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The disadvantage of winning an election[*]

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Abstract

This paper analyzes the problem that an incumbent faces during the legislature when deciding how to react to popular initiatives or policy proposals coming from different sources. We argue that this potential source of electoral disadvantage that the incumbent obtains after being elected can jeopardize the reelection possibilities of the incumbent. We analyze the decision of the incumbent when facing reelection and we characterize the conditions under which the advantages that the incumbent obtains can overcome the disadvantages. Finally, we use the results of this analysis to discuss some implications of the use of mechanisms of direct democracy like referenda and popular assemblies on electoral competition.

Keywords: Incumbency advantage, Referenda, Popular initiatives, Elections.
JEL-codes: D7, H1.

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1 Introduction

The incumbency advantage is a well documented phenomenon, according to which an incumbent politician is more likely to be reelected than a challenger candidate. Empirical studies, such as Gelman and King (1990) and Lee (2008), estimate the probability of success of an incumbent when facing reelection in the US House. Their results provide strong evidence in favor of the existence of an incumbency advantage. Other studies have analyzed the specific reasons for this advantage. They typically assume that incumbents have better ways to influence voters' decisions than challengers, and that they can do so through different mechanisms such as redistricting (Levitt and Wolfram 1997, Cox and Katz 2002), seniority (McKelvey and Reizman 1992), informational advantages (Krehbiel and Wright 1983), access to campaign resources (Goodliffe 2001, Jacobson 2001), legislative irresponsibility (Fiorina 1989) or pork barrel politics (Cain, Ferejohn and Fiorina 1987, Ansolabehere, Snyder and Stewart 2000).

Ansolabehere and Snyder (2002) measured the incumbency advantage in all state executives and found similar empirical support for the incumbent advantage. However, they argue that this advantage does not have its origin in the strategic choices made by incumbents but in their innate characteristics. In this line, Bevia and Llavador (2009) show that only good quality incumbents may enjoy an advantage and Asworth and de Mesquita (2008) show that on average incumbents' quality and ability are higher than the challengers'. Gowrisankaran, Mitchell and Moro (2006) find that incumbents face weaker challengers than candidates that face open seats and Stone, Maisel and Maestas (2004) find that incumbents' personal qualities deter strong challengers from running for office.

The present paper provides an explanation to the phenomenon of incumbency advantage that focuses on a mechanism that can potentially constrain elected politicians. After winning an election the incumbent is supposed to decide and implement some policies. These choices may turn out to be costly in terms of probability of reelection. Hence, the incumbent might be facing some restrictions on the policies that he can optimally implement if he wants to stay in office one more term. In this paper, we show how these disadvantages may be overcome by the advantages, and we also derive the conditions under which this may not be the case.

We have in mind a specific type of issues and policy choices as the origin of this incumbency disadvantage: The outcomes of different forms of citizen direct political participation. The outcomes of processes like referenda, citizens' initiatives or popular assemblies can constraint incumbents because the policy they will finally implement on these issues will factor into the voters' evaluation of the incumbent's performance. The challenger may have a greater chance of winning in theses cases unless the incumbent is ready to compromise. Our claim is that an incumbent facing this kind of situations has an initial disadvantage compared, not only to an incumbent who does not receive any proposal or does not have to call a referenda, but also compared to the challenger. The challenger is not required to react to the outcome of any of these processes of participation. In fact, the position of the challenger does not allow him to do anything with respect to it. Given this, it is to be expected that the incumbent's potential disadvantage will be larger the more policy motivated the incumbent is.

In order to perform the analysis of the incumbent's decision we build a formal model of electoral competition with two candidates, two issues and three stages. In the first stage of the game, the incumbent faces an exogenously given policy proposal on an issue, the *popular issue*, on which he has to make a policy choice. The implementation of a policy choice on the popular issue takes place during the legislature and before the beginning of the electoral campaign. In the second stage of the game, both candidates announce simultaneously their policy platforms on a different issue, the *electoral issue*. The electoral issue is defined in the same way as in most models of electoral competition; the candidates' choices in this issue represent their campaign promises. Finally, in the third stage of the game, voters vote for their most preferred candidate.

The model presented includes two different types of asymmetries. First, voters evaluate different candidates in different ways. We assume that voters use all the information they have available in order to decide to whom to give their vote. Thus when evaluating the incumbent, in addition to considering his campaign promises, they have to take into account his choice on the popular issue during the legislature. This information is not available for the challenger given that he was not in office when the policy proposal was announced.

The second asymmetry relates to how the two issues are treated. Citizens' will evaluate the incumbent performance on the popular issue by comparing his policy choice with the proposal they made to him. And they will evaluate candidates on the electoral issue by comparing their own preferred policy with each candidate's political platform. Moreover, they will assign different weights to the incumbent's choices on each one of the issues. Since only the incumbent is evaluated on the popular issue, the weight that citizens attribute to the electoral issue constitutes a measure of the degree of political competition between the two candidates.

Since the decisions on the two dimensions of the model are made sequentially, one at each stage, we can solve it as a one dimensional model within each stage. However, given the asymmetry implied by the decisions on the popular issue made in the first stage, the standard median voter analysis does not apply at the second stage. Indeed, in some cases we may obtain that voters with ideal points at the two extremes of the distribution decide to vote for the incumbent or for the challenger. This kind of situations does not arise in equilibrium, but it needs to be considered for the equilibrium analysis.

The optimal policy choices of the incumbent in both issues reflect the incumbent's trade-off between his own policy preferences and his benefit from being re-elected. For all parameter values the incumbent has a strategy that allows him to be reelected. The question is whether this winning strategy is always optimal from the incumbent's point of view. And the answer is no.

There are some instances where the incumbent prefers to forgo reelection and guarantee to himself a good payoff in terms of policies. For this to happen we need three conditions to be satisfied: (1) the incumbent must care enough about policy (that is, the value he obtains from holding office must be low enough); (2) there must exist an intense enough conflict of interests between the voters and the incumbent regarding the popular issue; and (3) there is a very strong electoral competition on the electoral issue. Under these circumstances, the policy implemented by the incumbent on the popular issue coincides with his ideal point, that is, the incumbent does not bear any utility cost from the policy implemented on the popular issue; and the winning policy in the electoral competition stage is close to the median voter's ideal point. The intuition for this result is as follows: the incumbent has a disadvantage coming from the popular issue whenever he does not satisfy the voters' demand on that issue. And he will enjoy the largest advantage at the electoral competition stage when he fully satisfies them. However, this is a costly strategy for a policy motivated candidate. If the incumbent is policy motivated he will choose the winning strategy that is cheapest in terms of policy from his view point. And when this strategy is too costly, either because the conflict of interests with voters in the popular issue is intense or because competition with the challenger is very strong in the electoral issue, he will decide to give up on reelection.

Otherwise, in equilibrium the incumbent chooses a winning strategy that consists of a combination of policies that depend on the weight that voters assign to his performance on each issue. In this case, we study the incentives of the incumbent to implement policies that are close to the voters' demands. The larger the weight that the voters assign to the electoral issue, the more the incumbent will satisfy the voters on the electoral issue. However, this is not the case for the popular issue. The incumbent fully satisfies the voters' demands only for intermediate levels of the weight that they assign to that issue. This is because the incumbent does not compete with the challenger in the popular issue. Hence, when citizens care a lot about it, the incumbent can implement a policy closer to his ideal one in this issue without jeopardizing his reelection; this cannot happen in the electoral issue where the incumbent has to compete against the challenger candidate.

The remainder of paper is organized as follows. In the next section we discuss two real political mechanisms the analysis outlined above can apply to, namely referenda and participatory democracy. Section 3 describes the formal model. Section 4 presents the results and Section 5 discuss them in the light of the mechanisms mentioned above. The last section of the paper offers some concluding remarks.

2 Two sources of incumbency disadvantage

Many models of elections assume that after an election takes place, the incumbent can choose to implement any policy that he likes. His policy choice will depend only on his objective function. Then, if the incumbent is mostly policy oriented he will tend to choose a policy close to his ideal point. But in real life this is not always the case. When an incumbent is deciding which policies to implement, he has to take into account that some policy choices might have a large negative effect on his chances of reelection. Citizens will factor in these choices into their evaluation of the incumbent's performance. Notice that jeopardizing his reelection is not optimal for the incumbent even when he cares about policy because the policy implemented in case he loses will be worse from his point of view than the policy that he could have chosen if he had won the election. Then, when incumbents have policy preferences, they will have to compromise on some dimension if they want to be reelected.

In this paper we will refer to two different types mechanism that can generate incumbency disadvantage: referenda and participatory democracy. There are some empirical studies that have shown that these mechanisms of direct democracy are very effective in satisfying voters preferences, and overall increasing the voters wellbeing[1]. The characteristics that both referenda and participatory democracy have in common are: (1) there is an issue that is considered very important for a significant part of the population; (2) there is a policy proposal received by the incumbent on this issue; (3) the incumbent has to make a decision regarding that issue: either he implements a particular policy or does not implement any policy on that issue; (4) there is a significant proportion of voters that may base their voting decision on

[1] See Frey (1994), Frey and Bohnet (1993) and Frey and Slutzer (2000).

that issue[2]. Next, we will elaborate more on how these mechanisms fit in our main argument.

2.1 Referenda and popular initiatives

Referenda may be mandatory or facultative. They are mandatory if the law (usually the constitution) directs authorities to hold referenda on specific matters. This is normally the case for amending constitutions, impeaching heads of state, ratifying international treaties, etc. Otherwise, when they are facultative, they may be initiated at the will of a public authority or at the will of some organized group of citizens (in this case it is also known as a popular initiative). By the nature of their effects, referenda may be either binding or non-binding. A non-binding referendum is merely consultative or advisory. It is left to the government or legislature to interpret its results and they may even choose to ignore them. A binding referendum forces the incumbent to implement its policy outcome. It may also require the support of a supermajority of votes cast or a minimum turn-out of the electorate (Herrera and Mattozzi, 2010).

If a non-binding referendum of any of these types is called during the time of the legislature, the incumbent will have to react to it with a given policy implementation or by choosing to ignore it. If he chooses not to implement the policy corresponding to the referendum outcome he may be punished by the voters. The relevance of this policy choice on the voters' when deciding whether to re-elect the incumbent will depend on the proportion of voters to whom this issue is relevant.

A referendum initiated by the incumbent himself might have a weaker effect on the voters' reaction than a referendum that originates with a popular initiative. The first type of referendum requires a more complicated strategy from the incumbent because he has to decide whether and when it is optimal to call it. The analysis of these strategies is beyond the scope of this paper[3].

[2] Of course, the policy proposal that comes out of any popular initiatives will not represent a decision problem for the incumbent in systems where the proposals received are considered as binding. In those instances the incumbent does not have a choice. He must implement the proposed policy. This policy implementation might harm the incumbent's payoffs, but he will not be facing the kind of problem that we aim to analyze here.

[3] See Xefteris (2008) for an analysis of the incumbent's decision about when to call a referendum.

2.2 Participatory democracy

Some proposals received by the incumbent that have their origin in an organized group of citizens do not need to induce a referendum. This is the case in systems of participatory democracy, an extended version of the system of representative democracy, that allows citizens to make policy proposals through popular assemblies. A real case of participatory democracy can be found in the town meetings of New England, a form of local government practiced since colonial times. It can also be found in the village governance system in the Indian states of Kerala and West Bengal and in the participatory budgeting system of many Latin-American cities. It has also been applied to school, university, and public housing budgets. The implications of a participatory democracy system on the behavior of citizens and politicians and on policy outcomes are analyzed in Aragones and Sánchez-Pagés (2009).

In all these cases, popular assemblies emerge as a governance mechanism because citizens, or at least a group of them, are interested on a certain issue, normally a local one, and they would like certain policies being implemented. These policies are decided through deliberation and discussions in meetings and assemblies. Because they care enough about these issues, their expected benefits from participating in the process may overcome the costs of coordinating and then a policy proposal will emerge from these process and sent to the incumbent. As a matter of fact, citizens' participation in these real experiences is substantial[4]. The incumbent in turn has formally complete discretion regarding the policies he can implement. However, because the support to these policy proposals is significant within the population, the incumbent's chances of being re-elected will partially depend on his policy choice on that issue.

3 The model

We assume that electoral competition takes place across two dimensions, denoted by x and y. Each dimension is represented by a unit interval of the real line $[0, 1]$. Dimension x represents the electoral issue and dimension y represents the popular issue. There are two candidates: the incumbent and the challenger. The game proceeds in three stages. The first stage takes place during the legislature: the incumbent receives a policy proposal on the popular issue and given that, he has to implement a policy on that issue.

[4] In Porto Alegre, a Brazilian city of 2 million inhabitants, around 15% of the population participates in the annual process of participatory budgeting.

Both the policy proposed to the incumbent and the policy implemented by him on the popular issue are common knowledge for all candidates and all voters. The second stage is the electoral campaign: both candidates make policy announcements simultaneously on the electoral issue. Again all policy announcements are common knowledge by all candidates and all voters. It is assumed that the winner will implement the announced policy on that issue. In the third stage of the game the election takes place: voters decide whether to reelect the incumbent or vote for the challenger. The winner is selected by majority rule and implements the policy announced on the electoral issue.

3.1 Candidates

The two candidates are denoted by L and R. Candidate L is assumed to be the incumbent. Candidates have single peaked preferences over the electoral issue. Without any loss of generality we assume that on the electoral issue the ideal point of candidate L is represented by $x_L = 0$ and the ideal point of candidate R is represented by $x_R = 1$. We assume that the incumbent has single-peaked preferences over the popular issue that are independent of his preferences on the electoral issue. We assume that the incumbent's ideal point on the popular issue is represented by $y_L = 0$. Given the features of the game we analyze, it is not necessary to specify the properties of the preferences on the popular issue for the challenger.

Let us denote by $y(L)$ the policy chosen by the incumbent on this issue during the legislature. The choice of the incumbent over the popular issue is made and implemented during the legislature. Thus at the time of the electoral campaign this choice has already been made and it is taken as given. At the end of the legislature, elections take place and we assume that they are represented by a standard model of electoral competition on the electoral issue: the incumbent and the challenger simultaneously announce policy platforms on the electoral issue, represented by $x(L)$ and $x(R)$ respectively. We assume full commitment, that is, the winner of the election will implement the policy he announced during the campaign on the electoral issue.

We assume that candidates have preferences over policies but they are also office-motivated. Candidates' payoffs are represented by the following utility functions that depend on the policy choice by the incumbent on the popular issue and the policy announcements of both candidates on the electoral issue:

$$\begin{aligned} V_L\left(y\left(L\right), x\left(L\right), x(R)\right) &= -\left|y_L - y\left(L\right)\right| + \pi_L\left(K - \left|x_L - x\left(L\right)\right|\right) - \left(1-\pi_L\right)\left|x_L - x\left(R\right)\right| \\ V_R\left(y(L), x(L), x\left(R\right)\right) &= \left(1-\pi_L\right)\left(K - \left|x_R - x\left(R\right)\right|\right) - \pi_L\left(\left|x_R - x\left(L\right)\right|\right) \end{aligned}$$

where $\pi_L = \pi_L(y(L), x(L), x(R))$ represents the probability that candidate L wins the election, and $1 - \pi_L$ denotes the probability that candidate R wins the election. The probability with which the incumbent is reelected depends on the policy choices made during the legislature and the policy announcements made during the campaign.

K is assumed to be a non-negative number that represents the utility of holding office. $K = 0$ implies that candidates do not obtain any extra utility from holding office, they only derive utility from the policy implemented. In this case we would have two candidates that are only policy motivated. On the other hand, the larger the value of K the more candidates value being in office. Thus for larger values of K candidates care more about winning than about the policies they need to implement or commit to in order to win. By increasing the value of K we obtain that candidates become purely office motivated.

There is an asymmetry embedded in the definition of the candidates' payoff functions. The incumbent obtains a negative payoff whenever he implements a policy on the popular issue that does not coincide with his ideal point on this issue. While the challenger's payoffs are not affected by the incumbent's policy choice on the popular issue. This assumption is justified because the challenger cannot do anything with respect to the policy implementation on the popular issue, since this takes place during the legislature when he does not have any implementation power. Therefore, he does not have to suffer any cost from that policy choice.

For simplicity, we have assumed that the incumbent cares equally about the two issues. Introducing a parameter in the incumbent's payoff function that represents the relative weight that each issue has on the incumbent overall payoffs would not change the main qualitative results obtained.

3.2 Voters

There is an infinite number of voters. Voters have single-peaked preferences over the electoral issue x. We assume that their ideal points are uniformly distributed over this issue x, thus the ideal point of the median voter on the electoral issue is $x_m = \frac{1}{2}$. We do not model the voters' preferences on the popular issue y. Let the ideal point of society in issue y be denoted by $y_m > 0$. The parameter y_m is considered exogenous in our model. It is to be interpreted as the outcome of a referendum or a process of participatory democracy. As a simplifying assumption we consider that it represents the will of all citizens, and therefore it has the support of all the constituency.

Notice that since the ideal point of the incumbent on the popular issue is assumed to be $y_L = 0$, the value of y_m measures the magnitude of the conflict of interest between the incumbent and the citizens with respect to the popular issue.

When facing the election, voters observe the policies announced by both candidates on the electoral issue, $x(L)$ and $x(R)$, take into account the policy implemented by the incumbent on the popular issue, $y(L)$, and decide whether to reelect the incumbent. Voters use all the information available in order to evaluate the two candidates. Since they have different kinds of information about each candidate, their decision rule must exhibit some sort of asymmetry.

We assume that voter i evaluates the incumbent according to the following function:

$$U_i(L) = -(1-\mu)|y_m - y(L)| - \mu |x_i - x(L)|.$$

where μ is a parameter that measures the relative weight that voters assign to the electoral issue with respect to the popular issue. We assume that $0 \leq \mu \leq 1$. The parameter μ thus measures the importance of the electoral issue over the popular issue. Values of μ close to one are to be interpreted as if the society considers that the popular issue is not very important, thus their payoffs would not be much affected by the incumbent's policy choice on that issue. Values of μ close to zero mean that the popular issue is regarded as very important from the voters' point of view and their payoffs will be largely affected by the incumbent's policy choice. Observe that voters evaluate the policy implemented on the popular issue comparing it to the policy proposed initially, and they evaluate the electoral issue comparing it to their own ideal point.

The challenger's performance on the popular issue cannot be evaluated, since he has not been able to do anything on that issue during the present legislature. Thus voters can only evaluate the challenger according to his policy promises on the electoral issue. The reader may argue that the challenger could also make promises on the popular issue that would be implemented in case he wins the election. We are assuming here that the popular issue refers to one-off issues or to issues on which the incumbent policy choices are impossible or too costly to reverse, like an annual budget, abortion legislation, participation in a war or signing an international treaty. Formally, voter i evaluates the challenger according to the following function:

$$U_i(R) = -|x_i - x(R)|.$$

Therefore, voter i will vote for candidate L if and only if

$$-(1-\mu)|y_m - y(L)| - \mu |x_i - x(L)| \geq -|x_i - x(R)|. \quad (1)$$

The lower the value of μ the more weight past choices have on the evaluation of the incumbent. That in turn will affect electoral competition, to the extent that a citizen with ideal point $x_i = x(L)$ will vote for candidate R whenever

$$\mu \leq 1 - \frac{|x_i - x(R)|}{|y_m - y(L)|}.$$

This shows that the existence of a policy proposed on the popular issue imposes a severe constraint on the incumbent's choices. If the distance between the incumbent's choice and the citizens' ideal point on the popular issue is large enough, it may be the case that the set of voters that decide to vote for the incumbent becomes non-connected.

Given this rule, the incumbent is reelected if an only if the set of voters that prefer the incumbent to the challenger contains a majority of the population. We assume that if there is a tie the incumbent is reelected.

This specification encompasses as particular cases some standard models of two-party competition. If $\mu = 1$, that is, voters care only about the electoral issue, we have a standard model of electoral competition. In this case, for very large values of K candidates are purely opportunistic and the model describes a downsian framework. Instead, for relatively small values of K, candidates behave as mostly policy motivated, and our model reproduces Wittman (1983) model of electoral competition. On the other hand, the case of $\mu = 0$ boils down to a more general version of our previous work on participatory democracy (Aragonès and Sánchez-Pagés, 2009).

Thus, we have set up a game in three stages: First, the incumbent implements a policy on the popular dimension. In the second stage, both candidates simultaneously announce policy platforms and in the third stage citizens vote to reelect or not the incumbent. In the next section we analyze the equilibrium of this game for all values of the parameters K, μ and y_m.

4 Equilibrium results

In order to solve the game described above we look for its subgame perfect equilibrium, solving the game by backward induction. Thus we start analyzing the electoral stage, taking as given the choice of the incumbent on the popular issue.

Citizens evaluate the incumbent according to his performance on the popular issue, so he does not enter the new election on the same grounds as the challenger. However, it would be wrong to conclude that the incumbent is indeed at a disadvantage. When the incumbent expects to face a tough competition on the electoral issue, he can soften it by conceding more on the popular issue. Similarly, when the incumbent expects to face a soft competition on the electoral issue, he can compensate his payoffs by not satisfying the voters on the popular issue. This is a strategic move that only the incumbent may afford. The following lemma illustrates this point.

Lemma 1 *If* $x(L) = x(R)$, *then* L *obtains at least* $1 - 2|y(L) - y_m|$ *of the votes and* R *obtains at most* $2|y(L) - y_m|$.

All proofs can be found in the appendix. This lemma shows how the presence of the popular issue affects the electoral competition. When both candidates choose the same position on the electoral issue, that is when $x(L) = x(R)$, only citizens at a distance of at least $|y(L) - y_m|$ from the policy proposed by both candidates vote for the incumbent. Thus, it is possible that the vote of the extremists is captured by the incumbent given a a specific performance on the popular issue. It also implies that the incumbent' chances of winning are higher if he does not depart too much from the society's most preferred policy on the popular issue. As a matter of fact, the threshold on this distance is critical in ascertaining whether the incumbent has an advantage. The next proposition describes the policy choices on the popular issue that guarantee the incumbent reelection in equilibrium.

Proposition 2 *If* $|y(L) - y_m| < 1/4$, *then* L *wins in equilibrium.*

When the incumbent decides to satisfy the citizens with his policy choice on the popular issue, the advantage that he obtains guarantees the existence of a wining strategy at the electoral stage. If the policy choice of the popular issue is close enough to the policy proposal, then it is also optimal for the incumbent to use the strategy that guarantees a sure reelection. That is what he will do in equilibrium.

On the other hand, if the incumbent does not satisfy the electorate with his performance on the participatory issue he will suffer a disadvantage at the electoral stage. In this case the incumbent cannot always guarantee a wining strategy at the electoral stage, and even when he can use a wining strategy, he prefers to lose in equilibrium. This is what the next proposition shows.

Proposition 3 *If* $|y(L) - y_m| > 1/4$, R *wins in equilibrium.*

The two previous propositions show that the incumbent obtains a decisive advantage only when she concedes enough to citizens on the popular issue. If, on the contrary, she departs considerably from y_m then she is doomed to lose reelection. In order to solve the problem of the incumbent when choosing what policy to implement on the popular issue, we need to fully characterize the equilibrium of the political competition stage. The following two propositions describe the equilibrium strategies used by the winner of the election in equilibrium. These strategies define the equilibrium policy outcome of the electoral stage as well. First we find the equilibrium outcomes of the electoral stage for the case in which the incumbent is reelected in equilibrium. In this case, the equilibrium policy outcome coincides with the strategies used by the incumbent in equilibrium at the electoral stage.

Proposition 4 *If $|y(L) - y_m| \leq \frac{1}{4}$, then L's equilibrium strategies at the electoral stage are:*

i) $x^*(L) = 0$ *if* $|y(L) - y_m| \leq \frac{1-3\mu}{4(1-\mu)}$

ii) $x^*(L) = \frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu}|y(L) - y_m|$ *if* $|y(L) - y_m| \geq \frac{1-3\mu}{4(1-\mu)}$

This proposition illustrates the type of trade-off the incumbent faces when he tries to win the election. The more he pleases the electorate on the popular issue, the more able he will be to implement his preferred policy in the electoral issue. In particular, the incumbent is able to guarantee his reelection by implementing his ideal point on the electoral issue if he satisfies voters enough on the popular issue. In order to be able to be reelected by proposing his ideal point on the electoral issue the incumbent will have to concede more on the popular issue the larger the value of μ since $\frac{1-3\mu}{4(1-\mu)}$ decreases with μ.

Otherwise, if he implements a policy on the popular issue that departs significantly from the policy proposal, then in equilibrium he still wins but in this case his equilibrium strategy implies that he has to compromise on the electoral issue to some extent. In this latest case, in order to guarantee a sure reelection the policy in the electoral issue that he has to announce will lie between his ideal point and the median voter's ideal point, and it will be closer to the median voter's ideal point the larger the distance between the policy implemented by the incumbent and the policy proposed on the popular issue. This equilibrium policy choice will also be closer to the median voter's ideal point the tougher the competition at the electoral stage, that is the larger the value of μ[5] . In the limit, as the competition at the electoral stage intensifies, i.e. μ increases, the policy announced by the incumbent on the electoral issue approaches the median voter's ideal point. Similarly, as the popular issue becomes more important, i.e. μ decreases, the policy announced by the incumbent on the electoral issue approaches the incumbent's ideal point.

The next proposition describes the equilibrium outcome of the electoral stage for the case in which the incumbent decides to forgo reelection in equilibrium. In this case the equilibrium policy outcome coincides with the strategies used by the challenger at the electoral stage in equilibrium.

Proposition 5 *If $|y(L) - y_m| > \frac{1}{4}$, then R's equilibrium strategy at the electoral stage is $x^*(R) = \frac{1}{2} + \frac{1-\mu}{1+\mu}|y(L) - y_m|$.*

When the incumbent has departed significantly from the citizens' ideal point in the popular issue, he prefers to lose the election and in the resulting equilibrium the challenger chooses a moderate policy in order to win. Observe that $x^*(R)$ is decreasing in μ so, as before, the tougher the competition at the electoral stage the closer the policy outcome will be to the median voter's ideal point. And the larger the distance between the policy proposal and the policy implemented on the popular issue the closer the policy outcome on the electoral issue will be from the challenger's ideal point.

After solving for the equilibrium strategies of the electoral stage of the game, we move backward in order to find the incumbent's best responses for the first stage of the game given the payoffs obtained from the continuation of the game.

Proposition 6 *L's best wining strategies are:*

(i) $y^*(L) = \max\left\{y_m - \frac{1-3\mu}{4(1-\mu)}, 0\right\}$ *and* $x^*(L) = 0$ *if* $\mu \leq \frac{1}{3}$

(ii) $y^*(L) = y_m$ *and* $x^*(L) = \frac{3\mu-1}{4\mu}$ *if* $\frac{1}{3} \leq \mu \leq \frac{1}{2}$

(iii) $y^*(L) = \max\left\{y_m - \frac{1}{4}, 0\right\}$ *and* $x^*(L) = \frac{1}{2}$ *if* $\mu \geq \frac{1}{2}$.

First, it is evident that as competition on the electoral issue becomes more intense, i.e. μ increases, the incumbent needs to select a platform closer to

[5] Straightfoward calculations show that $\frac{\partial x^*(L)}{\partial \mu} = \frac{1}{\mu^2}\left(\frac{1}{4} - |y(L) - y_m|\right) \geq 0$.

the median voter's ideal policy. But more surprisingly, there exists a non-monotonicity on the gap between the policy she implements in the participatory issue and the proposal of the assembly. When the participatory issue is important (low μ) the incumbent is virtually facing no opposition. Actually, when competition on the electoral issue is rather soft ($\mu \leq \frac{1}{3}$) the incumbent can win by implementing her most preferred policy on the electoral issue. As competition on the electoral issue becomes tougher, the concession on the participatory issue needed to achieve his reelection increases.

As μ becomes larger the incumbent faces a new decision. He can either please the citizens on the popular issue by implementing their proposed policy y_m and in return choose a policy close to her ideal one on the electoral one, or alternatively he can go for the median voter on that dimension and select a policy as close as possible to his own ideal policy on the popular issue. For intermediate levels of μ the first option pays off because electoral competition is still relatively soft so he can implement a policy relatively close to his ideal one on the electoral issue. That winning electoral promise will be larger (less favorable for the incumbent, but always smaller than $\frac{1}{4}$), the tougher electoral competition becomes, that is, the larger the value of μ. However for very tough electoral competition, i.e. $\mu > \frac{1}{2}$, the incumbent will prefer the second option and he will compromise substantially on the electoral issue, that is, he has to implement the median voter´s ideal point. Figure 1 depicts the incumbent´s winning strategy in both stages of the game as a function of μ.

[Insert Figure 1 here]

Next we find the incumbent's best losing strategy and the corresponding best response of the challenger.

Lemma 7 *The incumbent best losing strategies are* $y^*(L) = 0$ *and* $x^*(L) = 1/2$ *which in turn implies that* $x^*(R) = \frac{1}{2} + \frac{1-\mu}{1+\mu} y_m$.

The last step of the analysis amounts to characterize when the incumbent will prefer to win the election. The parameters that determine whether the incumbent prefers to win the election are: the policy proposed on the popular issue, y_m, the incumbent's value for holding office, K, and the relative weight that voters assign to the different issues, μ. In particular, we have that if the policy proposed on the popular issue is close enough to the ideal point of the incumbent on this issue, then the incumbent prefers to win for all values of K and all values of μ. Otherwise, if the preferences of the incumbent on the popular issue are not aligned with the policy proposal on this issue, then the incumbent may decide to forgo the reelection. In this case, he will do so only when he is mostly policy motivated (for low values of K). The tougher electoral competition is, the bigger is the range of values of K that induces the incumbent to forgo the election. Intuitively, the more intense electoral competition and the more costly is to please voters in the popular issue, the more likely is that the incumbent will prefer to lose.

Proposition 8 *The incumbent wins in equilibrium*

(i) When $y_m \leq \frac{1}{4}$ *for any* $K \geq 0$ *and any* $0 \leq \mu \leq 1$.

(ii) When $\frac{1}{4} \leq y_m \leq \frac{3}{8}$ *if and only if* $K > \max\left\{\frac{2\mu}{1+\mu}y_m - \frac{1}{4}, 0\right\}$

(iii) When $y_m \geq \frac{3}{8}$ *if and only if*

$$K > \begin{cases} \max\left\{\frac{2\mu}{1+\mu}y_m + \frac{5\mu-3}{4(1-\mu)}, 0\right\} & \text{if } \mu \leq \frac{1}{2} \\ \frac{2\mu}{1+\mu}y_m - \frac{1}{4} & \text{if } \mu \geq \frac{1}{2} \end{cases}$$

As we have proven before, when the preferences of the incumbent on the popular issue are aligned with those of the society ($y_m \leq \frac{1}{4}$) the incumbent always prefers to use a winning strategy. When that is not the case we find that for some combinations of values for K and μ the incumbent may prefer to forgo reelection. That happens only for large enough values of μ, and for small enough values of K. The area under the curves in Figure 2 correspond to the area of the parameter space where the incumbent is not reelected in equilibrium.

[Insert figure 2 about here]

Incumbents that are mostly policy motivated, i.e. have low values of K, might suffer a disadvantage from being in office. They may find it too costly to make a policy investment that would guarantee their reelection when their preferences are not aligned with society's preferences. The cost of being reelected may also be too high when the degree of competition on the electoral issue is high, i.e μ close to 1. In that case, the incumbent will have to propose a very moderate policy on the electoral issue if he wants to beat the challenger. Outside these cases, citizens' proposals can be used by the incumbent to provide him with a definitive advantage in political competition and make him be reelected.

5 Discussion

The success of representative democracy relies on the willingness of incumbents to deliver policies that satisfy the preferences of voters. The incentives that such a system offers to politicians often do not go in this direction. Incumbents that are policy motivated, as opposed to office motivated, do not take into account voters' preferences, and they often ignore them. Mechanisms of direct democracy can create a bridge between candidates and voters (1) by transmitting information about the voters' preferences to candidates, and (2) by offering incentives to incumbents to satisfy citizens' policy proposals. Our results show that these mechanisms will tend to create an incumbency advantage although incumbents will not be reelected if there exists a substantial disalignment between them and the voters' views. At the light of these results, we will now revisit the two mechanisms of citizen participation described in Section 2.

5.1 Referenda

There is empirical evidence showing that voters welfare increases when voters make use of mechanisms of direct democracy. Econometric cross-section studies for Switzerland reveal that policy choices regarding provision of public goods correspond better with the voters' preferences in those cantons where mechanisms of political participation are more extensively developed. Public expenditures are ceteris paribus lower in communities where the taxpayers themselves can decide on such matters.

One could argue that the use of mechanisms of referenda to achieve this is unnecessary because it is unlikely that the views of elected representatives will be substantively different from those of voters. However, empirical evidence suggests on the contrary that disalignment between the incumbent and the citizens is a likely scenario, especially in the case of local public services, as shown by Agreen, Dahlberg and Mork (2006) for a sample of Swedish municipalities. For the case of Switerzland, Frey and Bohnet (1993) report that:

> "In September 1992, the citizens of Switzerland turned down two proposals seeking to increase substantially the salaries and the staff of Swiss members of Parliament. Both issues would have become law without Swiss voters taking the optional referendum, and both issues would clearly have been to the benefit of the elected officials."

In addition, the two referenda called in Switzerland to decide whether the country should join the UN and the UE in 1986 and 1992 respectively yielded a rejection of these proposals despite the strong backing of all major political parties[6]. This is a more general trend. A 39% of the referenda held in Switzerland between 1948 and 1990 yielded results that opposed the views of the Parliament.

Referenda allow voters to destroy the agenda control of politicians and bring implemented policies closer to what satisfies voters' preferences. One could think that if these direct democracy systems are so effective in selecting policies, then lobbies would have strong incentives in manipulating them. However, Frey and Bohnet (1993) argue that lobbying is less successful when these systems of direct democracy are in place. They show that in Switzerland, even if pressure groups and the political class are united they cannot always have their way particularly on important issues.

5.2 Participatory democracy

Real life experiences of participatory democracy have mainly materialized in processes of "Participatory Budgeting" at the city level. This is the case of nearly two hundred Brazilian municipalities where direct democracy, in the form of popular assemblies, coexists with formal political parties and local elections. The most famous experience of Participatory budgeting was initiated in 1989 in the city of Porto Alegre after the Workers' Party, leaded by Inacio Lula da Silva, gained power. It was then extended to the state level Rio Grande del Sul in 1998 but was abruptly terminated after the Workers' Party lost the election of 2002.

After the first few years after the Participatory Budgeting system was implemented, critics raised issues about the system being used as partisan instrument by the Workers' Party. As a matter of fact, the party had won all municipal elections since 1989 by wide margins. The present paper argues that the incumbent party will enjoy an advantage if it satisfies voters' preferences. Most studies argue that that was the case, as suggested by the enhanced levels of income redistribution and the patterns of citizen participation in the process (Aragones and Sanchez-Pages, 2009). However, the system did not yield the reelection of the Workers' Party candidate in the 2002 state level election. What was the difference between these two scenarios?

[6] The referendum of Switzerland joining the United Nations resulted in a rejection by 76% of the voters. A 50% percent of the population voted against Switzerland becoming part of the European Economic Area.

As argued by Goldfrank and Schneider (2006), one key issue was the degree of political competition. A the city level, the Workers' Party held strong support in Porto Alegre, and it is likely that the popular issue was dominant in voters' minds when casting their vote in subsequent elections. However, at the state level, the Worker's party faced a much stronger opposition from the rest of parties. That would correspond in our model with a higher level of μ. Our results would suggest that the scenario in which the incumbent party does not enjoy an advantage would be much more likely to arise in this case. Goldfrank and Schneider (2006) computed the difference between promised investments and actual investments completed for each municipality under the Workers' Party rule and then estimated a strong negative effect of these dashed expectations on the share of municipal votes of the party in the 2002 election. That showed that the departure between actual policy choices and proposals in an issue likely to correspond to the popular issue in our model, severely undermined the advantage that he incumbent party could have potentially enjoyed in these municipalities.

6 Concluding remarks

The contribution of this paper is to show how the incumbency advantage coexists with an incumbency disadvantage. This is the case when the reaction of the incumbent to the outcome of different mechanisms of citizen political participation factors into the citizens' evaluation of the incumbent's performance.

We have characterized conditions under which the disadvantages generated by these mechanisms are compensated by the advantages and the incumbent can still run as favorite in the electoral campaign. In all these cases, the incumbent has to adjust his policy choices in order to accommodate the policy proposals he receives, and the final policy outcome is close to the policy outcome most preferred by society. But this is not always the case. When the policy demands made by society are too costly from the incumbent's point of view, the incumbent may decide to forgo reelection. In this case the final policy outcome is bad from the voters' point of view.

From the results of our analysis we conclude that policy proposals that are not aligned with the policy preferences of the incumbent will have a negative impact on the voters payoffs, and policy proposals that are aligned with the policy preferences of the incumbent will have a positive impact on the voters payoffs. Therefore, it is optimal for the voters to submit policy demands that do not put too much pressure on the incumbent.

Notice that this analysis applies only to policy proposals that are supported by a large enough proportion of the electorate and that are made on issues relevant to a substantial part of the population. Under these conditions, these proposals constitute a potential threat to the incumbent at the voting stage. In general, policy proposals may be put forward by a an organized group of citizens, the government, a party in the opposition, a lobby, etc. The results of polls and surveys on important issues like abortion, terrorism, immigration or military intervention on foreign countries may also be considered as sources of policy proposals. Each of these cases will have different consequences over the strategic behavior of all agents in the following stages of the game and over the final outcomes. In particular, depending on the origin of the proposal, the intensity of the voters response will be different and the balance between the incumbent's incentives to react to it will also change.

The elaboration and submission of a policy proposal through the call of a referendum, a popular initiative or a citizens' assembly is very costly for voters in terms of time and effort, in addition to the cost of coordinating actions. Thus, it will only be optimal for a group of citizens to engage in such a project when the expected benefits of it are large enough, that is, when the expected change in the incumbent's actions improves the voters' final payoff enough to compensate the cost of the process. And this can only happen on issues over which an important part of the society has strong preferences. The effect of the policy proposal on the voters' final payoffs would be larger if the incumbent's policy preferences on the issue are weak (that is, no policy implementation is expected on an issue that is regarded as very important by the electorate), or when there is a clear conflict of preferences between the incumbent and the society. In these cases it is reasonable to assume that voters will invest time and effort in elaborating a policy proposal to be submitted to the incumbent on those issues for which the voters preferences are very intense and the incumbents' preferences are very weak. That is, if they think that the incumbent is planning to make a more or less satisfactory policy choice on a certain issue, voters will not go through the trouble of organizing and making a proposal. On the other hand, if voters think that the incumbent is not going to act on an issue that they regard as important in a satisfactory way, then they will have incentives to submit a proposal and use it as a threat.

A novel feature of our approach is that the model we build combines elements of both retrospective voting and prospective voting. Voters use retrospective voting to evaluate the performance of the incumbent with respect to the popular issues. And voters use prospective voting to evaluate the campaign promises that candidates announce during the electoral campaign. In order to use all the information available to them at the time of voting, voters combine these two different kinds of evaluations in a unique payoff function.

We have assumed that voters use an asymmetric rule in order to evaluate the candidates. The reason is that we have identified two different kinds of asymmetries that we had to take into account: (1) only the incumbent is responsible for the policy implemented on the popular issue, and (2) there is a policy proposal made only on the popular issue. Thus, we have assumed that voters assign different weights to the two issues, they evaluate the incumbent according to his performance on the two issues and the challenger only according to the electoral issue. We could relax this assumption by assuming that both candidates are evaluated according to both issues. The evaluation of the challenger with respect to the popular issue would just become an exogenous parameter given that the challenger cannot implement any policy during the legislature. This parameter would represent the performance of the challenger on the popular issues in the past. We could also assume that the weights that voters assign to the different issues are different depending on the candidate that they are evaluating. In this case, we would have that the incumbent is more likely to have an advantage the larger it is the weight that the voters assign to the challenger on the electoral issue.

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7 Appendix

Proof of Lemma 1. If $x(L) = x(R)$ then $-(1-\mu)|y_m - y(L)| - \mu|x_i - x(L)| \geq -|x_i - x(R)|$ becomes $|y_m - y(L)| \leq |x_i - x(R)|$. Thus L obtains votes from all i such that are at a distance from $x(R) = x(L)$ of at least $|y(L) - y_m|$. This means that R obtains at most $2|y(L) - y_m|$ votes, therefore L obtains at least $1 - 2|y(L) - y_m|$ votes. Notice that in this case R obtains exactly $2|y(L) - y_m|$ if $|y(L) - y_m| \leq x(L) = x(R) \leq 1 - |y(L) - y_m|$. ■

Proof of Proposition 2. First suppose that $\{y(L), x(L), x(R)\}$ is an equilibrium outcome such that $x(R) = x(L)$. Then R cannot win because by the previous lemma R at most can obtain $2|y(L) - y_m| < 1/2$ votes.

Next suppose that $\{y(L), x(L), x(R)\}$ is an equilibrium outcome such that $x(R) \neq x(L)$ and R wins. Then we must have

$$U_L(y(L), x(L), x(R)) = -y(L) - x(R).$$

Consider that L chooses instead $x'(L) = x(R)$. Then by the previous lemma L obtains at least $1 - 2|y(L) - y_m| > 1/2$ votes. Thus L wins and his utility is

$$U_L(y(L), x(R), x(R)) = -y(L) + K - x(R) > -y(L) - x(R),$$

In this case L prefers to win and has a winning strategy. Thus R cannot win in equilibrium. ■

Proof of Proposition 3. First suppose that $|y(L) - y_m| > 1/2$. If L is winning in equilibrium with $x(L)$ and $x(R)$, then consider $x'(R)$ such that $x'(R) = x(L)$ and notice that in this case R obtains more than half of the total vote. Thus R can win the election in this using this strategy, and it is also optimal for R to do so since he obtains an extra payoff of K and his deviation does not involve any change in the policy implemented. The reason is that if $x(L) \leq 1/2$ then R obtains a vote share equal to $x(L) + \min\{1 - x(L), |y(L) - y_m|\}$ which is a majority. Similarly if $x(L) \geq 1/2$ then R obtains $1 - x(L) + \min\{x(L), |y(L) - y_m|\}$ which is also a majority. Thus L cannot win in equilibrium with $|y(L) - y_m| > 1/2$.

Next suppose that $|y(L) - y_m| \in (\frac{1}{4}, \frac{1}{2})$

If $x(L) \in \left[\frac{1}{2} - |y(L) - y_m|, \frac{1}{2} + |y(L) - y_m|\right]$ then R can defeat it with $x(R) = x(L)$. In this case R obtains a vote share of $2|y(L) - y_m| > 1/2$ which allows R to win. R prefers to do so since by mimicking L he obtains an extra payoff of K and his deviation does not involve any change in the policy implemented..

If $x(L) \leq \frac{1}{2} - |y(L) - y_m|$ then R can defeat L with $x(R) \in (\frac{3-2\mu}{4}, \frac{3}{4})$. To show this, note that the set of supporters of R is the interval $[\frac{x(R)+\mu x(L)}{1+\mu} - \frac{1-\mu}{1+\mu}|y(L) - y_m|, 1]$ whenever $x(R) > (1-\mu)(1 - |y(L) - y_m|) + \mu x(L)$. In addition, this number of voters constitutes a majority if and only if $x(R) < \frac{1+\mu}{2} + (1-\mu)|y(L) - y_m| - \mu x(L)$. This defines an interval of platforms that R can use to defeat L. Given the restrictions on $|y(L) - y_m|$ and the assumption on $x(L)$, this interval is at least as large as the interval $(\frac{3-2\mu}{4}, \frac{3}{4})$. Hence, any platform in this interval guarantees R a victory against $x(L)$. Note again that R prefers to win rather than to let L win because, in addition to obtaining K, the policy outcome is closer to his ideal point.

If $x(L) \geq \frac{1}{2} + |y(L) - y_m|$ then the best winning policy for R is $x(R) = \mu x(L) + (1-\mu)(\frac{1}{2} + |y(L) - y_m|)$. We show this by following the same procedure as above to define the set of R's supporters and then check when it constitutes a majority. Next we need to see whether R actually uses this winning strategy. For this to be the case it need to hold that

$$
\begin{aligned}
K-1+\mu x(L)+(1-\mu)(\frac{1}{2}+|y(L)-y_m|) &> -1+x(L) \\
\Leftrightarrow \quad x(L) &< \frac{K}{1-\mu}+\frac{1}{2}+|y(L)-y_m|\,.
\end{aligned}
$$

Hence, L will not able to win with a $x(L)$ in $(\frac{1}{2}+|y(L)-y_m|\,,1]$ if $K>(1-\mu)(\frac{1}{2}-|y(L)-y_m|)$. If $K<(1-\mu)(\frac{1}{2}-|y(L)-y_m|)$ we need to check whether L prefers to win the election with such rightist policy. The best case scenario for L if he wants to win is when $x(L)=\frac{K}{1-\mu}+\frac{1}{2}+|y(L)-y_m|\,.$ In that case, his payoff is just $-y(L)+K-\frac{K}{1-\mu}-\frac{1}{2}-|y(L)-y_m|\,.$ The best case scenario for L if in the contrary he decides to lose is to set $x(L)=\frac{1}{2}+|y(L)-y_m|$ given that that forces R to choose the same policy. His payoff is just $-y(L)-\frac{1}{2}-|y(L)-y_m|\,,$ so he actually prefers to lose. ■

Proof of Proposition 4. From the previous proposition we know that in this case L wins in equilibrium. Suppose that $x(L)$ and $x(R)$ is an equilibrium outcome such that L wins and $x(R)<x(L)$. Then we must have $U_L(y(L),x(L),x(R))=-y(L)+K-x(L)$. Consider that L chooses instead $x'(L)=x(R)$. Then by lemma 1 L obtains at least $1-2|y(L)-y(A)|>1/2$ votes and his utility is $U_L(y(L),x(R),x(R))=-y(L)+K-x(R)$.

Notice that $U_L(y(L),x(R),x(R))=-y(L)+K-x(R)>-y(L)+K-x(L)=U_L(y(L),x(L),x(R))$ since we assumed that $x(R)<x(L)$. Thus, $x(L)$ and $x(R)$ such that $x(R)<x(L)$ cannot be part of an equilibrium strategy and we must have $x(L)\leq x(R)$.

Let us first characterize the sets of voters that vote for candidate L given $y(L)\,,\,x(L)$ and $x(R)$.

The set of voters with $x_i<x(L)$ that vote for L is given by all x_i such that

$$x_i<\frac{x(R)-\mu x(L)}{1-\mu}-|y(L)-y_m|\equiv\underline{x_i}.$$

Similarly, the set of voters with $x_i>x(R)$ that vote for L is given by all x_i such that

$$x_i>\frac{x(R)-\mu x(L)}{1-\mu}+|y(L)-y_m|\equiv\overline{x_i}$$

Since by proposition 2 $\frac{x(R)-\mu x(L)}{1-\mu}>x(R)$ then we have that $\overline{x_i}>x(R)$. Notice that if $\underline{x_i}<0$ then $\overline{x_i}<1$ for all $|y(L)-y_m|<\frac{1}{2}$.

Finally, the set of voters with $x(L)<x_i<x(R)$ that vote for L is given by all x_i such that

$$x_i<\frac{x(R)+\mu x(L)}{1+\mu}-\frac{1-\mu}{1+\mu}|y(L)-y_m|\equiv\widetilde{x_i}$$

Since by proposition 2 $\frac{x(R)+\mu x(L)}{1+\mu} < x\left(R\right)$ then we have that $\widetilde{x_i} < x\left(R\right) < \overline{x_i}$. However, the comparison between $\underline{x_i}$ and $\widetilde{x_i}$ is not clear-cut. We have that $\underline{x_i} < \widetilde{x_i} < x(L)$ if and only if

$$x(R) - x(L) < (1-\mu)\left|y\left(L\right) - y_m\right|.$$

Thus, two cases can emerge:

Case 1: If $x(R) - x(L) \geq (1-\mu)\left|y\left(L\right) - y_m\right|$ then we have that the votes that L obtains are given by $\widetilde{x_i} + \max\left\{0, 1 - \overline{x_i}\right\}$.

Case 2: If $x(R) - x(L) < (1-\mu)\left|y\left(L\right) - y_m\right|$ then we have that the votes that L obtains are given by $\max\left\{0, \underline{x_i}\right\} + \max\left\{0, 1 - \overline{x_i}\right\}$.

Case 1: If $x(R) - x(L) \geq (1-\mu)\left|y\left(L\right) - y_m\right|$

In this case we have that the votes that L obtains are given by $\widetilde{x_i} + \max\left\{0, 1 - \overline{x_i}\right\}$.

Suppose in the first place that $x(L) = 0$. Then the number of votes that L receives are

$$\#L = \begin{cases} 1 - \left|y\left(L\right) - y_m\right| - \frac{x(R)}{1-\mu} & \text{if } x\left(R\right) < (1-\mu)\left|y\left(L\right) - y_m\right| \\ 1 - \frac{2\mu}{1-\mu^2}x(R) - \frac{2}{1+\mu}\left|y\left(L\right) - y_m\right| & \text{if } x\left(R\right) \in \left[(1-\mu)\left|y\left(L\right) - y_m\right|, (1-\mu)(1 - \left|y\left(L\right) - y\right.\right. \\ \frac{x(R)}{1+\mu} - \frac{1-\mu}{1+\mu}\left|y\left(L\right) - y_m\right| & \text{if } x\left(R\right) > (1-\mu)(1 - \left|y\left(L\right) - y_m\right|) \end{cases}$$

that attains a minimum when $x\left(R\right) = (1-\mu)(1 - \left|y\left(L\right) - y_m\right|$. The number of votes in that case is greater than $\frac{1}{2}$ if and only if

$$\left|y\left(L\right) - y_m\right| \leq \frac{1-3\mu}{4(1-\mu)}.$$

Note that if this holds, $x(L) = 0$ is a winning strategy for L. Otherwise, there exists a platform $x(R)$ that can defeat $x(L) = 0$.

Second, suppose that $\frac{1-3\mu}{4(1-\mu)} < \left|y\left(L\right) - y_m\right| > \frac{1}{4}$. Let us first show that any platform $x(L) \in (0, \frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu}\left|y\left(L\right) - y_m\right|)$ can be defeated by $x(R) = \frac{3-\mu}{4}$. First, note that we are in Case 1 since

$$x(R) - x(L) > (1-\mu)\left|y\left(L\right) - y_m\right| \Leftrightarrow x(L) < \frac{3-\mu}{4} - (1-\mu)\left|y\left(L\right) - y_m\right|$$

and in addition we have by assumption that

$$x(L) < \frac{3\mu - 1}{4\mu} + \frac{1-\mu}{\mu}\left|y\left(L\right) - y\left(A\right)\right| < \frac{3-\mu}{4} - (1-\mu)\left|y\left(L\right) - y\left(A\right)\right|$$

where the last inequality follows from simple algebra. One can also show that our assumption on $x(L)$ also implies that $\overline{x_i} > 1$ which means that the number of votes obtained by L is just $\widetilde{x}_i$ which in turn is smaller than $\frac{1}{2}$ if and only if

$$x(L) < \frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu}\left|y(L) - y_m\right|,$$

which holds by assumption. Hence, L is defeated if he chooses a platform in that interval. From the remainder, let us now show that $x(L) = \frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu}\left|y(L) - y_m\right|$ is a dominant strategy.

Again case we have to consider two cases:

1. Suppose that $x(R) > \frac{3\mu-1}{4\mu} - \frac{1-\mu^2}{\mu}\left|y(L) - y_m\right|$. In that case, the number of citizens who vote for the incumbent are given by $\min\{1, \overline{x_i}\} - \widetilde{x}_i$. We need to consider two subcases depending on the value of the extremes of this interval.

   i. If $\overline{x_i} > 1$ then R gets $1 - \widetilde{x}_i$ votes and wins the election if and only if

   $$\widetilde{x}_i < \frac{1}{2} \rightarrow x(R) < \frac{3-\mu}{4}$$

   Since $\overline{x_i} > 1$ if and only if $x(R) > \frac{3-\mu}{4}$ then this case cannot arise.

   ii. If $\overline{x_i} < 1$ then R gets $\overline{x_i} - \widetilde{x}_i$ votes. This number of votes is greater than $\frac{1}{2}$ if and only if $x(R) > \frac{3-\mu}{4}$. Since $\overline{x_i} < 1$ if and only if $x(R) < \frac{3-\mu}{4}$ again this case is not possible.

2. Suppose instead that $x(R) < \frac{3\mu-1}{4\mu} - \frac{1-\mu^2}{\mu}\left|y(L) - y_m\right|$. This means necessarily that $\overline{x_i} < 1$ and that the challenger collects votes in $(\max\{0, \underline{x_i}\}, \overline{x_i})$. We need to consider then two different subcases:

   i. If $\underline{x_i} < 0$ the challenger gets $\overline{x_i}$ votes and wins if and only if $x(R) \geq \frac{1+\mu}{4}$. But this leads to a contradiction because

   $$\frac{1+\mu}{4} > \frac{3\mu-1}{4\mu} - \frac{1-\mu^2}{\mu}\left|y(L) - y_m\right| \Leftrightarrow \frac{1-\mu}{4(1+\mu)} > -\left|y(L) - y_m\right|.$$

   ii. If $\underline{x_i} > 0$ then R gets $\overline{x_i} - \underline{x_i} = 2\left|y(L) - y_m\right|$ votes. So here R cannot win either.

Thus R cannot win the election for any $x(R)$ he may choose. Still, observe that $x(R) = \frac{3-\mu}{4}$ is a dominant strategy for her.

Since we have shown that L wins in equilibrium when $\left|y(L) - y_m\right| \leq \frac{1}{4}$, we have that L's most preferred best response is an equilibrium strategy. ■

Proof of Proposition 5. First, suppose that $|y(L) - y_m| > 1/2$. If $x(L) > x(R)$ in equilibrium, consider $x'(R)$ such that $x'(R) = x(L)$ and notice that: 1) in this case R obtains more than $|y(L) - y_m|$ votes, that is, more than half of the total; and 2) the equilibrium policy outcome is larger, therefore better off for R'. Thus this is a profitable deviation for R and it implies that $x(L) > x(R)$ cannot hold in equilibrium.

Since we know that in equilibrium $x(L) \leq x(R)$ R's best winning strategy is defined by $\overline{x}_i > 1$ and $\widetilde{x}_i < \frac{1}{2}$.This implies that

$$\overline{x_i} = \frac{x(R) - \mu x(L)}{1-\mu} + |y(L) - y_m| > 1$$

and

$$\widetilde{x}_i = \frac{x(R) + \mu x(L)}{1+\mu} - \frac{1-\mu}{1+\mu}|y(L) - y_m| < \frac{1}{2}$$

Thus the set of winnings strategies for R is defined by

$$(1-\mu)(1 - |y(L) - y_m|) + \mu x(L) < x(R) < \frac{1+\mu}{2} + (1-\mu)|y(L) - y_m| - \mu x(L)$$

and among them R prefers the largest one $x(R) = \frac{1+\mu}{2} + (1-\mu)|y(L) - y_m| - \mu x(L)$.

The best response for L in this case is the largest possible value of $x(L)$. So that R's best response to it corresponds to its smallest possible value. Since in equilibrium we need to have $x(L) \leq x(R)$ then $x(L) \leq \frac{1+\mu}{2} + (1-\mu)|y(L) - y_m| - \mu x(L)$ implies $x(L) \leq \frac{1}{2} + \frac{1-\mu}{1+\mu}|y(L) - y_m|$. Thus in equilibrium $x(L) = x(R) = \frac{1}{2} + \frac{1-\mu}{1+\mu}|y(L) - y_m|$.

Now suppose that $1/4 < |y(L) - y_m| < 1/2$. If $x(L) \in \left[0, \frac{1}{2} + |y(L) - y_m|\right]$ then R's best response, as in the previous proposition, is defined by $\overline{x}_i > 1$ and $\widetilde{x}_i < \frac{1}{2}$.

This implies that

$$\overline{x_i} = \frac{x(R) - \mu x(L)}{1-\mu} + |y(L) - y_m| > 1$$

and

$$\widetilde{x}_i = \frac{x(R) + \mu x(L)}{1+\mu} - \frac{1-\mu}{1+\mu}|y(L) - y_m| < \frac{1}{2}$$

Thus the set of winnings strategies for R is defined by $(1-\mu)(1 - |y(L) - y_m|) + \mu x(L) < x(R) < \frac{1+\mu}{2} + (1-\mu)|y(L) - y_m| - \mu x(L)$

and among them R prefers $x(R) = \frac{1+\mu}{2} + (1-\mu)|y(L) - y_m| - \mu x(L)$

And the best response for L in this case is the largest possible value of $x(L)$. Since in equilibrium we need to have $x(L) \leq x(R)$ then $x(L) \leq \frac{1+\mu}{2} +$

$(1-\mu)\left|y(L)-y_m\right|-\mu x(L)$ implies $x(L)\leq\frac{1}{2}+\frac{1-\mu}{1+\mu}\left|y(L)-y_m\right|.$ Thus for $x(L)\in\left[0,\frac{1}{2}+\left|y(L)-y_m\right|\right]$ R's best response is $x(R)=\frac{1}{2}+\frac{1-\mu}{1+\mu}\left|y(L)-y_m\right|.$

Given that if $x(L)\in\left[\frac{1}{2}+\left|y(L)-y_m\right|,1\right]$ we have that R's best response is $x(R)\in\left[\frac{1}{2}+\left|y(L)-y_m\right|,1\right]$, and for $x(L)\in\left[0,\frac{1}{2}-\left|y(L)-y_m\right|\right)$ we have that R's best response is $x(R)=\frac{1}{2}+\frac{1-\mu}{1+\mu}\left|y(L)-y_m\right|<\frac{1}{2}+\left|y(L)-y_m\right|,$ this implies that L's optimal strategy will not be in $\left[\frac{1}{2}+\left|y(L)-y_m\right|,1\right].$

Therefore the equilibrium if $\frac{1}{4}<\left|y(L)-y_m\right|<\frac{1}{2}$ is given by $x(L)=x(R)=\frac{1}{2}+\frac{1-\mu}{1+\mu}\left|y(L)-y_m\right|.$ ■

Proof of Proposition 6. Let us start with the case when $\left|y(L)-y_m\right|\leq\frac{1-3\mu}{4(1-\mu)}$.Notice that it can emerge if and only if $\mu\leq\frac{1}{3}$. In that case the incumbent's payoff is increasing with $\left|y(L)-y_m\right|$ so his most preferred value of $y(L)$ in this range corresponds to $y(L)=y_m-\frac{1-3\mu}{4(1-\mu)}$. We already know from previous results that in this case that he will then set $x^*(L)=0$

When $\frac{1-3\mu}{4(1-\mu)}\leq\left|y(L)-y_m\right|\leq\frac{1}{4}$, after plugging the incumbent's equilibrium platforms in the electoral issue, it is possible to rewrite his payoff as

$$V_L=-y_m+K-\frac{3\mu-1}{4\mu}-\frac{1-2\mu}{\mu}\left|y(L)-y_m\right|, \tag{2}$$

which is decreasing with $\left|y(L)-y_m\right|$ as long as $\mu\leq\frac{1}{2}$ and increasing otherwise. In the former case, L's most preferred value of $y(L)$ corresponds to the minimal value of $\left|y(L)-y_m\right|$ in this range, that is, $y(L)=\max\left\{y_m-\frac{1-3\mu}{4(1-\mu)},y_m\right\}.$ Hence, if $\mu\leq\frac{1}{3}$ he will set again $y(L)=y_m-\frac{1-3\mu}{4(1-\mu)}$ (and then $x^*(L)=0$) whereas if $\frac{1}{3}\leq\mu\leq\frac{1}{2}$ he must set $y(L)=y_m$ which in turn implies that $x^*(L)=\frac{3\mu-1}{4\mu}$.

The third case occurs when $\mu\geq\frac{1}{2}$. Then (2) is increasing with $\left|y(L)-y_m\right|.$ Thus while staying in this range his most preferred value of $y(L)$ corresponds to the one that maximizes $\left|y(L)-y_m\right|$, that is, $y(L)=y_m-\frac{1}{4}$, that from previous results it implies $x^*(L)=\frac{1}{2}$. ■

Proof of Proposition 7. We know from previous results that if the incumbent decides to lose by setting $\left|y(L)-y_m\right|>\frac{1}{4}$, the challenger will win the election and set $x(R)=\frac{1}{2}+\frac{1-\mu}{1+\mu}\left|y(L)-y_m\right|.$ In that case, the incumbent receives the payoff

$$V_L=-y_m-\frac{1}{2}-\frac{2\mu}{1+\mu}\left|y(L)-y_m\right|,$$

which is increasing in $\left|y(L)-y_m\right|.$ Thus while staying in this range, his most preferred value of $y(L)$ corresponds to the one that maximizes $\left|y(L)-y_m\right|$, that is, $y^*(L) = 0$, which implies that the challenger's best response in this case is $x^*(R) = \frac{1}{2} + \frac{1-\mu}{1+\mu} y_m$. ■

Proof of Proposition 8. Previous results show that since $y_m < \frac{1}{4}$ implies that $|y(L) - y_m| < \frac{1}{4}$ then L prefers to win in this case.

If $y_m \geq \frac{1}{4}$, if the incumbent decides to lose then he receives a payoff equal to

$$V_L = -\frac{1}{2} - \frac{1-\mu}{1+\mu} y_m.$$

If the incumbent decides to use his best winnings strategy then he receives a payoff equal to

when $\mu \leq \frac{1}{3}$ his payoff boils down to

$$V_L = -y_m + K - \frac{3\mu - 1}{4(1-\mu)} \text{ if } \mu \leq \frac{1}{2} \tag{3}$$

and

$$V_L = -y_m + K - \frac{1}{4} \text{ if } \mu \geq \frac{1}{2} \tag{4}$$

Thus, when $\mu \geq \frac{1}{2}$ he prefers to use his winning strategy as long as $-y_m + K - \frac{1}{4} \geq -\frac{1}{2} - \frac{1-\mu}{1+\mu} y_m$, that is, for

$$K > \frac{2\mu}{1+\mu} y_m - \frac{1}{4}$$

Notice that this value is strictly positive for all values of $\mu \in [0,1]$ as long as $y_m > \frac{3}{8}$. For $\frac{1}{4} \leq y_m \leq \frac{3}{8}$ we will have that the incumbent will decide to use a winning strategy for all values of K whenever $\frac{2\mu}{1+\mu} y_m - \frac{1}{4} > 0$, that is, $\mu > \frac{1}{8y_m - 1}$. Notice that the incumbent decides to win for all K whenever $y_m = \frac{1}{4}$. Furthermore, the incumbent always decides to forgo reelection for some positive values of K whenever $y_m > \frac{3}{8}$.

Similarly, when $\mu \leq \frac{1}{2}$ he prefers to use his winning strategy as long as $-y_m + K - \frac{3\mu-1}{4(1-\mu)} \geq -\frac{1}{2} - \frac{1-\mu}{1+\mu} y_m$, that is, for

$$K > \frac{5\mu - 3}{4(1-\mu)} + \frac{2\mu}{1+\mu} y_m,$$

Notice that this value is strictly negative for small values of μ (in particular for all $\mu \leq \frac{1}{3}$). For those values the incumbent decides to win the election for all K. The set of values of K for which the incumbent decides to use a winning strategy is smaller for larger values of μ in this area. ■

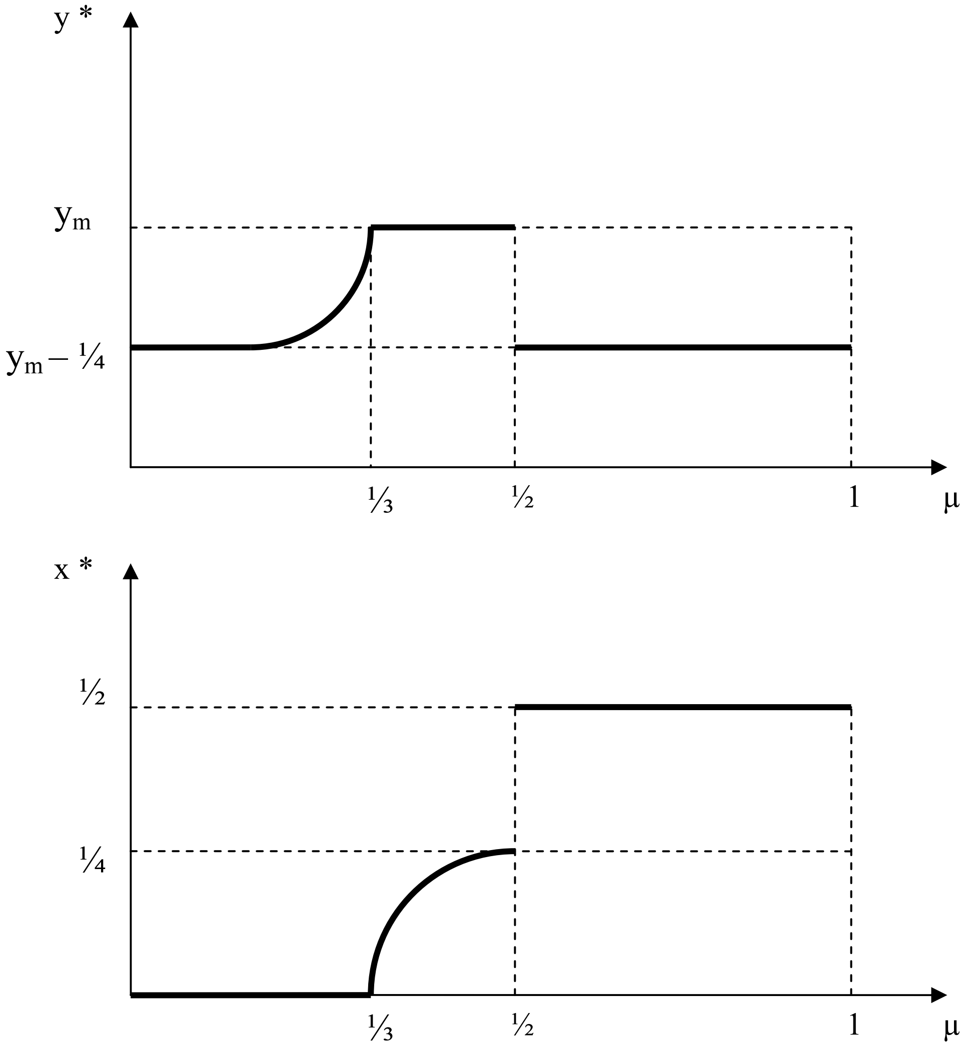

Figure 1: Incumbent's best winning strategies.

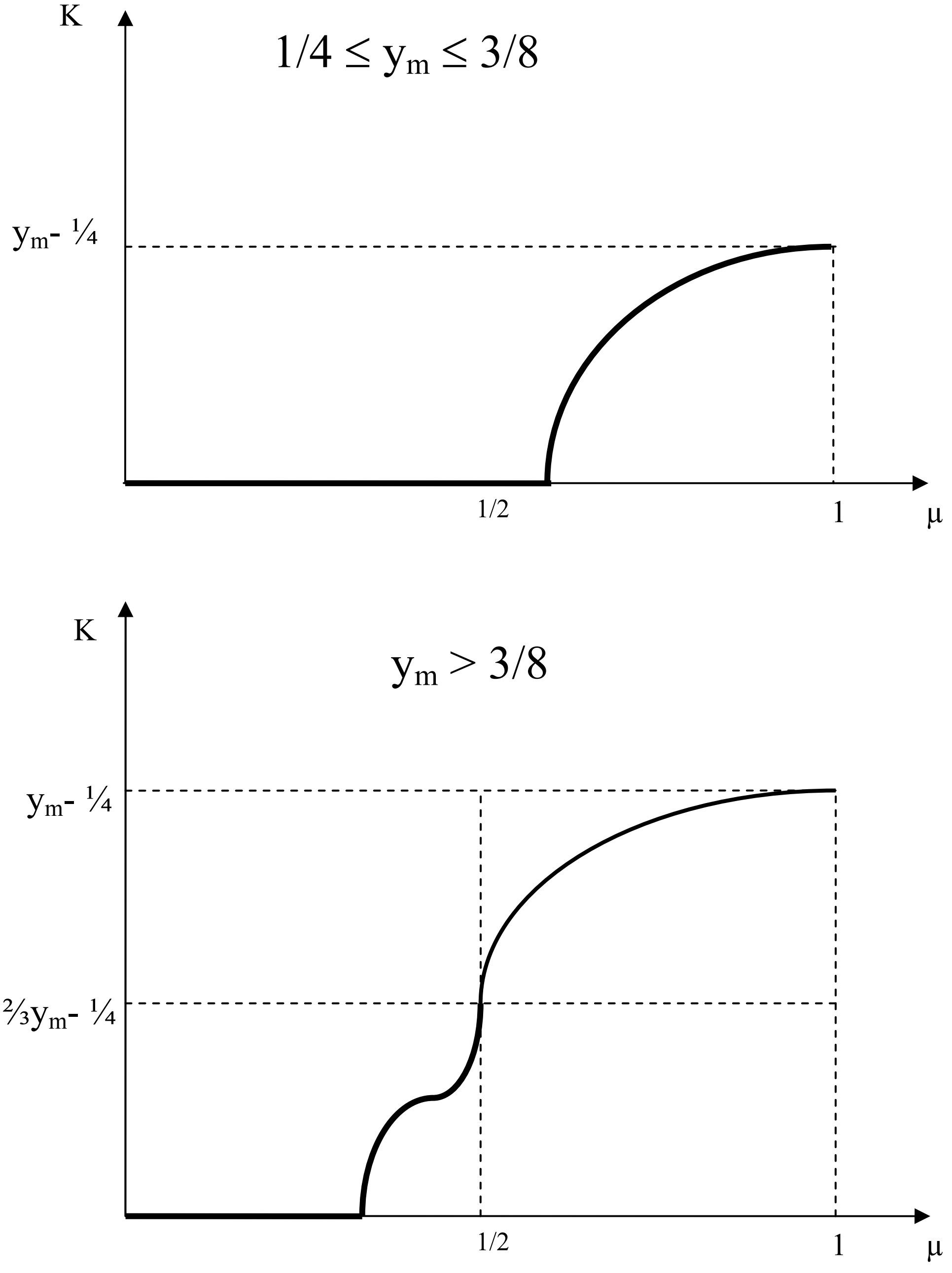

Figure 2 : Minimal values of K for which the incumbent prefers to use a winning strategy in equilibrium.