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A two-sector growth model with institutional saving and investment

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Abstract

This paper develops a two-sector growth model in which institutional investors play a significant role. A necessary and sufficient condition is established under which these investors own the entire capital stock in the long run. The dependence of the long-run growth rate on the behaviour of such investors, and the effects of a productivity increase are analysed.

JEL classification. O41, O43

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1 Introduction

In Pasinetti's [1] famous post-Keynesian growth model two classes of agent, workers and capitalists, save constant proportions of their income. On a balanced growth path the rate of profit is independent of the workers' savings propensity. Meade (1963)[2] and Samuelson and Modigliani (1966)[3] prove an "anti-Pasinetti" theorem which establishes the existence of an alternative balanced growth path on which pure capitalists cease to exist and all capital is owned by workers. Kaldor (1966)[4] proposed an alternative insitutional setting for post-Keyensian growth theory in which large capitalist corporations play an important role in savings and investment decisions. This kind of corporate economy is described in Marris (1964) [5], Marris and Wood (1971) [6], Wood (1975) [7] and Eichner (1976, 1985) [8] [9]. Moss (1978) [10] extends Pasinetti's analysis to a corporate economy by dividing Kaldor's (1966) household sector into workers and financial capitalists whose income arises only from financial capital. O'Connell (1985, 1995) [11] [12] develops an alternative approach to the corporate economy, showing that the "anti-Pasinetti" theorem does not hold when firms re-invest a proportion of their profits. Commendatore (1999) [13] extends the Post-Keynesian growth model to a corporate economy, analysing the effects of firm and shareholder behaviour. Feld'man (1928) [14] and Mahalanobis (1953/4) [15] analyse the effects of investment allocation on economic growth. This analysis is extended to a multi-sector model by Araujo and Teixeira (2011) [16].

In this paper we consider a two-sector model in which institutional investors such as pension funds, unit trusts, insurance companies have an important role. It reflects Pasinetti's idea that workers must own the capital to which their savings have given rise, but also acknowledges that, in a modern capitalist economy, these savings are typically mediated by institutions such as pension funds. Dinenis and Scott (1993) [21] argue that pension funds are a major vehicle for personal long-run saving in the UK economy. They report that such funds controlled over £250bn of funds in 1989, their total net assets constituting 38% of personal sector net financial wealth. These funds owned 23% of UK equity, 21% of British government securities and 18% of British holdings of foreign equity. Apilado (1972) [20] investigates whether pension savings in the US economy between 1955 and 1970 are a substitute for other forms of saving. He concludes that they were in fact an addition to other forms of saving and that, via an increase in total saving, generated an increase in the growth rate.

Van Groezen *et. al.* (2007) [17] develop a two-sector growth model with a capital intensive commodity sector (with endogenous growth) and a labour intensive services sector. They analyse the effects on economic growth of a switch to a more funded pension scheme. In this model increased savings resulting from the pension reform generate higher growth in a closed economy provided capital and labour are not strong substitutes. However, the opposite is true for a small open economy. Hachon (2010) [19] analyses the effect of the structure of pension systems on the growth rate. He contrasts "purely

Beveridgian" pension systems, where every agent receives the same pension, with "purely Bismarckian" systems, where pensions depend on agents' wages. Hachon's focus is on the redistributional effects of pensions, in similar vein to a paper of Docquier and Paddison (2003) [18].

2 Structure of the model

In an economy with institutional investors, investment and hence growth are likely to be influenced by the decisions of such investors, but under modern capitalism we also observe high-tech firms which re-invest most of their profits directly and rarely pay dividends. We refer to capital accumulated from retained profits as "corporate capital", and that accumulated through institutional investment as "institutional capital". Two questions which arise naturally in this setting are:

- Will institutional investors eventually own the entire capital stock?
- Will the long run growth rate depend on the behaviour of those institutional investors and, if so, how?

Both these questions are analysed below.

The model assumes two different production sectors. Sector 1 consistes of high-tech, capital intensive firms (e.g. electronics producers such as Apple) which invest all their profits, and also obtain investment from outside institutional investors. It produces an output Q_1 using labour L_1 and capital K_1. Sector 2 consist of medium-tech, less capital intensive firms (e.g. consumer durable producers) whose investment expenditure comes exclusively from outside institutional investors. It produces an output Q_2 using labour L_2 and capital K_2.Total output of the economy will be denoted $Q = Q_1 + Q_2$; total labour employed in the economy will be denoted $L = L_1 + L_2$; total capital employed in the economy will be denoted $K = K_1 + K_2$. Both factors are assumed perfectly mobile, equalising wage and profit rates between the two sectors. Capital is assumed fully employed, but there may be unemployed labour in the economy.

Outside institutional investors receive income based on wages (e.g. pension contributions) and from profits earned on their portion of the capital stock. They invest a proportion, s, of their income, of which a share, θ, goes to sector 1 and $1 - \theta$, goes to sector 2.

We establish conditons under which the growth rate of the economy is independent of the institutional investors' behaviour: in this case the share of the capital stock funded from retained profits remains strictly positive. There is also a balanced growth path along which the growth rate depends on the behaviour of institutional investors; in this case the share of the capital stock funded from retained profits disappears in the long run and the entire capital stock is owned by institutions.

3 Wage and profit rates

Sector 1 consists of high-tech firms with capital-output ratio $k_1 = \frac{K_1}{L_1}$ and output-labour ratio $q_1 = \frac{Q_1}{L_1}$. Sector 2 consists of medium-tech firms with capital-output ratio $k_2 = \frac{K_2}{L_2}$ and outout-labour ratio $q_2 = \frac{Q_2}{L_2}$. It will be assumed that:

$$q_1 > q_2 \text{ and } k_1 > k_2 \tag{1}$$

Togther these inequalities imply that:

$$\frac{K_1}{L_1} > \frac{K_2}{L_2} \tag{2}$$

Wage profit-frontiers can readily be derived for the two sectors. Let w denote the wage rate and r the profit rate. Then:

$$Q_1 = wL_1 + rK_1 \Rightarrow \tag{3}$$
$$1 = \frac{w}{q_1} + rk_1 \tag{4}$$

And:

$$Q_2 = wL_2 + rK_2 \Rightarrow \tag{5}$$
$$1 = \frac{w}{q_2} + rk_2 \tag{6}$$

The two wage-profit frontiers are illustrated in figure 1.

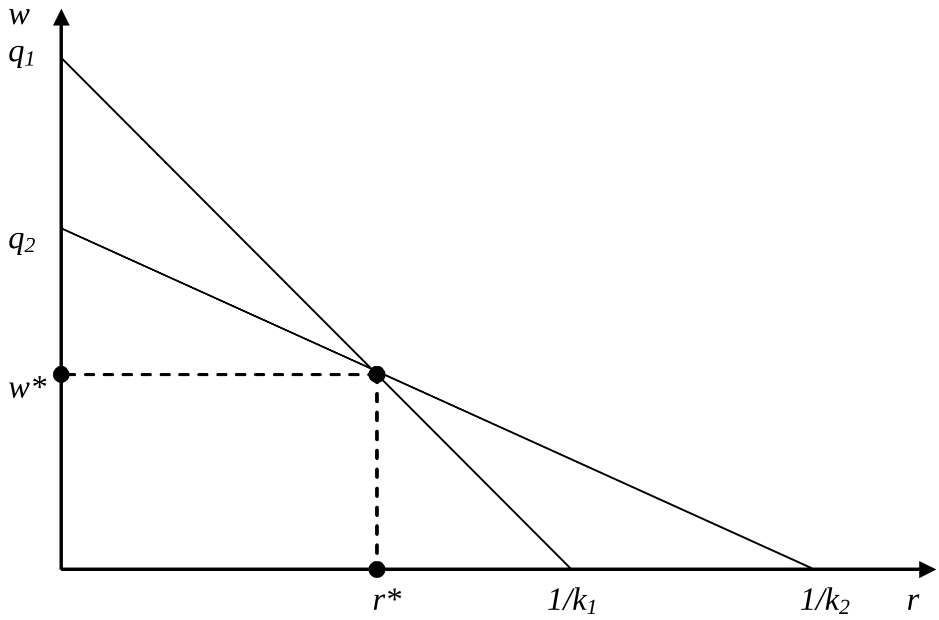

Figure 1. Wage-profit frontiers for sectors 1 and 2.

Inter-sectoral mobility of the two factors ensures that wage and profit rates are determined at the intersection of the two frontiers yielding:

$$w^* = \frac{k_2 - k_1}{k_2/q_1 - k_1/q_2} \tag{7}$$

and:

$$r^* = \frac{q_2 - q_1}{k_2 q_2 - k_1 q_1} \tag{8}$$

Note that the wage-profit frontiers do not assume full employment of labour: the availability of labour is never a constraint on growth. Capital is assumed fully employed, and both factors are assumed instantaneously and costlessly mobile between the two sectors.

4 Capital accumulation

Sector 1 (hi-tech) firms will be assumed to re-invest all their profits and also to receive a share θ of institutional investment. So let $K_1 = X + Y$ where

X = "corporate capital" (i.e. that portion of sector 1 capital funded from retained profits) and Y = "institutional capital" (i.e. that portion of sector 1 capital funded by outside institutions). Sector 2 (medium-tech) firms will be assumed to fund their capital accumulation entirely from outside institutional sources. For notational consistency let $Z = K_2$. Institutional investors own a portion $Y + Z$ of the capital stock. Assume all capital depreciates at a rate δ.

Institutional investors will be assumed to have an income V, consisting of a proportion t of the wage bill (e.g. pension contributions) and the profits they earn on their portion of the capital stock. We therefore have:

$$V = tw^* [L_1 + L_2] + r^* [Y + Z] \tag{9}$$

Using the definitions of section 3 together with equation Y, this yields:

$$V = t \left[\frac{w^* K_1}{k_1 q_1} + \frac{w^* K_2}{k_2 q_2} \right] + r^* [Y + Z] \tag{10}$$

Using the definitions above, this yields:

$$V = t \left[\frac{w^*(X + Y)}{k_1 q_1} + \frac{w^* Z}{k_2 q_2} \right] + r^* [Y + Z] \tag{11}$$

It is now simple to derive accumulation equations for X, Y and Z:

$$\dot{X} = (r^* - \delta)X \tag{12}$$

$$\dot{Y} = (1 - \theta)sV - \delta Y \tag{13}$$

$$\dot{Z} = \theta sV - \delta Z \tag{14}$$

Substituting (11) into (13) and (14) and rearranging gives:

$$\dot{Y} = [(1-\theta)s \left[\frac{tw^*}{k_1 q_1} + r^* \right] - \delta] Y + (1-\theta)s \left[\frac{tw^*}{k_2 q_2} + r^* \right] Z + \frac{(1-\theta)stw^*}{k_1 q_1} X \tag{15}$$

$$\dot{Z} = \theta s \left[\frac{tw^*}{k_1 q_1} + r^* \right] Y + [\theta s \left(\frac{tw^*}{k_2 q_2} + r^* \right) - \delta] Z + \frac{\theta stw^*}{k_1 q_1} X \tag{16}$$

Equations (12), (15) and (16) constitute a linear dynamical system in X, Y and Z. Equation (12) can solved independently to give:

$$X(t) = X(0)e^{(r^* - \delta)t} \tag{17}$$

Now define the following shares in the total capital stock:

$$x = \frac{X}{K};\ y = \frac{Y}{K};\ z = \frac{Z}{K} \tag{18}$$

We focus on conditions necessary and sufficient for the institutional investors to own the whole economy in the steady state (that is $x = 0$, or $\frac{Y+Z}{K} = 1$, in the steady state).

5 Dynamics of the model

We first establish:

Proposition 1 *The dynamical system consisiting of equations (12), (15) and (16) converges to a steady state in which $x > 0$ and the rate of growth is given by $g = r^* - \delta$, if and only if $\frac{stw^*}{(1-s)}\left[\frac{1-\theta}{k_1 q_1} + \frac{\theta}{k_2 q_2}\right] \leq r^*$.*

Proof. The dynamical system clearly has g as one eigenvalue. Let the other two eigenvalues be λ_1 and λ_2. The determinant of the dynamical system is the product of the eigenvalues and the trace is their sum. Thus we may write:

$$\lambda_1 \lambda_2 = \delta(\delta - (1-\theta)a_2 - \theta a_3) \tag{19}$$

$$\lambda_1 + \lambda_2 = (1-\theta)a_2 + \theta a_3 - 2\delta \tag{20}$$

where:

$$a_2 = s\left(\frac{tw^*}{k_1 q_1} + r^*\right) \text{ and } a_3 = s\left(\frac{tw^*}{k_2 q_2} + r^*\right) \tag{21}$$

The solution of these equations, which are symmetric in λ_1 and λ_2, is:

$$\lambda_1 = -\delta \tag{22}$$

$$\lambda_2 = (1-\theta)s\left(\frac{tw^*}{k_1 q_1} + r^*\right) + \theta s\left(\frac{tw^*}{k_2 q_2} + r^*\right) - \delta = \tag{23}$$

$$stw^*\left(\frac{1-\theta}{k_1 q_1} + \frac{\theta}{k_2 q_2}\right) + sr^* - \delta \tag{24}$$

Solutions of the dynamical system take the form:

$$
\begin{aligned}
X(t) &= X(0)e^{gt} && (25)\\
Y(t) &= b_1 e^{\lambda_1 t} + b_2 e^{\lambda_2 t} + b_3 e^{gt} && (26)\\
Z(t) &= c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} + c_3 e^{gt} && (27)
\end{aligned}
$$

It follows that the system will tend to steady state growth at a rate g if and only if $\lambda_2 \leq g$, since $\lambda_1 < 0$. We have $g = r^* - \delta$, so that $\lambda_2 \leq g$ if and only if:

$$
\begin{aligned}
(1-\theta)s\left[\frac{tw^*}{k_1 q_1} + r^*\right] + \theta s\left[\frac{tw^*}{k_2 q_2} + r^*\right] &\leq r^* \Longleftrightarrow && (28)\\
stw^*\left[\frac{1-\theta}{k_1 q_1} + \frac{\theta}{k_2 q_2}\right] &\leq (1-s)r^* \Longleftrightarrow && (29)\\
\frac{stw^*}{(1-s)}\left[\frac{1-\theta}{k_1 q_1} + \frac{\theta}{k_2 q_2}\right] &\leq r^* && (30)
\end{aligned}
$$

which is the required condition. From equations (25), (26) and (27) it is clear that, if $\lambda_2 \leq g$, $x = \dfrac{X}{K} \longrightarrow \dfrac{X(0)}{b_3 + c_3 + X(0)} > 0$ as $t \to \infty$. That is $x > 0$ in the steady state, as required. ■

So, provided the condition (30) of proposition 1 is satisfied, the steady state growth rate is independent of the behaviour of institutional investors, and the long-run share of corporate capital in the total capital stock is non-zero.

We now establish the converse of Proposition 1.

Proposition 2 *The dynamical system consisting of equations (12), (15) and (16) converges to a steady state in which $x = 0$, and the rate of growth is given by $\lambda_2 = stw^*\left(\dfrac{1-\theta}{k_1 q_1} + \dfrac{\theta}{k_2 q_2}\right) + sr^* - \delta$, if and only if condition (30) is violated.*

Proof. By the argument of Proposition 1, $\lambda_2 > g$ if and only if condition (30) is violated. Then, from equations (25), (26) and (27), the steady state growth rate must be equal to λ_2. Moreover, since $\lambda_2 > g$ and $\lambda_1 < 0$, $x = \dfrac{X}{K} \to 0$ as $t \to \infty$. That is, $x = 0$ in the steady state, as required. ■

So, if the condition (30) of proposition 1 is violated, the steady state growth rate does depend on the behaviour of institutional investors. In particular it increases with t (proportion of the wage bill received by institutional investors), s (the invested proportion of institutional investors' income) and θ (the proprtion of that investment that goes to sector 1 (high tech) firms).

6 Numerical simulations

A simple Matlab program was written to simulate the dynamical system of section 5 above. Initial conditions and parameters were set as follows: $\delta = 0.02$, $k_1 = 3.5$, $k_2 = 2.5$, $q_1 = 10.5$, $q_2 = 10.0$, $X(1) = 45.0$, $Y(1) = 25.0$, $Z(1) = 30.0$. This implies equilibrium wage and profit rates of $w^* = 8.936$ and $r^* = 0.043$ respectively. The economy simulated is thus one which, initially contains a significant high tech sector, with accumulation financed prodominantly from retained profits. Initially, it also includes a medium tech sector, financed by institutional investors. Table 1 shows values of x (share of corporate capital in total capital stock) at $time = 20$ and the growth rate at $time = 20$ for various values of the parameters. The first four lines of the table relate to parameter combinations which satisfy condition (30), so that the long-run share of corporate capital in the total capital stock (x) is non-zero, and the long run growth is independent of the investors' parameters (it is equal to $r^* - \delta = 0.023$). The next four lines of the table relate to parameter combinations which violate condition (30), so that the long-run share of corporate capital in the total capital stock (x) is zero and the long run growth rate does depend on investors' parameters, according to equation (24).

s	t	θ	$x(20)$	growth rate at $time = 20$	LR value of x	LR growth rate
0.2	0.05	0.3	0.58	0.0097	0.93	$r^* - \delta =$
0.2	0.05	0.9	0.58	0.010	0.93	
0.2	0.20	0.3	0.51	0.016	0.71	0.023
0.2	0.20	0.9	0.51	0.018	0.68	
0.8	0.05	0.3	0.40	0.028	0.0	0.026
0.8	0.05	0.9	0.40	0.032	0.0	0.028
0.8	0.20	0.3	0.22	0.060	0.0	0.059
0.8	0.20	0.9	0.21	0.071	0.0	0.059

Table 1.

Figures 2 - 5 show output for the simulations described in the first four lines of table 1, corresponding to a low institutional savings ratio (0.2). In each case the share of corporate capital in sector 1 trends upwards towards an upper limit between 0.93 and 0.68. The long-run growth rate is independent of institutional investors' parameters, being equal to $r^* - \delta = 0.023$. Figures 6 - 9 show output for the simulations described in the last four lines of table 1, corresponding to a high institutional savings rate (0.8). In each case thw share of corporate capital trends downwards towards zero. The long run growth rate now depends on institutional investors' parameters. It is significantly higher than the long-run growth rate of figures 2 - 5, ranging from 0.026 to 0.059.

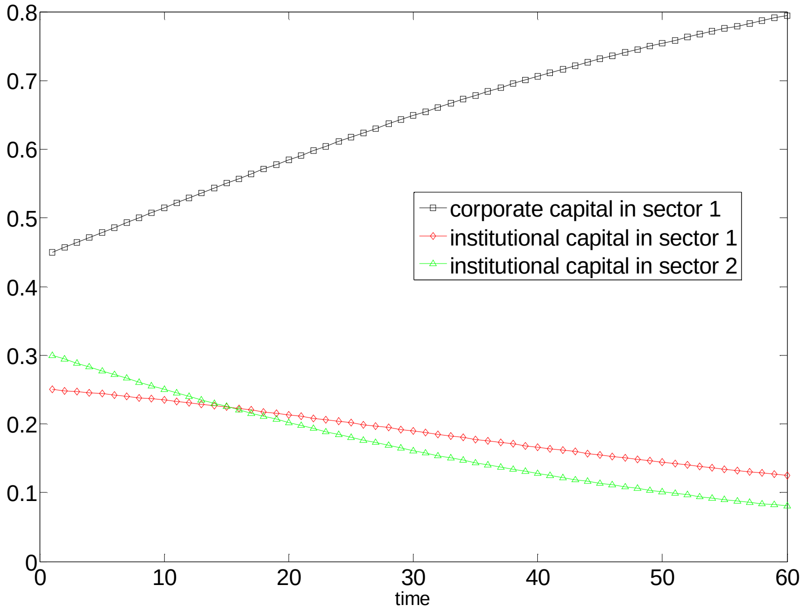

Fig 2. Output of simulation for $s = 0.2, t = 0.05, \theta = 0.3$

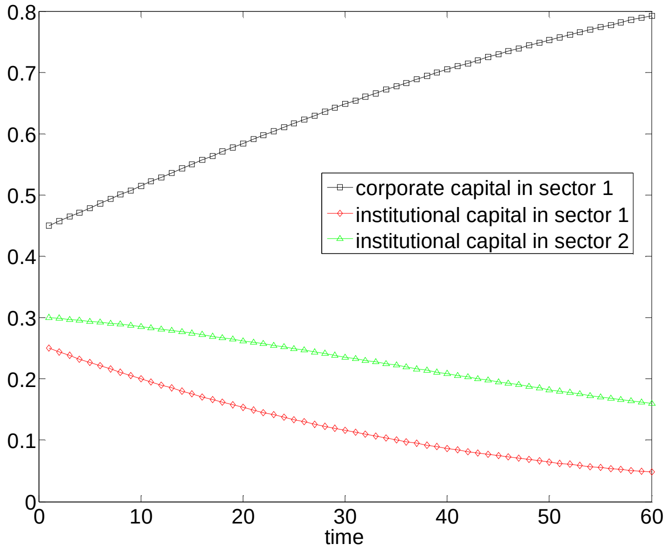

Fig. 3 Output of simulation for $s = 0.2, t = 0.05, \theta = 0.9$

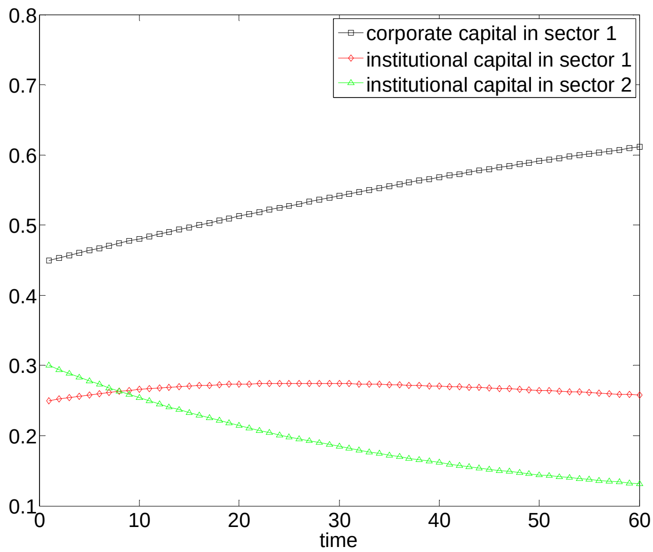

Fig. 4 Output of simulation for $s = 0.2, t = 0.2, \theta = 0.3$

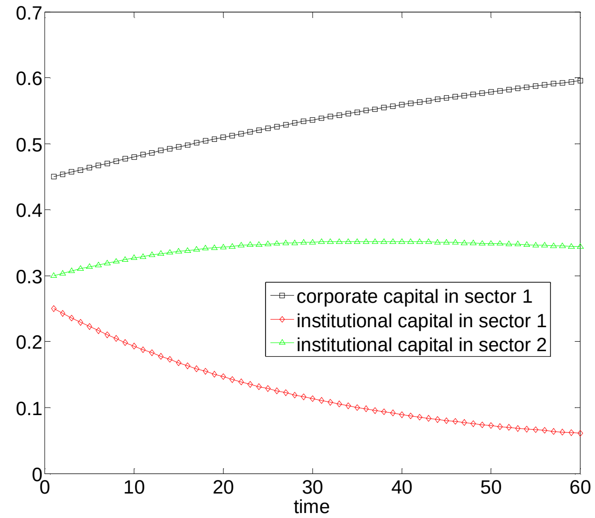

Fig. 5. Output of the simulation for $s = 0.2, t = 0.2, \theta = 0.9$

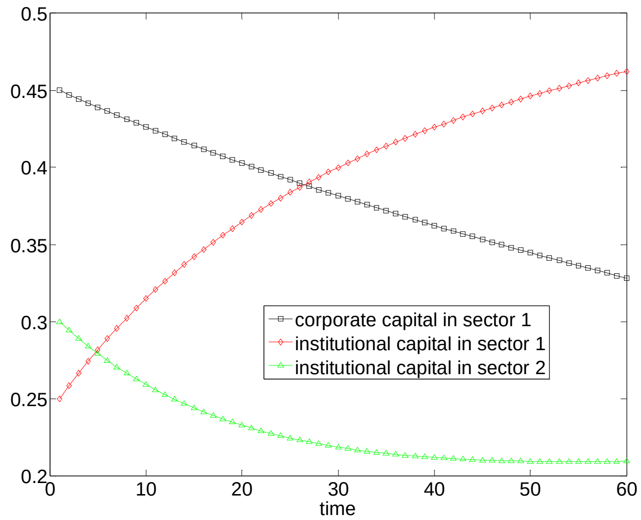

Fig 6. Output of the simulation for $s = 0.8, t = 0.05, \theta = 0.3$

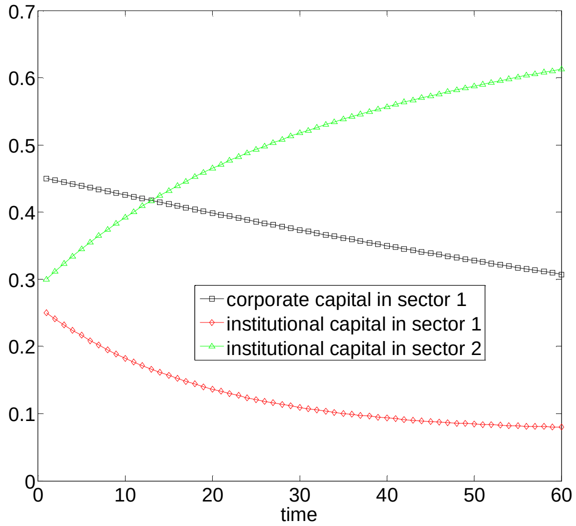

Fig. 7. Output of simulation for $s = 0.8, t = 0.05, \theta = 0.9$

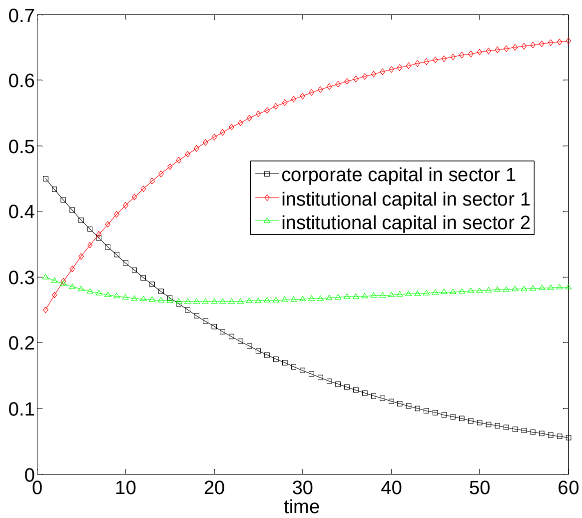

Fig. 8. Output of simulation for $s = 0.8, t = 0.2, \theta = 0.3$

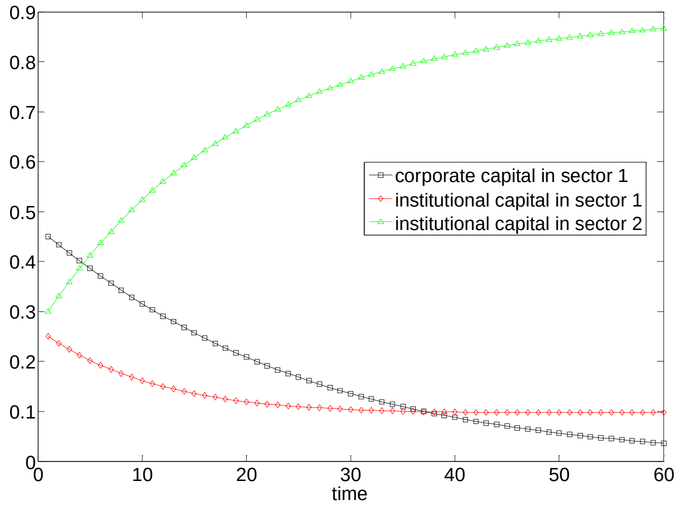

Fig. 9. Output of simulation for $s = 0.8, t = 0.20, \theta = 0.9$

7 Increase in productivity

We now consider the effect of an increase in labour productivity in sector 1 (high-tech) with all else (including labour productivity in sector 2) held constant. Figure 2 depicts the wage-profit frontiers for the two sectors. An increase in labour productivity in sector 1 from q_{11} to q_{12} increases the profit rate from r^* to r^{**} and decreases the wage rate from w^* to w^{**}. It also affects the income of institutional investors and the proportions of that income arising from the two sectors of the economy. The combined effect on corporate capital over time is shown in figures 11 and 12. For the high productivity simulations, q_1 is set at 12.5; for the low productivity simulations q_1 is set at 10.5. In the former case the wage rate is 6.67 and the profit rate is 0.133. In the latter case the wage rate and profit rate are as in section 6 above, 8.936 and 0.043 respectively. Figure 11 depicts the simulation output for $s = 0.2, t = 0.05, \theta = 0.3$, corresponding to line 1 of table 1, in which the share of corporate capital increases much more rapidly in the low productivity case. Figure 12 depicts the simulation output for $s = 0.8, t = 0.2, \theta = 0.9$, corresponding to line 8 of table 1, in which the share of corporate capital declines much more rapidly in the high productivity case.

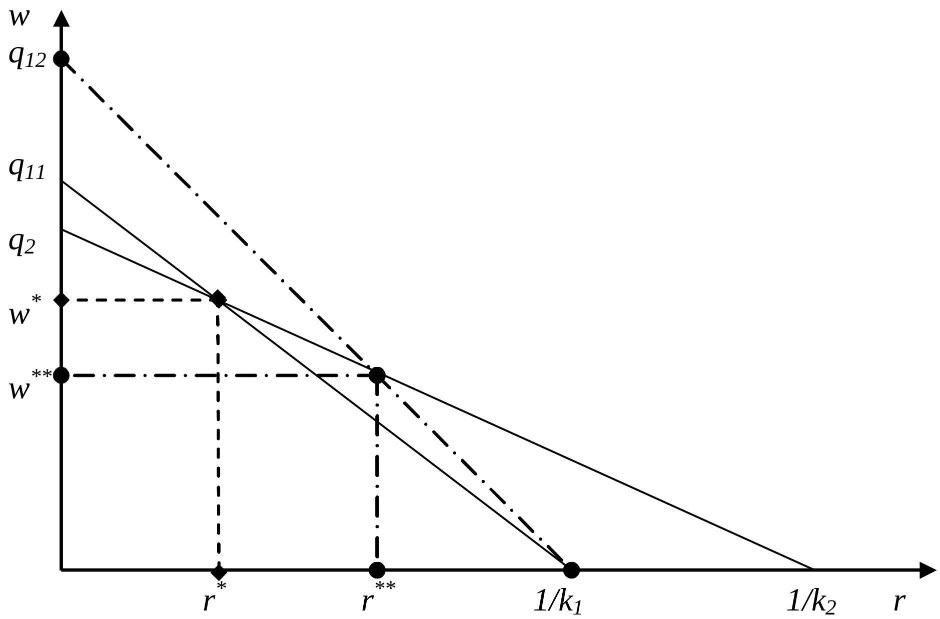

Fig. 10. Wage-profit frontiers for low and high productivity in sector 1.

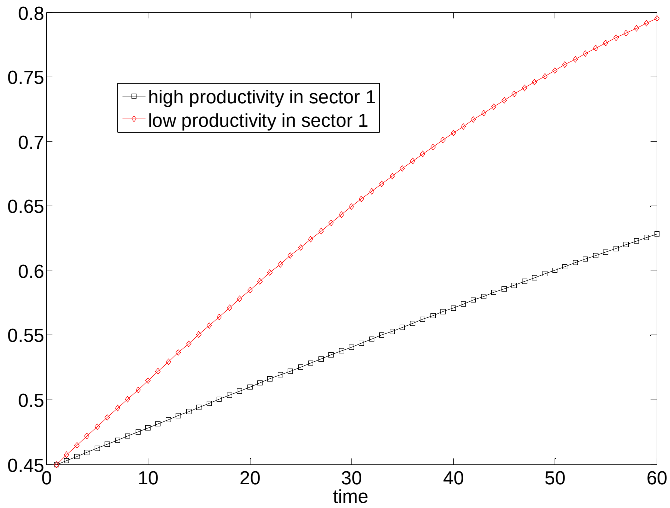

Fig.11 Share of corporate capital for high and low productivity in sector 1

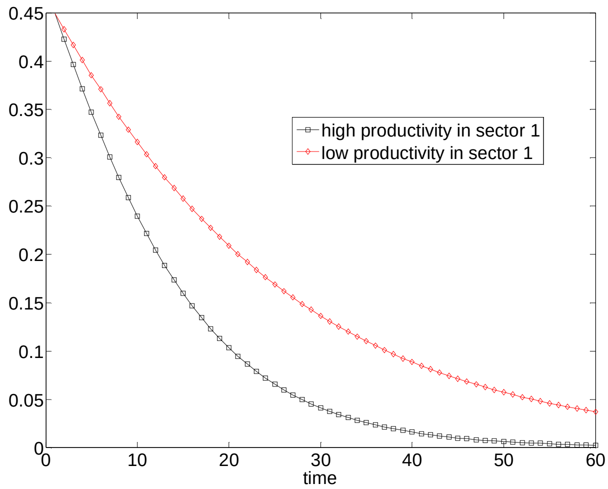

Fig. 12 Share of corporate capital for high and low productivity in sector 1

8 Conclusions

We analyse an economy in which corporate saving is undertaken by high-tech firms with high capital/labour ratios and correspondingly high labour productivity. Institutional saving is undertaken by pension funds, insurance companies and unit trusts which invest in high and medium tech firms (the latter having lower capital/labour ratios and correspondingly lower labour productivity). In high profit economies, the steady state growth rate is independent of the behaviour of institutional investors and the steady state share of corporate capital is non-zero (i.e. the institutions do not own the whole economy in the long run). In low profit economies the steady state growth rate does depend on the behaviour of institutional investors. In particular it increases with t (proportion of the wage bill received by institutional investors), s (the invested proportion of institutional investors' income) and θ (the proportion of that investment that goes to sector 1 (high tech) firms). Moreover, the steady state share of corporate capital is zero (i.e. the insitutions do own the entire economy in the long run). That share declines more rapidly or rises more slowly if labour productivity rises in sector 1.

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