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Technical progress and product reliability under competition and monopoly

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Abstract

Technical progress lowers costs and prices but appears to have an ambiguous effect on product reliabilty. This paper presents a simple model which explains this observation.

Acknowledgements. Precursors of this paper have been presented in seminars at the University of Melbourne (Australia), University of Waikato (New Zealand), University of Canterbury (New Zealand), Hamilton College/Colgate University joint seminar (USA) and Queen's University (Canada). I am grateful for the invitations to those seminars, and for the constructive comments made. I am also indebted to Les Oxley and Richard Watt, and to the Carnegie Foundation for a research grant which supported this research. The usual disclaimer applies.

1 Introduction

It is a commonplace observation that technical progress over time lowers production costs and prices, but the effect of technical progress on product[1] reliabilty is more ambiguous. Cars are more reliable now than they were fifty years ago, but trains are not necessarily more punctual. Silicon chips (for example memory chips used in personal computers) are cheaper (adjusting for improved performance) than they were ten years ago, but fail relibility checks at about the same rate (see Pecht, Radjojcic and Rao[1] for a discussion of

[1] Throughout the paper the term "product" will mean "product or service".

chip reliabilty). Technical progress has presumably improved production and quality control processes, so these observations are surprising. Figure 1 shows warranty data for Hewlett Packard between 2003 qnd 2010. Quarterly claims, the accrual rate and the claims rate can all be thought of as proxies for product reliability. Taking the three measure together there is no clear trend of reliability over time.

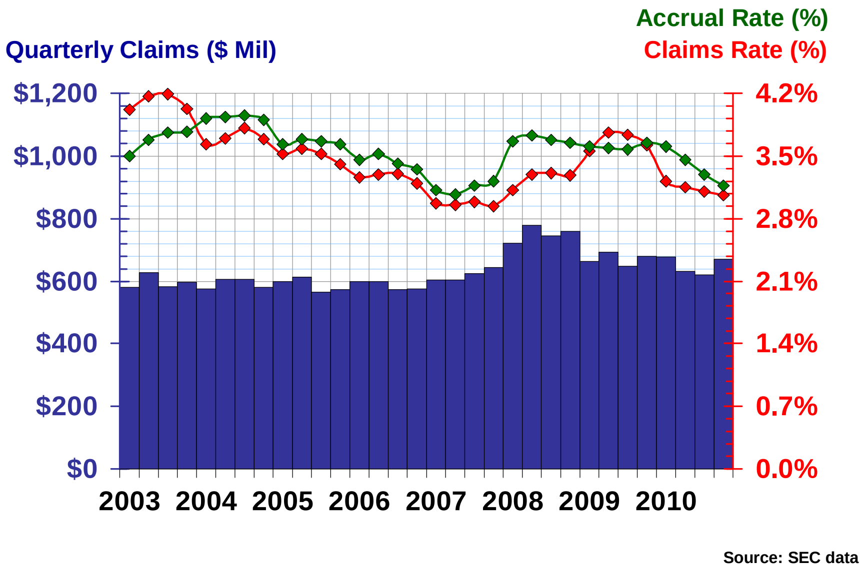

Fig 1. Reliability proxies for Hewlett Packard 2003 - 2010

Figure 2 shows reliabilty proxies (warranty accruals as a percentage of automotive revenue) for six car manafacturers between 2003 an 2011. Again there is no discernible trend over time.

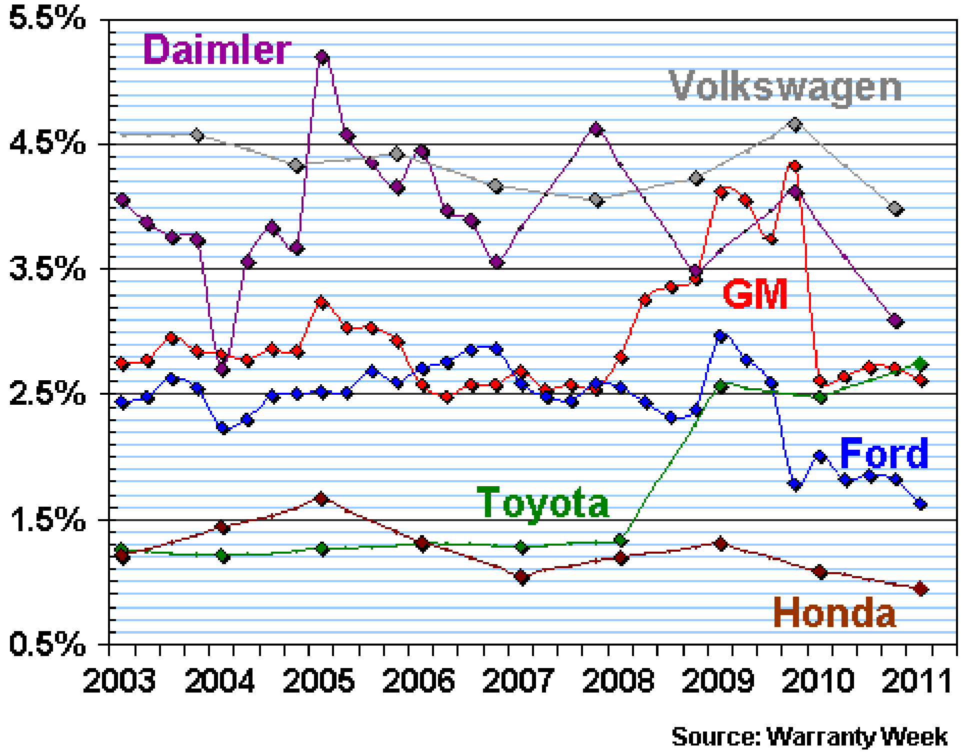

Fig 2. Reliabilty proxies for six car manafacturers, 2003 - 2011

This paper develops a simple model of product reliability which provides a possible expalnation for these "stylised facts". The paper draws a sharp distinction between quality and reliability. It defines the latter as the objective probability (frequency) of output failure, a probability which the firm can choose, for example via the quality control process, the choice of technique or the design of output. The paper argues that this approach comes closer to describing the process by which reliability is actually determined than does the conventional output quality approach, which is more relevant to aspects of production other than reliability. For example, the conventional analysis explains well why some computers have fast processors or large hard drives, but not why they break down so often.

2 Modelling Reliability

In the model presented here "reliability" will be defined as "the probability of an event disliked by consumers not occurring". Examples of such an event are:

A: A consumer durable (e.g. computers, cars, washing machines, televisions etc.) breaking down within some given time period.

B: An intermediate good such as a silicon chip (or other component) failing to function correctly (here the "consumer" is another firm).

C: A train or plane not arriving within some predetermined time period of its scheduled arrival time.

D: Electricity, gas, or water supplies or telephone services being interrupted for more than some predetermined period. Note that commercial contracts for these services may allow some interruption of service without penalty (e.g. the commercial supply of gas). Matsukawa and Fujii (1994)[2] study Japanese electricity consumers and show, among other things, that they face a trade-off between price and reliability of electricity supply.

The probability referred to here is an objective frequency. For example, if a computer manufacturer produces 100,000 computers each year and 93,000 do not break down within a given time period (say one year) the reliability of these computers is 0.93. It will be assumed that the firm chooses reliability (as defined here), for example via output design, choice of technique and quality control procedures. Thus the computer firm knows for sure that 7,000 of its computers will break down within the year, but it neither knows nor cares which 7,000 they will be. Now suppose the firm offers a one-year warranty with its computers, promising compensation in the event of a breakdown. In this model the firm faces no uncertainty concerning its profits: it knows its revenue and production costs, it knows that there will be 7,000 claims under the warranty (though not which customers will make them) and it knows how much it will have to pay out per claim (that can either be treated as endogenous or imposed by a regulator). There is therefore no uncertainty about its profits.

It will be assumed that consumers (who, in the case of intermediate goods mentioned in point B above, will be other firms) have no knowledge about individual products or services but do know the reliability of each firm's output (in the sense defined here). Consumers read Which? magazine or Consumer Reports or obtain information on reliability from other sources. For example in the UK, the Strategic Rail Authority publishes information on the average punctuality of the different rail operators.

Supplying firms will be assumed to have the same information. This is a plausible assumption because it is usually impossible or extremely costly for firms to obtain information on each example of its output before it is sold. Firms will be assumed to vary reliability (as defined here), for example via product design, choice of technique and quality control procedures. It will be assumed that higher reliability entails higher production costs. Thus, the computer manufacturer will be able to reduce (or increase) the number of breakdowns in a given time period without knowing (or caring) which computers will break down and which will not. It will therefore be assumed to know the reliability of its output (as defined here), without knowing which examples of its output will break down.

In this model the firm faces no uncertainty, though this is not true of consumers, who are assumed to be risk averse. This assumption is readily justified, for example, in the consumer durables market, where each consumer typically owns one example of the good and is thus extremely concerned at the prospect of its breaking down. The firm, by contrast, supplies many examples of the good, and may well find it profitable to operate a risk-pooling warranty scheme. Under these assumptions there arises a demand, on the part of consumers, for insurance. This might, as mentioned above, be provided in the form of a product warranty offered by the firm, or an insurance policy provided jointly with the product. In the case of intermediate goods, "warranties" may be thought of as compensation clauses built into standard supply contracts. A similar interpretation applies to services such as electricity and gas. In the case of transport services, it is clearly possible for suppliers to offer compensation to dissatisfied passengers. Throughout the paper attention will be confined to voluntarily offered warranties or compensation, though the model is readily modified to include legally compelled compensation. It could also be modified to cover more than one undesired event (e.g. different degrees of product breakdown), or to cover product hazard and safety issues. Warranties, whether voluntary or legally compelled, have an important bearing on decisions affecting reliability because the higher the reliability of a firm's marketed output, the lower the warranty costs experienced by the firm (ceteris paribus). In the model developed in this paper, warranties play the role of allocating risk, and providing an incentive to supply reliable output, in contrast to much of the existing literature on warranties, where they have a signalling role.

As noted in section 1, the model developed in this paper differs sharply from that presented in the literature on product quality. That literature deals either with search goods or with experience goods. In the former case both supplier and consumer know all relevant characteristics of each good or service before sale takes place (e.g. the computer has a 5.0GHz processor and an 500Gb hard drive, the train has a good restaurant etc.). See for example Mussa and Rosen (1978)[3], and Matthews and Moore (1987)[4] for models of this type. In the case of experience goods, there is an asymmetry of information. "Nature" dictates all relevant characteristics of each good or service to the supplier before sale, but these are unknown to the consumer at that stage. (E.g. the computer's hard drive will fail in the first year, the train will be 2 hours late). The supplier's problem is thus one of signalling. Perhaps by means of advertising, or offering a warranty or compensation deal, the supplier of high quality output seeks to signal his high quality to consumers in a credible way. See, for example, Grossman (1981)[5], Milgrom and Roberts (1982, 1986)[6][7], Kreps and Wilson (1982)[8], Klein and Leffler (1981)[9], Shapiro (1983)[10], and McClure and Spector (1991)[11] for models of this type. Neither of these approaches is of much use in analysing reliability. Search good models assume too much information on both sides of the market, while experience good models assume too much information on the supply side and not enough on the demand side. Moreover, the latter kind of model is based on an exogenously given quality level, while reliability (as defined in this paper) will be determined endogenously by the supplier's decisions, as discussed above.

A standard problem, often assumed away in the literature, is that of moral hazard on the part of consumers. If consumers can themselves influence the probability or size of a claim under the warranty, for example by failing to take proper care of the good during consumption, then the economic role of warranties may be reduced. See, for example McKean (1970)[12], Oi (1973)[13], Priest (1981)[14] and Goering (1997)[15], who discusses the problem of moral hazard facing a durable goods monopolist. For simplicity moral hazard will be assumed away in this paper. It should be noted that the model presented here focuses on reliability and warranties, deliberately suppressing some other aspects of the markets discussed above. For example, it is essentially a static model, and is not intended to deal with the issue of dynamic consistency in durable goods markets. Moreover, it is a model of symmetric information. In such a model nothing can be gained by admitting the possibility of repeat purchasing, since neither side of the market can learn anything useful about the other.

3 The Demand Side

In the model of this paper, consumers' preferences have three distinct aspects.

- The consumer's preference for the good in its un-broken-down state. This varies across consumers and is exogenous.
- The consumer's degree of risk aversion. For convenience this is assumed constant across consumers and is exogenous.
- The probability of the good not breaking down within some given time period (i.e. the reliability of the good). Subjective and objective probabilities are, by definition, identical in this model. It is an essential feature of the model that this probability is endogenous (determined by firms' decisions) and the same for all consumers.

The details of consumers' utility functions are developed below.

The demand side of the market will be assumed to consist of z consumers, each consuming a single unit of the output. Each consumer has a different reservation price, and hence the market demand curve is downward sloping. For simplicity we take z to be a strictly positive real variable. Each consumer has a money budget M available and pays a price p for the output. As discussed in section 2, two states of the world are assumed: either the undesired event occurs or it does not. In the latter case the z'th consumer receives a stream of services which she values at $f(z)$ (perhaps generated by a durable good). Note that $z > 0$ and $f'(z) < 0$. In the former case the consumer values the stream of services at zero, but the firm makes a voluntary warranty (or compensation) payment of β to her. Costs of writing and enforcing the warranty (or compensation) contract are ignored. Thus the z'th consumer receives income stream:

$$x = M - p + f(z) \tag{1}$$

if the undesired event does not occur and:

$$y = M - p + \beta \tag{2}$$

if it does.

The reliability of a product will be defined as in section 2, as the objective probability (R) of the undesired event not occurring. Consumers are assumed to be risk-averse maximisers of expected utility. As discussed in section 2, it will be assumed that consumers are fully informed about reliability (R), so that the subjective probability of the undesired event not occurring is equal to the objective probability (R). Of course R is determined endogenously by the management decisions of the firm.

The z'th consumer maximises expected utility:

$$V = RU(M - p + f(z)) + (1 - R)U(M - p - \beta) \tag{3}$$

Clearly $U'(.) > 0$, and, to ensure risk aversion, it is assumed that $U''(.) < 0$ (i.e. the function $U(.)$ is assumed strictly concave).

Note that the z'th consumer is indifferent between consuming and not consuming when:

$$V = RU(M - p + f(z)) + (1 - R)U(M - p - \beta) = U(M) \tag{4}$$

since $U(M)$ is the expected utility she would get by not consuming the output (she will be referred to as the "marginal consumer"). Equation (4) generates, for given values of R and β, a relationship between p and z, namely the market demand curve. Each consumer has a different reservation price, and thus the market demand curve slopes downwards (see figure 1). Note that R and β are determined by the decisions of the firm, so that consumers can be thought of as consuming a "bundle" consisting of a stream of services (perhaps provided by a durable good), its reliability and the warranty deal. They are not able to "unbundle" these three things. If the firm raises R or β the demand curve will shift upwards, except that, when a full "money back" warranty is offered ($\beta = p$), the marginal consumer will be indifferent as to whether the undesired event occurs or not (since $x = y$ when $\beta = p$). In this case, as R changes, the demand curve will rotate about the equilibrium, which will itself be immune to variations in R. Note also that the $f(z)$ curve must be steeper than the demand curve (see figure 3) because it is the relationship between p and z which would hold *if β were continually kept equal to p* (this is clear from equation (4)). Accordingly the function $p = f(z)$ will be referred to as the *pseudo-demand curve.* Of course the demand curve proper is defined ceteris paribus (i.e. holding everything constant except p and z).

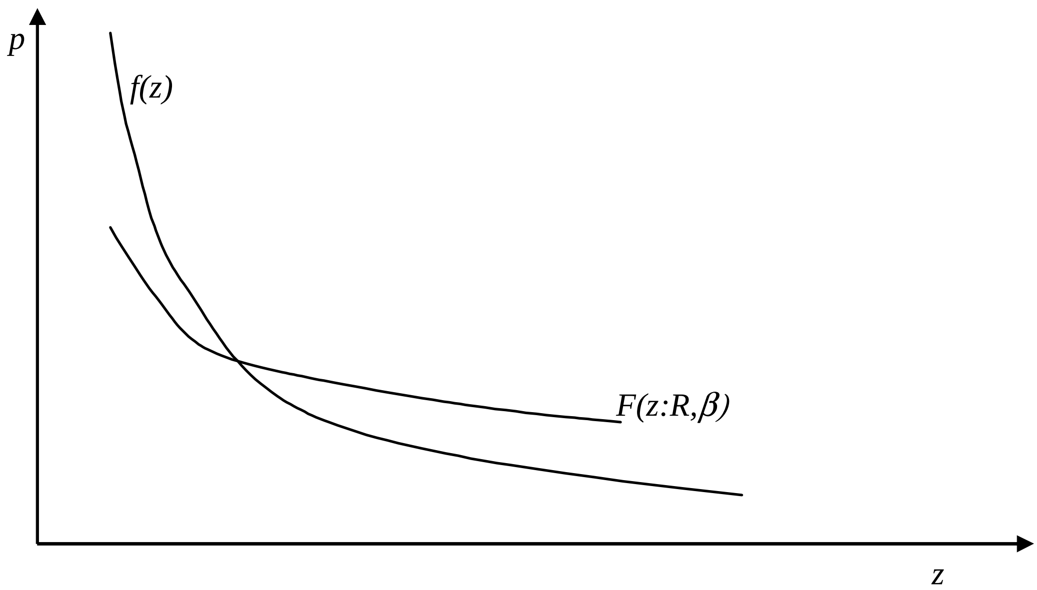

Fig 3. The distribution of tastes (or pseudo-demand curve) $f(z)$ and the market demand curve $F(z : R, \beta)$

4 The Supply Side

Firms' costs will depend on the reliability of their output for a number of different reasons.

1. Reliability can be designed into the output. Higher reliability designs will, in general, be more costly to produce than lower reliability ones.

2. Techniques of production can be adopted which generate higher reliability. Techniques generating higher reliability output will, in general, be more costly to operate than those generating lower reliability output.

3. The stringency of quality control can be varied. Stricter quality control will, in general, raise reliability, but will also raise scrap or rework costs.

4. Higher reliability will reduce the number of claims under the warranty and hence, for a given warranty payment, reduce warranty costs.

It is to be expected that technical progress would affect product design, production techniques and the costs of quality control, having an overall effect of reducing costs at any given level of reliabilty. This is captured in the cost function specified below.

The model developed here formalises the firm's costs by assuming that production costs are increasing in the reliability (R) of output, and by incorporating warranty costs into the firm's profit-maximising decision. Average and marginal production costs, at a given reliability level and state of technology, will be assumed constant. Note that z is the firm's output and t the state of technology. Adopting the assumptions set out above a suitable production cost function is:

$$zC(R,t) \tag{5}$$

where $C_R(R,t) > 0, C_t(R,t) < 0$ and $C_{RR}(R,t) > 0$ for $0 < R < 1$. The $C(R)$ function is depicted in figure 4.

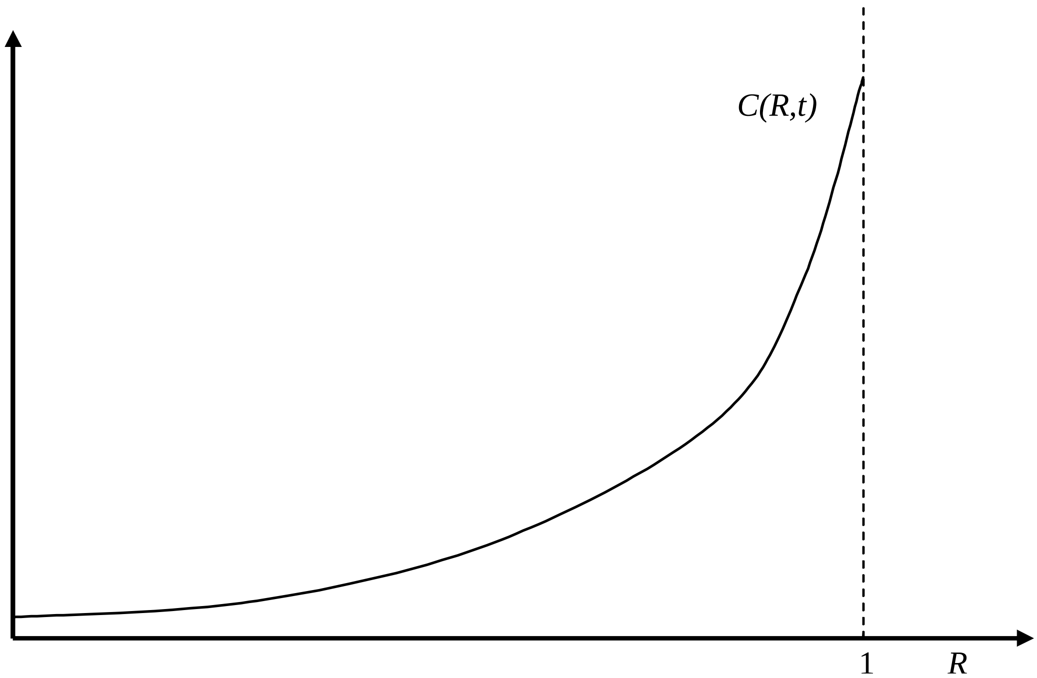

Fig 4. The cost of reliability function $C(R,t)$

The number of times that the undesired event occurs is clearly $z(1-R)$, and thus warranty or compensation costs are given by:

$$\beta z(1-R) \tag{6}$$

Thus the firm maximises the profit function:

$$\pi = pz - zC(R) - \beta z(1-R) \tag{7}$$

5 Competitive Equilibrium

As discussed in section 4 above, average and marginal costs (at given levels of R an β) are assumed constant. The number of firms in competitve equilibrium is therefore left undetermined and maximising industry profits is equivalent to maximising firm profits. Free entry (or Bertrand competiton) will ensure that equilibrium profits are zero. There are no non-convexities, externalities or public goods in the model, and a competitive equilibrium is therefore Pareto-efficient. Formally, a competitive equilibrium is a 4-tuple (p, z, R, β) with the following two defining characteristcs:

1. Each consumer's expected utilty is maximised subject to the constraint that industry profits are non-negative (this constraint will turn out to be binding).

2. The 4-tuple (p, z, R, β) satisfies equation (4) (the market demand curve).

To characterise a competive equilibrium we take a Lagrange multiplier λ for the profit constraint and form the Lagrangian:

$$L = RU(x) + (1-R)U(y) + \lambda[pz - zC(R,t) - \beta z(1-R)] \tag{8}$$

We now derive first-order conditions for an interior solution of this problem. First note the definitions of equations (1) and (2) ($x = M - p + f(z)$ and $y = M - p + \beta$).

Proposition 1 *In competitive equilibrium the zero profit constraint is binding.*

Proof. Differentiating first with respect to p:

$$-RU'(x) - (1-R)U'(y) + \lambda z = 0 \Rightarrow \lambda z = RU'(x) + (1-R)U'(y) > 0 \quad (9)$$

But $z > 0$,hence $\lambda > 0$ and it follows by complementary slackness that the zero profit constraint is binding. ∎

We now establish that, in competitive equilibrium, a full "money back" warranty is offered. Thus risk neutral firms fully insure risk-averse consumers and the allocation of risk is efficient.

Proposition 2 *In competitive equilibrium $f(z) = p = \beta = C_R(R,t)$ and thus a full "money back" warranty is offered.*

Proof. First differentiate the Lagrangian with respect to β:

$$L_\beta = (1-R)U'(y) - \lambda z(1-R) = 0 \quad (10)$$

Equations (9) and (10) together yield: $(1-R)U'(y) = (1-R)(RU'(x) + (1-R)U'(y) \Rightarrow U'(y) = RU'(x) + (1-R)U'(y) \Rightarrow RU'(y) = RU'(x) \Rightarrow x = y$ ($R \neq 1$ and $R \neq 0$ because we are seeking an interior solution: $U'(.)$ is invertible because $U''(.) < 0$). Hence, given the definitions of x and y, we have:

$$\beta = f(z) \quad (11)$$

in competitive equilibrium. Now differentiate the Lagrangian with respect to R:

$$L_R = U(x) - U(y) + \lambda(-zC_R(R,t) + \beta z) = 0 \quad (12)$$

But, in competitive equilibrium, $x = y$, and $z > 0$, because we are seeking an interior solution. Hence equation (12) yields:

$$\beta = C_R(R,t) \quad (13)$$

Now note that a competitive equilibrium must satisfy the market demand curve (equation (4)). But we have $x = y$ in competitive equilibrium so equation (XX) yields: $RU(x) + (1-R)U(x) = U(M) \Rightarrow U(x) = U(M) \Rightarrow x = M$ (noting that $U'(.)$ is invertible because $U''(.) < 0$). Hence, from the definition of x we have:

$$p = f(z) \quad (14)$$

in competitive equilibrium. Now combining equations (11), (13) and (14) it is straightforward to establish that in competitive equilibrium:

$$f(z) = p = \beta = C_R(R,t) \quad (15)$$

and thus that a full "money back" warranty is offered. ∎

6 The effects of technical progress

Technical progress is modelled as an increase in t, given the properties of the cost of reliabilty function $C(R,t)$ assumed in section X above, namely:

1. $C_R(R,t) > 0$ (for $0 < R < 1$)
2. $C_t(R,t) < 0$ (for $0 < R < 1$)
3. $C_{RR}(R,t) > 0$ (for $0 < R < 1$)

We first establish an expression for $\dfrac{dR}{dt}$.

Proposition 3 $\dfrac{dR}{dt} = \dfrac{C_t(R,t) - RC_{Rt}(R,t)}{RC_{RR}(R,t)}$

Proof. From Proposition 1, the zero profit condition is binding in competitive equilibrium. We therefore have: $pz = zC(R,t) + \beta z(1-R) \Rightarrow p = C(R,t) + \beta(1-R) \Rightarrow C(R,t) = \beta R = C_R(R,t)R$ (from equation 15). Differentiating totally with respect to t yields: $C_R(R,t)\dfrac{dR}{dt} + C_t(R,t) = R\left[C_{RR}\dfrac{dR}{dt} + C_{Rt}(R,t)\right] + C_R(R,t)\dfrac{dR}{dt}$. Hence: $C_t(R,t) = RC_{RR}\dfrac{dR}{dt} + RC_R(R,t)$. Re-arranging yields the required result. ■

Given Proposition 3 and properties 1 - 3 above, it is clear that the sign of $\dfrac{dR}{dt}$ depends on the sign of $C_t(R,t) - RC_{Rt}(R,t)$. In particular:

$$C_t(R,t) - RC_{Rt}(R,t) > 0 \Rightarrow \frac{dR}{dt} > 0 \text{ and } C_t(R,t) - RC_{Rt}(R,t) < 0 \Rightarrow \frac{dR}{dt} < 0 \tag{16}$$

We now adopt a particular form for the cost of reliability function:

$$C(R,t) = e^{-t}D(R) \tag{17}$$

Where $D(R) > 0$, $D'(R) > 0$ and $D''(R) > 0$. Hence:

$$C_R(R,t) = e^{-t}D'(R) > 0;\ C_t(R,t) = -e^{-t}D(R) = -C(R,t) < 0 \tag{18}$$
$$C_{RR}(R,t) = e^{-t}D''(R) > 0;\ C_{Rt}(R,t) = -e^{-t}D'(R) < 0 \tag{19}$$

and conditions 1, 2 and 3 above are satisfied. The conditions of (16) can now be written as:

$$D(R) < RD'(R) \Rightarrow \frac{dR}{dt} > 0;\ D(R) > RD'(R) \Rightarrow \frac{dR}{dt} < 0 \qquad (20)$$

Interpreting t as time, it is now possible to analyse the behaviour of reliability over time for different $D(R)$ functions.

Example 1. Let $D(R) = aR^2 + b$ where $a > b > 0$. Then $RD'(R) = 2aR^2$. The functions $D(R)$ and $RD'(R)$ are shown in figure 5 together with the resulting dynamics of R (derived from the conditions of (20). In this case there is a knife-edge level R^* of reliabilty. If R is initially above $R^* = \sqrt{\frac{b}{a}}$ technical progress will always raise reliability but if R is initially below R^* technical progress will always lower reliability.

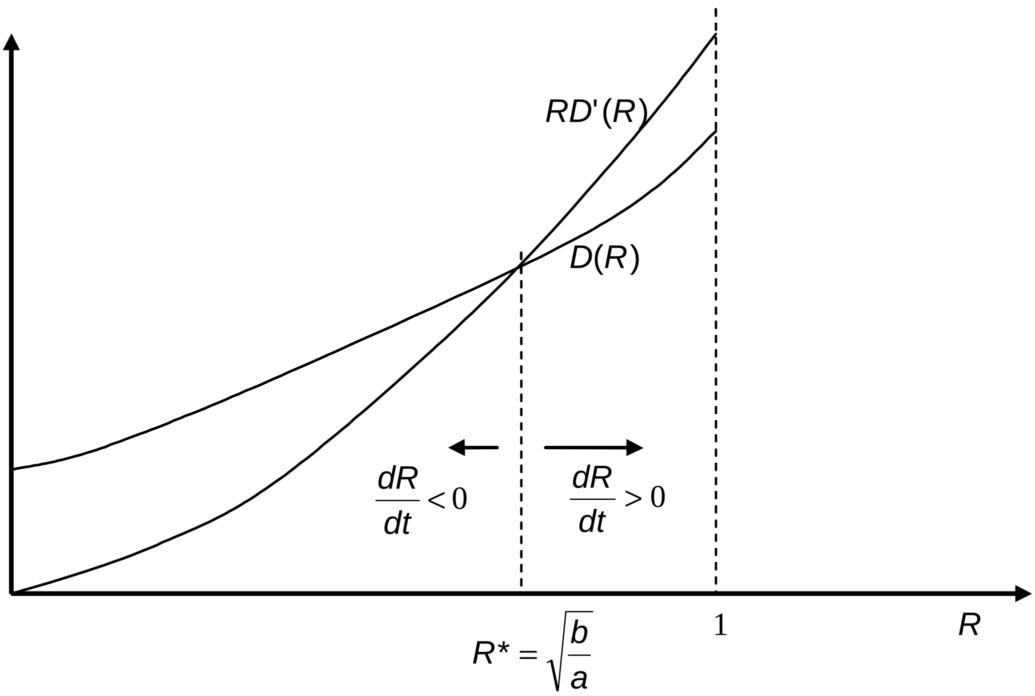

Fig 5. The dynamics of R for the function $D(R) = aR^2 + b$ where $a > b > 0$

Example 2. Now consider the same $D(R)$ function as in Example 1, but take $b > a > 0$. The functions $D(R)$ and $RD'(R)$ are shown in figure 6 together with the resulting dynamics of R (derived from the conditions of (20). In this case technical progress will always lower reliability. $D(R) = aR^2 + b$ where $a > b > 0$.

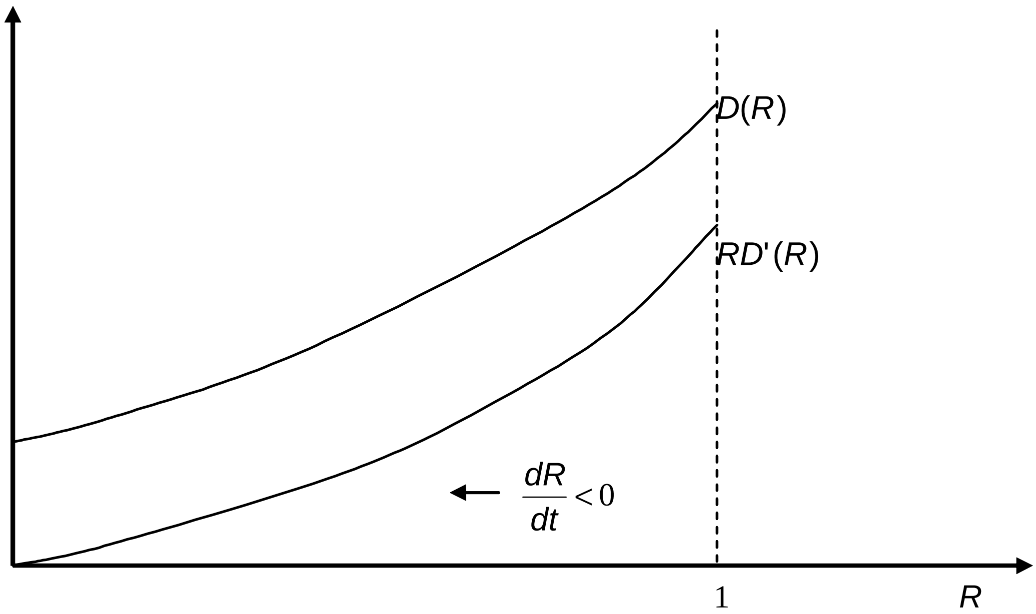

Fig 6. The dynamics of R for the function $D(R) = aR^2 + b$ where $b > a > 0$

Example 3. Now let $D(R) = R(1-R)^{-1}$. In this case products of zero reliability cost nothing to produce, while perfect reliability ($R = 1$) attracts infinite cost. Then $RD'(R) = R(1-R)^{-1} + R^2(1-R)^{-2} = D(R) + [D(R)]^2 > D(R)$ for $0 < R < 1$. The functions $D(R)$ and $RD'(R)$ are shown in figure 7 together with the resulting dynamics of R (derived from the conditions of (20). In this case technical progress will always raise reliability.

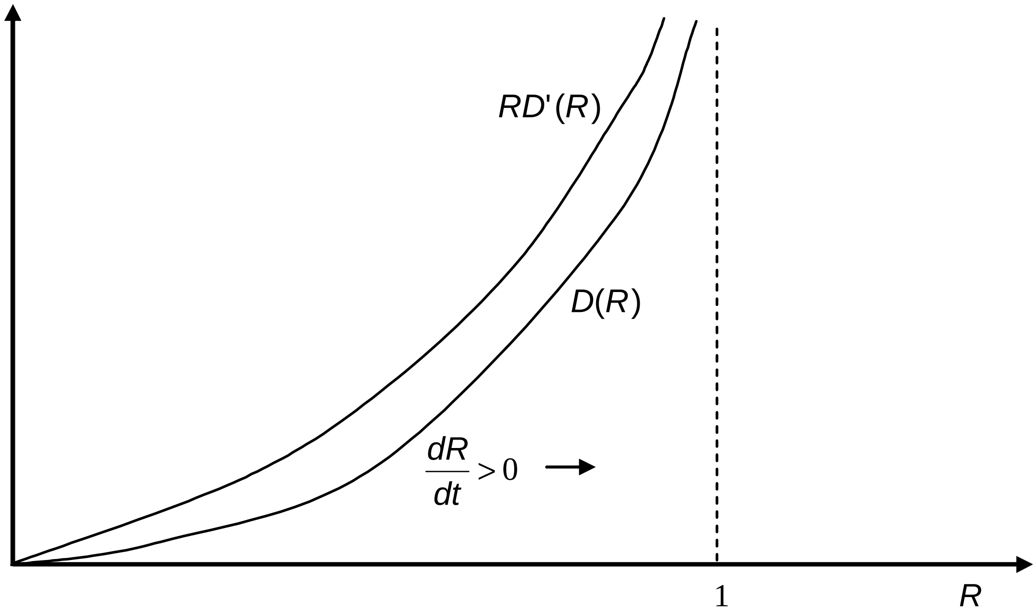

Fig 7. The dynamics of R for the function $D(R) = R(1-R)^{-1}$

7 Monopoly Equilibrium

In monopoly equilibrium there is a single supplier, maximising his profits, subject to the voluntary participation constraint. This is the constraint that each consumer obtains at least as much expected utility from purchasing the output as from not doing so. Mathematically it is simply:

$$RU(M-p+f(z))+(1-R)U(M-p-\beta)\geq U(M) \tag{21}$$

In monopoly equilibrium z is determined at a level which makes this constraint bind (i.e. the z'th. consumer is the marginal consumer, who is just on the point of leaving the market, and z is the monopolist's total output). A monopoly equilibrium is easily characterised by taking a Lagrange multiplier (μ) for the constraint (21) (noting equation (7), which specifies the monopolist's profits) and forming the Lagrangian:

$$M = pz - zC(R,t) - \beta z(1-R) + \mu(RU(M-p+f(z)) + (1-R)U(M-p-\beta) - U(M)) \quad (22)$$

Differentiating (22) yields first-order conditions for an interior solution. First, note the definitons of equations (1) and (2) ($x = M - p + f(z)$ and $y = M - p + \beta$). It is now straightforward to establish:

Proposition 4 *In monopoly equilibrium the voluntary participation constraint binds. That is: $U(M) = RU(x) + (1-R)U(y)$.*

Proof. Differentiating first with respect to p:

$$z + \mu(-RU'(x) - (1-R)U'(y)) = 0 \quad (23)$$

It follows that $\mu \neq 0$, since we seek an interior solution ($z > 0$). Hence, by complemenatry slackness, the voluntary participation constraint must bind. ■

We now establish:

Proposition 5 *In monopoly equilibrium $f(z) = p = \beta = C_R(R,t)$ and thus a full "money back" warranty is offered.*

Proof. Differentiating with respect to β yields:

$$-z(1-R) + \mu(1-R)U'(y) = 0 \quad (24)$$

Hence $z = \mu U'(y)$. So now (23) implies $R\mu[U'(y) - U'(x)] = 0 \Rightarrow x = y \Rightarrow$

$$\beta = f(z) \quad (25)$$

But the voluntary participation constraint binds, so $U(y) = U(x) = U(M) \Rightarrow x = y = M \Rightarrow$

$$p = \beta = f(z) \quad (26)$$

Differentiating now with respect to R yields:

$$-zC_R(R,t) + \beta z + \mu[U(x) - U(y)] = 0 \quad (27)$$

But we have $x = y$ in monopoly equilibrium, hence:

$$\beta = C_R(R,t) \quad (28)$$

Combining (26) and (28) we have (in monopoly equilibrium):

$$f(z) = p = \beta = C_R(R,t) \tag{29}$$

■

We now establish:

Proposition 6 *In monopoly equilibrium:*

$$\frac{Rzf'(z)}{p} = \frac{C(R,t)}{C_R(R,t)} - R \tag{30}$$

Proof. First differentiate the Lagrangian (22) with respect to z:

$$p - C(R,t) - \beta(1-R) + \mu RU'(x)f'(z) = 0 \tag{31}$$

Hence, using (23) and noting that $x = y$ in monopoly equilibrium„ we have: $p + zRf'(z) = C(R,t) + \beta(1-R)$. Using Proposition 5 we obtain: $zRf'(z) = C(R,t) - RC_R(R,t)$. Re-arranging, using Proposition 5 again, yields the required result. ■

Now note from section 3 that $f(z)$ is the demand curve which would face the monopolist when he offers a full "money back" warranty (i.e. maintains $\beta = p$), which he will do in equilibrium, as Proposition Y shows. Accordingly the function $f(z)$will be referred to as the *pseudo demand curve*. Thus the expression:

$$\eta = \frac{1}{\epsilon} = \frac{zf'(z)}{p} \tag{32}$$

defines the *pseudo elasticity of demand* (ϵ). It is the elasticity of demand which arises when the monopolist continually offers a full "money back" warranty.

We now establish:

Proposition 7 *In monopoly equilibrium:*

$$\frac{dR}{dt} = \frac{(1+\eta)RC_{R,t}(R,t) - C_t(R,t)}{\eta C_R(R,t) + (1+\eta)RC_{RR}(R,t)} \tag{33}$$

Proof. Write (30) as: $R\eta = \dfrac{C(R,t)}{C_R(R,t)} - R$. Re-arranging yields: $(1+\eta)RC_R(R,t) = C(R,t)$. Now differentiate totally w.r.t t. This yields:

$$(1+\eta)[R(C_{RR}\frac{dR}{dt} + C_{Rt}) + C_R\frac{dR}{dt}] = C_R\frac{dR}{dt} + C_t \tag{34}$$

Re-arranging (34) yields the required result. ■

Corollary When the pseudo-elasticity of demand is greater than -1 (i.e. when the pseudo demand curve is inelastic) it is easy to show that, adopting the functional form of (17) and the consequent inequalities (18) and (19), technical progress under monopoly always reduces reliabilty.

Proof. First subsitute the functional form of (17) into (33). this yields:

$$\frac{dR}{dt} = \frac{-(1+\eta)RD'(R) + D(R)}{\eta D'(R) + (1+\eta)RD''(R)} \tag{35}$$

Now simply note that $0 > \epsilon > -1 \Rightarrow (1+\eta) < 0$, then inspection of (35) yields $\dfrac{dR}{dt} < 0$. ■

When $\epsilon < -1$ it is necessary to adopt a particular functional form for $D(R)$. Adopting the functional form of Example 1 yields:

Example 4. Let $D(R) = aR^2 + b$ where $a, b > 0$. Then $D'(R) = 2aR$ and $D''(R) = 2a$. Substituting in (35), it is easy to show that: $\epsilon > -2 \Rightarrow \dfrac{dR}{dt} < 0$ and $\epsilon < -2 \Rightarrow \dfrac{dR}{dt} > 0$.

8 Conclusions

Technical progress reduces costs and prices but has an amibiguous effect on product reliability. The model of this paper treats product reliability as the objective probabilty (frequency) of "product failure", a probability chosen by firms under conditions of symmetric but imperfect information. The paper thus sharply differentiates "reliability" from "quality", usually analysed as a product characteristic dictated by "Nature" under conditions of asymmetric information, thereby generating a signalling problem. In the model of this paper firms supply many examples of their product while consumers demand just one (or none). In this situation firms (competitive or monopolistic) naturally supply a "money back" warranty along with the product, thus insuring risk-averse consumers. The model permits an analysis of the effects of technical progress on reliability under competition and monopoly. For the competitive case three possibilities are illustrated: (1) there exists a threshold level of reliabilty: if reliability is initially above this level, technical progress increases it, if reliabilty is initially below this level, technical progress reduces it: (2) technical progress always increases reliabilty: (3) technical progress always reduces reliabilty. For the monopolistic case the outcome depends on the pseudo-elasticity of demand (i.e. the elsaticity which would arise when the monopolist offers a full money-back warranty, as he would in equilibrium). When the pseudo-demand curve is inelastic, technical progress under monopoly always reduces reliabilty. When the pseudo-elasticity of demand is less than -1 the outcome depends on the form of the cost of reliability function. In general, the more elastic the pseudo-demand curve, the more likely is technical progress to increase reliability.

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