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A Long Run Structural Macroeconometric Model of the UK*

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Abstract

A new modelling strategy is introduced which provides a practical approach to incorporating long-run structural relationships, suggested by economic theory, in an otherwise unrestricted VAR model. The strategy is applied in the construction of a small quarterly macroeconomic model of the UK, estimated over the period 1965q1-1995q4 in eight core variables: domestic and foreign outputs, domestic and foreign prices (both measured relative to oil prices), the nominal effective exchange rate, nominal domestic and foreign interest rates and real money balances. The aim is to develop a core model with a transparent and theoretically coherent foundation. Tests of restrictions on the long-run relations of the model are presented and the dynamic properties of the model are discussed.

Keywords: Long-Run Structural VAR, A Core UK Model, Macroeconomic Modelling, Persistence Profiles.

JEL Classifications: C32, E24

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1 Introduction

Over the past two decades, there has been a growing interest in developing macroeconomic models with transparent theoretical foundations and flexible dynamics that fit the historical time series data reasonably well. The modelling framework described in the present paper, along with the work of King et al. (1991) and Mellander et al. (1992), represent the first steps towards this aim. However, our work is distinguished from these earlier contributions in three respects. First, we develop a long-run framework suitable for modelling a small open macroeconomy like the UK. The six-variable model of King et al. is a closed economy model perhaps suitable for modelling an economy such as the US. The four-variable model of Mellander et al. is made open only through a terms of trade variable added to the three-variable consumption-investment-income model analysed by King et al. and does not have a rich enough structure for the analysis of many open economy macroeconomic problems of interest. Second, we describe a new strategy which provides a practical approach to incorporating the long-run structural relationships suggested by economic theory in an otherwise unrestricted vector autoregressive (VAR) model. And third, we employ new econometric techniques in the construction of the model and in the testing of the long-run properties predicted by the theory. The description of the modelling work not only provides one of the first examples of the use of these techniques in an applied context, but also includes a discussion of some bootstrap experiments designed to investigate the small-sample properties of the tests employed. Hence, the paper reinforces the arguments of King et al. and Mellander et al. by emphasising the (statistical and economic) importance of the long-run in macroeconometric modelling. But it also provides a practical demonstration of how this can be done through the development a quarterly “core” macroeconometric model for the UK economy estimated over the period 1965q1-1995q4.

The model contains transparent and theoretically coherent long-run properties of the type exhibited by Real Business Cycle models. The long-run relations are derived rigorously from production, trade, arbitrage, solvency and portfolio balance conditions, and these are then embedded in an otherwise unrestricted VAR model.1 The model comprises five domestic variables whose developments are widely regarded as essential to a basic understanding of the behaviour of the UK macroeconomy; namely, aggregate output, the relative domestic price level (relative to oil prices), the nominal interest rate, the exchange rate and real money balances. The model also contains foreign output, foreign interest rates and a foreign price variable measured once again relative to oil prices.

The empirical analysis of the core model provides insights into the functioning of the UK macroeconomy from two perspectives. First, the econometric methodology that has been developed provides the means for testing formally the validity of (over identifying) restrictions implied by specific long-run structural relations without imposing “incredible” restrictions on the short-run coefficients.2 The ability to test rigorously the validity of long-

1The role of the long-run in macroeconometric modelling is discussed in Gannett et al. (1999) where the structural cointegrating VAR approach to modelling followed in this paper is compared to other approaches found in the literature.

2For a discussion of the econometric methodology see Pesaran and Shin (1999) and Pesaran, Shin and Smith (1999) and the references cited therein.
run restrictions implied by economic theory within the context of a small and transparent, but reasonably comprehensive, model of the UK macroeconomy is an important step towards an evaluation of the long-run underpinnings of alternative macro theories. Second, the paper provides insights into the dynamic response of the macroeconomy as it reacts to new events, focusing on the contribution of the embedded long-run relationships to these dynamics. To this end, we compare the statistical performance of the model with a benchmark model which omits the long-run relationships. We also present persistence profiles based on our estimated model which illustrate the speed with which disequilibria in the various long-run relationships are eliminated.

We shall also briefly discuss how our modelling approach relates to the recent developments in macroeconometrics literature, as set out, for example, in the papers by Bernanke (1986), Christiano and Eichenbaum (1992), Cochrane (1998), Crowder et al. (1999) and Wickens (1999). In these, the effects of particular types of shocks (typically those associated with monetary policy changes) are considered. Such an analysis requires the use of a priori identifying restrictions on the short-run dynamics as well as on the long-run relationships of the model. Different approaches to the identification of the short-run dynamics in macroeconometric models have been attempted, but so far no consensus has emerged.\(^3\) In contrast, identification of the long-run relationships and the extent to which the economy deviates from its long-run equilibrium is less controversial. These deviations will be referred to as the “long-run structural disturbances”. Not all of the variables involved in the long-run relationships suggested by economic theory are observable, however, and in writing the long-run relationships in terms of observable variables, “long-run reduced form disturbances” are derived as a function of the long-run structural disturbances. It is these long-run reduced form disturbances which are embedded within an otherwise unrestricted VAR to obtain a model with a potentially sophisticated dynamic structure (unconstrained by economic theory) but which incorporates the long-run restrictions that are suggested by economic theory. This procedure does not contradict the possibility that an explicitly-formulated model of the short-run could be described and used alongside the theory of the long-run, but neither does it necessarily contribute to such a theory. Moreover, in describing the relationships between the long-run structural and long-run reduced form disturbances, the procedure highlights some of the difficulties which will arise in formulating the economic theory of the short-run and the problems involved in interpreting the effects of shocks in general, and in the analysis of impulse responses in particular.\(^4\)

The plan of the paper is as follows: Section 2 describes a long-run theoretical framework for macroeconomic modelling of a small open economy such as the UK, and derives testable restrictions on the long-run relations. Section 3 outlines how the long-run relations are embodied in a Vector Error Correction model. Section 4 describes the empirical analysis underlying the construction of the core model, discusses the results obtained from testing its long-run properties and comments on the dynamic properties of the estimated model. Section 5 provides some concluding remarks.

\(^3\)For a recent statement of the problem, see Pagan (1999).
\(^4\)The problems associated with the identification of the effects of specific shocks are discussed in Levchenkova et al. (1998) and elsewhere.
2 A Framework for Long-Run Macromodelling

In this section, we outline the theoretical basis of our approach to macroeconomic modelling of a small open economy such as the UK. It begins with a rigorous derivation of the long-run steady-state relationships expected to prevail between the main variables in the core model. The analysis emphasises arbitrage conditions and stock-flow equilibria and, as such, corresponds to many of the long-run properties of the RBC and large macroeconometric models.

There are two main theoretical approaches to the derivation of long-run, steady state relations of a core macroeconomic model. One possibility is to start with the inter-temporal optimization problems faced by “representative” households and firms and solve for the long-run relations using the Euler first-order conditions and the stock-flow constraints. Given the typically non-quadratic form of the utility and production functions and the linear forms of the constraints, the resultant relations of the model economy are generally highly non-linear and are usually approximated by log-linear relations (the real business cycle literature follows this methodology). The long-run relations are then obtained by ignoring expectational errors and assuming that the model economy is stationary and ergodic in certain variables, such as growth rates, capital per effective worker and asset-income ratios. An alternative approach, and the one which is followed here, is to work directly with the arbitrage conditions which provide inter-temporal links between prices and asset returns in the economy as a whole. The arbitrage conditions, however, must be appropriately modified to allow for the risks associated with market uncertainties.

The above two approaches are closely related and yield similar results as far as the long-run relations are concerned. The main difference between them lies in their treatment of short-run dynamics. The inter-temporal optimization approach purports to be based on a complete short-run dynamic theory: the intrinsic dynamics of the model, generated by the model’s intertemporal elements, are usually complemented by extrinsic dynamics through the specification of dynamic stochastic processes for taste and technology variables. The strength of the inter-temporal optimization approach lies in the explicit identification of macroeconomic disturbances as innovations (shocks) to processes generating tastes and technology. However, this is achieved at the expense of often strong assumptions concerning the form of the underlying utility and production functions. Further, despite the efforts to capture the short-run dynamics explicitly, the models do abstract from the dynamic effects of many types of adjustment costs, of learning, and of aggregation across agents with heterogeneous information. The evidence presented by Christiano and Eichenbaum (1992) and Kim and Pagan (1995), for example, suggests that many of the parameter restrictions underlying the current vintage of RBC models are not supported by the data. In contrast, the approach advocated in this paper, by focussing on long-run theory restrictions and leaving the short-run dynamics largely unrestricted (in the context of a VAR model), provides a much more flexible modelling strategy.

We begin our derivation of long-run relations with a specification of the sectoral disaggregation that we will be using, and the associated accounting identities.
2.1 Stock-Flow Relations and Accounting Identities

To introduce the notations and to define the concepts, we use with the following stock identities:

\[ \tilde{D}_t = \tilde{H}_t + \tilde{B}_t, \]  
\[ \tilde{F}_t = E_t \tilde{B}^*_t - (\tilde{B}_t - \tilde{B}^*_t), \]  
\[ \tilde{L}_t = \tilde{H}_t + \tilde{B}^*_t + E_t \tilde{B}^*_t, \]  

where \( \tilde{D}_t \) is net government debt, \( \tilde{H}_t \) is the stock of high-powered money, \( \tilde{B}_t \) is the stock of domestic bonds issued by the government, \( \tilde{F}_t \) is the net foreign asset position of the economy, \( \tilde{B}^*_t \) is the stock of foreign assets held by domestic residents, \( \tilde{B}^*_t \) is the stock of domestic assets held by domestic residents, and \( \tilde{L}_t \) \( = \tilde{D}_t + \tilde{F}_t \) is the stock of financial assets held by the private sector.\(^5\) All the stocks are measured at the beginning of period \( t \). Nominal magnitudes are denoted with a ‘~’ and are expressed in Pounds Sterling, except \( \tilde{B}^*_t \) which is expressed in foreign currency. \( E_t \) is the effective exchange rate, defined as the domestic price of a unit of foreign currency at the beginning of period \( t \), so that an increase in the exchange rate represents a depreciation of the home country currency. It is assumed that the government holds no foreign assets of its own.

We also have the output-expenditure flow identity:

\[ \tilde{Y}_t = \tilde{C}_t + \tilde{I}_t + \tilde{G}_t + (\tilde{X}_t - \tilde{M}_t), \]  

where \( \tilde{Y}_t \) is gross domestic product, \( \tilde{C}_t \) consumption expenditures, \( \tilde{I}_t \) investment expenditures, \( \tilde{G}_t \) government expenditures, \( \tilde{X}_t \) is expenditures on exports and \( \tilde{M}_t \) expenditures on imports, all are in current market prices and expressed in Pounds Sterling. The private sector disposable income is defined by

\[ \tilde{Y}^d_t = \tilde{Y}_t - \tilde{T}_t + R_t \tilde{B}^*_t + E_t \tilde{B}^*_t, \]  

where \( \tilde{T}_t \) represents taxes net of transfers to the private sector, \( R_t \) is the nominal interest rate on domestic assets held from the beginning to the end of period \( t \), and \( \tilde{B}^*_t \) is the nominal interest rate paid on foreign assets during period \( t \).

The core model economy’s stock-flow relationships are:

\[ \Delta \tilde{D}_{t+1} = \tilde{C}_t + R_t \tilde{B}^*_t - \tilde{T}_t \]  
\[ \Delta \tilde{L}_{t+1} = \tilde{Y}^d_t - \tilde{C}_t - \tilde{I}_t + (E^{-}_t - E^{-}_t) \tilde{B}^*_t \]  
\[ \Delta \tilde{F}_{t+1} = \tilde{X}_t - \tilde{M}_t + NFA_t + (E^{-}_t - E^{-}_t) \tilde{B}^*_t \]  

where \( NFA_t = E_t R_t^* \tilde{B}^*_t - R_t^* (\tilde{B}_t - \tilde{B}^*_t) \) is net factor income from abroad, and \( E^{-}_t \) stands for exchange rate expectations formed on the basis of publicly available information at time \( t \). Hence, the term \( (E^{-}_t - E^{-}_t) \tilde{B}^*_t \) is the (expected) revaluation of foreign assets held by domestic residents accruing through exchange rate appreciation in period \( t \).\(^6\) Note that, since \( \tilde{L}_t = \tilde{D}_t + \tilde{F}_t \), any two of (2.6)-(2.8) implies the third.

\(^5\)It is assumed that foreign asset holdings of domestic residents and domestic holdings of foreign residents are composed of government bonds only.

\(^6\)In most formulations of stock-flow relationships the asset revaluation term is either ignored or is approximated by an \textit{ex post} counterpart such as \( (E_t - E_{t-1}) \tilde{B}^*_t \). But for consistency with the arbitrage (equilibrium) conditions to be set out in Section 2.3 below, we prefer to work with the \textit{ex ante} asset revaluation term.
2.2 Production Technology and Output Determination

We assume that, in the long-run, aggregate output is determined according to the following constant returns to scale production function in labour (denoted by $N_t$) and capital stock (denoted by $K_t$):

$$\frac{\dot{Y}_t}{P_t} = F(K_t, A_t N_t) = A_t N_t F\left(\frac{K_t}{A_t N_t}, 1\right),$$

(2.9)

where $P_t$, is a general price index, $\dot{Y}_t/P_t$ is real aggregate output, and $A_t$ stands for an index of labour-augmenting technological progress, assumed to be composed of a deterministic component, $a_0 + gt$, and a stochastic mean-zero component, $u_{at}$:

$$\ln(A_t) = a_0 + gt + u_{at}. \quad (2.10)$$

The process generating $u_{at}$ is likely to be quite complex and there is little direct evidence on its evolution. But a few studies that have used patent data or R&D expenditures to directly analyse the behaviour of $u_{at}$ over the course of the business cycle generally find highly persistent effects of technological disturbances on output. (Discussed in Fabiani (1996) and the references cited therein). The indirect evidence on $u_{at}$, obtained from empirical analysis of aggregate output, also corroborates this finding and generally speaking does not reject the hypothesis that $u_{at}$ contains a unit root. (See, for example, Nelson and Plosser (1982) and, for the UK, Mills (1991)).

We further assume that the fraction of the population which is employed at time $t$, $\lambda_t = N_t/POP_t$, is a stationary process such that

$$N_t = \lambda POP_t \exp(\eta_{at}), \quad (2.11)$$

where $POP_t$ is population at the end of period $t$ and $\eta_{at}$ represents a stationary, mean-zero process capturing the cyclical fluctuations of the unemployment rate around its steady state value, $1 - \lambda$.\footnote{Notice that the assumption that the unemployment rate, $1 - (L_t/POP_t)$, is stationary in effect rules out long-run hysteresis effects in the unemployment process. To accommodate such effects in a satisfactory manner requires consideration of non-linear dynamic models which is beyond the scope of the present paper.} Under the above assumptions and using the relations (2.9), (2.10) and (2.11) it now readily follows that

$$y_t = a_0 + \ln(\lambda) + gt + \ln(f(\kappa_t)) + u_{at} + \eta_{at}, \quad (2.12)$$

where $y_t = \ln\left(\frac{\dot{Y}_t/P_t}{POP_t}\right)$ is the logarithm of real per capita output, $\kappa_t = K_t/A_t N_t$ is the capital stock per effective labour unit, and $f(\kappa_t) = F(K_t/A_t N_t, 1)$ is a well behaved function in the sense that it satisfies the Inada conditions. (See, for example, Barro and Sala-i-Martin (1995, p.16)).

Assuming the aggregate saving rate is monotonic in $\kappa_t$ then, under certain other mild regularity conditions, Binder and Pesaran (1999) show that, irrespective of whether the process generating $u_{at}$ is stationary or contains a unit root, $\kappa_t$ converges to a globally attracting, invariant steady state probability distribution; namely $\kappa_t \rightarrow \kappa_\infty$, where $\kappa_\infty$ is a time-invariant
random variable with a non-degenerate probability distribution function. Hence, in the long-run the evolution of per capita output will be largely determined by technological process, with $E(\Delta \ln(y_t)) = g$. Also whether $y_t$ contains a unit root crucially depends on whether there is a unit root in the process generating technological progress.

Given the small and open nature of the UK economy, it is reasonable to assume that, in the long-run, $A_t$ is determined by the level of technological progress in the rest of the world; namely

$$A_t = \gamma A^*_t \exp(\eta_{kt}),$$

(2.13)

where $A^*_t$ represents the level of foreign technological progress, $\gamma$ captures productivity differentials based on fixed, initial technological endowments, and $\eta_{kt}$ represents stationary, mean zero disturbances capturing the effects of information lags or (transitory) legal impediments to technology flows across different countries, for example. Assuming that per capita output in the rest of the world is also determined according to a neoclassical growth model, and using a similar line of reasoning as above, we have

$$y_t - y^*_t = \ln(\gamma) + \ln(\lambda/\lambda^*) + \ln \left\{ f(\kappa_t)/f^*(\kappa^*_t) \right\} + \eta_{kt} + (\eta_{kt} - \eta_{kt}^*),$$

(2.14)

where foreign variables are shown with a “star”. Similarly to $\kappa_t$, the foreign capital stock per effective labour unit, $\kappa^*_t$, also tends to a time invariant probability distribution function, and hence under the assumption that $A^*_t$ (or $A_t$) contain a unit root, $(y_t, y^*_t)$ will be cointegrated with a cointegrating vector equal to $(1, -1)$.

The above stochastic formulation of the neoclassical growth model also has important implications for the determination of the real rate of return, which we denote by $\rho_t$. Profit maximisation on the part of firms ensures that, in the steady-state, $\rho_t$ will be equal to the marginal product of capital, so that

$$\rho_t = f'(\kappa_t),$$

(2.15)

where $f'(\kappa_t)$ is the derivative of $f(\kappa_t)$ with respect to $\kappa_t$. Since $\kappa_t \rightarrow \kappa_\infty$, it therefore follows that $\rho_t \rightarrow f'(\kappa_\infty)$; thus establishing that the steady state distribution of the real rate of return will also be ergodic and stationary. This result allows us to write

$$1 + \rho_{t+1} = (1 + \rho) \exp(\eta_{\rho,t+1}),$$

(2.16)

where $\eta_{\rho,t+1}$ is a stationary process normalized so that $E(\exp(\eta_{\rho,t+1}) \mid \Omega_t) = 1$, and where $\Omega_t$ is the publicly available information set at time $t$. This normalization ensures that $\rho$ is in fact the mean of the steady state distribution of real returns, $\rho_t$, given by $E[f'(\kappa_\infty)]$.

### 2.3 Arbitrage conditions

The first set of arbitrage conditions to be considered here are included in many macroeconomic models in one form or another. They are the (relative) Purchasing Power Parity (PPP), the Fisher Inflation Parity (FIP), and the Uncovered Interest Parity (UIP) relationships. We consider each of these in turn.

Purchasing Power Parity is based on the presence of goods market arbitrage, and captures the idea that the price of a common basket of goods will be equal in different countries.

[6]
when measured in a common currency. Information disparities, transportation costs or the effects of tariff and non-tariff barriers are likely to create considerable deviations from (absolute) PPP in the short-run and, with the likely exception of information disparities, these might persist indefinitely. However, if the size of these influences has a constant mean over time, then the common currency price of the basket of goods in the different countries will rise one-for-one over the longer term, and this is captured by the (weaker) concept of ‘relative PPP’. The primary explanation of long-run deviations from relative PPP is the ‘Harrod-Balassa-Samuelson (H-B-S) effect’ in which the price of a basket of traded and non-traded goods rises more rapidly in countries with relatively rapid productivity growth in the traded goods sector.\(^8\) Deviations from PPP might also be observed because real exchange rates are measured using price indices which involve different baskets of commodities across countries. In this case, real shocks which cause changes in the relative price of particular commodities will have differential impact on countries’ prices, and deviations from PPP remains consistent with goods market arbitrage.

Following these arguments, we express relative PPP as

\[
P_{t+1} = E_{t+1}P^*_t \left\{ \left(\frac{P^*_t}{P^*_t} \right) \theta \exp(\eta_{p_{pp},t+1}) \right\},
\]

where \(P^*_t\) is the foreign price index, \(P^*_t\) is the oil price index, and the term in brackets captures the deviations from PPP. Here, \(\eta_{p_{pp},t+1}\) is assumed to follow a stationary (or possibly trend-stationary) process capturing short-run variations in transport costs, information disparities, and the effects of tariff and non-tariff barriers. The errors \(\eta_{p_{pp},t+1}\) could be conditionally heteroscedastic, although this is unlikely to be very important in quarterly macro-models. The effects of differential productivity growth rates in the traded and non-traded goods sectors at home and abroad, accommodating the H-B-S effect, can be captured by assuming that \(\eta_{p_{pp},t+1}\) contains a trend. Since the large changes in oil prices occurred after UK oil reserves had been discovered, the UK can be considered an oil-producer, and the (potential) direct effect of changes in the relative price of oil on the UK’s real exchange rate arising from this fact is acknowledged by including the relative oil price variable, \(P^*_t/P^*_t\) in (2.17).\(^9\) Of course, one might doubt that changes in relative oil prices would have a permanent effect on real exchange rates over long horizons, in which case \(\theta = 0\). However, even in this case, the relative oil price variable could still affect real exchange rates over prolonged periods, given the size of the oil price changes in recent years, because of differential speeds of adjustment to the productivity shock in different economies.\(^10\)

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\(^8\)See Obstfeld and Rogoff (1996, Ch.4) and Rogoff (1996) for further discussion of this effect and alternative modifications to PPP.

\(^9\)This approach is advocated in Chauduri and Daniel (1998), for example. The inclusion of the relative oil price term in (2.17) can also be justified with reference to the H-B-S effect. Certainly, (relative) oil price changes have a pervasive effect on productivity, and these might have a differential effect in the traded and non-traded sectors of different economies. (See Bruno and Sachs (1984) or Perron (1989), among others, for discussion of the role of the 1973 oil price shock in the worldwide slowdown in productivity).

\(^10\)Distinguishing whether these effects are permanent or transitory is likely to be difficult using available datasets. However, the importance of explicitly taking into account the effects of oil price changes on the dynamics of real exchange rates has been widely acknowledged in applied work; see, for example, Johansen and Juselius (1992).
The FIP relationship captures the equilibrium outcome of the arbitrage process between holding bonds and investing in physical assets. Denoting the real rate of return on physical assets over the period $t$ to $t + 1$ by $\rho_{t+1}$, and denoting inflation expectations over the same period by $(P_{t+1}^{e} - P_{t})/P_{t}$, we have

$$
(1 + R_{t}) = (1 + \rho_{t+1}) \left(1 + \frac{P_{t+1}^{e} - P_{t}}{P_{t}}\right) \exp(\eta_{fip,t+1}) \\
= (1 + \rho_{t+1}) \left(1 + \frac{\Delta P_{t+1}}{P_{t}}\right) \exp(\eta_{fip,t+1}),
$$

where $\eta_{fip,t+1}$ is the risk premium, capturing the effects of money and goods market uncertainties on risk-averse agents. We assume that $\eta_{fip,t+1}$ follows a stationary process with a finite mean and variance. Also recall that in the context of the neoclassical growth model the real rate of interest (which we take to be the same as the real rate of return on capital) follows a stationary process; see (2.15) and (2.16).

The third arbitrage condition is based on the UIP relationship, which captures the equilibrium outcome of the arbitrage process between holding domestic and foreign bonds. In this, any differential in interest rates across countries must be offset by expected exchange rate changes to eliminate the scope for arbitrage. The presence of transactions costs, risk premia and speculative effects provide for the possibility of short-run deviations from UIP, and we therefore define the Interest Rate Parity relationship (IRP) as follows:

$$
(1 + R_{t}) = (1 + R_{t}^{*}) \left(1 + \frac{E_{t+1}^{c} - E_{t}}{E_{t}}\right) \exp(\eta_{kip,t+1}) \\
= (1 + R_{t}^{*}) \left(1 + \frac{\Delta E_{t+1}}{E_{t}}\right) \exp(\eta_{kip,t+1}),
$$

where $\eta_{kip,t+1}$ is the risk premium associated with the effects of bonds and foreign exchange uncertainties on risk-averse agents. As before, we shall assume that $\eta_{kip,t+1}$ is stationary and ergodic.\(^{11}\)

In the absence of direct observations on inflation and exchange rate expectations and for the purpose of long-run modelling, we write

$$
P_{t+1}^{e} = P_{t+1} \exp(\eta_{fip,t+1}^{e}), \text{ and } E_{t+1}^{c} = E_{t+1} \exp(\eta_{kip,t+1}^{c})
$$

and assume that the expectations errors $\eta_{h_{t+1}, i = p, e}$ follow stationary processes. The assumption that the errors $\eta_{fip,t+1}^{e}$ and $\eta_{kip,t+1}^{c}$ are stationary seems quite plausible and is consistent with a wide variety of hypotheses concerning the expectations formation process.\(^{12}\)

\(^{11}\)As noted above, the relationships in (2.18) and (2.19) can also be derived from Euler equations obtained from consumer and producer optimisation in an intertemporal model of an economy with well behaved preferences and technologies.

\(^{12}\)This assumption is consistent with the Rational Expectations Hypothesis (REH), for example. However, it is much less restrictive than the REH, and can accommodate the possibility of systematic expectational errors in the short-run, possibly due to incomplete learning.
Using the relations in (2.20) and (2.16), the FIP and the IRP relations in terms of the observables can be written as:

\[
\ln(1 + R_t) = \ln(1 + \rho) + \ln \left(1 + \frac{\Delta P_{t+1}}{P_t} \right) + \eta_{fip,t+1} + \eta_{p,t} + \eta_{i,t+1}, \quad (2.21)
\]

and

\[
\ln(1 + R_t) = \ln(1 + R_t^*) + \eta_{\Delta e,t+1} + \eta_{airp,t+1} + \eta_{c,t+1}, \quad (2.22)
\]

where \(\eta_{\Delta e,t+1} = \Delta \ln(E_{t+1})\). The log-linear version of the PPP relationship in (2.17) is also given by

\[
\ln(P_{t+1}) = \ln(P_{t+1}^*) + \ln(E_{t+1}) + \theta \ln \left(\frac{P_{t+1}}{P_{t+1}^*} \right) + \eta_{ppp,t+1}. \quad (2.23)
\]

### 2.4 Long-run solvency requirements

In addition to the arbitrage conditions, the economy is also subject to the long-run solvency constraint obtainable from the stock-flow relationships given by (2.6)-(2.8). In order to ensure the long-run solvency of the private sector asset/liability position, we assume

\[
\tilde{L}_{t+1}/\tilde{Y}_t = \mu \exp(\eta_{y,t+1}), \quad (2.24)
\]

where \(\eta_{y,t+1}\) is a stationary process, so that the ratio of total financial assets to the nominal income level is stationary and ergodic. Expression (2.24) captures the idea that domestic residents are neither willing nor able to accumulate claims on, or liabilities to, the government and the rest of the world which are out of line with its current and expected future income. This condition, in conjunction with assumptions on the determinants of the equilibrium portfolio balance of the private sector assets and on import and export determination, provide two further long-run relations which we shall include in our core model of the macroeconomy.

In modelling the equilibrium portfolio balance of private sector assets, we follow Branson’s (1977) Portfolio Balance Approach. From (2.3), we note that the stock of financial assets held by the private sector consists of the stock of high-powered money plus the stock of domestic and foreign bonds held by domestic residents. Given this adding-up constraint, we specify two independent equilibrium relationships relating to asset demand; namely, those relating to the demand for high-powered money and for foreign assets. These relationships are characterised in our model by the following:

\[
\frac{\tilde{H}_{t+1}}{L_t} = F_h \left( \frac{Y_t}{P_t}, \rho_{b,t+1}, \rho_{b,t+1}^*, \frac{\Delta P_{t+1}}{P_t}, t \right) \exp(\eta_{h,t+1}), \quad F_{h1} \geq 0, F_{h2} \leq 0, F_{h3} \leq 0, F_{h4} \leq 0, \quad (2.25)
\]

and

\[
\frac{\tilde{F}_{t+1}}{L_t} = F_f \left( \frac{Y_t}{P_t}, \rho_{b,t+1}, \rho_{b,t+1}^*, \frac{\Delta P_{t+1}}{P_t}, t \right) \exp(\eta_{f,t+1}), \quad F_{f1} \leq 0, F_{f2} \leq 0, F_{f3} \geq 0, F_{f4} \geq 0, \quad (2.26)
\]
where

\[ \rho_{h,t+1} = \frac{(1 + R_t)}{\left( 1 + \frac{F_{h,t+1} - F_{t}}{F_{t}} \right)} - 1, \quad \text{and} \quad \rho_{h,t+1}^* = \frac{(1 + R_t^*)}{\left( 1 + \frac{E_{h,t+1} - E_t}{E_t} \right)} - 1 \]  \hspace{1cm} (2.27)

are respectively the expected real rates of return on domestic and foreign bonds (both measured in domestic currency), \( \eta_{h,t} \) is a stationary process which captures the effects of various factors that contribute to the short-run deviations of the ratio of money balances to total financial assets from its long-run determinants, and where \( \eta_{f,t} \) is the corresponding stationary process capturing the effects of short-run deviations of the ratio of foreign assets to total financial assets from its long-run position. The determinants of the ratio of money to total financial assets in (2.25) include the real output level, to capture the influence of the transactions demand for money, and the expected real rates of return on the three alternative forms of holding financial assets; namely domestic bonds, foreign bonds and high-powered money. We have also specified a deterministic trend in \( F_h(\cdot) \) to allow for the possible effect of the changing nature of financial intermediation, and the increasing use of credit cards in settlement of transactions on the convenience value of money. One would expect a downward trend in \( H/L \), reflecting a trend reduction in the proportion of financial assets held in the form of non-interest bearing high-powered money over time. The determinants of the ratio of foreign assets to total financial assets in (2.26) are the same, with the decision to hold assets in the form of bonds mirroring that relating to holding assets in the form of money.

In view of the IRP relationship of (2.19), it is clear that, in the steady state, domestic and foreign bonds become perfect substitutes, and their expected rates of return are equal. Similarly, given the FIP relationship of (2.18) the real rates of return on (both) domestic and foreign bonds are equal to the (stationary) real rate of return on physical assets in the steady state. Hence, the asset demand relationships of (2.25) and (2.26) can be written equally as:

\[ \frac{\bar{H}_{t+1}}{L_t} = F_{h1} \left( \frac{Y_t}{P_t}, R_t, t \right) \exp(\eta_{h,t+1}), \quad F_{h1} \geq 0, \quad F_{h2} \leq 0, \]  \hspace{1cm} (2.28)

and

\[ \frac{\bar{F}_{t+1}}{L_t} = F_{f1} \left( \frac{Y_t}{P_t}, R_t, t \right) \exp(\eta_{f,t+1}), \quad F_{f1} \leq 0, \quad F_{f2} \geq 0, \]  \hspace{1cm} (2.29)

where the effects of the short-run deviations from IRP and FIP are now subsumed into the more general stationary processes \( \eta_{h,t+1} \) and \( \eta_{f,t+1} \) and where the effects of the expected real rate of return on non-interest bearing money holdings (i.e. minus the expected inflation rate) are captured by the domestic nominal interest rate (again making use of (2.18)). Note that this final effect implies that different rates of inflation, and hence different levels of nominal interest rates, could change the equilibrium portfolio composition, depending on the responsiveness of the asset demands to the relative returns on the three assets, so that changes in nominal rates of interest can potentially have lasting real effects.\(^{13}\)

\(^{13}\)The possibility of the “super-non-neutrality” of monetary policy arising through this route is discussed in Buitner (1980), for example.
2.4.1 Imports and exports

The conditions (2.24) and (2.29), when taken with assumptions on import and export determination, provide an equilibrium condition between the real exchange rate, domestic and foreign outputs and the interest rate. To see this, note first that (2.24) and (2.29) yield the following expression for the ratio of net foreign assets (measured in domestic currency) to the nominal output level:

\[ \frac{\tilde{F}_{t+1}}{\tilde{Y}_t} = \mu F_t \left( \frac{Y_t}{P_t}, R_t, t \right) \exp(\eta_{Mt,t+1} + \eta_{b,t+1}). \]  

(2.30)

Next, we note that the stock-flow relationship (2.8) can be used in conjunction with the definition of the country’s net foreign asset position in (2.2) to write

\[ \tilde{F}_{t+1} = \tilde{X}_t - \tilde{M}_t + \tilde{F}_t + E_t \tilde{R}_t^e \tilde{B}_t^e - \tilde{R}_t (\tilde{B}_t - \tilde{B}_t^e) + (E_t^e - E_t) \tilde{B}_t^e \]

\[ = \tilde{X}_t - \tilde{M}_t + (1 + R_t) \tilde{F}_t - E_t \tilde{B}_t^e \left[ R_t - R_t^e - \frac{\Delta E_{t+1}^e}{E_t} \right]. \]

Dividing through by nominal income, and writing the various ratios in per capita terms, we obtain

\[ \frac{F_{t+1}}{Y_t} = \frac{X_t - M_t}{Y_t} + (1 + R_t) \left( \frac{F_t}{Y_{t-1}} \right) \left( \frac{Y_{t-1}}{Y_t} \right) + \frac{E_t B_t^e}{Y_t} \left[ R_t - R_t^e - \frac{\Delta E_{t+1}^e}{E_t} \right], \]

(2.31)

where \( Y_t = \tilde{Y}_t / POP_t \), \( X_t = \tilde{X}_t / POP_t \), \( F_{t+1} = \tilde{F}_{t+1} / POP_t \), ..., and \( POP_t \) is population. Let \( g_t \) denote the growth of per capita output and note that \( Y_t / Y_{t-1} = (1 + g_t)(1 + \Delta P_t / P_{t-1}) \).

Hence,

\[ \frac{F_{t+1}}{Y_t} = \frac{X_t - M_t}{Y_t} + \frac{(1 + R_t)}{(1 + g_t)(1 + \Delta P_t / P_{t-1})} \left( \frac{F_t}{Y_{t-1}} \right) + \frac{E_t B_t^e}{Y_t} \left[ R_t - R_t^e - \frac{\Delta E_{t+1}^e}{E_t} \right]. \]

(2.32)

Now, under our assumptions, \((1 + g_t) \) and \((1 + R_t)/(1 + \Delta P_t / P_{t-1}) \) both tend to stationary processes with constant means, \(1 + g \) and \(1 + \rho \), respectively, and the term in square brackets, \( R_t - R_t^e - [\Delta E_{t+1}^e / E_t] \), either becomes negligible or itself tends to a stationary process. Recalling from (2.30) that the value of \( \tilde{F}_{t+1} / \tilde{Y}_t \) depends on \( Y_t / P_t, R_t \) and \( t \), the solvency condition and the relationships describing the determinants of the ratio of foreign to total financial assets provides, through (2.32), a long-run relationship between \((X_t - M_t) / Y_t \), the ratio of the nominal trade balance to real income, \( Y_t / P_t \), and the interest rate, \( R_t \). We represent this result by the following:

\[(X_t - M_t) / Y_t = F_b \left( Y_t / P_t, R_t, t \right) \exp(\eta_{bt+1}), \quad F_{b1} \leq 0, \quad F_{b2} \geq 0, \]  

(2.33)

where \( \eta_{bt+1} \) is a stationary process. To complete our derivations we further assume that real per capita imports \((M_t / P_t) \) and exports \((X_t / P_t) \) are determined according to the following relations:

\[ \frac{X_t}{P_t} = F_x \left( \frac{Y_t}{P_t}, \frac{E_t P_t^*}{P_t}, \frac{P_t^*}{P_t} \right) \exp(\eta_{xt}), \quad F_{x1} > 0, \quad F_{x2} > 0, \quad F_{x3} \leq 0 \]  

(2.34)

\[ \frac{M_t}{E_t P_t^*} = F_m \left( \frac{Y_t}{P_t}, \frac{E_t P_t^*}{P_t}, \frac{P_t^*}{P_t} \right) \exp(\eta_{mt}), \quad F_{m1} > 0, \quad F_{m2} < 0, \quad F_{m3} \leq 0. \]
where $\eta_{lt}$ and $\eta_{mt}$ are stationary processes with zero means. In the long-run, real per capita exports are assumed to depend on real activity levels abroad, $Y_t^*/P_t^*$, and on the relative price of goods abroad compared to those at home, while real per capita imports depend on domestic real per capita output and relative prices. The relative price of oil could also exert an influence on import and export demand independent of the effect of oil prices on domestic and foreign price levels, following the same line of reasoning discussed above in relation to the PPP relationship. The stationary processes $\eta_{lt}$ and $\eta_{mt}$ characterize the short-run departure of exports and imports from their long-term determinants. Using (2.33) and (2.34), we obtain

$$F_x \left( \frac{Y_t^*}{P_t^*}, \frac{E_t}{P_t}, \frac{P_t^*}{P_t^*} \right) \exp(\eta_{lt}) - \frac{E_t}{P_t} F_m \left( \frac{Y_t}{P_t}, \frac{E_t}{P_t}, \frac{P_t^*}{P_t} \right) \exp(\eta_{mt})$$

$$= \frac{Y_t}{P_t^*} F_b \left( \frac{Y_t}{P_t^*}, R_t, t \right) \exp(\eta_{lt+1}).$$

(2.35)

This provides a complicated, non-linear relationship relating the real exchange rate to the domestic and foreign outputs, the interest rate, relative oil prices and the various stationary terms depicting disequilibria in the remainder of the economy.

### 2.4.2 Liquidity (Real Money Balances)

The long-run relationship capturing the joint effect of the solvency conditions and the determinants of the ratio of liquid assets as a ratio of total financial assets is the mirror image of the foreign asset/output condition derived above. Combining (2.28) and (2.24) we obtain

$$\frac{\tilde{H}_{t+1}}{Y_t} = \frac{H_{t+1}}{Y_t} = \mu F_h (Y_t, R_t, t) \exp(\eta_y + \eta_d t + 1),$$

(2.36)

where $H_t = \tilde{H}_t / POP_{t-1}$. This provides the final long-run relationship to be considered in our core model of the UK macroeconomy.

### 2.5 Log-linear approximation of the core model

The translation of the steady state relationships described above into a form that can be examined empirically within the context of a VAR model requires a log-linear approximation. In the above discussion, we have considered six long-run equilibrium relationships, as described in (2.23), (2.22), (2.14), (2.35), (2.36) and (2.21), in the following 9 variables: $p_t = \ln(P_t), p_t^* = \ln(P_t^*), p_t^* = \ln(P_t^*), e_t = \ln(E_t), y_t = \ln(Y_t / P_t), y_t^* = \ln(Y_t^* / P_t^*), r_t = \ln(1 + R_t), r_t^* = \ln(1 + R_t^*),$ and $h_t - y_t = \ln(H_{t+1} / P_t) - \ln(Y_t / P_t) = \ln(H_{t+1} / Y_t)$. The six log-linearised long-run relations of the core model are given by

$$p_t - p_t^* - e_t = a_{10} + a_{11} t + \beta_1 (p_t^* - p_t^*) + \epsilon_{1, t+1},$$

(2.37)

$$r_t - r_t^* = a_{20} + \epsilon_{2, t+1},$$

(2.38)

\[14\text{ For expositional simplicity, we have chosen to denote } \ln(H_{t+1} / P_t) \text{ by } h_t, \text{ rather than } h_{t+1}. \text{ Recall that } H_{t+1} \text{ relates to the stock of high powered money at the beginning of period } t + 1.\]
\[ y_t - y_t^* = a_{30} + \epsilon_{3,t+1}, \]  
\[ p_t - p_t^* - \varepsilon_t = a_{40} + a_{41} t + \beta_{33} r_t + \beta_{43} y_t + \beta_{48} (p_t^* - p_t^*) + \epsilon_{4,t+1}, \]  
\[ h_t - y_t = a_{50} + a_{51} t + \beta_{33} r_t + \beta_{53} y_t + \epsilon_{5,t+1}, \]  
\[ r_t - \Delta p_{t+1} = a_{60} + \epsilon_{6,t+1}, \]  

where \( a_{30} = \ln(1 + \rho), \) and \( \beta_{48} = -\theta. \) We have allowed for intercept and trend terms (when appropriate) in order to ensure that \( \epsilon_{i,t+1}, i = 1, 2, \ldots, 6, \) have zero means.\(^{15}\)

For the long-run relations (2.37)-(2.39), the (long-run) reduced-form disturbances, \( \epsilon_{i,t+1}, \) are related to the (long-run) structural disturbances, the \( \eta_t^* \)'s, in the following manner:

\[ \epsilon_{1,t+1} = \eta_{pp,t} - a_{10} - a_{11} t, \]  
\[ \epsilon_{2,t+1} = \eta_{hp,t+1} + \eta_{c,t+1} + \eta_{\Delta c,t+1} - a_{20}, \]  
\[ \epsilon_{3,t+1} = \eta_{dt} + (\eta_{dt} - \eta_{dt}^*) + (\eta_{dt} - \eta_{dt}^*), \]

but there is no simple relationship between \( \epsilon_{4,t+1} \) and the other structural disturbances, \( \eta_{ct}, \eta_{ht}, \) and \( \eta_{bt,t+1} \) in (2.40).\(^{16}\) For the long-run relationships of (2.41) and (2.42), we have

\[ \epsilon_{5,t+1} = \eta_{ht} + \eta_{dt}, \]  
\[ \epsilon_{6,t+1} = \eta_{tp,t+1} + \eta_{ct,t+1} + \eta_{pt,t+1}. \]

The above relationships between the long-run structural disturbances, \( \eta_t^* \)'s, and the long-run reduced form disturbances, \( \epsilon_t^* \)'s, clearly show the difficulties involved in identifying the effects of changes in particular structural disturbances on the dynamic behaviour of the macroeconomy. For example, \( \epsilon_{6,t+1} \) is composed of the three structural disturbances, \( \eta_{tp,t+1}, \eta_{ct,t+1}, \eta_{pt,t+1}, \) representing the different factors that could be responsible for disequilibria between inflation and interest rates. In general, without further \textit{a priori} restrictions, the effect of \( \eta_t^* \)'s cannot be identified: firstly, there are many more long-run structural disturbances than there are long-run reduced form disturbances; and, secondly, there is no reason to believe that the \( \eta_t^* \)'s are not themselves contemporaneously correlated. Empirical analysis at best enables us to identify the effect of changes in the long-run reduced form disturbances on the evolution of the macroeconomy towards its long-run equilibrium, although, as we discuss below, even identification of the effects of specific changes in these long-run reduced form disturbances will typically require further identifying restrictions based on an explicit model of short-run decision-making. Given the difficulties associated with a fully structural impulse response analysis, in this paper we focus on the persistence profiles of the effects of system-wide shocks, defined as innovations in linear functions of the reduced-form or structural disturbances. For a further discussion see Sections 4.2 and 4.3 below.

\(^{15}\)From (2.35), economic theory suggests that both \( y_t \) and \( y_t^* \) enter the trade balance relationship given by (2.40). However, given the cointegrating relationship in (2.39) it is clear that the single variable \( y_t \) is sufficient to capture the separate effects of \( y_t \) and \( y_t^* \) in (2.40), in which case the coefficient \( \beta_{45} \) will be a function of the income elasticity of demand for imports and exports. The substitution of \( y_t^* \) in (2.40) means that \( \epsilon_{4,t+1} \) is a function of the structural disturbances which comprise \( \epsilon_{3,t+1} \) as well as \( \eta_{ct}, \eta_{ht}, \) and \( \eta_{bt,t+1}. \)

\(^{16}\)In the case of \( \epsilon_{2,t+1} \) we have taken account of the effect of the exchange rate depreciation on the interest rate differential since, as we shall see below, the hypothesis that \( \eta_{\Delta c,t+1} \) is stationary cannot be rejected.
3 Econometric Formulation of the Core Model

In order to embody the long-run relations (2.37) to (2.42) within a suitable macroeconometric model, it is important that the orders of integration of the core variables in the theoretical model are ascertained. More specifically, given the econometric methods we employ, we wish to ensure that the variables used in the empirical analysis can reasonably be argued to be $I(1)$ variables. The variables under consideration are $y_t$, $y_t^s$, $r_t$, $r_t^s$, $e_t$, $h_t - y_t$, $p_t$, $p_t^s$, and $p_t^f$. A detailed description of these variables is given in Table 1. The data is quarterly, seasonally adjusted covering the period 1963q1-1995q4, although to ensure that all regressions are comparable (irrespective of the order chosen for the underlying dynamic model), all estimations are carried out over the period 1965q1-1995q4 (124 observations). The Augmented Dickey-Fuller (ADF) test statistics, computed over the sample period 1965q1-1995q4, for the levels and first differences of the core variables are reported in Table 2. The results suggest that it is reasonable to treat $e_t$, $r_t$, $r_t^s$, $y_t$, $y_t^s$, and $h_t - y_t$ as $I(1)$ variables. For these variables the unit root hypotheses is rejected when applied to their first differences, but provide no evidence with which to reject the unit root hypothesis when the tests are applied to their levels. We obtained similar results using the Phillips and Perron (1988) tests.17

There is, however, some ambiguity regarding the order of integration of the price variables. Application of the $ADF(s)$ tests to $\Delta p_t$ and $\Delta p_t^s$ yields mixed results: the hypothesis that there is a unit root in the domestic and foreign inflation rates is rejected for low orders of augmentation (namely, for $s = 0$ and 1), but not for higher orders. The application of the Phillips and Perron test rejects the unit root hypothesis when applied to $\Delta p_t^s$, but not when applied to $\Delta p_t$. Overall the available data is not informative as to whether domestic and foreign prices are $I(1)$ or $I(2)$. However, if we consider the two price variables relative to the oil price series, i.e. $p_t - p_t^o$ and $p_t^s - p_t^o$ respectively, then the unit root hypothesis in the first differences of these variables is clearly rejected, whilst at the same time the unit root hypothesis in the levels of these variables cannot be rejected. So we can conclude that these transformed variables are unambiguously $I(1)$. This provides a strong argument for the use of these relative price variables in the econometric formulation of the core model. As discussed in Haldrup’s (1998) review of the econometric analysis of $I(2)$ variables, the (inappropriate) treatment of $I(2)$ variables as $I(1)$ variables in cointegrating VAR models involving mixtures of $I(1)$ and $I(2)$ variables can have pervasive effects on the properties of estimators, so that the use of the untransformed variables in our econometric work is potentially problematic. Given the widespread ambiguity on the order of integration of many variables, and given that the methods appropriate for the analysis of systems involving mixtures of $I(1)$ and $I(2)$ variables are still to be fully developed, Haldrup suggests that it is often useful to transform time series a priori to obtain variables that are unambiguously $I(1)$ rather than dealing with mixtures of $I(1)$ and $I(2)$ variables directly. In the case of the core variables under consideration, this is achieved by working with the relative price variables $p_t - p_t^o$ and $p_t^s - p_t^o$ rather than the three price levels $p_t$, $p_t^s$ and $p_t^o$ separately. There is, however, an important downside to this strategy: although the FIP relationship is part of the model suggested by economic theory in the previous section, appearing both as a relationship in its own right in (2.42) and as an element in the derivation of a number of the other long-run relationships.

17The results for the Phillips and Perron unit root tests are available from the authors on request.

[14]
given in (2.37)-(2.41), its inclusion in the core econometric model would run the risk of mixing \( I(1) \) and \( I(2) \) variables in the same econometric model which, for technical reasons, we cannot handle at the present time. In what follows, we therefore choose not to include the FIP relationship in our empirical analysis.

Thus our core macroeconomic model comprises the following five long-run equilibrium relationships in eight \( I(1) \) variables, as described in (2.37), (2.38), (2.39), (2.40) and (2.41).

\[
(p_t - p^*_t) - (p_t^* - p^*_t) - \epsilon_t = a_{10} + a_{11}t + \beta_{18}(p_t^* - p^*_t) + \epsilon_{1,t+1}, \tag{3.46}
\]
\[
r_t - r_t^* = a_{20} + \epsilon_{2,t+1}, \tag{3.47}
\]
\[
y_t - y_t^* = a_{30} + \epsilon_{3,t+1}, \tag{3.48}
\]
\[
(p_t - p^*_t) - (p_t^* - p^*_t) - \epsilon_t = a_{40} + a_{41}t + \beta_{43}r_t + \beta_{45}y_t + \beta_{48}(p_t^* - p^*_t) + \epsilon_{4,t+1}, \tag{3.49}
\]
\[
h_t - y_t = a_{50} + a_{51}t + \beta_{53}r_t + \beta_{55}y_t + \epsilon_{5,t+1}. \tag{3.50}
\]

The five long-run relations of the core model, (3.46) - (3.50), can be written more compactly as

\[
\epsilon_t = \beta' z_{t-1} - (a_0 - a_1) - a_1 t, \tag{3.51}
\]

where

\[
z_t = (p_t - p^*_t, \epsilon_t, r_t, r_t^*, y_t, y_t^*, h_t - y_t, p_t^* - p^*_t)', a_0 = (a_{10}, a_{20}, a_{30}, a_{40}, a_{50})', a_1 = (a_{11}, 0, a_{41}, a_{51}), c_t = (\epsilon_{11}, \epsilon_{21}, \epsilon_{31}, \epsilon_{41}, \epsilon_{51})',
\]

and

\[
\beta = \begin{pmatrix}
1 & -1 & 0 & 0 & 0 & 0 & 0 & -(1 + \beta_{18}) \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
1 & -1 & -\beta_{43} & 0 & -\beta_{45} & 0 & 0 & -(1 + \beta_{48}) \\
0 & 0 & -\beta_{53} & 0 & -\beta_{55} & 0 & 1 & 0
\end{pmatrix}. \tag{3.52}
\]

In modelling the short-run dynamics, we follow Sims (1980) and others and assume that departures from the long-run relations, \( \epsilon_t \), can be approximated by a linear function of a finite number of past changes in \( z_{t-1} \). In doing this, we partition \( z_t = (y_t', p_t^* - p^*_t)' \) where \( y_t = (p_t - p^*_t, \epsilon_t, r_t, r_t^*, y_t, y_t^*, h_t - y_t)' \). Here, \( p_t^* - p^*_t \) is considered to be a ‘long-run forcing’ variable for the determination of \( y_t \), in the sense that changes in \( p_t^* - p^*_t \) have a direct influence on \( y_t \), but changes in \( p_t^* - p^*_t \) are not affected by the presence of \( \epsilon_t \), which measure the extent of disequilibria in the UK economy. The concept of long-run forcing is somewhat weaker than the notion of “Granger causality”, and allows for possible feedback effects of past changes in \( y_t \) into \( p_t^* - p^*_t \). (See Pesaran, Shin and Smith (1999) for more details.\textsuperscript{18})

\textsuperscript{18}The treatment of foreign prices and foreign oil prices as long-run forcing represents a generalisation of the treatment of the variable in some previous applications of cointegrating VAR analyses (e.g. Johansen and Juselius (1992), or Pesaran and Shin (1996)), where the oil price change is treated as a strictly exogenous \( f(0) \) variable. The approach taken in the previous

\[\text{[15]}\]
literature excludes the possibility that there exist cointegrating relationships which involve the oil price level, while the approach taken here allows the validity of the hypothesised restriction to be tested, and for the restriction to be imposed if it is not rejected. Our decision to treat foreign output as endogenous follows from the arguments advanced in the previous section in which both domestic and foreign output are driven by a common stochastic trend measuring technological progress. Given the simultaneity of the determination of $y_t$ and $y_t^*$, and the possibility of feedbacks, it is appropriate to model the two output variables symmetrically. Similar arguments also hold for domestic and foreign interest rates. The endogenous treatment of foreign output and interest rates involves loss of efficiency in estimation if they were in fact long-run forcing or strictly exogenous, but we consider this to be less serious than following the alternative route of treating these variables as exogenous if this turned out to be false.

Following our modelling strategy, we embody $\varepsilon_t$ in an otherwise unrestricted VAR($s-1$) in $\Delta y_t$:

$$\Delta y_t = b_0 - \alpha \varepsilon_t + \sum_{i=1}^{s-1} \Gamma_i \Delta z_{t-i} + \psi \Delta (p_t^s - p_t^e) + u_t,$$

where $b_0$ and $u_t$ are $7 \times 1$ vectors of fixed intercepts and serially uncorrelated shocks, respectively, $\alpha$ is a $7 \times 5$ matrix of error-correction coefficients, $\{\Gamma_i, i = 1, 2, \ldots, s-1\}$ are $7 \times 8$ matrices of short-run coefficients, and $\psi$ is a $7 \times 1$ vector representing the impact effects of changes in relative oil prices on $\Delta y_t$. Using equation (3.51), we have

$$\Delta y_t = c_0 + \alpha a_1 t - \alpha \xi_t + \sum_{i=1}^{s-1} \Gamma_i \Delta z_{t-i} + \psi \Delta (p_t^s - p_t^e) + u_t,$$

where $c_0 = b_0 + \alpha (a_0 - a_1)$, is a $7 \times 1$ vector unrestricted intercepts, and $\xi_t = \beta z_{t-1}$ are the error correction terms. The above specification embodies the economic theory’s long-run predictions by construction, in contrast to the more usual approach where the starting point is an unrestricted VAR model, with some vague priors about the nature of the long-run relations.

Estimation of the parameters of the core model, (3.54), can be carried out using the long-run structural modelling approach described in Pesaran and Shin (1999) and Pesaran, Shin and Smith (1999). It is based on a modified and generalised version of Johansen’s (1991,1995) maximum likelihood approach to the problem of estimation and hypothesis testing in the context of vector autoregressive error correction models. With this approach, having selected the order of the underlying VAR model (using model selection criteria such as the Akaike Information Criterion (AIC) or the Schwartz Bayesian Criterion (SBC)), we test for the number of cointegrating relations among the 8 variables in $z_t$. When performing this task, and in all the subsequent empirical analysis, we work in the context of a VAR model with unrestricted intercepts and restricted trend coefficients. In terms of (3.54), we allow the intercepts $c_0$ to be freely determined but restrict the trend coefficients so that $\alpha a_1 = \Pi \gamma$, where $\Pi = \alpha \beta$ and $\gamma$ is an $8 \times 1$ vector of unknown coefficients. These restrictions ensure that the solution of the model in levels of $z_t$ will not contain quadratic

\[\text{[16]}\]

\[\text{[16]}\]
trends.\textsuperscript{20} We then compute Maximum Likelihood (ML) estimates of the model’s parameters subject to exact and over-identifying restrictions on the long-run coefficients. Assuming that there is empirical support for the existence of five long-run relationships, as suggested by theory, exact identification in our model requires five restrictions on each of the five cointegrating vectors (each row of $\beta$), or a total of twenty-five restrictions on $\beta$. These represent only a subset of the restrictions suggested by economic theory as characterized in (3.52), however. Estimation of the model subject to all the (exact- and over-identifying) restrictions given in (3.52) enables a test of the validity of the over-identifying restrictions, and hence the economic theory, to be carried out.

4 Estimation and Testing of The Core Model

In this section, we present the estimation results for the core model of the UK economy in the eight variables $p_t - p^*_t$, $e_t$, $r_t$, $r^*_t$, $y_t$, $y^*_t$, $h_t - y_t$, and $p^*_t - p^*_t$. All estimations are carried out over the period 1965q1-1995q4. The first stage of our modelling sequence is to select the order of the underlying VAR in these variables. Here we find that a VAR of order two appears to be appropriate when using the AIC as the model selection criterion, but that the SBC favours a VAR of order one. We proceed with the cointegration analysis using a VAR(2), on the grounds that the consequences of over-estimation of the order of the VAR is much less serious than under-estimating it (see Kilian (1997)).

Using a VAR(2) model with unrestricted intercepts and restricted trend coefficients, and treating the relative foreign price variable, $p_t^* - p_t$, as weakly exogenous for the long-run parameters (or ‘long-run forcing’), we computed the ‘trace’ and ‘maximal eigenvalue’ statistics.\textsuperscript{21} These statistics, together with their associated 95% and 90% critical values, are reported in Table 3. Two sets of critical values are provided. The asymptotic critical values from Pesaran, Smith and Shin (1999), denoted by ‘cv\textsubscript{as}’, and the bootstrapped values denoted by ‘cv\textsubscript{b}’.\textsuperscript{22} The latter are based on a VAR(2) model estimated over the period 1965q1-1995q4 in the first differences of the core variables using 10,000 replications.\textsuperscript{23} The bootstrapped critical values are likely to be more appropriate in small samples. As has been shown by

\textsuperscript{20}By an analogous argument, it should be noted that the analysis of a model in which oil price inflation is taken as exogenous has the undesirable property that the effects of oil prices on the model differs according to the number of cointegrating relations assumed to exist among the variables. The treatment of oil prices in this paper, which allows oil prices to enter into the cointegrating vectors, eliminates this possibility. This provides a further important argument in favour of our approach compared to the treatment of oil prices in some of the earlier papers discussed above.

\textsuperscript{21}The numerical algorithms for the computation of cointegration test statistics in the presence of $I(1)$ exogenous variables are provided in Pesaran, Shin and Smith (1999).

\textsuperscript{22}Recently, MacKinnon, Haug and Michelis (1999), have also produced asymptotic critical values for models with exogenous $I(1)$ variables that are based on more replications and are arguably more accurate. In the present application, the results are robust to which of these two sets of asymptotic critical values are used.

\textsuperscript{23}For each replication, a simulated dataset was obtained by first generating a set of 124 draws from a multivariate Normal distribution with the variance-covariance taken to be the same as that obtained using the residuals from the estimated VAR(2) model of the first-differences. These were used, in conjunction with the estimated VAR(2) model, to obtain a simulated dataset of the same length and with the same dynamic properties and contemporaneous error correlation structure as the original data.
Gonzalo (1994), Haug (1996) and Abadir et al. (1999), for example, asymptotic critical values may not be valid for VAR models with a relatively large number of variables, unless samples are sufficiently large. The bootstrapped critical values are computed as a first-order check on the validity of the tests based on asymptotic critical values. In the event, the same conclusion is drawn on the basis of the trace and the maximal eigenvalue statistics using either set of critical values. In all tests, we reject the null hypotheses that \( r = 0, 1, 2 \) and \( 3 \) at the 5 per cent level of significance, and reject \( r = 4 \) at the 10 per cent level of significance. We cannot reject the null of hypothesis that \( r = 5 \) using any set of critical values.\(^{24}\) These results are encouraging, as they provide empirical support for the long-run theory’s prediction; namely, that there should be five cointegrating relations among the 8 variables in \( z_t \).

In view of the empirical evidence supporting the hypothesis of \( r = 5 \), we estimated the model subject to the following twenty five exact-identifying restrictions on the cointegrating vectors:

\[
\beta' = \begin{pmatrix}
1 & \beta_{12} & 0 & 0 & 0 & \beta_{16} & 0 & -(1 + \beta_{18}) \\
\beta_{21} & 0 & 1 & \beta_{24} & 0 & 0 & \beta_{27} & 0 \\
0 & 0 & \beta_{33} & 0 & 1 & \beta_{36} & 0 & \beta_{38} \\
1 & -1 & -\beta_{43} & 0 & -\beta_{45} & 0 & 0 & -(1 + \beta_{48}) \\
0 & 0 & -\beta_{53} & 0 & -\beta_{55} & 0 & 1 & \beta_{58}
\end{pmatrix}, \tag{4.55}
\]

associated with \( z_t = (p_t - p_t', e_t, r_t, r_t^*, y_t, y_t^*, h_t - y_t, p_t^* - p_t')' \). The first vector (the first row of \( \beta' \)) relates to the (modified) \( PPP \) relationship defined by (3.46) and is normalised on \( p_t - p_t' \); the second relates to the \( IRP \) relationship defined by (3.47) and is normalised on \( r_t; \) the third relates to the “output gap” relationship defined by (3.48) and is normalised on \( y_t; \)^{25} the fourth relates to the trade balance relationship defined by (3.49) and is normalised on \( p_t - p_t' \); and the fifth is the real money balance relationship defined by (3.50), normalised on \( h_t - y_t \).

Having exactly identified the system, we then tested the over-identifying restrictions predicted by the long-run theory outlined in Section 2. There are fifteen unrestricted parameters in (4.55), and six in (3.52), yielding a total of nine over-identifying restrictions. In addition, working with a cointegrating VAR with restricted trend coefficients, there are potentially five further parameters on the trend terms in the five cointegrating relationships. The economic theory of Section 2 provided no justification for a time trend in the \( IRP \) or output gap relationship, and the imposition of zeros on the trend coefficients in these relationships provides a further two over-identifying restrictions. The absence of a trend in the \( PPP \) relationship is also consistent with the theory of Section 2, as is the restriction that \( \beta_{45} = 0 \) (i.e. there is no independent effect of relative oil prices on the trade balance relationship) and that \( \beta_{55} = 0 \) (so that equation (3.50) is effectively a relationship explaining the velocity of circulation of money). These final three restrictions, together with those which are intrinsic

\(^{24}\)Our preference is to use the trace statistic for inference given the Monte Carlo evidence presented in Cheung and Lai (1993). According to this evidence, in small samples the cointegration tests based on the trace statistic tend to be more robust to skewness and excess kurtosis in the innovations as compared to the tests based on the maximum eigenvalue statistic.

\(^{25}\)Our use of the term “output gap relationship” to describe (3.48) should not be confused with the more usual use of the term which relates more specifically to the difference between a country’s actual and potential output levels (although clearly the two uses of the term are related).
to the theory of Section 2, mean that there are just six parameters to be freely estimated in
the cointegrating relationships and provide a total of fourteen over-identifying restrictions
on which the core model is based and with which the validity of the economic theory can be
tested. The estimated core model is described in detail in Sections 4.1 and 4.2 below and the
log-run relationships are given in (4.56)-(4.60).

The log-likelihood ratio (LR) statistic for jointly testing the fourteen over-identifying
restrictions takes the value 39.78. The statistic is asymptotically distributed as a χ² variate
with 14 degrees of freedom, and the relevant critical values are 23.68 at 95 per cent and 21.06
at 90 per cent levels of significance. The asymptotic p-value is 0.0001 so that the 14 restric-
tions are rejected by the data at conventional significance levels. Taken literally, therefore,
the above results suggest that the economic theory motivating the restrictions is rejected by
the data. However, in view of the relatively large dimension of the underlying VAR model,
the number of restrictions considered and the available sample size, it is important that the
validity of the asymptotic critical values are evaluated by means of bootstrap techniques.
Although the bootstrapped small sample corrections proved unimportant as far as the co-
tegrating rank tests were concerned, the same need not apply to tests of the over-identifying
restrictions which concern us here.26

However, the use of standard bootstrap techniques in the case of the present application
is prohibitive since, for each replication of the bootstrap procedure, one needs to compute
the maximum likelihood estimators of the cointegrating relations subject to over-identifying
restrictions. This generally involves considerable convergence problems which we have not
been able to resolve in an automated manner, as required in a bootstrap exercise. However,
an indication of the size of the bias can be obtained through a bootstrap exercise which in-
volves the estimation of a related model using the OLS regression method. In this exercise,
the test relates to the validity of 20 over-identifying restrictions imposed on our core model.
The restrictions include the 14 based on economic theory plus 6 additional restrictions im-
posed on the parameters of the core model that are freely estimated, where it is assumed that
these take the parameter values of the estimated version of the core model shown in (4.56)-
(4.60) below.27 The exercise is based on 5000 bootstrapped replications of the LR statistic
testing the 20 restrictions. For each replication, a new sample of data is generated (of the
same length as the original data set) on the assumption that the estimated version of the core
model is the true data-generating process, using the observed initial values of each variable,
the estimated model, and a set of random innovations.28 The test of the over-identifying
restrictions is carried out on each of the replicated data sets and the empirical distribution
of the test statistic is derived across all replications. Figure 1 illustrates the distribution
of the simulated test statistic, obtained using the non-parametric bootstrap, alongside the

26 The extent of the small sample bias of chi-squared tests in (small) cointegrating VAR models is illustrated
by Greedenhoff and Jacobson (1998), for example, who suggest the use of bootstrap methods for this reason.
27 Clearly, there are no freely-estimated parameters in the cointegrating relationships when these restric-
tions are imposed. With the error correction terms assumed known, the cointegrating VAR model can be
estimated using OLS and a bootstrap exercise can be readily undertaken.
28 These innovations can be obtained as draws from a multivariate Normal distribution chosen to match the
observed correlation of the estimated reduced form errors (termed a ‘parametric bootstrap’) or by resampling,
with replication, from the estimated residuals themselves (a ‘non-parametric bootstrap’).
$\chi^2_{30}$ distribution.\footnote{In view of important departures from Normality in the estimated model’s residuals, the non-parametric bootstrap is more appropriate here. However, similar critical values are obtained with the parametric bootstrap.} This shows that the relevant critical values for the joint tests of the 20 over-identifying restrictions are 54.21 at the 10 per cent significance level and 59.26 at the 5 per cent level. This compares with 28.41 and 31.41, the corresponding critical values of the Chi-squared distribution with 20 degrees of freedom. Clearly, the bootstrapped critical values are very different from the asymptotic critical ones, and suggest that the appropriate small sample critical value for the original 14 theory-based restrictions will be very much higher than the asymptotic values. Seen in this light, the LR statistic of 39.78 may not be large enough to reject the over-identifying restrictions implied by (and consistent with) the theory of Section 2.

### 4.1 The Long-Run of the Core Model

The estimated long-run relations, which incorporate all the restrictions suggested by the theory in Section 2 are summarised below:

\[
(p_t - p_t^*) - (p_t^* - p_t^\circ) - e_t = 4.6386 - 0.0856 (p_t^* - p_t^\circ) + \hat{\epsilon}_{1,t+1} \tag{4.56}
\]

\[
r_t - r_t^* = 0.0050 + \hat{\epsilon}_{2,t+1} \tag{4.57}
\]

\[
y_t - y_t^* = 4.6091 + \hat{\epsilon}_{3,t+1} \tag{4.58}
\]

\[
(p_t - p_t^*) - (p_t^* - p_t^\circ) - e_t = 10.5992 + 0.0071 t + 11.5501 r_t - 1.5244 y_t + \hat{\epsilon}_{4,t+1} \tag{4.59}
\]

\[
h_t - y_t = 0.5295 - 0.0102 t - 20.3253 r_t + \hat{\epsilon}_{5,t+1} \tag{4.60}
\]

The bracketed figures are asymptotic standard errors. The first equation, (4.56) describes our modified PPP relationship. As elaborated in Section 2, this relationship is based on a linear combination of the real exchange rate and the relative oil price variable, with the coefficient on the relative oil price variable, \(\theta\), taking the (statistically significant) value of 0.0856 (0.0336). We found unambiguous evidence using our data set that no cointegrating relationship exists between \(p_t - p_t^*\) and \(e_t\). This finding is consistent with much of the literature examining the co-movements of exchange rates and relative prices, and the use of relative oil prices in obtaining a modified PPP relationship is consistent with results reported in Chauduri and Daniel (1998).

The second cointegrating relation, defined by (4.57), is the IRP condition. This includes an intercept, which can be interpreted as the deterministic component of the risk premia associated with bonds and foreign exchange uncertainties. Its value is estimated at 0.0050, implying a risk premium of approximately 2.0 per cent per annum. The empirical support we find for the IRP condition, namely that \(r_t - r_t^* \sim I(0)\) is in accordance with the results obtained in the literature, and is compatible with UIP, defined by (2.22). However, under the UIP hypothesis it is also required that a regression of \(r_t - r_t^*\) on \(\eta_{\Delta c,t+1} = \Delta \ln(E_d,t+1)\) has a unit coefficient, but this is not supported by the data.

The third long-run relationship, given by (4.58), supports the Output Gap (OG) relationship with per capita domestic and foreign output (measured by the total OECD output)
levels moving in tandem in the long-run. It is noteworthy that the co-trending hypothesis cannot be rejected; i.e. the coefficient of the deterministic trend in the output gap equation is zero. This suggests that average long-run growth rate for the UK is the same as that in the rest of the OECD. This finding seems, in the first instance, to contradict some of the results obtained in the literature on the cointegrating properties of real output across countries. Campbell and Mankiw (1989), Cogley (1990) and Bernard and Durlauf (1995), for example, consider cointegration among international output series and find little evidence that outputs of different pairs of countries are cointegrated. However, our empirical analysis, being based on a single foreign output index, does not necessarily contradict this literature, which focuses on pairwise cointegration of output levels. The hypothesis advanced here, that $y_t$ and $y_t^*$ are cointegrated, is much less restrictive than the hypothesis considered in the literature that all pairs of output variables in the OECD are cointegrated.\footnote{See Lee (1998) for further discussion of cross-country interdependence in growth dynamics.}

The Trade Balance ($TB$) relation (4.59), derived from the long-run solvency constraint, captures the fact that the level of debt or liability with respect to the rest of the world can only be accumulated in line with the country’s ability to pay as measured by its output. Debt accumulation is associated with current and capital account deficits. In turn, current account deficits are associated primarily with the level of competitiveness and relative domestic and foreign demand, while the capital account reflects the relative rates of return on domestic and foreign assets.\footnote{Recall that the IRP and OG relations suggest we drop both the $y_t^*$ and $r_t^*$ from the Trade Balance relationship, so that the estimated coefficients on $y_t$ and $r_t$ represent an amalgam of effects relating to $y_t^*$ and $y_t$, and to $r_t^*$ and $r_t$, respectively.} Although theory does not impose particular a priori restrictions on the signs of the coefficients of the $TB$ relationship, the negative estimate obtained for the coefficient on $y_t$ and the positive estimate obtained for the coefficient on $r_t$ in (4.59) seem sensible. They suggest that rising domestic output levels or falling domestic interest rates, which might both be reasonably thought to coincide with balance of payments deficits, are compatible with equilibrium only if the real exchange rate, $e_t + p_t^* - p_t$, is depreciating.

Finally, consider the Real Money Balance ($RMB$) relationship given by (4.60). Estimating this relationship, we could not reject the hypothesis that the elasticity of real money balances with respect to real output is equal to unity, and therefore (4.60) in fact represents a M0 velocity equation. The $RMB$ equation, however, contains a deterministic downward trend, representing the steady decline in the money-income ratio in the UK over the period 1965-1995, arising primarily from the technological innovations in financial intermediation. There is also strong statistical evidence of a negative interest rate effect on real money balances. This long-run specification is comparable with the recent research on the determinants of the UK narrow money velocity reported in, for example, Breedon and Fisher (1996).

### 4.2 Error Correction Specification of the Core Model

Table 4 presents the estimates of the seven error correction equations associated with the endogenous variables of the core model. These equations seem to fit reasonably well. Notice that our measures of fit, the adjusted $R^2$, refer to the variations of the changes in the endogenous variables explained by regressions where all the regressors (except for that which
is long-run forcing) are lagged by at least one period. Therefore, the reported $\hat{R}^2$ values are free of the spurious fit and simultaneity biases that tend to plague (or at least used to plague) time series econometrics.

In order to evaluate the equations of the core model, we also present, in Table 5, a set of benchmark univariate ARMA models estimated separately on the first differences of the seven endogenous variables in the model. These benchmark equations are constructed without reference to economic theory and provide representations of the variables which are based purely on their time series properties. Comparison with the estimated core model allows us to consider how much, if at all, the explanatory power and potential forecasting ability of the model is improved by the adoption of the long-run structural modelling approach.\footnote{Of course, such a comparison does not take into account the value of the structural interpretation and understanding provided through the use of a model based on explicit economic theory.} For each variable, the benchmark model is chosen from the class of ARMA($s,q$), $s,q = 0,1,\ldots, 4$ models using the $AIC$.\footnote{The $\hat{R}^2$ reported for the $\Delta(p_t - p_t^*)$ equation in Table 4 refers to $\Delta p_t$, which is directly comparable to the $\hat{R}^2$ obtained for the univariate benchmark model which is estimated on $\Delta p_t$.}

The overall picture presented by the estimated equations of the core model given in Table 4 is that the model fits the data well and has satisfactory diagnostic statistics. The parameters of the disequilibrium terms show that the long-run relations make an important contribution in most equations and that the disequilibrium terms provide for a complex and statistically significant set of interactions and feedbacks across variables. Apart from the $\Delta \epsilon_t$ and $\Delta(h_t - y_t)$ equations, which have no feedbacks from any of the disequilibrium terms, all the error correction equations have at least two statistically significant feedbacks (working at the 95\% significance level). The $\Delta y_t^*$ and $\Delta r_t$ equations have two significant error correction terms, namely, $[\hat{e}_{2.t}, \hat{e}_{3.t}]$ and $[\hat{e}_{1.t}, \hat{e}_{4.t}]$ respectively; and the $\Delta(p - p^*)_t$, $\Delta y_t$ and $\Delta r_t^*$ equations each have three, namely $[\hat{e}_{2.t}, \hat{e}_{3.t}, \hat{e}_{4.t}]$ and $[\hat{e}_{1.t}, \hat{e}_{4.t}, \hat{e}_{5.t}]$ respectively. The PPP, IRP and RMB disequilibrium terms are significant in two, three and respectively of the seven error correction equations, whereas the $O\!G$ and $T\!B$ disequilibrium terms both appear in two of the error correction equations.

Comparison of the model in Table 4 with the univariate ARMA specifications of Table 5 shows that the error correction equation for domestic prices (relative to oil prices) explains around 78 per cent of the total variation in $\Delta p_t$. This compares with 70 per cent for the corresponding benchmark model.\footnote{Clearly, the exchange rate (in logs) appears to follow a random walk with a drift, a finding consistent with the usual findings for this variable.} For the domestic output growth equation, we have $\hat{R}^2 = 0.21$ as compared with only 0.056 per cent for the benchmark model. The fit of the foreign output growth equation is even higher; 37 per cent of the total variation in $\Delta y_t^*$ is explained, compared to 23 per cent for the benchmark model. Similar results are also obtained for the domestic and foreign interest rate equations, which explain 23 per cent and 36 per cent of total variations in domestic and foreign interest rate changes, respectively, compared to 0.0 and 22 per cent for the benchmark models. The real money balance equation accounts for 14 per cent of the total variation in $\Delta(h_t - y_t)$, compared to 6 per cent for the benchmark model. Only the exchange rate equation (perhaps not surprisingly) fails to show good explanatory power, accounting for just 3 per cent of the total variation in $\Delta \epsilon_t$ compared to 2 per cent for the benchmark model.\footnote{Clearly, the exchange rate (in logs) appears to follow a random walk with a drift, a finding consistent with the usual findings for this variable.}
The diagnostic statistics of the equations in Table 4 reject the presence of serial correlation in the residuals of all equations, with the exception of the $\Delta r_t^\ast$ equation. But the assumption of normally distributed errors is rejected in the case of all the error correction equations. This is hardly surprising, considering the three major hikes in oil prices experienced during the estimation period. There is evidence of heteroscedasticity in four of the equations, $\Delta (p_t - p_t^\ast)$, $\Delta r_t$ and $\Delta r_t^\ast$ and the functional form statistics reject at the 5 per cent level for the $\Delta r_t$ and $\Delta r_t^\ast$ equations. Generally speaking, however, the diagnostics statistics are satisfactory, and the equations of Table 4 appear to capture well the time series properties of the main macroeconomic aggregates in the UK over the period since the mid-1960s.

The above analysis clearly demonstrates the advantages of the long-run structural modelling in terms of fit, parsimony and consistency with long-run macro-theory. However, without further restrictions on the contemporaneous linkages between the different variables in the model, the coefficients of the error correction specifications (in Table 4) and the impulse response properties of the model to shocks cannot be identified. (See Pesaran and Shin, 1999). The short-run identification problem can be approached from a number of different perspectives, all of which can be characterized in the context of the following structural Vector Error Correction Model (VECM) associated with our long run core model defined by (3.54):

$$A_0 \Delta y_t = A_0 c_0 + A_0 \alpha \alpha_1 t - A_0 \alpha \beta z_{t-1} + \sum_{i=1}^{s-1} A_0 \Gamma_i \Delta z_{t-i} + A_0 \psi \Delta (p_t^\ast - p_t^\ast) + A_0 u_t,$$

(4.61)

where $A_0$ is the $(7 \times 7)$ matrix of contemporaneous correlations across the seven endogenous variables of the core model. For a given choice of $A_0$, the “structural” shocks are given by $\varepsilon_t = A_0 u_t \sim IID (0, \Omega)$. This model is structural in the sense that it explicitly allows for instantaneous interactions between the endogenous variables through the contemporaneous coefficient matrix $A_0$ and through the contemporaneous correlations between the structural innovations as captured by non-zero off-diagonal terms in $\Omega$. The presence of $r$ cointegrating relationships among the variables $z_t$ implies rank restrictions on $\Pi = A_0 \alpha \beta = A_0 \Pi$, but since $A_0$ is non-singular it does not imply any further restrictions on the contemporaneous coefficients, $A_0$ and $\Omega$. It is clear, therefore, that without a priori restrictions on $A_0$ and/or $\Omega$ it would not be possible to give economic meanings to the estimates of the loading coefficients, $\tilde{\alpha} = A_0 \alpha$, or to identify economically meaningful impulse response functions to shocks.\textsuperscript{35} The restrictions on $A_0$ that are necessary for identification of these structural effects requires a tight description of the decision-rules followed by economic agents, incorporating information on agents’ use of information and the timing of the flow of information. Starting with the seminal work of Sims (1980), a number of alternative (short-run) identification schemes have been advanced in the literature, without a clear consensus. See, for example, Bernanke (1986), Christiano and Eichenbaum (1992), Cochrane (1998), Crowder

\textsuperscript{35}Such an analysis will typically also require orthogonality restrictions to be placed on the $\Omega$ matrix so that the effect of the specified shock can be isolated from the potential contemporaneous effects of other structural shocks. While not ruling out the possibility that these orthogonality restrictions exist, the discussion of the sources of the long-run structural disturbances of the model outlined in Section 2 of the paper suggest that it might be difficult to justify such restrictions in many cases.
et al. (1999) and Wickens (1999). A comprehensive application of these alternative identification strategies to our model is beyond the scope of the present paper and will be pursued elsewhere. (See Garratt et al., 2000). In the rest of this section, we shall confine our short-run analyses to a much more limited range of questions; namely, the dynamic properties of the reduced form model and the effects of shocks to the cointegrating relations, captured by the Persistence Profiles introduced in Pesaran and Shin (1996), that are invariant to the choice of A0.

4.3 Short-Run Dynamics and Persistence Profiles

To examine the time profile of the effect of shocks on the cointegrating relations (known as the Persistence Profiles, PP), we assume that both oil price changes and foreign price relative to the oil price changes are strictly exogenous. After some experimentation, we arrived at the following specification for \( \Delta p_t^* \) estimated over the period 1965q1-1995q4:

\[
\Delta p_t^* = .01678 + \hat{u}_t, \quad (4.62)
\]

\[
\hat{\sigma}_{uo} = .1676, \quad \chi^2_{SC}[4] = 1.58, \quad \chi^2_N[2] = 6361.9,
\]

To model \( \Delta(p^*_t - p^*_t) \) over the same period, we estimated autoregressive distributive lag models, \( ARDL(s_1, s_2) \), in \( \Delta(p^*_t - p^*_t) \) and \( \Delta p^*_t \) for all orders \( s_1 \leq 4 \) and \( s_2 \leq 4 \), and then selected \( s_1 \) and \( s_2 \) using the Akaike model selection criteria. The result was the following \( ARDL(2, 2) \) specification:

\[
\Delta(p^*_t - p^*_t) = .0022 + .5848 \Delta(p^*_t - p^*_t-1) + .2155 \Delta(p^*_t - p^*_t-2) - .9603 \Delta p^*_t + .5737 \Delta p^*_t-1 + .2108 \Delta p^*_t-2 + \hat{u}_t, \quad (4.63)
\]

\[
\bar{R}^2 = .856, \quad \hat{\sigma}_{us} = .0043, \quad \chi^2_{SC}[4] = 3.86, \quad \chi^2_{F}[1] = .32, \quad \chi^2_N[2] = 336.4, \quad \chi^2_H[1] = .042.
\]

The model for oil price changes is a simple geometric random walk with a small positive drift, although the drift coefficient is not statistically significant. Despite its simplicity, the hypothesis that the residuals of the oil price equation are serially uncorrelated cannot be rejected. There is, however, clear evidence of non-normal errors, primarily reflecting the three major oil price changes experienced during the period under consideration. As is to be expected, oil price changes are highly volatile: the estimated standard error of the oil price equation, \( \hat{\sigma}_{uo} = .1676 \), is 4.8 times the standard error of the exchange rate equation, and almost 19 times the standard error of the output equation.

The equation for foreign price changes relative to the oil price, \( \Delta(p^*_t - p^*_t) \), is marginally more complicated by comparison, allowing for the feedback effects from past changes in the foreign price relative to the oil price and oil prices. It has a high degree of explanatory power, explaining 86 per cent of the total variations in the rate of change of foreign prices relative to the oil price, and passes the diagnostic tests for residual serial correlation, functional form

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36 In the absence of a fully-specified economic model of the short-run, impulse response analysis is best conducted using the Generalised Impulse Response function, developed in Koop et al. (1996) and Pesaran and Shin (1998), which does not place a structural interpretation on the effect of shocks.

[24]
misspecification, and heteroscedasticity. However, it again fails the normality test. Finally, it is worth emphasizing that by assumption the disturbances in the above two equations, $u_t^\gamma$ and $u_t^\alpha$, are distributed independently of the errors of the core model, $u_t$, defined in (3.54).

The Persistence Profiles (PP) of the five long-run relations of the model are plotted in Figures 2 and 3. These illustrate the time profile of the effects of system-wide shocks to each of the long-run relationships. These plots are scaled, so that the value of the profiles are unity on impact and tend to zero as the time horizon tends to infinity if the relation under consideration is cointegrated. The PPs provide information on the speed with which the different relations in the model, once shocked, will return to their long-run equilibria. These are frequently non-monotonic and in these cases the PPs also provide interesting insights on the dynamic response of the system to disequilibria. To obtain a simple summary of a profile, it is often convenient to report the ‘half-life’ measure, which describes the time horizon over which the profile falls to 0.5. Given that the profile starts at unity and falls to zero, this gives a simple indication of the speed of adjustment for a profile and makes comparisons across profiles straightforward. It is also useful to provide confidence intervals for this measure, obtained using the same bootstrap procedure used to obtain the small sample critical values for the test of the over-identifying restrictions (i.e. based on 5000 replications of PPs where each replication is based on a data set generated under the assumption that the core model is the true data-generating process). The measures corresponding to 0.2 and 0.01, showing 80% and full adjustment, are also sometimes useful where the adjustment to equilibrium slows markedly over longer time horizons.

Figures 2 and 3 show the PPs for the five cointegrating relations of the core model. The former Figure provides a comparison of the speeds of adjustment to the different equilibria, while the latter shows the associated confidence intervals generated through the bootstrap exercise. Figure 2a shows the PP for the first two long-run relationships; namely the (modified) PPP and IRP relations. In the case of the modified PPP, the profile shows a steady decline towards its equilibrium value, with a half-life of 3.4 quarters but a relatively wide 95% confidence interval of (1.8, 9.6). Approximately 80 per cent of the adjustment taking place within 9 quarters, and the full adjustment taking about five years to complete. The fact that the persistence profile of the modified PPP relation tends to zero is in line with our earlier conclusion (based on formal statistical tests) that the modified PPP relationship is in fact cointegrating. Notice, however, that the speed of convergence of the modified PPP towards its equilibrium is much faster than the ones reported in the literature for the (unmodified) PPP. The existing results put the half life of deviations from PPP at about four years for the major industrialised countries (see, for example, Johansen and Juselius (1992), Pesaran and Shin (1996), or Rogoff (1996)), while for the modified PPP this is much shorter. The PP profile of the IRP relationship shows a more rapid rate of adjustment towards its long-run value, with a half-life of 1.5 quarters (1.0, 3.5), with approximately 80 per cent of the adjustment having been completed within 6 quarters and full adjustment occurring within three to four years. These results are consistent with those found in the literature.

Figure 2b plots the PP for the OG, TB and RMB relations. The OG profile suggests a relatively slow speed of adjustment. Although it displays a half-life of 2.0 quarters, the

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37In this application the system wide shocks are taken to include the shocks to the exogenous equations (4.62) and (4.63) as well as the shocks to the endogenous variables in the core model.
associated 95% confidence interval of (0.8, 6.0) is wide, and the profile shows that it takes a further 9 quarters for 80 per cent of the adjustment to occur, and takes approximately five years for the full adjustment to take place.\footnote{The slow speed of adjustment suggested by this analysis is comparable to those implied by Barro and Sala-i-Martin’s (1995) analyses of international output series. However, the inferences are based on entirely different modelling frameworks. For example, Barro and Sala-i-Martin assume that output series are trend stationary and study convergence to a common trend growth rate. The present study assumes the output series are difference stationary and tests for cointegration between UK and OECD output series. For further discussion, see Lee \textit{et al.} (1997, 1998).} In comparison, the persistence profile of the TB relationship shows a much faster speed of adjustment; it shows a half-life of 1.5 (0.8, 2.3) with 80 per cent of the adjustments having occurred within 4 quarters. But it still takes around 3 years before the full effects of the system-wide shocks on the TB relation is completely wiped out. Finally, the persistence profile for the RMB relation initially shows a rapid rate of adjustment, with a half-life of 1.4 quarters (0.6, 2.9), but a slower movement thereafter, with 80 per cent of the adjustment occurring in 10 quarters, and full adjustment occurring after approximately five years. This may seem rather surprising at first, but may reflect the prolonged and persistent effect of technological innovations in financial markets that have taken place, particularly over the past two decades.

5 Conclusions

This paper provides an account of how macro-modelling can be undertaken following the long-run structural VAR modelling approach. It outlines a theoretical framework for the long-run analysis of an open macroeconomy; introduces a new modelling strategy providing a practical approach to incorporating theory-based long-run relationships in an otherwise unrestricted VAR; and presents the estimates and the tests we have carried out to construct a “core” macroeconometric model of the UK. The description of our approach to modelling starts with an explicit statement of a set of long-run relationships between the variables of interest, as derived from macroeconomic theory, taking account of a number of key long-term arbitrage and solvency conditions. These long-run relationships are embedded within an otherwise unrestricted VAR model in eight “core” variables, augmented appropriately by intercepts and linear trends. The VAR model is estimated over the period 1965q1-1995q4, subject to the theory restrictions on the long-run coefficients using recently developed econometric techniques. Such a long-run structural model has the advantage shared by all VAR models that it is able to capture complicated dynamic relationships in the data, but it also incorporates theory-consistent long-run properties in a transparent manner. Hence, the approach is capable of providing a structural understanding of the macroeconomy through the estimation of a small and transparent, but reasonably comprehensive, model of the UK macroeconomy.

An important feature of our modelling approach is that it provides the means for testing formally the validity of restrictions suggested by economic theory in the context of a complete macroeconomic model. The economic theory elaborated in the paper, and the statistical considerations for the empirical application, suggest that there are five long-run cointegrating relationships among the eight core variables of the macro-model, and the statistical tests
provided little evidence with which to reject this view. Under the assumption that there are indeed five long-run relationships, we obtained a model in which the freely-estimated parameters take sensible signs and are of plausible orders of magnitude, including the intercept estimated for the Interest Rate Parity relationships and the parameters of the modified PPP relationship, the trade balance relationship and the real money balance relationship. Further, we tested for the validity of fourteen (over-identifying) restrictions, eleven of which were intrinsic to the theory, and three of which were consistent with the theory and were included for reasons of parsimony. Using likelihood ratio tests, we were not able to reject these over-identifying restrictions, and from this we conclude that the estimated model is both theory and data consistent, at least as far as our data sample is concerned.

A further advantage of our approach, also illustrated in the paper, is in the modelling of the short-run dynamics. The cointegrating VAR model is not only able to provide a reasonably flexible characterisation of the short-run dynamics of the macroeconomy but, by making explicit the link with the long-run relationships suggested by economic theory, it also enables us to consider explicitly the links between “structural” and “observable” disturbances. We noted that an analysis of the response of the macro variables over time to economically-meaningful structural shocks is not possible without the imposition of further restrictions. These would need to be based on an explicitly-defined economic theory of the short-run reactions of economic agents, which is beyond the scope of the present paper. However, it is still possible to identify the effects of system-wide shocks on the cointegrating relations, as characterised by the Persistence Profiles. For example, the estimated profiles clearly illustrate the differential speeds of response to the disequilibria involving financial variables (e.g. IRP, in which 80% of adjustment takes six quarters to complete) compared to those involving real magnitudes (e.g. the output gap relationship, in which 80% of the adjustment takes two and a half years to complete).
Table 1: List of Variables and their Descriptions in the Core Model

$y_t$: natural logarithm of the UK real per capita GDP at market prices ($1990 = 100$).

$p_t$: natural logarithm of the UK Producer Price Index ($1990 = 100$).

$r_t$: is computed as $r_t = 0.25 \ln(1 + R_t/100)$, where $R_t$ is the 90 day Treasury Bill average discount rate per annum.

$h_t$: natural logarithm of UK real per capita M0 money stock ($1990 = 100$).

$e_t$: natural logarithm of the nominal Sterling effective exchange rate ($1990 = 100$).

$y_t^*: $natural logarithm of the foreign (OECD) real per capita GDP at market prices ($1990 = 100$).

$p_t^*: $natural logarithm of the foreign price index ($1990 = 100$). The index is an import weighted average of 42 countries' price indices (where the countries are the OECD, oil producing and a number of other countries with relatively large values of imports and exports with the UK).

$r_t^*: $is computed as $r_t^* = 0.25 \ln(1 + R_t^*/100)$, where $R_t^*$ is the weighted average of 90 day interest rates per annum in the United States, Germany, Japan and France.

$p_t^o$: natural logarithm of oil prices, measured as the Average Price of Crude Oil.

$t$: time trend, taking the values 1, 2, 3, ..., in 1965q1, 1965q2, 1965q3, ..., respectively.

Notes: For the data sources and a detailed description of the construction of foreign prices and interest rates see the Data Appendix in Garratt et al. (2000).
Table 2: Augmented Dickey-Fuller Unit Root Test Applied to Variables in the Core Model; 1965q1-1995q4

<table>
<thead>
<tr>
<th>Variable</th>
<th>ADF(0)</th>
<th>ADF(1)</th>
<th>ADF(2)</th>
<th>ADF(3)</th>
<th>ADF(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta y_t)</td>
<td>-11.23</td>
<td>-7.63</td>
<td>-4.97α</td>
<td>-1.73</td>
<td>-3.51</td>
</tr>
<tr>
<td>(\Delta y_t^\ast)</td>
<td>-6.75</td>
<td>-4.93α</td>
<td>-4.07</td>
<td>-3.81</td>
<td>-3.86</td>
</tr>
<tr>
<td>(\Delta r_t)</td>
<td>-10.00α</td>
<td>-7.35</td>
<td>-7.03</td>
<td>-5.67</td>
<td>-5.85</td>
</tr>
<tr>
<td>(\Delta r_t^\ast)</td>
<td>-6.74α</td>
<td>-6.28</td>
<td>-4.72</td>
<td>-4.47</td>
<td>-4.13</td>
</tr>
<tr>
<td>(\Delta \epsilon_t)</td>
<td>-9.43α</td>
<td>-7.92</td>
<td>-6.74</td>
<td>-5.65</td>
<td>-5.63</td>
</tr>
<tr>
<td>(\Delta (h_t - y_t))</td>
<td>-12.22α</td>
<td>-8.62</td>
<td>-6.33</td>
<td>-5.40</td>
<td>-5.89</td>
</tr>
<tr>
<td>(\Delta p_t)</td>
<td>-3.34α</td>
<td>-3.06</td>
<td>-2.68</td>
<td>-2.58</td>
<td>-2.51</td>
</tr>
<tr>
<td>(\Delta p_t^\ast)</td>
<td>-4.98</td>
<td>-3.45</td>
<td>-2.56α</td>
<td>-2.57</td>
<td>-2.60</td>
</tr>
<tr>
<td>(\Delta p_t^2)</td>
<td>-10.79α</td>
<td>-8.46</td>
<td>-6.35</td>
<td>-5.59</td>
<td>-5.41</td>
</tr>
<tr>
<td>(\Delta^2 p_t)</td>
<td>-12.48</td>
<td>-9.68α</td>
<td>-7.71</td>
<td>-6.59</td>
<td>-6.74</td>
</tr>
<tr>
<td>(\Delta^2 p_t^\ast)</td>
<td>-16.30</td>
<td>-12.62α</td>
<td>-8.42</td>
<td>-6.73</td>
<td>-6.53</td>
</tr>
<tr>
<td>(\Delta (p_t - p_t^\ast))</td>
<td>-11.18α</td>
<td>-8.85</td>
<td>-6.66</td>
<td>-5.95</td>
<td>-5.81</td>
</tr>
<tr>
<td>(\Delta (p_t^2 - p_t^\ast^2))</td>
<td>-11.03α</td>
<td>-8.76</td>
<td>-6.66</td>
<td>-5.87</td>
<td>-5.71</td>
</tr>
</tbody>
</table>

Notes: When applied to the first differences, augmented Dickey-Fuller (1979, ADF) test statistics are computed using ADF regressions with an intercept and \(s\) lagged first-differences of dependent variable, while when applied to the levels, ADF statistics are computed using ADF regressions with an intercept, a linear time trend and \(s\) lagged first-differences of dependent variable, with the exception of the following variables: \(\Delta p_t\), \(\Delta y_t\), \(\Delta y_t^\ast\), \(r_t\) and \(r_t^\ast\) where only an intercept was included in the underlying ADF regressions. The relevant lower 5 per cent critical values for the ADF tests are -2.88 for the former and -3.45 for the latter. The symbol “α” denotes the order of augmentation in the Dickey-Fuller regressions chosen using the Akaike Information Criterion, with a maximum lag order of four.
Table 3: Cointegration Rank Statistics for the Core Model

\((p_t - p_t^o, \epsilon_t, r_t, r_t^*, y_t, h_t - y_t, p_t^* - p_t^o)\)

(a) Trace Statistic

<table>
<thead>
<tr>
<th>(H_0)</th>
<th>(H_1)</th>
<th>Test Statistic</th>
<th>95% Critical Values</th>
<th>90% Critical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r = 0)</td>
<td>(r = 1)</td>
<td>231.74</td>
<td>163.01</td>
<td>153.34</td>
</tr>
<tr>
<td>(r \leq 1)</td>
<td>(r = 2)</td>
<td>170.02</td>
<td>128.79</td>
<td>121.63</td>
</tr>
<tr>
<td>(r \leq 2)</td>
<td>(r = 3)</td>
<td>126.50</td>
<td>97.83</td>
<td>92.98</td>
</tr>
<tr>
<td>(r \leq 3)</td>
<td>(r = 4)</td>
<td>85.88</td>
<td>72.10</td>
<td>68.65</td>
</tr>
<tr>
<td>(r \leq 4)</td>
<td>(r = 5)</td>
<td>47.69</td>
<td>49.36</td>
<td>47.89</td>
</tr>
<tr>
<td>(r \leq 5)</td>
<td>(r = 6)</td>
<td>21.05</td>
<td>30.77</td>
<td>20.34</td>
</tr>
<tr>
<td>(r \leq 6)</td>
<td>(r = 7)</td>
<td>7.87</td>
<td>15.44</td>
<td>14.88</td>
</tr>
</tbody>
</table>

(b) Maximum Eigenvalue Statistic

<table>
<thead>
<tr>
<th>(H_0)</th>
<th>(H_1)</th>
<th>Test Statistic</th>
<th>95% Critical Values</th>
<th>90% Critical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r = 0)</td>
<td>(r = 1)</td>
<td>61.73</td>
<td>52.62</td>
<td>49.63</td>
</tr>
<tr>
<td>(r \leq 1)</td>
<td>(r = 2)</td>
<td>43.51</td>
<td>46.97</td>
<td>44.42</td>
</tr>
<tr>
<td>(r \leq 2)</td>
<td>(r = 3)</td>
<td>40.62</td>
<td>40.89</td>
<td>38.71</td>
</tr>
<tr>
<td>(r \leq 3)</td>
<td>(r = 4)</td>
<td>38.20</td>
<td>34.70</td>
<td>33.01</td>
</tr>
<tr>
<td>(r \leq 4)</td>
<td>(r = 5)</td>
<td>26.64</td>
<td>28.72</td>
<td>27.60</td>
</tr>
<tr>
<td>(r \leq 5)</td>
<td>(r = 6)</td>
<td>13.19</td>
<td>22.16</td>
<td>21.32</td>
</tr>
<tr>
<td>(r \leq 6)</td>
<td>(r = 7)</td>
<td>7.87</td>
<td>15.44</td>
<td>14.88</td>
</tr>
</tbody>
</table>

Notes: The underlying VAR model is of order 2 and contains unrestricted intercepts and restricted trend coefficients, with \(p_t^* - p_t^o\) treated as exogenous \(I(1)\) variable. The statistics refer to Johansen's log-likelihood-based trace and maximal eigenvalue test statistics and are computed using 124 observations for the period 1965q1-1995q4. 'cv\(a\)' and 'cv\(b\)' are respectively the asymptotic and boot-strapped critical values. The former are taken from Pesaran, Shin and Smith (1999), and the latter are generated using a Monte Carlo procedure as described in the text.
Table 4: Error Correction Specification for the Core Model

<table>
<thead>
<tr>
<th>Equation</th>
<th>$\Delta(\beta_{it} - \beta_{it}^{*})$</th>
<th>$\Delta\delta_{t}$</th>
<th>$\Delta\tau_{t}$</th>
<th>$\Delta\tau_{t}^{*}$</th>
<th>$\Delta\gamma_{t}$</th>
<th>$\Delta\delta_{t}^{*}$</th>
<th>$\Delta(h_{t} - y_{t})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{e}_{1, t}$</td>
<td>0.007 (0.020)</td>
<td>-0.119 (0.085)</td>
<td>-0.030 (0.006)</td>
<td>-0.008 (0.003)</td>
<td>0.120 (0.029)</td>
<td>0.017 (0.012)</td>
<td>-0.078 (0.041)</td>
</tr>
<tr>
<td>$\hat{e}_{2, t}$</td>
<td>-0.031 (0.242)</td>
<td>0.355 (1.05)</td>
<td>-0.019 (0.083)</td>
<td>0.007 (0.003)</td>
<td>0.880 (0.315)</td>
<td>6.161 (1.155)</td>
<td>-0.236 (0.507)</td>
</tr>
<tr>
<td>$\hat{e}_{3, t}$</td>
<td>0.135 (0.034)</td>
<td>-0.241 (1.45)</td>
<td>-0.117 (0.011)</td>
<td>-0.006 (0.003)</td>
<td>-0.134 (0.044)</td>
<td>-0.016 (0.021)</td>
<td>0.006 (0.070)</td>
</tr>
<tr>
<td>$\hat{e}_{4, t}$</td>
<td>-0.063 (0.097)</td>
<td>0.132 (1.16)</td>
<td>0.041 (0.009)</td>
<td>0.015 (0.004)</td>
<td>0.043 (0.034)</td>
<td>-0.016 (0.017)</td>
<td>0.049 (0.056)</td>
</tr>
<tr>
<td>$\hat{e}_{5, t}$</td>
<td>0.009 (0.015)</td>
<td>0.001 (0.003)</td>
<td>0.006 (0.002)</td>
<td>0.005 (0.000)</td>
<td>-0.081 (0.019)</td>
<td>-0.045 (0.009)</td>
<td>0.001 (0.031)</td>
</tr>
<tr>
<td>$\Delta(p_{it} - p_{it}^{*})$</td>
<td>0.465 (0.060)</td>
<td>0.322 (0.391)</td>
<td>0.003 (0.104)</td>
<td>0.015 (0.118)</td>
<td>0.351 (0.038)</td>
<td>0.054 (0.018)</td>
<td>-0.480 (0.039)</td>
</tr>
<tr>
<td>$\Delta(r_{it} - 1)$</td>
<td>-0.013 (0.027)</td>
<td>0.266 (1.19)</td>
<td>0.010 (0.000)</td>
<td>0.006 (0.004)</td>
<td>-0.004 (0.006)</td>
<td>-0.004 (0.001)</td>
<td>-0.127 (0.007)</td>
</tr>
<tr>
<td>$\Delta(r_{it} - 1)^{*}$</td>
<td>0.123 (0.300)</td>
<td>-0.114 (1.32)</td>
<td>0.154 (1.01)</td>
<td>-0.083 (0.04)</td>
<td>0.563 (0.398)</td>
<td>-0.204 (0.195)</td>
<td>-0.335 (0.120)</td>
</tr>
<tr>
<td>$\Delta(\gamma_{it} - 1)$</td>
<td>-0.465 (0.024)</td>
<td>2.300 (2.70)</td>
<td>0.312 (0.207)</td>
<td>0.063 (0.088)</td>
<td>0.781 (0.313)</td>
<td>-1.810 (1.289)</td>
<td>1.311 (0.905)</td>
</tr>
<tr>
<td>$\Delta(h_{it} - 1)$</td>
<td>0.005 (0.094)</td>
<td>-0.007 (0.408)</td>
<td>-0.029 (0.031)</td>
<td>-0.001 (0.015)</td>
<td>-0.063 (0.123)</td>
<td>-0.012 (0.060)</td>
<td>-0.190 (0.198)</td>
</tr>
<tr>
<td>$\Delta(h_{it} - 1)^{*}$</td>
<td>-0.162 (0.160)</td>
<td>-0.335 (0.695)</td>
<td>0.022 (0.053)</td>
<td>0.076 (0.052)</td>
<td>0.014 (0.208)</td>
<td>-0.111 (0.102)</td>
<td>0.404 (0.335)</td>
</tr>
<tr>
<td>$\Delta(r_{it} - y_{it} - 1)$</td>
<td>0.106 (0.053)</td>
<td>0.264 (0.231)</td>
<td>0.012 (0.018)</td>
<td>-0.010 (0.003)</td>
<td>0.082 (0.039)</td>
<td>-0.035 (0.034)</td>
<td>0.187 (0.112)</td>
</tr>
<tr>
<td>$\Delta(p_{it} - p_{it}^{<em>})^{</em>}$</td>
<td>1.031 (0.004)</td>
<td>-0.019 (0.019)</td>
<td>-0.002 (0.001)</td>
<td>-0.001 (0.0007)</td>
<td>-0.014 (0.006)</td>
<td>0.003 (0.0003)</td>
<td>-0.021 (0.0009)</td>
</tr>
<tr>
<td>$\Delta(p_{it} - p_{it}^{<em>})^{</em>}$</td>
<td>-0.418 (0.083)</td>
<td>-0.528 (0.400)</td>
<td>-0.001 (0.030)</td>
<td>0.006 (0.013)</td>
<td>-0.026 (0.121)</td>
<td>-0.057 (0.039)</td>
<td>0.521 (0.194)</td>
</tr>
</tbody>
</table>

$R^{2}$ | .778 (0.030) | 0.225 (0.363) | 0.212 (0.212) | 0.374 (0.142) | 0.075 | 0.57 |

Benchmark $R^{2}$ | .609 | 0.021 | 0.000 | 0.223 | 0.556 | 0.226 | 0.507 |

Notes: The five error correction terms are given by

$$
\hat{e}_{1, t+1} = p_{it} - p_{it}^{*} - \delta_{t} - 0.0856 (p_{it}^{*} - p_{it}^{*}) - 4.6386,
$$

$$
\hat{e}_{2, t+1} = r_{it} - r_{it}^{*} - 0.0050,
$$

$$
\hat{e}_{3, t+1} = y_{it} - y_{it}^{*} + 4.6091,
$$

$$
\hat{e}_{4, t+1} = p_{it} - p_{it}^{*} - 11,5501 r_{it} + 1.5244 y_{it} - 0.0071 t - 10.5002,
$$

$$
\hat{e}_{5, t+1} = h_{it} - y_{it} + 20.3253 r_{it} + 0.0102 t + 0.5295.
$$

Standard errors are given in parenthesis. "*" indicates significance at the 10% level, and "††" indicates significance at the 5% level. The diagnostics are chi-squared statistics for serial correlation (SC), functional form (FF), normality (N) and heteroscedasticity (H). "*" The $R^{2}$ for the $\Delta(p_{it} - p_{it}^{*})$ equations refers to the $\Delta p_{it}$ equation. The benchmark $R^{2}$ statistics are taken from ARMA models for each of our variables, see Table 5.
Table 5: Benchmark ARMA(s,q) Models for the Core Endogenous Variables

<table>
<thead>
<tr>
<th>Equation</th>
<th>$\Delta y_t$</th>
<th>$\Delta y_s$</th>
<th>$\Delta y_2$</th>
<th>$\Delta y_s^2$</th>
<th>$\Delta y_4$</th>
<th>$\Delta y_5$</th>
<th>$\Delta (y_t - y_s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>.003*</td>
<td>.007*</td>
<td>-.000</td>
<td>-.000</td>
<td>.000*</td>
<td>.005*</td>
<td>-.013*</td>
</tr>
<tr>
<td>AR order</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>s = 1</td>
<td>.835*</td>
<td>-</td>
<td>-</td>
<td>-.285*</td>
<td>.148</td>
<td>.122</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(16.91)</td>
<td></td>
<td></td>
<td>(1.70)</td>
<td>(1.29)</td>
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Notes: The ARMA(s,q) specification is selected on the basis of maximising the Akaike Information Criterion. t-statistics are given in parenthesis. “*” indicates significance at the 5% level, and “†” indicates significance at the 10% level.
Figure 1. Distributions of the Bootstrapped Statistic for the Test of Twenty Over-identifying Restrictions and the $\chi^2_{20}$ Variate
**Figure 2a.** Persistence Profiles of the Modified Purchasing Power Parity (PPP) and Interest Parity Relations (IRP) in the Core Model.

**Figure 2b.** Persistence Profiles of the Output Gap (OG), Trade Balance (TB) and Real Money Balance (RMB) Relations in the Core model.
Figure 3. Persistence Profiles and their 95% Empirical Confidence Intervals.
References


[R3]