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Labour Contracts, Equal Treatment and Wage-Unemployment Dynamics

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Abstract

This paper analyses a model in which firms cannot pay discriminate based on year of entry to a firm, and develops an equilibrium model of wage dynamics and unemployment. The model is developed under the assumption of worker mobility, so that workers can costlessly quit jobs at any time. Firms on the other hand are committed to contracts. Thus the model is related to Beaudry and DiNardo (1991). We solve for the dynamics of wages and unemployment, and show that real wages do not necessarily clear the labor market. Using sectoral productivity data from the post-war US economy, we assess the ability of the model to match actual unemployment and wage series. We also show that equal treatment follows in our model from the assumption of at-will employment contracting.

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1 Introduction

This paper develops a model in which firms cannot pay discriminate based on year of entry to a firm—there are no “cohort effects”— and develops an equilibrium model of wage dynamics and unemployment. The model is developed under the assumption of worker risk aversion, and also mobility, so that workers can costlessly quit jobs at any time. Firms on the other hand are risk neutral and are committed to contracts. Firms have to trade-off the desire to insure their risk-averse workers against the need to respond to market conditions to not only prevent their workers from quitting but, because of equal treatment of workers, also to take advantage of states of the world where labor is cheap. We solve for the dynamics of wages and unemployment when the only exogenous variable is productivity shocks, and show that real wages exhibit a downward stickiness, due to the desire to insure incumbent workers. The equal treatment assumption prevents firms from cutting wages for new entrants, so that in periods with adverse shocks the wage may not fall sufficiently to clear the labor market. We argue that even our rudimentary model, when fed sectoral productivity shocks from the post-war U.S. economy, gives a reasonably good account of unemployment and wage movements.

The idea that internal equity considerations can play a part in wage rigidity is by no means novel. Truman Bewley has argued recently that it is a key feature constraining wage cuts for new hires in recessions. In his story, because wage cuts for incumbents will have such a negative impact on morale, firms avoid them under all but extreme circumstance; at the same time while new hires may be willing to work at a lower wage than that paid to incumbents, paying them less would disrupt internal equity and so their wages will be set at the same level as incumbents' (controlling for experience, etc.):

> New employees, in contrast, feel it is inequitable to be paid according to a scale lower than the one that applied to colleagues that were hired earlier. For this reason, downward pay rigidity for new hires exists only because the pay

of existing employees is rigid. (Bewley (1999a))

Bringing in workers at *higher* pay than incumbents is even more problematic; thus while—in contrast to the primary sector—he found evidence that new hires are sometimes paid a lower rate than incumbents in the secondary sector, even there, paying new hires *more* than incumbents is deemed to be very disruptive (Bewley (1999b, p. 320)).

Bewley's account mainly concentrates on the question of why firms do not cut wages in recession. But it raises the important question, which we attempt to answer, of how forward looking firms take into account the fact that such constraints may arise in the future: for example, a firm, anticipating this downward wage rigidity, may temper wage increases in better times. Or in more generality, and supposing that firms can offer long-term contracts, the firm must take into account these equal treatment constraints which will prevent it bringing in new hires at a low wage in downturns, and also prevent the firm hiring at a higher wage than that offered to incumbents when the labor market is tight. To our knowledge, the dynamic implications of equal treatment have not been analyzed elsewhere.[1]

The linking of the pay of new hires to that of incumbents means that wage rigidity also has real allocational implications. Obviously wage rigidity for incumbents need not imply deviations from Arrow-Debreu outcomes so long as hiring is at the efficient level (in our model workers only separate for exogenous reasons). We show however that (under certain conditions) firms hire up to the point where the real wage equals the marginal product of labor; to the extent then that wages do not correspond to market-clearing levels hiring will be inefficient; in fact we show that this occurs only in the direction of wages being too high leading to inefficiently low employment and an excess supply of labor.

[1] Our model differs from Bewley's account in that the motive to temper incumbent wage cuts arises from to the desire to insure workers, rather than directly from worker morale considerations. We do not view this contracting perspective to be necessarily inconsistent with his account however. The morale effects due to wage cuts that he documents might be considered to be the response by the workforce to a perception that the firm has violated an implicit insurance contract: low worker morale might be regarded as a punishment mechanism used to sustain the implicit contract.

The paper builds on the seminal contribution of Beaudry and DiNardo (1991) (hereafter BD). They develop a model of labor contracting where a risk-neutral firm offers insurance to risk-averse employees but, following Holmstrom (1983), there is no worker commitment (perfect mobility). Wages follow a ratchet-like process, rising when productivity is higher than previously, but staying constant otherwise; they show that the current wage is determined by the tightest labor market during a worker's tenure. In testing, this perfect mobility model does better than two alternatives: a spot market model in which current unemployment determines wages, and a full commitment model in which unemployment at the time of hiring is the determining factor. Subsequent research (McDonald and Worswick 1999, Grant 2003, Shin and Shin 2003, Devereux and Hart 2005) has largely confirmed these results over different periods and using different datasets, although both Grant, and Devereux and Hart, find more of a role for the current unemployment rate than did BD. Although the economic environments are distinct, in essence our theoretical model deviates from theirs only in the imposition of equal treatment.

In BD, without the equal treatment assumption, each worker is treated independently, and the partial equilibrium analysis then boils down to a two-player game in which competition forces profits to zero (given their constant returns to scale technology). It follows that the labor market must always clear, since at the point of hiring there are no restrictions on wages. The downward wage rigidity in their perfect mobility model provides insurance to the worker but does not directly affect employment decisions. Here, by contrast, we do not allow firms to treat each worker separately, but each new cohort of hires must fit into an existing wage structure. Even though we find that the characterization of optimal contracts is in a number of respects similar to that in BD, the implications for employment are very different, as there will be episodes of involuntary unemployment. We also show that although very robust, the estimated business cycle effect on wages (i.e., through the minimum unemployment rate) in their estimations cannot explain very much of the movement of wages over the sample we look at (an extension of the one they examine). On the other hand, wage movements predicted by our model can explain much of this.

The idea that equal treatment can lead to wage rigidity has been argued in a union context by Carruth and Oswald (1987) and Gottfries (1992). In these papers, outsiders have reservation wages below any wage that insiders might receive even in "good" states of the world. Wages are kept constant in the face of rising demand to prevent too much surplus leaking to outsiders. More closely related, Thomas (2005) considers an essentially static model with risk neutral workers and unverifiable states; in combination with equal treatment this can lead to wage-stickiness *across* states in a given period (but not as here, over time).

There is little direct empirical evidence on the issue of equal treatment. The principal exception is a study of pay discrimination by Baker, Gibbs and Holmstrom (1994), who examined the pay of managerial employees in a single firm over time. They found that incumbents' pay tends to move together, but the pay of entrants is significantly more variable, suggesting that the pay of new hires may be more subject to outside conditions than that of incumbents. However, as discussed above, survey evidence in Bewley (1999b) suggests that violations of equal treatment are unusual, particularly in the primary sector. Similar findings exist for other countries: "Managers responded that hiring underbidders would violate their internal wage policy" (Agell and Lundborg (1999, p.7), based on a Swedish survey); in a British survey, Kaufman (1984) reported that almost all managers viewed bringing in similarly qualified workers at lower wage rates as "infeasible." Akerlof and Yellen (1990) argue that personnel management texts treat the need for equitable pay as virtually self-evident.

It is possible to derive some version of equal treatment from primitive hypotheses. For example, Moore (1983) shows that if it is necessary to retain at least one worker to train the new employees, then there is a unique von Neumann-Morgernstern stable set consisting of configurations in which all workers receive the same wage. We adopt a similar approach using the idea that pay differences may be exploited by employers to replace more expensive workers by cheaper ones, and that it is difficult to distinguish between voluntary quits and fires, or alternatively, labor law requires contracts to be "at will", so either party can dissolve the relationship without penalty (but, crucially, not vary the wage).

An outline of the paper is as follows. The model is presented and solved in Section 2. In 2.3 we show that equal treatment arises in equilibrium if labor contracts are "at will." Empirical evidence is considered in Section 3: in 3.1 we outline our strategy for using sectoral TFP data from the postwar US economy to simulate the model and generate predictions of unemployment movements; in 3.2 we argue that a simulated wage from our model gives a reasonable account for macroeconomic wage movements, and although we confirm BD's findings over a longer sample than they study, their approach cannot account for aggregate wage fluctuations. Finally Section 4 contains concluding comments.

2 The model

The model is as follows. There is a horizon T, $t = 1, 2, 3 \ldots T$, where $T \geq 2$ may be finite or infinite, and a single consumption good each period. All workers are assumed to be identical, apart from the date of entry into the economy (we abstract from any tenure or experience effects on productivity). Workers are risk averse with per period twice differentiable utility function $u(w)$, $u' > 0, u'' < 0$, where w is the income/consumption received within the period; it is assumed that they cannot make credit market transactions. There is no disutility of work, but hours are fixed so that workers are either employed or unemployed. Assume that if workers are not employed in a period, they receive some low consumption level $\underline{c} \geq 0$. There is a large (but fixed) number of identical risk-neutral firms. The firm has a diminishing returns technology where output is $f(N, s_t)$ with $\partial f / \partial N > 0, \partial^2 f / \partial N^2 < 0$, where N is labor input and s_t is the current productivity shock (the sole source of fluctuations). It is assumed that a firm must always employ some (minimum measure of) workers each period.[2] Workers and firms discount the future with respective factors $\beta_w, \beta_f \in (0,1)$. There is an exogenous separation probability of $(1-\delta)$, $\delta \in (0,1)$, each period, and separated workers must seek work elsewhere. Separation occurs at the end of a period so that separated workers who find a job in the following period do not suffer unemployment. Moreover, there are a large number of workers relative to the number of firms, and we normalize the ratio of workers to firms to be one each period.[3] We assume that the "spot wage" solution is always greater than the unemployment consumption level: $\partial F/\partial N(1, s_t) > \underline{c}$ all t.

The state of nature (productivity) s_t follows a Markov process, with initial value s_1, and countable state space S, but assume that from any state s only a finite number of states $r \in S$ are reachable next period with transition probabilities: $\pi_{sr} > 0$.[4] Let $h_t \equiv (s_1, s_2, \ldots, s_t)$ be the history at t. While the firm is committed to contracts, workers are not (although we relax this later). The labor market offers a worker currently looking for work (at the start of t) a utility (discounted to t) of $\chi_t = \chi(h_t)$. We assume symmetry between the situation of a worker who is currently employed and one who is searching for work, by assuming that a worker who either is separated from, *or* quits, their current employer at t gets $\chi(h_t)$. Thus a firm must offer at least $\chi(h_t)$ to prevent its workers from quitting, and this is also the minimum utility that must be offered to hire: We assume that the firm can hire any number of workers by offering at least χ_t (and cannot hire otherwise). So the labor market is modelled as being competitive.

Our strategy will be to construct an equilibrium under the working hypothesis that firms hire each period (so they replace at least some of those who are separated), and then

[2] This can be motivated by an assumption that firms cannot produce after a period of zero production.

[3] Thus we take the fraction of a firm's workforce leaving to be exactly $(1-\delta)$. If N was finite, then the fraction leaving a firm would be random, and it can be shown that the contract could be improved by conditioning on this. (An alternative assumption to N large would be to simply rule out contracts that condition on this fraction on the grounds that verification may be impossible.)

[4] We use a Markov process to fix ideas, although the arguments go through for more general stochastic processes.

later we will find a restriction on parameters under which hiring does indeed always occur. This working hypothesis will also imply that we can ignore layoffs, but formally we will state the optimization problem *imposing* no layoffs, to avoid complicating the statement of the problem. Then we shall construct the hiring equilibrium as a solution to this problem. Finally it will follow that the hiring equilibrium is also a solution to a problem in which layoffs are permitted.[5]

We work with a representative firm, and we shall use a $*$ superscript to denote equilibrium values. At the start of date 1, after s_1 is observed, firms commit to contracts $(w_t(h_t))_{t=1}^T = (w_1(h_1), w_2(h_2), w_3(h_3), \ldots)$, $w_t(h_t) \geq 0$, which we assume are not binding on workers. *We assume equal treatment:* a worker joining subsequently, at τ after history h_τ, is offered a continuation of this same contract: $(w_\tau(h_\tau)$, $w_{\tau+1}(h_\tau, s_{\tau+1})$,$w_{\tau+2}(h_\tau, s_{\tau+1}, s_{\tau+2}), \ldots)$. (This is to be contrasted with the case where discrimination is permitted: in that case a worker joining at τ is offered a contract which in principle may be unrelated to that offered to previous cohorts.) Let $V_t(h_t)$ denote the continuation utility from t onwards from the contract:

$$\begin{aligned} V_t(h_t) &= u(w_t(h_t)) + \\ & E\left[\sum_{t'=t+1}^{T} (\beta_w)^{t'-t}\left[\delta^{t'-t}u(w_{t'}(h_{t'})) + \delta^{t'-t-1}(1-\delta)\chi_{t'}\right] \mid h_t\right], \end{aligned} \tag{1}$$

where E denotes expectation, and the term involving $\chi_{t'}$ reflects the utility after exogenous separation. Each firm also has a planned employment path $(N_t(h_t))_{t=1}^T$, where $N_t(h_t) \geq 0$.

The problem faced by the firm is:

$$\max_{(w_t(h_t))_{t=1}^T,(N_t(h_t))_{t=1}^T} E\left[\sum_{t=1}^{T}(\beta_f)^{t-1}\left(f(N_t(h_t)) - N_t(h_t)w_t(h_t)\right)\right] \tag{Problem A}$$

subject to

$$V_t(h_t) \geq \chi(h_t) \tag{2}$$

[5] Thus given that the rate of separation is exogenous, movements in unemployment occur through changes in hiring. This is consistent with the evidence reviewed in Hall (2005) that shows that the separation rate is roughly constant. Although job losses rise during recessions, the increase is usually very small in relation to the normal levels of separations.

for all positive probability $h_t, T \geq t \geq 1$, and

$$N_t(h_{t-1}, s) \geq \delta N_{t-1}(h_{t-1}) \tag{3}$$

for all positive probability h_{t-1}, all $s \in S$ with $\pi_{s_{t-1}s} > 0$, $T \geq t \geq 2$. (2) is the *participation constraint* that says that at any point in the future the contract must offer at least what a worker can get by quitting, while (3) imposes that the firm may not layoff workers.[6]

The outside option is determined by the following in a symmetric equilibrium:

$$\chi_t = N_t^*(h_t)V_t^*(h_t) + (1 - N_t^*(h_t))U_t(h_t) \tag{4}$$

where $U_t(h_t)$ is the discounted utility of a worker who is unemployed at t, so $U_t(h_t) = u(\underline{c}) + \beta_w E\left[\chi_{t+1} \mid h_t\right]$, i.e., the utility from the reservation wage plus future utility from not having a job at the beginning of $t+1$.[7] There are two cases: if the labor market at time t clears, $N_t^*(h_t) = 1$, then from (4) it must offer the utility offered by other firms. In symmetric equilibrium, other firms are offering an identical contract, and so it is the utility associated with this, $V_t^*(h_t)$, which must be offered. If, on the other hand, there is excess supply of labor,[8] $N_t^*(h_t) < 1$, the outside opportunity will depend on the probability of getting a job, $N_t^*(h_t)$. (Recall that quitters, those exogenously separated at the end of the previous period, and the unemployed from the previous period, are all in the same position.)

Necessary conditions for an optimal contract can be characterized with the help of a simple variational argument. *This is the central idea explaining why there is a lower bound on the fall of real wages;* even if the labor market is slack at $t+1$, the firm will not want

[6] More precisely, (3) implies layoffs are not *needed*. However the definition of $V_t(h_t)$ in (1) implies that a worker remains with the firm unless exogenously separated, so together these two assumptions rule out layoffs. We show in the Appendix that our solution is robust to allowing layoffs.

[7] Clearly $\chi_t \geq U_t(h_t)$, since remaining unemployed is an option for workers (i.e., if $V_t^*(h_t) < U_t(h_t)$ then no workers would accept jobs and $N_t^*(h_t) = 0$). So if $V_t(h_t) \geq \chi_t$, unemployed workers are not better off refusing a job.

[8] Intuitively, the case of excess demand for labour cannot arise in equilibrium, as an infinitessimally small increase in the wage would cure the individual firm's supply problem. In contrast, because of equal treatment the case of excess supply can arise since workers cannot undercut.

to cut the wage too far because of the desire to insure incumbents. Once this point is reached, the wage will not fall faster no matter how low the supply price of outside workers (i.e., new hires will strictly want to work for the firm in this case). Suppose we are at h_t, let N_t and N_{t+1}^s denote the optimal employment levels after h_t and (h_t, s) respectively, and consider, starting from the optimal contract, reshuffling wages between t, and $t+1$ in state s, to backload them. Increase the wage at $t+1$ after state s by a small amount Δ, and cut the wage at t by x so as to leave the worker indifferent; do not change the contract otherwise:

$$\pi_{s_t s} \delta \beta_w u' \left(w_{t+1}(h_t, s) \right) \Delta - u' \left(w_t(h_t) \right) x \simeq 0.$$

This backloading satisfies all participation constraints since worker utility rises at $t+1$, and so from this point on constraints are satisfied, but also after h_t and earlier since utility is held constant over the two periods. The change in profits (viewed from h_t) is

$$-\pi_{s_t s} \beta_f N_{t+1}^s \Delta + N_t x \simeq -\pi_{s_t s} \beta_f N_{t+1}^s \Delta + \frac{\pi_{s_t s} \delta \beta_w u' \left(w_{t+1}(h_t, s) \right) N_t \Delta}{u' \left(w_t(h_t) \right)},$$

which is positive for Δ small enough unless

$$\frac{u' \left(w_{t+1}(h_t, s) \right)}{u' \left(w_t(h_t) \right)} \leq \frac{\beta_f N_{t+1}^s}{N_t \delta \beta_w}. \tag{5}$$

Since the change in profits cannot be positive by optimality of the original contract, (5) must hold: marginal utility growth cannot exceed a certain amount. Conversely, the reverse argument (frontloading), which would be profitable if the strict version of (5) holds, cannot be undertaken (only) if next period's participation constraint binds since utility falls at $t+1$, so the constraint would be violated. We summarize the necessary condition:

Lemma 1 *In an optimal contract with perfect mobility, (5) must hold; it can only hold strictly ($<$) if the participation constraint binds at (h_t, s).*

A way then to think about the evolution of an optimal contract is that there is a “target marginal utility growth rate”:

$$\frac{u'\left(w_{t+1}(h_t, s)\right)}{u'\left(w_t(h_t)\right)} = \frac{\beta_f N^s_{t+1}}{N_t \delta \beta_w} \tag{6}$$

which will be maintained, unless a binding participation constraint at $t+1$ forces it to be lower. Put differently, this puts a lower bound on how fast real wages can decline, but a tight labor market at $t+1$ can imply that wage growth is not against this bound. Note that this lemma applies whether or not the firm is hiring at t or $t+1$.

It is instructive to compare this with the BD model (in this context) which has symmetric discounting, so assume that $\beta_f = \beta_w$. The corresponding target (gross) “growth rate” in their model is 1: wages stay constant unless a binding participation constraint forces them to be higher. The only difference arises here because the term $N^s_{t+1}/N_t\delta$ reflects the number of new hires that will be made next period for each incumbent at t. The reason is the following: if discrimination is allowed (as in BD) then each worker is treated independently, so the risk-neutral firm would like to fully insure each worker by holding wages constant.[9] In the equal treatment model, wages would likewise be constant if the term $N^s_{t+1}/N_t\delta = 1$, that is, if none of the workers who separate are replaced. In this case the firm is only having to deal with the incumbents, so this corresponds to the discrimination case. Whenever $N^s_{t+1}/N_t\delta > 1$, however, the firm is taking on additional workers at $t+1$ who will receive the same wage as the incumbents; hence the future wage is taken into account with a larger weight by the firm than by the incumbent worker, and this imparts a downward bias to the future wage in comparison with the discrimination case.

To proceed, assume provisionally that firms always hire (at all h_t) in equilibrium. That is to say, we proceed on the supposition that the constraint (3) in problem A never binds in the solution. We characterize the solution if this is the case, and later find conditions on

[9] The exogenous separation probability affects firm and worker equally—the firm only has to pay the agreed upon wage next period with probability δ (times $\pi_{s_t s_{t+1}}$) and the worker only receives the wage with the same probability—so it nets out.

a specific parametrization for which the solution satisfies this property. Finally we verify that this is also a solution to the original problem.

Then employment is determined by a standard marginal productivity equation:

Lemma 2 *If in a symmetric equilibrium hiring takes place at every h_t, then $N_t^*(h_t)$ satisfies*

$$\partial F(N_t^*(h_t), s_t)/\partial N = w_t^*(h_t). \tag{7}$$

Proof. Suppose that $\partial F(N_t^*(h_t), s_t)/\partial N > w_t^*(h_t)$. It is feasible to increase current hiring holding the wage contract constant, and consider this as the only change to the firm's plan: An increase in current hiring by $\Delta > 0$, for Δ small enough, and holding the wage constant at $w_t^*(h_t)$, would lead to an increase in current profits. At the same time, holding employment at $t+1$ constant at $N_{t+1}^*(h_{t+1})$ in all states (so hiring falls by $\delta\Delta$), is feasible for Δ small enough given hiring is positive at $t+1$. Thus there is an increase in profits at t, and no change at other dates, contradicting profit maximization. A symmetric argument, using the fact that current hiring is positive so current hiring can be *reduced* by Δ, and that $t+1$ employment can be increased by $\delta\Delta$, rules out $\partial F(N_t^*(h_t), s_t)/\partial N$ $< w_t^*(h_t)$. ■

Suppose that at some t, the participation constraint binds. Then there must be full employment and the wage is determined by marginal productivity at full employment:

Lemma 3 *Consider a symmetric equilibrium in which hiring always occurs; then the participation constraint binds at h_t if and only if $N_t^*(h_t) = 1$; moreover if the constraint binds then $w_t^*(h_t) = \partial F(1, s_t)/\partial N$.*

Proof. (i) Suppose first that the participation constraint binds,

$$V_t^*(h_t) = \chi(s_t), \tag{8}$$

and suppose contrary to the lemma that $N_t^*(h_t) < 1$. Under the hiring hypothesis, we know from Lemma 2 that $\partial F(N_t^*(h_t), s_t)/\partial N = w_t^*(h_t) > \underline{c}$ by the assumption on $\underline{c}$ and diminishing marginal productivity (i.e., $w_t^*(h_t) \leq \underline{c}$ would imply $N_t^*(h_t) > 1$). Likewise, at any t' it is not possible that $w_{t'}^*(h_{t'}) \leq \underline{c}$ since there is no feasible employment level ($N \leq 1$) for which $\partial F(N, s_t)/\partial N \leq \underline{c}$, and so Lemma 2 would be contradicted. Consequently, a worker who gets a job at t receives strictly more current utility than the utility from being unemployed, and in the future receives no less no matter when (or if) she would get a job if unemployed today, given that she would receive $w_\tau^*(h_\tau)$ regardless of when she was hired; consequently an unemployed worker is strictly worse off than an employed. Hence quitting at t will lead to a utility strictly less than $V_t^*(h_t)$ as there is a positive probability of unemployment. This contradicts (8). The equilibrium wage follows directly from Lemma 2. (ii) Now suppose that $N_t^*(h_t) = 1$. Since all workers are employed, $\chi_t(h_t)$ is defined to be equal to $V_t^*(h_t)$, so the participation constraint binds. ■

We define $\underline{w}_s^* = \partial F(1, s)/\partial N$, which in view of the above lemma is the equilibrium wage when the participation constraint binds in state s. Then we can summarize: in a symmetric equilibrium with hiring, if at $t+1$ the participation constraint isn't binding, wages are updated according to (6); if it is binding, then $w_{t+1}^* = \underline{w}_{s_{t+1}}^*$.

2.1 Empirical Implementation

To proceed to an explicit solution, in order to facilitate the empirical analysis, we put more structure on the problem.[10] This will allow us to assert that the wage updating rule is of the following simple form: given w_t^* compute w_{t+1} under the hypothesis that the participation constraint at $t+1$ is not binding; if $w_{t+1} > \underline{w}_{s_{t+1}}^*$ then the hypothesis is confirmed and w_{t+1} is the equilibrium wage; otherwise the constraint *is* binding and the equilibrium wage will be at $\underline{w}_{s_{t+1}}^*$. The structure will also allow us to demonstrate sufficient conditions for the symmetric hiring equilibrium to exist.

[10] Essentially we need the problem faced by the firm to be concave; concave production and utility functions are not sufficient to guarantee this.

From henceforth assume each firm has technology given by, at time t,

$$F(N, s_t) = M_t + a_t N^{1-\alpha}/(1-\alpha), \tag{9}$$

where $\alpha > 0$, $\alpha \neq 1$, $M_t \geq 0$ and for $\alpha < 1$, $M_t = 0$. (M_t, a_t) will evolve according to a Markov process, with $\beta_f, \beta_w < \min\left\{E[a_{t+1}/a_t \mid M_t, a_t]^{-1}, E[M_{t+1}/M_t \mid M_t, a_t]^{-1}\right\}$. Note that for $\alpha > 1$, F has an upper bound given by M_t, which given that we are modelling short-run production functions at the establishment or plant level, may be appropriate. We also assume henceforth that workers have per-period utility functions of the constant relative risk aversion family with coefficient $\gamma > 0$, $\gamma \neq 1$, described by $u(c) = c^{1-\gamma}/(1-\gamma)$.[11] Finally we assume that $\alpha\gamma > 1$.

The "target" rate of wage growth (i.e., if unconstrained at $t+1$) is, from (6),

$$\frac{w_{t+1}}{w_t} = \left(\frac{\lambda N_t}{N_{t+1}}\right)^{\frac{1}{\gamma}}, \tag{10}$$

where $\lambda \equiv \frac{\delta \beta_w}{\beta_f}$. Under the hiring assumption, we also have that the marginal product of labor equals $a_t N_t^{-\alpha}$, so that using (7),

$$N_t = a_t^{\frac{1}{\alpha}} w_t^{-\frac{1}{\alpha}}. \tag{11}$$

Combining (10) and (11) yields an equation for the evolution of wages if unconstrained at $t+1$:

$$\frac{w_{t+1}}{w_t} = \lambda^{\frac{\alpha}{\alpha\gamma-1}} \left(\frac{a_{t+1}}{a_t}\right)^{\frac{-1}{\alpha\gamma-1}} \equiv \xi\left(\frac{a_{t+1}}{a_t}\right). \tag{12}$$

where the function $\xi(.)$ simplifies notation. Moreover if firms are constrained at $t+1$, then as $N_{t+1} = 1$, $w_{t+1} = \underline{w}^*_{s_{t+1}} = a_{t+1}$ (from Lemma 3). We can now state

Proposition 4 *In a symmetric equilibrium with positive hiring, wages will satisfy*

$$w^*_{t+1} = \max\left\{\xi\left(\frac{a_{t+1}}{a_t}\right) w^*_t, a_{t+1}\right\}, \tag{13}$$

where $w^*_1 = a_1$.

[11] For $\alpha = 1$, we can specify $F(N, s_t) = M_t + \log(N)$, and for $\gamma = 1$, $u(c) = \log(c)$; all results go through.

Proof. We have just shown that w^*_{t+1} must equal one of the arguments of the max operator, depending on whether or not the participation constraint binds at $t+1$. Suppose first that $\xi\left(\frac{a_{t+1}}{a_t}\right)w^*_t > a_{t+1}$, which given $\alpha\gamma > 1$, can be rewritten as $w^*_t > \left(a_t^{-1}a_{t+1}^{\alpha\gamma}\lambda^{-\alpha}\right)^{1/(\alpha\gamma-1)}$. Suppose that the participation constraint binds at $t+1$ (so $w^*_{t+1} = a_{t+1}$ and $N_{t+1} = 1$) contrary to assertion. Lemma 1 implies that $\frac{w^*_{t+1}}{w^*_t} \geq \left(\frac{\lambda N_t}{N_{t+1}}\right)^{\frac{1}{\gamma}}$ with equality unless the participation constraint binds at $t+1$. Thus $a_{t+1}/w^*_t \geq \left(\lambda a_t^{\frac{1}{\alpha}} w_t^{*-\frac{1}{\alpha}}/1\right)^{1/\gamma}$, or equivalently $w^*_t \leq \left(a_t^{-1}a_{t+1}^{\alpha\gamma}\lambda^{-\alpha}\right)^{1/(\alpha\gamma-1)}$. So we have a contradiction. Alternatively, suppose that $\xi\left(\frac{a_{t+1}}{a_t}\right)w^*_t < a_{t+1}$, and suppose that $w^*_{t+1} = \xi\left(\frac{a_{t+1}}{a_t}\right)w^*_t$. But this implies that labor demand exceeds unity, which is incompatible with equilibrium. Finally if $\xi\left(\frac{a_{t+1}}{a_t}\right)w^*_t = a_{t+1}$, then whether the participation constraint either binds or does not, w^*_{t+1} equals this common value. To show that $w^*_1 = a_1$, note that in an optimal contract the participation constraint binds at the initial date ($t = 1$): if it did not, the firm would increase profits by cutting $w_1(s_1)$ holding the remainder of the contract fixed, and would still satisfy all participation constraints. Thus by Lemma 3 $N^*_t(h_t) = 1$, so $w^*_1 = a_1$. ■

It should be stressed that (13) must hold in a symmetric equilibrium in which hiring always takes place; i.e., it is a necessary condition.

2.1.1 A Numerical Example

We present a two period example ($t = 1, 2$). Suppose that $f(N, a_t) = a_t LogN$ where a_t is the state of productivity at time t, with $a_1 = 1$, and a_2 taking values 1.1 and 0.9, each with probability 0.5, so productivity growth is $\pm 10\%$. Workers have a utility function $u(w) = -w^{-1}$, and $\underline{c} = 0.7$. Assume there is symmetric discounting and that the survival probability δ is 0.87.

A spot market solution $\tilde{w}_t$ would solve $\partial f(N, a_t)/\partial N = w_t$ at $N = 1$ (full employment), so that $\tilde{w}_t = a_t$. From the analysis below, the only difference in the equilibrium with firm commitment and equal treatment, $(w^*_t)^2_{t=1}$, is that $w^*_2(0.9) = 0.966 > 0.9 = \tilde{w}_t(0.9)$;

i.e., the wage in the bad state at $t = 2$ does not fall sufficiently to clear the labor market. Employment is determined by the standard wage equal marginal productivity condition, so that there is employment of 0.93 in the bad state, i.e., an unemployment rate of 7%, but full employment in the good state (and in period 1). Any attempt to cut $w_2^*(0.9)$ will lead to an increase in overall wage costs because the need to compensate period 1 hires for the extra wage variability more than offsets the fact that period 2 hires would be cheaper.

2.2 Parameter values for which hiring equilibrium exists

Using the above solution, the condition for hiring to occur at $t+1$ is

$$N_{t+1}^* = a_{t+1}^{\frac{1}{\alpha}} w_{t+1}^{*-\frac{1}{\alpha}} > \delta N_t^* = \delta a_t^{\frac{1}{\alpha}} w_t^{*-\frac{1}{\alpha}}. \tag{14}$$

When will the hiring condition (14) be satisfied, and when does the model predict outcomes other than spot market ones? The hiring condition requires $a_{t+1}^{\frac{1}{\alpha}} w_{t+1}^{*-\frac{1}{\alpha}} > \delta a_t^{\frac{1}{\alpha}} w_t^{*-\frac{1}{\alpha}}$; if firms are constrained at $t+1$ then $N = 1$ and hiring is positive; if they are not, then (12) holds, and after simplification the condition becomes

$$\frac{a_{t+1}}{a_t} > \lambda^{\frac{1}{\gamma}} \delta^{\frac{\alpha\gamma - 1}{\gamma}} = \delta^{\alpha} \left(\frac{\beta_w}{\beta_f}\right)^{\frac{1}{\gamma}}. \tag{15}$$

Consequently, provided (15) holds for all states reachable with positive probability from (any) a_t that occurs with positive probability, the wage path, given by (13), with associated employment levels given by (11), *is* an equilibrium. For $\beta_w = \beta_f$, condition (15) requires that the maximum rate of *fall* of productivity should be smaller than the exogenous turnover rate raised to the power of α.

To see when outcomes differ from spot outcomes, starting from full employment in some state a_t, we need the wage to fall by less than the spot wage. Thus we need, using $w_t = a_t$, from (12)

$$w_{t+1} = \lambda^{\frac{\alpha}{\alpha\gamma-1}} \left(\frac{a_{t+1}}{a_t}\right)^{\frac{-1}{\alpha\gamma-1}} a_t > a_{t+1}$$

which can be rewritten as

$$\frac{a_{t+1}}{a_t} < \lambda^{\frac{1}{\gamma}}. \tag{16}$$

Since $\delta^{\frac{\alpha\gamma-1}{\gamma}} < 1$ (from $\alpha\gamma > 1$), (15) and (16) are compatible: there exist shocks $\frac{a_{t+1}}{a_t}$ sufficiently small ("bad") that the contract wage does not fall enough to maintain full employment, but not sufficiently small that hiring falls to zero.

Although we have found a unique solution to the necessary conditions under the hiring assumption, we have not yet shown that if this solution satisfies (14), then this is sufficient for it to be an equilibrium. This is established in Appendix A. where we consider a relaxed version of the problem faced by a potential deviant firm and show that this cannot improve on the putative equilibrium; it follows that a deviant cannot do better in a more constrained version.

2.3 Endogenizing the Equal Treatment Constraint

So far we have simply imposed equal treatment as a constraint. In the absence of this constraint, a firm will offer a lower cost contract to new hires in bad states of the world than the continuation of incumbents' contracts. Suppose however that courts cannot distinguish between a voluntary quit and one that is enforced by the employer, for example by making working conditions unpleasant, or alternatively by dismissing workers on the basis of minor contract violations. Alternatively it may be that the law stipulates that employment contracts must be "at will".[12] Thus we assume that a worker's contract specifies wages over time, but either firm or worker can terminate it at any point. Then the firm will have an incentive to replace incumbents by cheaper new hires in bad states. Given that workers will anticipate this, it does not follow that the ability to pay discriminate is advantageous to firms.

In Appendix B we show that if pay discrimination occurs, the effect of a cohort being ousted by a cheaper one can be replicated by a contract in which the incumbent cohort is

[12] The doctrine of at-will employment recognises "that where an employment was for an indefinite term, an employer may discharge an employee 'for good cause, for no cause, or even for cause morally wrong, without being thereby guilty of legal wrong'." (Wisconsin Supreme Court, 1983).

retained but paid according to the continuation of the new hires' contract; since the new hires' contract must satisfy the participation constraint, and since the incumbents who are ousted at t in this fashion will receive exactly χ_t, the incumbents cannot be worse off. (New hires brought in at on a different contract than that of incumbents can be allocated a continuation of the latter contract.) In this manner a new contract satisfying equal treatment can be constructed which is at least as good as the original contract which does not satisfy equal treatment. Hence a firm cannot suffer by *committing* to equal treatment, and we can show that the solution to the model derived above remains an equilibrium in an environment with no equal treatment requirement but with at-will contracts.

A related argument has been made in the insider-outsider context by, amongst others, Gottfries (1992) (see also Carmichael (1983) and MacLeod and Malcomson (1989)).[13] There is some evidence for this concern existing among incumbent workers when faced with the possibility of two-tier wages, see, e.g., Bewley (1999b, p. 146).

2.4 Worker Commitment

We assumed that workers are not committed to contracts, and hence it is the ex post mobility of workers which drives the wage dynamics. Suppose we drop the assumption that workers can costlessly quit the firm, for example by assuming that there is a mobility cost suffered if a worker changes jobs. Because of equal treatment, very little changes. If there is a symmetric equilibrium with mobility costs in which firms hire every period, then it must be identical to a symmetric hiring equilibrium with ex post mobility since the same participation constraint needs to be satisfied each period—if the continuation contract offers enough to hire a new worker, then it will also offer enough to prevent a worker from

[13] Gottfries and Sjostrom (2000) allow the firm to fix a termination payment for workers, payable irrespective of who initiates termination, which in principle should allow outsiders to be brought in at lower pay without creating incentives for replacement of insiders; on the other hand it increases turnover as insiders are more willing to leave. It is shown that the turnover effect may stop the firm from offering termination payments. A similar argument can be made here if we allow for on-the-job search (which does not, *per se,* affect our equilibrium) so that termination payments may induce incumbents who find a job elsewhere to leave, and if we introduce sufficiently convex turnover costs.

leaving. However the converse may not be true: an equilibrium with fully mobile workers may not be one with mobility costs. It may pay firms to choose *not to hire* in some periods (to avoid increases in wages) and let $V_t(h_t)$ fall below $\chi(h_t)$. In the mobility case a firm doing this will lose its incumbent workers too, something by assumption it wants to avoid. The two cases *will* coincide if however we additionally assumed that a firm must always hire some workers to replace separated workers; this could be justified if there are 'key' workers who cannot be replaced by reallocating incumbents and new workers must be hired and trained in these jobs; hence the participation constraint must be satisfied at each date.

3 Empirical Evidence

In this section we examine the evidence in support of our theory using both unbalanced panel data from the Panel Study on Income Dynamics (PSID) and macroeconomic data from the Bureau for Labor Statistics (BLS). We start by assessing the success of our theory in explaining unemployment. Then we use the PSID to assess the relative success of BD's empirically successful contracting model and our own in explaining macroeconomic wage movements over the cycle.

3.1 Macroeconomic Evidence: US Postwar Unemployment

In this subsection we assess how well our model fits US post war aggregate unemployment data from the BLS and the US Abstract of Statistics. In the one sector model studied above, unemployment falls to zero whenever the productivity shock is not too bad. Using a multisector model in which each sector is subject to idiosyncratic productivity shocks we will obtain more realistic unemployment levels because it is less likely that all labor markets will simultaneously clear; moreover when the aggregate productivity shock is positive, there will be more sectors with low unemployment and consequently aggregate employment is likely to be lower. Naturally this exercise depends on how well correlated the sectoral shocks are.

In order to get some realistic predictions from our model, we use actual U.S. manufacturing industry multifactor productivity processes for 17 sectors plus a residual non manufacturing sector, as provided by the BLS and then aggregate the model's predictions made for each of these sectors. This simultaneously fixes the degree of shock correlation, and also allows us to generate simulated unemployment and wage series which can be directly compared to the data. We make the extreme assumption that each sector is otherwise independent, so that the sectoral labor markets are completely segmented.[14] As we shall see, even though the model is lightly parametrized (two degrees of freedom for wages and three for unemployment), feeding it these sectoral shocks leads to unemployment and wage predictions that correspond reasonably well to the data.

As Proposition 4 makes clear, given knowledge of the model's parameters, given an initial time period where there was full employment and given a TFP series it is possible to generate the sectoral "real wage" series that would be predicted by our theory. We note that we are able to solve the model on this basis because of the convenient property that the solution depends only on actual realizations of the random processes, and not on their distributions. It is then possible to derive the corresponding implications for unemployment (rates).

We generate separate predicted wage and unemployment series for 17 manufacturing sectors and one residual non manufacturing "sector", and then aggregate using each sector's employment shares. To implement our simulations we need to calibrate the rate of change in real wages when firms are unconstrained (and productivity is unchanged), $\lambda^{\frac{\alpha}{\alpha\gamma-1}}$, and γ and α, the parameters governing (relative) risk aversion and the curvature of the production function respectively. For the coefficient of relative risk aversion, γ, we

[14] We use this data as it is the only sectoral TFP series available for such a long time scale and collected on a consistent basis; TFP data for other broad sectors such as services are only available from the early 70's onwards. It is also extreme to assume that these sectors map exactly into genuinely distinct and separate labor markets. Nonetheless we work with what is available to us and accept that what we are able to do will be more of an indicative rather than rigorous empirical exercise.

use the value 1.2 which is in the standard range for simulations and for α we use 1.4. This translates to a short-run elasticity of demand for labor of approximately -0.7. Estevão and Wilson (1998) analyzing BLS manufacturing data for a similar period that we study, found a short-run demand elasticity ranging between close to zero and -0.71 with aggregate data, and of between -0.5 and -0.89 at the 4-digit industry level for manufacturing.[15] In fact the wage solution depends only on two composite parameters, $\alpha\gamma$ and $\lambda^{\frac{\alpha}{\alpha\gamma-1}}$. Thus varying α and γ but keeping their product constant does not affect the solution for wages provided we hold $\lambda^{\frac{\alpha}{\alpha\gamma-1}}$ constant; the unemployment series will vary with $-1/\alpha$ however, as this measures the elasticity of labor demand by which $w_t/a_t > 1$ (i.e., the extent to which wages are too high for market clearing) translates into unemployment. Thus a lower value for α will magnify fluctuations in sectoral unemployment. We set $\lambda^{\frac{\alpha}{\alpha\gamma-1}}$ to be 0.98 (equivalently, $\lambda \approx .99$), which will lead to a distribution (depending on productivity shocks) of real wage declines when the constraint is not binding centred around 2% per year.[16,17] Individual predicted wage series were generated for each of the 17 two digit manufacturing sectors for which TFP data are available from the BLS and for the residual sector (whose TFP is constructed as the weighted difference of total nonfarm business TFP and manufacturing TFP, all in logs). Treating each sector as a separate economy

[15] Hamermesh (1993) reports that a lower elasticity, around -0.3, is typical.

[16] Elsby (2005) charts the distribution of real wage changes in the PSID over a relatively low inflation period (so surprise inflation is less likely to lead to unanticipated real wage falls), 1983-1992; real wage falls rarely exceed about 6%, with a spike around 2-4%. Given that the data includes displaced workers who will receive wage cuts in their new jobs, our choice of the upper end of this range seems reasonable. Likewise Christophides and Stengos (Apr 2003) find from Canadian wage contract data in the unionized sector that most real wage reductions in the 1990s were of the order of 1-2%.

[17] Alternatively we could calibrate this term by calibrating its constituent parts $\beta_f, \beta_w, \delta, \alpha$ and γ. Certainly, given that annual turnover in the PSID is as high as 30%, this is likely to lead to a lower value for $\lambda^{\frac{\alpha}{\alpha\gamma-1}}$, which in turn would make labor markets more likely to clear. On the other hand, a richer thory would be likely to lead to a number of offsetting elements. First, plant turnover, from which we abstracted, would have the opposite effect from worker turnover on the target wage change. Secondly, it may be that much of the observed turnover is intentional in the sense workers are planning to leave when an appropriate opportunity comes along (particularly in the secondary sector) and are unlikely to be retained even by an appropriate wage policy. Such turnover should not enter into the expression for target wage growth. It is actually only separations which are *unanticipated* by workers who are not in the above category which matters for determing the target wage growth. (This can be seen in an extreme case; suppose that 30% of the workforce plans to leave at the end of the current period, to be replaced, and the other 70% will stay if wages are as good as elsewhere. Then the wage would in fact stay constant—if $\beta_f = \beta_w$ and assuming next period's participation constraint does not bind—since both stayers and the firm trade-off marginal wage changes at the two dates equally. The relevant δ is one.)

we used the model and the relevant TFP series to generate a simulated unemployment series for each sector. An aggregate unemployment index was then constructed as the weighted average of the individual sector simulations with weights given by employment shares.[18] The results for actual aggregate unemployment and model unemployment are graphed in Figure 1. The simulated model unemployment series appears to do quite well. In particular the volatility of actual unemployment is reasonably well matched as are the peaks and troughs of the actual series. Finally regressing actual unemployment (u) on the model predicted unemployment ($\widehat{u}$) gives (standard errors in brackets):

$$u_t = \underset{(.128)}{const + .734\widehat{u}_t} \qquad R^2 = .42 \qquad t = 1955, \ldots, 2001.$$

This confirms what Figure 1 indicates, namely that there is a highly significant relationship between the actual and predicted series[19]. Finally as a robustness check on the correlation coefficient between u and $\widehat{u}$ of 0.65, we allowed λ to vary between 0.98 and 0.995, α, γ between 1.1 and 2, and found that the correlation coefficient varies between 0.59 and 0.66.

3.2 Macroeconomic Evidence from the PSID

We now assess the ability of the model's predicted wage series—using the same calibration as above—to explain movements in aggregate wages garnered from the PSID. One advantage of using the PSID for this purpose is that it was used by BD and this allows us to replicate their analysis (Appendix C confirms their results on our longer sample) and assess the relative success of their key variable against our model prediction in explaining macro wage movements. Another advantage is that the aggregate annual wage we extract has been purged of the effects of changes from year to year in worker characteristics. By

[18] The (fixed) employment weights were taken from the middle year of the sample and the manufacturing sector as a whole was assumed to be 50% larger than the residual sector - roughly consistent with the average relative actual sizes over the period.

[19] We have added 4% to our simulated series to allow for a constant level of frictional unemployment.

contrast the BLS aggregate wage series may move purely because of compositional changes of the working labor force over the business cycle. Given that our theory makes predictions for a representative individual the PSID panel is in many ways more appropriate benchmark target than is the BLS aggregate data.[20] There is a problem with this however. Our theory explains macroeconomic movements in wages purely in terms of productivity shocks. A glance at Figure 2 shows that between 1968 and 1993 real wages (non farm private sector) and aggregate TFP (likewise private nonfarm) have opposite trends—real wages fall whilst TFP rises—a general feature of postwar US data. But as is well known, there is an increasing discrepancy between total (wage plus nonwage) compensation and wages, due largely to sharp rises in company medical and pension, etc. benefits. If we look at total worker compensation (Figure 2 again) we see this clearly. Interestingly total compensation has roughly the same trend as TFP. In our model wages are driven largely by the demand for labor, which depends on total compensation not just its wage element. In what follows therefore we adjust annual real wage measures extracted from the PSID to allow for non wage compensation. As a final check on our empirical results we attempt to match the model's wage predictions with wage estimates from the PSID allowing for different trends via detrending and as we shall see it does not substantially affect our main results.

We collected data from the PSID for the years 1968 to 1993—encompassing the BD years of 1976-84. We collected data on private sector employees' hourly wage and a basic set of characteristics: gender, age, education, occupation, tenure (in months), race and state of residence. For the macro variates we use the annual CPI and monthly aggregate unemployment rates as reported by the BLS. Whilst we did not collect data on all of the BD characteristics[21] we have arguably the most important and most frequently recorded ones. Unlike BD (but not Grant (2003)) we do not exclude women and individuals who

[20]The BLS Employment Cost Index represents an attempt to measure year to year wage movements whilst controlling for changes in the year to year composition of the labour force. However this series starts relatively recently and a complete set of ECI figures are unavailable for the years in our sample.

[21]BD also have union, marriage and industry dummies

were in the workforce prior to 1947—this reflects our desire to be as comprehensive as possible in order to be able to generate macroeconomic results using the data later on.

The differences in data collection make it impossible for us to replicate BD's results exactly but we check whether the broad features of our sample are in line with theirs, and in Appendix C we report results confirming their basic findings over our longer sample. Table 1 gives sample means and standard errors of our and BD's main variates for the BD years. The table shows that we have nearly 30% more data points than do BD and that average wages in our sample are around 11% lower than in BD. Both of these differences are largely though not wholly down to the inclusion of women (excluding women, for example, gives an average log wage less than 2% below BD's). Average tenure is a little higher in BD but their key variable, minimum unemployment rate during job tenure (henceforth we refer to this variable as just "$\min u$") is rather lower than in our data. We should expect some differences here as we did not adopt BD's adjustment method. Instead we simply use the PSID variate "number of months with current employer" without adjustment.[22]

The predicted wage series for 1968-1993 from our model was an input into the analysis of unemployment undertaken above. (Recall that it was a weighted average of the model's predicted wages for 17 manufacturing sectors and a residual sector.) To apply PSID data to our macro analysis we must do two things. First, because it reports wages not total compensation, some adjustment must be made when matching it with our simulated series. Second one has to take control for the effect of the changes in yearly characteristics (e.g., the proportion of professionals in the year) which vary quite markedly over the sample years. We deal with the second issue first. We may write the following empirical model for PSID observations, w_{it} :

$$w_{it} = \pi^{'} c_{it} + \alpha \widetilde{m}_{it} + \alpha \overline{m}_t + \theta^{'} x_t + \varepsilon_t + \epsilon_{it}, \tag{17}$$

where w_{it} are individual i's log of wages deflated by the annual CPI in year t $(i = 1, ...n_t)$,

[22] For 1968 to 1974 the only tenure related question in the PSID refers to length of time in job rather than with employer which is somewhat ambiguous.

c_{it} is a $k \times 1$ vector of individual i's characteristics (6 occupation dummies, sex, 3 race dummies, tenure, tenure squared, age, age squared, state of residence and 8 education dummies) at time t, x_t is a vector of variables that have direct common influence on PSID wages (trend, cyclical variates, etc., to be specified below) with $\pi = (\pi_1, \pi_2 ...)$ and $\theta = (\theta_1, \theta_2 ...)$ being a conformable vector of parameters, and $\widetilde{m}_{it} = m_{it} - \overline{m}_t$ with m_{it} being the BD measure of individual $i's$ tightest labor market (i.e., the minimum unemployment rate) during his current job tenure at time t ("$\min u$") and $\overline{m}_t$ being the sample mean of m_{it} in year t.[23] The errors $\varepsilon_t = (\varepsilon_1, \varepsilon_2 ... \varepsilon_T)$ and $\epsilon_{it} = (\epsilon_{11} ... \epsilon_{n_1 1}, \epsilon_{12} ... \epsilon_{n_2 2} ... \epsilon_{1T} ... \epsilon_{n_T T})$ are assumed to be mean zero $i.i.d.$ Taking annual averages of (17) gives the "macro" model for PSID wages as

$$\overline{w}_t = \pi^{'} \overline{c}_t + \alpha \overline{m}_t + \theta^{'} x_t + \varepsilon_t + O_p(n_t^{-1}), \tag{18}$$

where $\overline{c}_t = \sum_{i=1}^{n_t} \frac{c_{it}}{n_t}$ contains the annual means of c_{it}. In effect this means that the only role of characteristics in the macro model is to allow for year to year compositional changes in the panel. If for example the 1968 data had a preponderance of professionals but the 1969 data was dominated by unskilled workers, we would expect a drop in wages that reflects a combination of the change in composition and the wage differential between unskilled and professional workers. From the viewpoint of our macro theory we are really only interested in α and θ in (18) because $\pi^{'} \overline{c}_t$ merely picks up aggregate wage movements associated with changes in the mix of characteristics in any particular year in the PSID. Explicitly we wish to analyse the relative importance of $\overline{m}_t$ and rival macro variables in x such as trend, the simulated wages from our model and TFP. One obvious and direct way to do this would be to treat (18) as a simple regression (assuming that the $O_p(n_t^{-1})$ terms are negligible) but there are two problems with this. First we have over 60 characteristics plus at least two further (macro) regressors but only 26 annual data points. Second, the left hand side variable $\overline{w}_t$ excludes non wage benefits (pension, health insurance etc.) which, from arguments given previously, are the relevant compensation measure in the

[23] Constant terms are subsumed in the characteristic dummies and for simplicity are suppressed in the notation.

theory. The answer to the first conundrum is to exploit the cross section of the panel to obtain estimates of π and to the second problem is to adjust wages for non wage benefits using aggregate data. We achieve these aims via two stage estimation. In the first stage we estimate (17) but allow $\alpha\overline{m}_t+\theta^{'}x_t+\varepsilon_t$ to be absorbed into year dummies. We therefore estimate the model

$$w_{it} = \pi^{'}c_{it} + \alpha\widetilde{m}_{it} + \sum_{t=1}^{T}\beta_t D_t + \epsilon_{it}, \tag{19}$$

where D_t takes the value 1 in year t but zero otherwise.[24] All of the macro effects are now captured in the estimates $\widehat{\beta}_t$ so that we can write

$$\widehat{\beta}_t = \alpha\overline{m}_t + \theta^{'}x_t + \varepsilon_t + O_p(n_t^{-1}). \tag{20}$$

To adjust for non wage benefits we add the BLS measure of the log of the ratio of aggregate total compensation to wages (r_t) to the left hand side of (20), and denote this by $\widehat{\tau}_t \equiv \widehat{\beta}_t + r_t$.

There is reasonably strong evidence that both $\widehat{\tau}_t$ and $\widehat{\beta}_t$ are trend stationary - ADF statistics (with one augmentation) for the detrended series were borderline significant for $\widehat{\tau}_t$ at -3.41 and more robustly significant for $\widehat{\beta}_t$ at -3.65 respectively when compared with the critical value of -3.41. It is therefore reasonable to proceed under the trend stationarity assumption and estimate

$$\widehat{\tau}_t \equiv \widehat{\beta}_t + r_t = \alpha\overline{m}_t + \theta^{'}x_t + \varepsilon_t + O_p(n_t^{-1}) \tag{21}$$

free of restrictions. In particular we do not impose the restriction that the coefficient on $\overline{m}_t$ be equal to α in (19)—this allows $\overline{m}_t$ the freedom to have a macroeconomic impact that is separate from and unconstrained by its cross sectional/within year effects on individual workers. Finally and again as a robustness check we also run versions of (21) with $\widehat{\beta}_t$ as the LHS variable. In BN's model, $\overline{m}_t$ explains $\widehat{\tau}_t$ (or $\widehat{\beta}_t$) and in our model, because of equal treatment, only the model predicted wage should matter once individual characteristics are controlled for.

[24] Of course we may relax the i.i.d. assumption at this stage and allow for general forms of heteroscedasticity.

Figure 3 plots the detrended PSID estimates of total compensation ($\widehat{\tau}_t^d$) together with detrended macro $\min u$ ($\overline{m}_t^d$) and detrended model predicted total compensation (w_t^{*d}). Visual inspection suggests that the model's predicted total compensation matches the dynamics of the PSID estimates rather better than does the macro measure of $\min u$. More serious is the apparent positive association of $\overline{m}_t$ with PSID total compensation estimates which is perverse: larger values of the yearly averaged $\min u$ should imply lower total compensation in the year not higher.

Table 2 gives regression results for a number of versions of (21). The regression of $\widehat{\tau}_t^d$ on $\overline{m}_t^d$ (first line) gives implausible results because the $\overline{m}_t^d$ variable is insignificant and incorrectly signed: the positive coefficient implies that an increase in average $\min u$ from one year to another will lead to higher worker compensation, not lower as the theory would suggest. By contrast and turning to lines 3 to 6 of the table the two regressions of $\widehat{\tau}_t^d$ on w_t^{*d} and $\widehat{\tau}_t^d$ on $\overline{m}_t^d$ and w_t^{*d} show the model's predicted series to be highly significant in explaining total compensation estimated from the PSID. On its own, it captures 55% of the variation in $\widehat{\tau}_t$ compared with 6% for $\overline{m}_t^d$ and has a coefficient very close to unity. Adding detrended aggregate TFP (y_t^d) to the regression (line 7) adds somewhat to the explanatory power of the equation but not the size nor significance of w_t^{*d}. Although w_t^{*d} is substantially more significant than TFP, this regression does suggest that our model is some way from being a comprehensive explanation of the dynamic movements in total compensation—not surprising given the simplicity and parsimony of the model.

For completeness we report in lines 9 to 14 the regression of detrended PSID time effects unadjusted for non-wage benefits ($\widehat{\beta}_t^d$) on $\overline{m}_t^d$ (line 9), on w_t^{*d}(line 11) and on w_t^{*d} and $\overline{m}_t^d$ (line 13). Overall the results are similar—w_t^{*d} is important and robustly significant whilst $\overline{m}_t^d$ is insignificant. In sum then our model predictions seem to track the dynamics of both PSID total compensation and PSID wage measures well.

Finally, as we have just seen, $\overline{m}_t^d$ is not significant in the regressions, despite the fact that $\min u$ is a statistically significant determinant of individual wages (see Appendix C). Grant (2003) points out that its importance in accounting for the time series variation in wages may not be great because the variation in $\min u$ over time is not very large. If we add year dummies to a basic BD regression to absorb macro effects (hence we regress log wages on characteristics, $\min u$ and year dummies) then we find, not surprisingly, that in both the short (BD) and long (68-93) samples $\min u$ explains less than 0.5% of the within year variation in log wages and accounts for less than 1% of the explained variance of wages in the pooled regression. More important from the macroeconomic viewpoint is $\min u$'s contribution to the year by year/macro movements in log wages. Trend deviations in mean $\min u$ explain only 3.8% of the variation in the trend deviations of log wages (see Table 2). It must of course be stressed that BD's model is not formulated to explain macroeconomic phenomena, but to test alternative theories of contracting. Indeed, in cross section regressions, not reported in detail, $\min u$ remains significant. Thus it appears successful in explaining differentials between workers within a year, but it does not satisfactorily explain year to year movements in real wages.

4 Closing Comments

This paper has analyzed a model in which firms cannot pay discriminate based on year of entry to a firm. The trading-off of wage insurance for incumbents against the desire to be flexible in the hiring wage paid to new hires leads to wages which do not always clear the labor market. On the other hand, the need to hire means that wages have to respond to sufficiently positive shocks, so that wages in the long-run respond to productivity movements. We find that these two features imply that the model gives a reasonable account of unemployment and compensation in recent US history.

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5 Appendix A: Sufficiency

Assume that the solution satisfying the necessary condition (13) satisfies (14) (recall that (15) guarantees this). We show that this is a solution to Problem A; moreover it is a solution to the problem *where layoffs are permitted.* We shall consider a relaxed version of the problem faced by a potential deviant firm (i.e., where $(\chi_t)_{t=1}^{T}$ is fixed at the putative equilibrium levels) and show that this cannot improve on the putative equilibrium and use this to demonstrate that a deviant cannot do better in Problem A , nor when allows layoffs are allowed. Layoff pay is ruled out for simplicity[25] and we assume that a laid-off worker receives χ_t.

We deal with the case $T < \infty$.[26] We consider the problem as formulated earlier, but in which the firm has no employment constraints, so that it solves Problem A without the constraint (3) (that is, it can costlessly reduce its workforce at any time, and only has to respect the participation constraints, which do not take into account layoffs, this despite the fact that a worker in calculating his utility from the contract should take into account the layoff possibility). We call this Problem A^R. We also consider the problem in which layoffs are permitted (these could be cohort dependent), but in which workers do factor in layoff probabilities into their calculations; this is the natural economic problem and we call it Problem B (for brevity's sake we omit its statement).

Consider the static problem of maximizing profits given that workers receive utility u, so that $w = ((1-\gamma)\,u)^{1/(1-\gamma)}$. Substituting from (11) for N (this must hold in the static problem), yields profits of

$$\Pi\,(u, a_t) \equiv M_t + \frac{a_t^{\frac{1}{\alpha}}\,\alpha\,((1-\gamma)\,u)^{-\frac{1-\alpha}{\alpha(1-\gamma)}}}{1-\alpha}. \tag{22}$$

[25] Allowing for layoff pay raises the possibility that the firm may want to replace its entire workforce in bad states in order to benefit from low outside wages. In the absence of layoff pay the incumbents will factor this into their calculations when deciding whether to join the firm, and there can be no benefit from this to the firm (since an equivalent policy would be to retain the incumbents and to offer them a continuation utility of χ_t). If the firm could insure the incumbents it lays off, however, then the picture is less clear cut. Nevertheless, when a bad shock occurs, if the firm is downsizing, as is likely to be the case, it cannot benefit from replacing its workforce. Alternatively, we could introduce turnover costs which would render such a strategy unprofitable.

[26] If $T = \infty$, then by the assumption on δ_f, we can show that profits are finite and standard arguments can be used to extend the finite horizon argument.

As $\alpha\gamma > 1$, this is a strictly concave function of u. We can formulate Problem A^R faced by the firm as:

$$\max_{(u_t(h_t))_{t=1}^{T}} E\left[\sum_{t=1}^{T}(\beta_f)^{t-1}\Pi\left(u_t(h_t), a_t\right)\right] \qquad \text{(Problem A}^R\text{)}$$

subject to

$$\tilde{V}_t\left(h_t\right) \geq \chi\left(h_t\right) \tag{23}$$

for all positive probability $h_t, T \geq t \geq 1$, where

$$\begin{aligned} \tilde{V}_t\left(h_t\right) &= u_t(h_t) + \\ & E\left[\sum_{t'=t+1}^{T}\left(\beta_w\right)^{t'-t}\left[\delta^{t'-t}u_t(h_{t'}) + \delta^{t'-t-1}\left(1-\delta\right)\chi_{t'}\right] \mid h_t\right]. \end{aligned} \tag{24}$$

Thus the maximand is strictly concave and the constraints are linear. The Slater condition is satisfied by, for all h_t, $u_t(h_t) = u(w^*(h_t) + \varepsilon)$, for $\varepsilon > 0$. Moreover it is straightforward to show that the Kuhn-Tucker conditions are satisfied at the putative equilibrium, hence the necessary conditions developed in the text are *sufficient* for existence in the relaxed problem.

Thus provided (14) holds, a solution to Problem A^R exists and coincides with the solution to Problem A, and is the one we identified. Consider now a feasible plan in Problem B which involves layoffs occurring. Suppose we implement the same wage (i.e., utility) plan in Problem A^R; (23) must hold given that any cohort facing a layoff probability in Problem B will get weakly less continuation utility than $\tilde{V}_t$. Since, given w_t, and hence u_t, per-period profits are maximized in Problem A^R, the solution to the latter must weakly dominate the solution to Problem B. Putting this together, if (14) holds, our solution is also a solution to Problem B.

6 Appendix B: "At Will" Contracting Implies Equal Treatment

We maintain the assumption that the firm can commit to wages, which here includes commitment to the contracts of cohorts yet to be hired, *but it cannot commit not to replace workers.* We show that it is optimal to commit to a single wage policy, and that our solution for the equal treatment case remains a solution with potential discrimination. For simplicity we treat the case of T finite (although the argument can be extended to $T = \infty$).

Potentially a contract now will depend on the entry date to the firm, so the wage at t of a cohort entering at $\tau \leq t$ is denoted w_t^τ, with N_t^τ the employment level at t; $N_t(h_t) \equiv \sum_{\tau=1}^{t} N_t^\tau(h_t)$ now denotes aggregate employment at the firm. We write a wage contract as $\omega = \left((w_t^\tau(h_t))_{\tau\leq t}\right)_{t=1}^{T}$, and an *employment plan* as $\nu = \left((N_t^\tau(h_t))_{\tau\leq t}\right)_{t=1}^{T}$ where $N_{t+1}^\tau \in [0, \delta N_t^\tau]$ for all $\tau \leq t$, $t = 1, \ldots, T-1$. We allow for layoffs: a $\tau-$cohort worker who is still employed at t is forced to leave at $t+1$ with probability $\left(1 - \frac{N_{t+1}^\tau}{N_t^\tau}\right)$ for $\tau < t$ (this assumes no quits at $t+1$); this combines both the exogenous separation rate and any enforced terminations. Given (ω, ν), $V_t^\tau(h_t; \omega, \nu)$ is the cohort's continuation utility from remaining with the firm at t which takes into account the termination and quitting possibilities (we define it for a worker at t who will *not* be laid off nor quit at t), defined recursively as:

$$V_t^\tau(h_t; \omega, \nu) = u(w_t^\tau) + \beta_w E\left[\frac{N_{t+1}^\tau}{N_t^\tau} \max\{V_{t+1}^\tau, \chi_{t+1}\} + \left(1 - \frac{N_{t+1}^\tau}{N_t^\tau}\right)\chi_{t+1} \mid h_t\right], \tag{25}$$

where $V_{T+1}^\tau, \chi_{T+1} \equiv 0$. If $V_t^\tau < \chi_t$ then the worker is better off quitting at t.[27] V_t^τ is defined after h_t such that $N_t^\tau(h_t) > 0$. If $N_t^\tau(h_t) = 0$ (i.e., the cohort is not employed on the equilibrium path after h_t), however, we need to define V_t^τ in case the firm deviates. For simplicity we assume that in this case workers are pessimistic and always assume that they will be replaced in the following period, although the results do not depend on this.[28]

Problem A of Section 2 can be reformulated with cohort dependent wages and employment levels. We call this Problem A' below. Given (ω, ν), define the set of *best employment plans* as

$$\Lambda(\omega, \nu) = \arg\max_{\tilde{\nu}} E\left[\sum_{t=1}^{T} (\beta_f)^{t-1}\left(f(\tilde{N}_t(h_t)) - \sum_{\tau=1}^{t} \tilde{N}_t^\tau(h_t) w_t^\tau(h_t)\right)\right]$$

subject to

$$\tilde{N}_t(h_{t-1}, s) \geq \delta \tilde{N}_{t-1}(h_{t-1}) \tag{26}$$

[27] For simplicity assume that no quits occur when $V_t^\tau = \chi_t$.

[28] So $V_t^\tau = u(w_t^\tau) + \beta_w E\left[\chi_{t+1} \mid h_t\right]$. The argument is hardest to prove under these pessimistic beliefs.

for all positive probability (h_{t-1}, s), $T \geq t \geq 1$; and to the cohort specific participation constraint, for all $\tau \leq t$, all positive probability $h_t, T \geq t \geq 1$, all τ such that $\tilde{N}_t^\tau > 0$:

$$V_t^\tau (h_t; \omega, \nu) \geq \chi_t (h_t) . \tag{27}$$

(27) restricts employment to cohorts satisfying the participation constraint *calculated on the assumption that ν will be implemented.* Then an optimum solves

$$\max_{(\omega,\nu)} E\left[\sum_{t=1}^{T}(\beta_f)^{t-1}\left(f(N_t(h_t)) - \sum_{\tau=1}^{t} N_t^\tau(h_t) w_t^\tau(h_t)\right)\right] \tag{Problem A'}$$

subject to

$$\nu \in \Lambda (\omega, \nu) . \tag{28}$$

(28) requires not only that the appropriate participation constraints are satisfied, but that the plan is credible, since the firm cannot commit to its employment policies; otherwise, for example, it may commit to paying high wages to a cohort at the end of their employment, and then replace the cohort with cheaper new hires when the high wages kick in.

Proposition 5 *A solution to Problem A (i.e., an optimum under equal treatment) is also a solution to Problem A' (i.e., when wages can vary across cohorts).*

Proof. The main argument is to show that the firm does not suffer by committing to a single wage contract.

(1) Let $(\tilde{\omega}, \tilde{\nu})$ be a solution to Problem A'. Suppose that $(\tilde{\omega}, \tilde{\nu})$ involves cohort 1 being (possibly partially) ousted or quitting at some h_t, $t > 1$ (i.e., $N_t^1 < \delta N_{t-1}^1$); by (26) this implies $N_t^t > 0$. We change the contract as follows: replace the continuation contract from h_t of cohort 1 by that of cohort t, and retain all of cohort 1, holding total employment N_t constant (feasible by (26)), *unless* the ousting is partial and the continuation contract from h_t of cohort 1 offers higher continuation utility than that of the new hire contract, in which case make no change. In the former case if on any continuation path after h_t cohort t is ousted or quits at $t' > t$, we repeat the exercise, replacing the (new) continuation contract from $h_{t'}$ of cohort 1 by that of cohort t'; in the latter case do the same but for cohort 1 instead (i.e., if the remainder of cohort 1 was replaced at $t' > t$ in the original

contract). This process is repeated until T. Denote by (the random variable) $\tau(t)$ the cohort at time t whose wage is selected by this contract replacement process (so that $\tau(t)$ $= 1$ until the first replacement occurs at time $t' > 1$, say, then $\tau(t) = t'$ from $t = t'$ until the next replacement, etc.).

Next, we replace the contract of each cohort $\tau > 1$ by the continuation of cohort 1's (new) contract, thus creating an equal treatment contract. And $\tilde{\nu}$ is changed as follows: in the previous step we held N_1^1 constant, and thereafter retained the entire cohort (so $N_{t+1}^1 = \delta N_t^1$). Now adjust N_2^2 so that total employment $N_2 \equiv N_2^1 + N_2^2$ is constant, thereafter retaining the entire cohort, and so on, so that N_t is unchanged at all t (feasible by (26)). Call this new contract and employment plan $(\hat{\omega}, \hat{\nu})$.

(2) We show that the cost of each new cohort under $(\tilde{\omega}, \tilde{\nu})$ is at least the cost of any retained incumbent cohort, so total costs cannot rise under $(\hat{\omega}, \hat{\nu})$, and as output is held constant, profits cannot fall; moreover participation constraints are satisfied. In more detail: We can define the minimum cost of retaining a member of cohort $\tau \le t$ at t recursively as follows. $C_T^\tau\left(h_T; \tilde{\omega}, \tilde{\nu}\right) = \tilde{w}_T^\tau$ if $\tilde{N}_T^\tau > 0$ and ∞ otherwise. $C_{T-1}^\tau\left(h_{T-1}; \tilde{\omega}, \tilde{\nu}\right) =$ $\tilde{w}_{T-1}^\tau + \delta\beta_f E\left[\min\{C_T^\tau\left(\tilde{\omega}, \tilde{\nu}\right), C_T^T\left(\tilde{\omega}, \tilde{\nu}\right)\} \mid h_{T-1}\right]$ if $\tilde{N}_{T-1}^\tau > 0$ and ∞ otherwise; in general, $C_t^\tau\left(\tilde{\omega}, \tilde{\nu}\right) = \tilde{w}_t^\tau + \delta\beta_f E\left[\min\{C_{t+1}^\tau\left(\tilde{\omega}, \tilde{\nu}\right), C_{t+1}^{t+1}\left(\tilde{\omega}, \tilde{\nu}\right)\} \mid h_t\right]$ if $\tilde{N}_t^\tau > 0$ and ∞ otherwise. This can be interpreted as the cost of retaining a τ-cohort worker given that in future she will be replaced by any cheaper cohorts, where potential replacement cohorts are restricted to those actually hired under $(\tilde{\omega}, \tilde{\nu})$. It must be the case that

$$C_t^\tau\left(\tilde{\omega}, \tilde{\nu}\right) \le C_t^t\left(\tilde{\omega}, \tilde{\nu}\right) \tag{29}$$

if $\tilde{N}_t^\tau > 0$, otherwise it would be cheaper to replace this cohort at t. To see this, start at T: clearly (29) holds since $\tilde{w}_T^\tau \le \tilde{w}_T^T$ for any employed cohort τ, assuming $\tilde{N}_T^T > 0$ so the participation constraint is satisfied for new hires (if $\tilde{N}_T^T = 0$ then $C_T^T\left(\tilde{\omega}, \tilde{\nu}\right) = \infty$, so again (29) holds). At $T-1$ suppose that $\tilde{N}_{T-1}^\tau > 0$, but that $C_{T-1}^\tau\left(\tilde{\omega}, \tilde{\nu}\right) > C_{T-1}^{T-1}\left(\tilde{\omega}, \tilde{\nu}\right)$ (so that $\tilde{N}_{T-1}^{T-1} > 0$). Then the fraction δ of cohort τ who survive to T, or their replacements at T (by (26)), are paid under $\tilde{\omega}$ at least $\min\{C_T^\tau\left(\tilde{\omega}, \tilde{\nu}\right), C_T^T\left(\tilde{\omega}, \tilde{\nu}\right)\}$, so that the cost discounted to $T-1$ is at least $C_{T-1}^\tau\left(\tilde{\omega}, \tilde{\nu}\right)$, while they could be replaced at $T-1$ at a cost of $C_{T-1}^{T-1}\left(\tilde{\omega}, \tilde{\nu}\right)$ (since only employed cohorts are used, this new employment plan satisfies (27)), hence costs are not minimized, so $\tilde{\omega} \notin \Lambda(\tilde{\omega}, \tilde{\nu})$, a contradiction. A similar argument applies at all earlier dates. Likewise, if cohort τ is partially replaced at t, (29) holds with equality. Now consider $(\hat{\omega}, \hat{v})$. Cohort 1's contract must satisfy[29]

$$C_t^1(\hat{\omega}, \hat{\nu}) \leq C_t^t(\tilde{\omega}, \tilde{\nu}) \tag{30}$$

at all t since $C_T^1(\hat{\omega}, \hat{\nu}) = C_T^{\tau(T)}(\tilde{\omega}, \tilde{\nu})$, and working backwards, $C_{T-1}^1(\hat{\omega}, \hat{\nu}) = C_{T-1}^{\tau(T-1)}(\tilde{\omega}, \tilde{\nu})$ as any states at T at which replacements occur $(\tau(T) = T)$ must satisfy $C_T^{\tau(T-1)}(\tilde{\omega}, \tilde{\nu}) \geq C_T^T(\tilde{\omega}, \tilde{\nu})$, etc. So $C_t^1(\hat{\omega}, \hat{\nu}) = C_t^{\tau(t)}(\tilde{\omega}, \tilde{\nu}) \leq C_t^t(\tilde{\omega}, \tilde{\nu})$ by (29) as $N_t^{\tau(t)} > 0$ by definition of $\tau(t)$. Now consider, at any t, net hires (after exogenous separation) $\tilde{N}_t - \delta\tilde{N}_{t-1}$. Because of (26) we can follow this group (declining in size at rate δ) through to T, allowing for replacements (if there is partial replacement then this may not be unique, but this is inconsequential). The per worker cost at t associated with this group is $C_t^t(\tilde{\omega}, \tilde{\nu})$ (it cannot be lower by definition of $C_t^t(\tilde{\omega}, \tilde{\nu})$; if it is greater costs are not being minimized). Hence under $(\hat{\omega}, \hat{\nu})$, the cost of each group of net hires of size $\hat{N}_t - \delta\hat{N}_{t-1} (= \tilde{N}_t - \delta\tilde{N}_{t-1})$ is no higher by (30), and so total costs are no higher. Finally, cohort 1's participation constraint is satisfied at all t under $(\hat{\omega}, \hat{\nu})$ since if complete replacement occurs under $(\tilde{\omega}, \tilde{\nu})$, the substitution contract in the construction must yield at least χ_t while the ousted or quitting cohort receives exactly χ_t, and if substitution is partial, the construction gives the higher of the two continuation utilities. Consequently the participation constraint is satisfied for all employed cohorts τ.

(3) Take a solution to Problem A, say (ω', ν') in the current notation. Any ν'' which satisfies (26) and (27) for (ω', ν') would also satisfy (2) and (3) for ω' in Problem A (if replacement occurs in ν'' this can be eliminated as above; if ν'' involves employment of a cohort not employed under ν', the participation constraint (2) is satisfied since it is satisfied for the equal treatment contract at all dates) and so cannot offer higher profits. Thus $\nu' \in \Lambda(\omega', \nu')$, so (ω', ν') is a candidate solution to Problem A'. Equally, since $(\hat{\omega}, \hat{\nu})$ satisfies (2) and (3), it is a candidate solution to Problem A so the solution to Problem A must yield at least the profits from $(\hat{\omega}, \hat{\nu})$ and hence from $(\tilde{\omega}, \tilde{\nu})$, the optimum. Thus (ω', ν') offers the same profits as $(\tilde{\omega}, \tilde{\nu})$ and is optimal in Problem A'. ■

[29] $C_t^1(\hat{\omega}, \hat{\nu})$ is just the cost from t associated with a continuously employed worker in the equal treatment contract $\hat{\omega}$ as there are no layoffs or quits.

Thus the solution to the equal treatment problem derived in Section 2 remains a solution without the equal treatment restriction provided contracts are "at will".[30]

7 Appendix C: Beadry-DiNardo Regressions

Several empirical studies (see Introduction) have largely confirmed the robustness of BD's main empirical findings that the minimum rate of unemployment since hiring is a statistically important determinant of the current wage of an individual. In particular Grant (2003) extends BD's analysis (using six cohorts from the National Longitudinal Surveys) to cover the time period 1966 to 1998. He finds that the significance and importance of $\min u$ is broadly robust with respect to the addition of fixed time effects and using subsamples selected on the basis of age, and sex, although current unemployment levels also have some explanatory power. Here, we perform a similar exercise on our data, which as already noted, considerably extends the BD years.

Table 3 gives the results for a BD style regression of log real hourly wages on characteristics (see Section 3.2), $\min u$ (m_{it}) and BD's two other "competitor" labor market condition variates, namely, the unemployment rate at the date of hiring (u_{0it}) and the current unemployment rate (un_t), for the BD years and our full sample.[31] Despite some differences in data construction and characteristics used we see that our results are quite close to those found by BD and particularly so for the key variate $\min u$ itself. In further regressions (not reported here but available on request), we follow Grant in finding that the results are qualitatively robust with respect to the addition of a trend and year dummies—$\min u$ is always negative, highly significant and with a coefficient value between -.02 to -.06 with u_{0it} and un_t (obviously we drop the latter when year dummies are added) poorly determined and often incorrectly signed.

Finally we report one further robustness test for the BD theory. When we apply a BD regression to *new hires only* (over the full time period), the coefficient on $\min u$

[30] I.e., under the parameter restrictions given by (15), so that (26) is not binding in the putative solution. Sufficiency can be handled as in Appendix A.

[31] BD also assess the impact of including fixed effects which are absent in our analysis. However they find that adding these terms changes the results little. Grant uses fixed effects in all specifications, and argues that their inclusion tends to increase the significance of current unemployment.

(which in this case is just u_{0it}, the current unemployment rate) falls to below -.01 and is insignificant. In our data this is a robust finding.

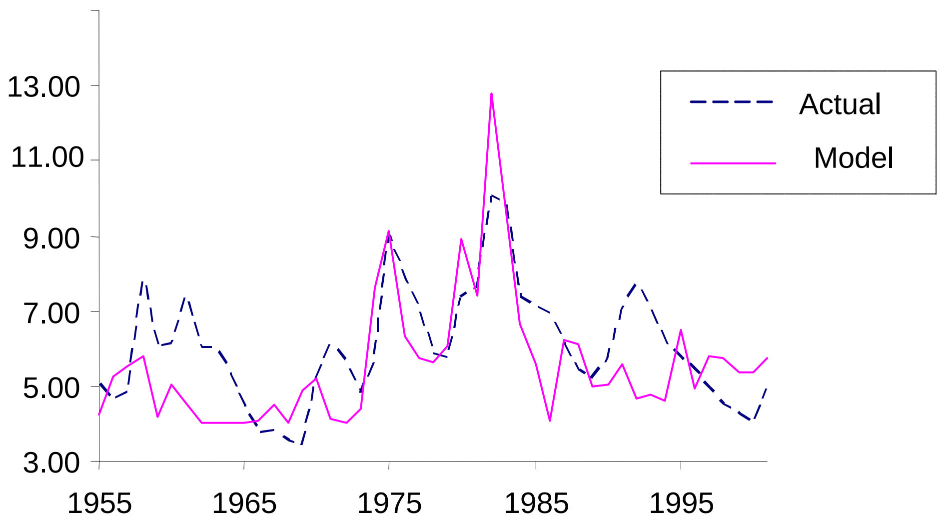

Figure 1: Actual versus model unemployment rates, 1955-2001.

Table 1: Data Means and Standard Deviations

	Our Sample 1976-1984	**BD'S Sample**
Log of Real Wages	1.01	1.12
	(.003)	(.004)
Min u	4.6%	4.2%
	(.009)	(.013)
Months in Post (Tenure)	81.7	83.0
	(.57)	(.66)
Percent White	37%	32%
	(.003)	(.003)

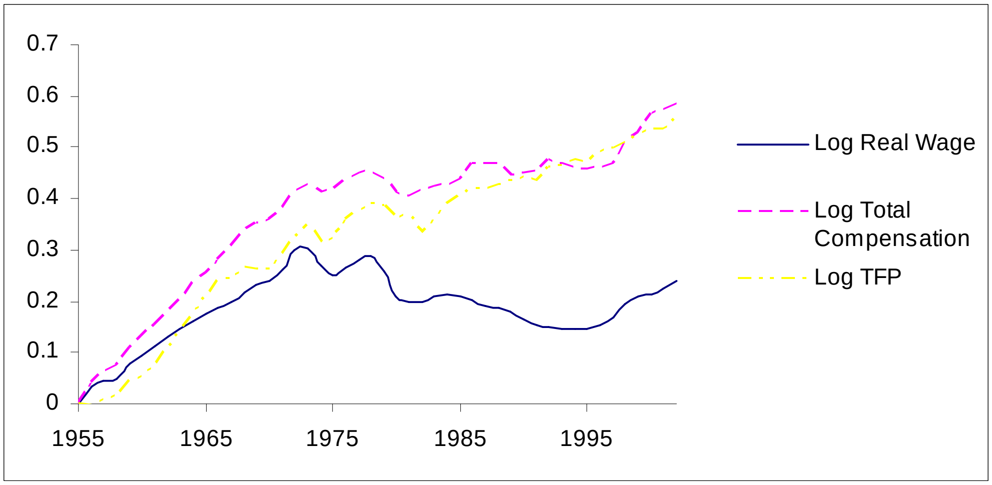

Figure 2: Logs of TFP, real wages and total compensation, 1955-2002

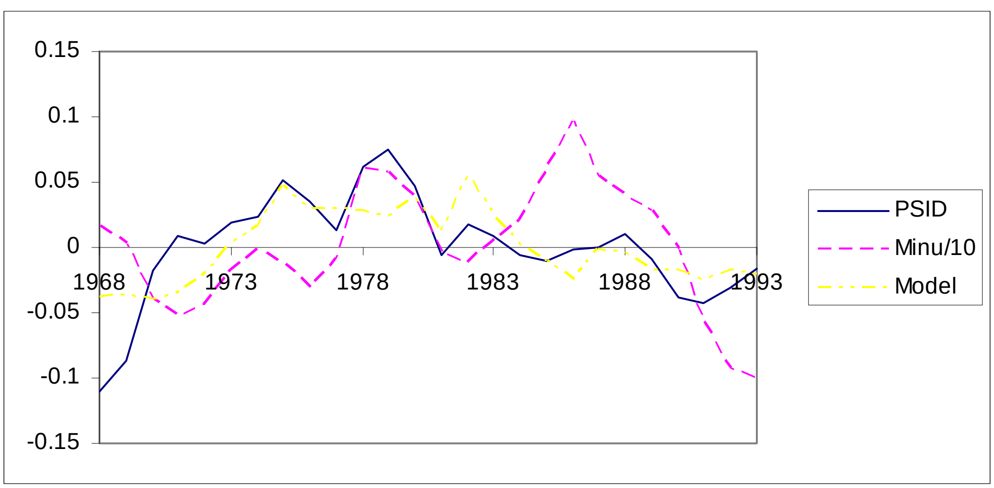

Figure 3: Detrended estimates of total compensation

Table 2: Annual Time Series Regressions 1968-1993

		$\overline{m}_t^d$	w_t^{*d}	y_t^d	R^2
LHS Variable	$\widehat{\tau}_t^d$	.021	-	-	.06
		(.017)	-	-	
		-	1.093	-	.55
		-	(.201)	-	
		.008	1.065	-	.56
		(.012)	(.209)	-	
		.003	1.007	.597	.65
		(.011)	(.192)	(.250)	
LHS Variable	$\widehat{\beta}_t^d$	.019	-	-	.04
		(.019)	-	-	
		-	1.153	-	.48
		-	(.244)	-	
		-.001	1.057	.809	.62
		(.013)	(.227)	(.296)	

Table 3: BD Pooled Regressions

	m_{it}	u_{0t}	u_t
BD Sample(1976-1984)	-.059	.013	.000
	(.006)	(.004)	(.002)
	-.045	-	-
	(.003)	-	-
Our Sample(1976-1984)	-.024	.011	.026
	(.004)	(.002)	(.003)
	-.022	-	-
	(.003)	-	-
Our Sample(1968-1993)	-.052	.004	.019
	(.002)	(.001)	(.001)
	-.033	-	-
	(.001)	-	-