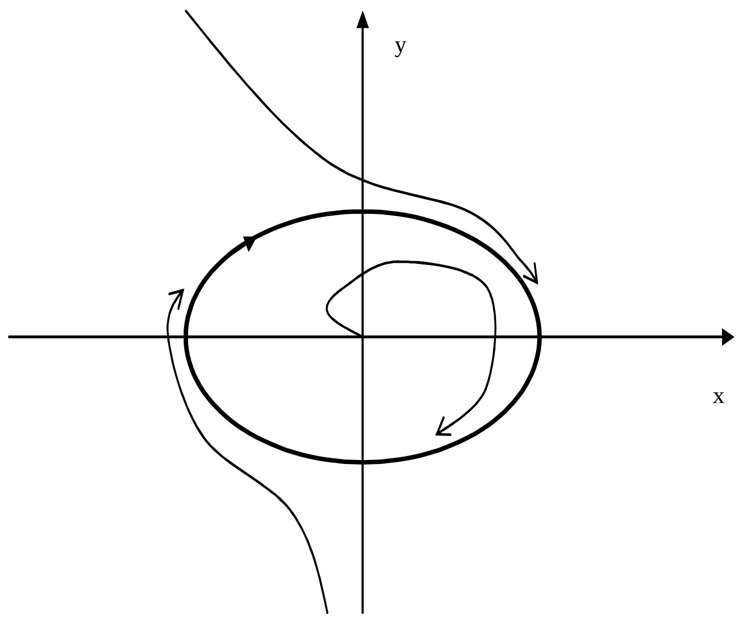
For further discussion of jump variables see Chiarell and Flaschel, 2000 [16]; Chiarella, 1986 [12] and Flaschel and Sethi, 1999 [20].

Clearly the macrodynamic orthodoxy involves far more than the innocuous RE Hypothesis. The Duhem-Quine problem reminds us that any test of a macrodynamic orthodoxy model would involve *jointly* testing all its assumptions, and the maximising, infinite horizon household, replete with jump variables, does far more work in these models than Rational Expectations. In fact macrodynamic orthodoxy models well illustrate a common and undesirable feature of economic models generally, namely reverse-engineering. Having decided, in effect, to retain the convergence property in a robust way, assumptions are added to generate this outcome regardless of their plausibility on theoretical or empirical grounds. Ironically, *divergence* is a robust property of macrodynamic orthodoxy models, so that observing *convergence* of the macroeconomy would satisfactorily refute this type of model. Macrodynamic orthodoxy practitioners seem reluctant to follow that research strategy however.

4 Non-linearity

Prior to 1980 macroeconomists had been convinced of the importance of non-linear models (e.g. Goodwin, 1951 [26]; Hicks, 1950 [29] and Desai, 1973 [18]), but the focus of their attention had been the trade cycle. Theorists of the macrodynamic orthodoxy were more sceptical however; for Lucas and Sargent, 1981 [32] "it is open to question whether for explaining the central features of the business cycle, there will be any big reward to fitting nonlinear models". However, for most macroeconomists the linear models of the macrodynamic orthodoxy were only ever intended as local approximations. These modellers were relying (sometimes without realising it) on a theorem due to Hartman which describes how to construct a local linear approximation to most nonlinear dynamical systems. The technique involved is the familiar one of calculating first partial derivatives at an equilibrium. Hartman's theorem then guarantees that the phase portrait of the linearised system is an approximation to the phase portrait of the original (non-linear) system. The theorem has important caveats however:

1. It does not apply if the linearised system is a centre (i.e. has purely imaginary eigenvalues and a phase portrait consisting of concentric closed orbits).



2. The approximation is only topological (homeomorphism) and

3. It only holds locally.

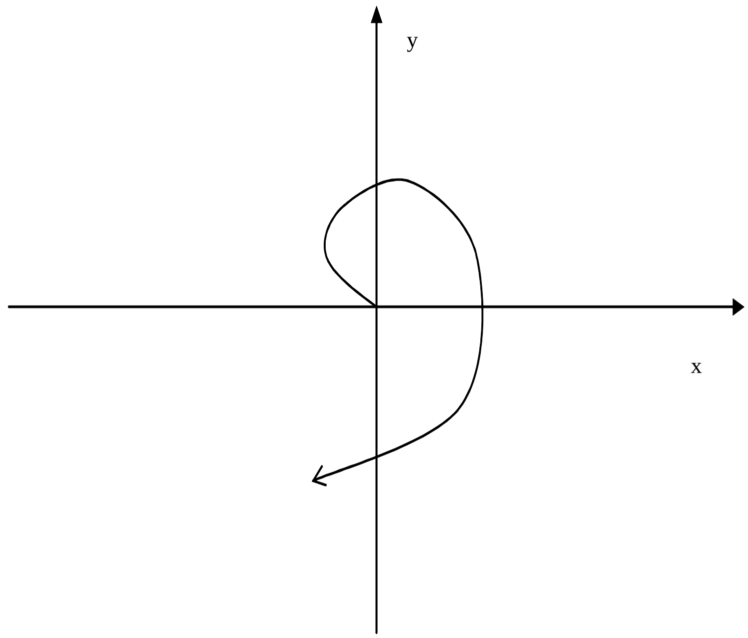
The key problem is point 3 above. Consider the (slightly) non-linear system:

$$\dot{x} = y \tag{9}$$

$$\dot{y} = -x + y - y^3 \tag{10}$$

Its only equilibrium is at the origin (it is an unstable equilibrium) and its phase portrait is that of a stable limit cycle (it is depicted in figure 2).

Solution paths in figure 2 tend towards the bold closed curve (the limit cycle): a macroeconomic model wth the dynamics of equations (9) and (10) displays regular cyclical behaviour. Its local linearisation, however, is very different. It is easily described by calculating the first partial derivatives of equations (9) and (10), yielding:



$$\overset{\bullet}{x} = y \tag{11}$$

$$\overset{\bullet}{y} = -x + y \tag{12}$$

The dynamical system of equations (11) and (12) is an unstable spiral; it is depicted in figure 3. A macroeconomic model with these dynamics diverges to infinity and appears to be of little economc interest. Note that *near the equilibrium* (the origin) the linearisation is a good approximation to the original nonlinear system. Globally however the linearisation is extremely misleading: far from exploding to infinity, the original model settles down to stable, persistent cycles.

This simple example illustrates a general point, namely that nonlinear dynamic models, analysed globally (i.e. not using local linearisation) offer a much richer menu of dynamic behaviour, of potential interest to the macro-economist, than do linear (or linearised) models. In addition to stable and unstable equilibria, centres and limit cycles, there also arises the possibility of strange (chaotic) attractors. Economic applications of chaos theory are discussed by Barnett and Chen, 1986 [1]; Baumol and Benhabib, 1989 [3];

Boldrin and Woodford, 1990 [8]; Frank and Stengos [21], Goodwin, 1990 [27]; Oxley and George, 2005 [39] and Strogatz, 1994 [42]. The use of non-linear modelling also admits the theory of bifurcations and catastrophes (see, for example, Benhabib and Nishimura, 1985 [5] and Perko, 1993 [40]). Catastrophe theory provides a way for jump variables to be analysed as an integral part of the model, rather than being tacked on to it as an unconvincing afterthought in the manner of the macrodynamic orthodoxy (see for example George, 1981 [23] and 1988 [24]) . Finally, note the interesting contribution of Chiarella, 1990 [13] and 1991 [14] to the debate on jump variables in macroeconomic models, and the many economic applications of nonlinear dynamics in George, Oxley and Potter (eds.), 2000 [25] and Puu, 1997 [41].

5 Conclusions

To achieve scientific validity macroeconomic models must make predictions which, at least in principle, are open to empirical refutation. A necessary condition for this is the property of robustness described in this paper. Most models of the macrodynamic orthodoxy make predictions based on the convergence of the macroeconomy to an equilibrium, but in this class of models convergence is non-robust while divergence is robust. A methodologically sound argument in this situation would be to argue that the macroeconomy is actually not divergent, therefore the macrodynamic orthodoxy should be rejected. This line of argument is rarely followed however: macroeconomists typically prefer to reverse-engineer their models to recover the robustness-of-convergence property. This entails extra assumptions which are either unacceptably *ad hoc* or extremely implausible.

We conclude that the Rational Expectations Hypothesis is a useful simplifying assumption which should be retained in macroeconomics, but that the simple linear (or linearised) models in which the hypothesis is typically embedded, should be consigned to the dustbin of history, along with the associated saddlepoint/jump variable fairy story. Macroeconomists could then turn their attention to nonlinear dynamic models, analysed globally, allowing them to place macrodynamics on a genuinely scientific basis.

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