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# A decentralized market for a perishable good\*

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March 3, 2006

## **Abstract**

We characterize the steady state of a market with random matching and bargaining, where the sellers' goods can perish overnight. Generically, the quantity traded is suboptimal, prices are dispersed and there is a dead-weight loss caused by excess supply or demand. In the limit as the cost of staying in the market tends to zero, only the amount of trade tends to the efficient level, the other two non-competitive characteristics remain. We discuss the implications of these findings on the foundations of competitive equilibrium and on the robustness of the results in the literature on durable-good markets.

## **1 Introduction**

The literature on dynamic decentralized markets for a homogeneous good assumes that the good traded does not lose its value over time: it is durable. Under this assumption, it has been shown that – in general – when information is complete and frictions are small, the outcome of strategic interaction in a steady-state market is approximately efficient (thus providing a non-cooperative foundation for competitive

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equilibrium)<sup>1</sup>. In practice, however, it is often the case that a delay of the transaction would imply a lower value placed on the item, at least in expected terms. There are two salient ways this can happen. The first one – depreciation – corresponds to the situation where there is a deterministic, gradually decreasing relation between an item’s age and its (expected) value. While this scenario is a good representation of some goods – say, a closed-end bond – the assumption of predictability is rather strong, and the analysis of the model is too complex, as items of different age co-existing in the market essentially make it into one of differentiated products. Instead, we concentrate on the other possibility, where decay is sudden and it occurs in a random manner. We call this phenomenon perishability.<sup>2</sup> Obvious cases are when the object itself may “malfunction” or “rot”. Alternatively, sometimes trading opportunities are only available in a given window of time. Finally, the decrease in value may come as a result of a change in preferences – as in the case of fashion goods – or a change in market structure – as in the case of industries with a high R&D component. These latter examples are special, as the shocks affecting identical items are (perfectly) correlated. This phenomenon is usually called obsolescence in the literature.

In short, the analysis of the trade of perishable goods is empirically relevant, and therefore called for. We re-examine the question of decentralized dynamic price formation when the good traded is perishable to some degree. Namely, each single item may become worthless in each period with some (given) probability.<sup>3</sup>

Our main finding is that a market for a perishable good is inherently inefficient, even as the search frictions vanish. Moreover, this inefficiency is not generated by the “wrong” trades taking place.<sup>4</sup> Rather, it stems from a queuing externality: the dead-

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<sup>1</sup>See Gale (1987, 2000).

<sup>2</sup>In management science, the corresponding notion would be (un)reliability.

<sup>3</sup>While we set up our model with random shocks that are independent, our analysis also captures the correlated case when the new good, which makes the old one obsolete, is replacing the old one immediately and that its supply and demand are also the same as those of the old one.

<sup>4</sup>This is what is driving the results of Gale and Sabourian (2005), who show that in a market

weight loss created by too many buyers entering the market. We begin by illustrating the issues with an example.

## 2 A simple example

Consider a market for fresh milk. There are 3 newly generated units of demand for milk every day with marginal valuations  $\{24, 20, 19\}$  and 3 newly generated units of potential supply with marginal costs  $\{15, 16, 23\}$ . The 3 potential buyers and 3 potential sellers must decide whether to enter a decentralized market that may include buyers from previous days who have yet to purchase milk. The milk turns sour overnight, so there are no milk units from previous days. Market participation costs the buyers  $c_b = 1$  and the sellers  $c_s = 0.1$  per day. The buyers and sellers in the market are randomly matched, and if the number of buyers and sellers is unequal, then the remainder do not trade. Matched traders first observe each other's valuations of the good and then engage in bilateral bargaining where a trader gets to make a take-it-or-leave-it offer to his trading partner and both traders are equally likely to play the role of proposer. If the offer is accepted, trade takes place and both agents leave the market. If the offer is rejected, then the buyer joins the matching pool the following day and the seller leaves the market. Starting at date 1, the efficient outcome is clearly where 2 buyers and 2 sellers enter and both offers are accepted. This is illustrated in Figure 1.

Since there will be no market participants who failed to trade, the same will be true on day, 2, 3 and so on. Hence the Pareto efficient flow of surplus per day will be  $24 - 15 + 20 - 16 - 2 \times 1 - 2 \times .1 = 10.8$ .

Now let us look at how this market actually evolves. Let  $V_i^B(b)$  be the value of 

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game of one-time entry (without a steady-state), heterogeneity of valuations is sufficient to cause inefficiency, as high valuation buyers may trade with high valuation sellers, and low valuation sellers with low valuation buyers, increasing the amount of trade but decreasing the aggregate gains from trade.

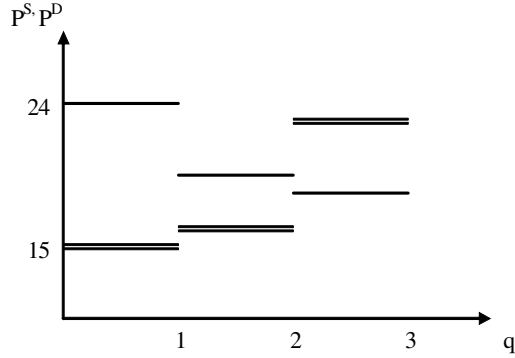


Figure 1: Supply and demand

participation in the market for a buyer with marginal valuation  $b$  starting from day  $i$ , and  $V_i^S(s)$  be the value of participation in the market for a seller with marginal cost  $s$  on day  $i$ . On day 1 a seller who makes a proposal will offer the buyer  $V_2^B(b)$  (that is, asking price  $b - V_2^B(b)$ ) because anything less will be rejected, providing that this does not involve a loss for the seller. A buyer who makes a proposal will offer the seller 0 (i.e. offer price  $s$ ) as the good is non-durable, providing that this gives the buyer at least  $V_2^B(b)$  as this is what the buyer will get if he fails to trade. Hence in a perfect Bayesian equilibrium

$$V_1^S(s) = \frac{1}{2}\pi_1^s E_b[\max\{b - s - V_2^B(b), 0\}] - 0.1 \quad (1)$$

$$V_1^B(b) = \frac{1}{2}\pi_1^b E_s[\max\{b - s, V_2^B(b)\}] + (1 - \frac{1}{2}\pi_1^b)V_2^B(b) - 1 \quad (2)$$

where  $\pi_1^s$  and  $\pi_1^b$  are the probabilities that the sellers and buyers, respectively, are matched on day one. Now consider the case where 2 buyers  $\{24, 20\}$  and 2 sellers  $\{15, 16\}$  enter each period and a buyer with valuation 19 enters only in period 1. Then we have a process that is stationary from the point of view of the buyers, since  $\pi_b = \frac{2}{3}$  and the sellers are the same each period. Solving (2) gives  $V^B(24) = 5.5$ ,  $V^B(20) = 1.5$  and  $V^B(19) = 0.5$ . So it is indeed optimal for the buyer with valuation 19 to enter the market on the first day if there is no future entry from a buyer with

valuation 19. The sellers' valuation on day  $i$  will be given by

$$V_i^S(s) = \frac{1}{2} \left[ \left(1 - \frac{1}{3^i}\right) \left(\frac{1}{2}(24 - s - 5.5) + \frac{1}{2}(20 - s - 1.5)\right) + \frac{1}{3^i}(19 - s - 0.5) \right] - 0.1 \quad (3)$$

which is positive for  $s = 15$  and  $s = 16$ . We also need to check that the other potential traders are happy with their non-participation. The seller with cost 23 will not wish to enter. She will not trade with a buyer with valuation 20 or 19, as this is less than her marginal cost, while a buyer with valuation 24 will not accept a price above  $23^5$ . Consider the case where the buyer with valuation 19 enters on day 2 but never again. Then  $V_2^B(24) = 4.5$ ,  $V_2^B(20) = 0.5$  and  $V_2^B(19) = -0.5$ . Since further entry in the future only makes matters worse, the buyer with valuation 19 will make a loss from entering in period 2. Hence, we have an equilibrium where a buyer with valuation 19 will enter in period 1 resulting in a permanent buyer queue. The equilibrium does not give rise to a steady state because  $V_i^S(s)$  is changing over time. The reason for this variation is that the probability that the buyer with  $b = 19$  is still in the market is diminishing over time. Eventually, the system approaches a steady state with  $\lim_{i \rightarrow \infty} V_i^S(15) = 1.65$  and  $\lim_{i \rightarrow \infty} V_i^S(16) = 1.15$ .

The flow of surplus per day in this steady state is  $5.5 + 1.5 + 1.65 + 1.15 = 9.8$ . Hence, ignoring the efficiency loss incurred by the fact that there is a point at which the buyer with valuation 19 trades (displacing a trade with a buyer of higher valuation), there is still a loss of 1 per day, relative to the efficient steady state. Now, at first sight, this significant loss may seem strange as this equilibrium does not appear to be terribly inefficient. With the exception of the first period, we have 2 buyers and 2 sellers entering every period and all offers are accepted, as in the efficient steady state. The only departure from the efficient steady state is the entry of a buyer with valuation 19 in period 1. However, this entry of one extra buyer significantly reduces the per period flow of surplus as *each* buyer now expects to incur the cost  $(1 + \frac{1}{3} + (\frac{1}{3})^2 \dots) = \frac{3}{2}$ , because with probability  $\frac{1}{3}$  they do not trade each day. In the efficient case the cost is

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<sup>5</sup>He will have at least a 1 in 2 chance of meeting the seller with a cost of 16 or less in the next period and so is guaranteed a continuation payoff of at least  $\frac{1}{2} \times \frac{1}{2} \times 8 - 1 = 1$ .

only 1, as they trade on the same day. Furthermore, this inflated cost is “necessary” in equilibrium, to prevent another buyer with valuation 19 from entering.

The inefficiency persists even if we reduce the costs towards zero, as in equilibrium this simply makes the queue longer. In the above example the non-participation condition for the buyer with valuation 19 is  $\frac{c_b}{\pi_i^b} > \frac{1}{2}(\frac{1}{2}(19 - 16) + \frac{1}{2}(19 - 15)) = \frac{7}{4}$ . Then a buyer with valuation 19 will not enter on day  $i$  if  $\pi_i^b = \frac{2}{i+2} < \frac{4c_b}{7}$ . Hence, in equilibrium, the last period in which a buyer with valuation 19 will enter is given by  $\max\{i \in I \mid i < \frac{14}{4c_b} - 2\}$ , where  $I$  is the set of integers. If we take  $c_b = 0.1$ , we obtain that we will have additional entry for the first 33 days. As above, the system will eventually approach a steady state, now with  $V^B(24) = 5$ ,  $V^B(20) = 1$ ,  $V^S(15) = 1.9$ ,  $V^S(16) = 1.4$ . Note that despite the lower cost of staying in the market the buyers are worse off as they have a lower probability of matching. This, in turn, improves the sellers’ bargaining position, leading to a higher steady-state payoff for them. The loss per day relative to the efficient steady state will be the waiting cost of the 33 unserved buyers: 3.3. In the limit as the participation costs go to zero the queue becomes infinitely long but the aggregate loss per day approaches  $2 \times \frac{7}{4} = 3.5$  in the limit. The limiting difference is higher as the relative weight of the two high-valuation buyers – whose participation costs are unavoidable – also converges to zero.

### 3 The general model

Take a market that evolves over (discrete) time, say, days. Buyers are seeking to purchase one unit of an indivisible, homogeneous good, and sellers have one unit for sale each. Any item that has not been sold during the day perishes overnight with probability  $d \in [0, 1]$ . Every day a new cohort of (a continuum of) potential buyers and sellers appear who (simultaneously) decide whether to enter the market – considering their outside option normalized to zero. The aggregate inverse demand function of each new cohort of potential buyers is  $P^d(\cdot)$ , assumed continuous and strictly decreasing, while the aggregate inverse supply function of each new cohort of

potential sellers is  $P^s(\cdot)$ , assumed continuous and strictly increasing.

The traders who decide to enter the market join those who have entered before but have yet to trade. Then, all the incumbent traders participate in an anonymous, one-to-one random matching process, where each trader on the same side of the market has equal probability of being matched ( $\pi_s$  and  $\pi_b$  for sellers and buyers, respectively)<sup>6</sup> and is matched to each trader on the other side of the market with equal probability. For simplicity, we also assume that the matching technology is efficient, that is, the traders on the short side of the market always find a partner. Matched traders first observe each other's valuations of the good and then engage in bilateral bargaining. Bargaining takes the following simple form: with probability  $\lambda \in (0, 1)$  the buyer –  $(1 - \lambda)$  the seller – makes a take-it-or-leave-it offer to his trading partner. If the offer is accepted, trade takes place and both agents leave the market. If the offer is rejected, then together with the unmatched traders they join the matching pool the following period (in the seller's case, provided her good has not perished).<sup>7</sup> Finally, market participation is costly: buyers and sellers incur a cost of  $c_b$  and  $c_s$ , respectively, per period while they are in the market.

## 4 The steady state and its characteristics

We wish to examine the steady-state behavior of this market, especially in terms of its efficiency. In other words, we are looking for a perfect Bayesian equilibrium, where the composition of the market and the amount traded is constant over time. We will compare this outcome to the efficient one. Denote the difference between the (inverse) demand and supply functions by  $G(\cdot)$ , that is,  $G(x) \equiv P^d(x) - P^s(x)$ . Then the (constrained) efficient flow of trade is given by  $x^e = G^{-1}(c_b + c_s)$ , that is,

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<sup>6</sup>In the general model we concentrate the analysis on the steady state, so the matching probabilities do not vary over time.

<sup>7</sup>We do not rule out that a trader might leave the market (to earn a gross payoff of zero). However, as will be seen later, this will never happen in equilibrium.



the quantity at which the surplus of the marginal participants is equal to the total cost of participating. As the costs approach zero,  $x^e \rightarrow x^c$ , which is the *static* market clearing equilibrium corresponding to the per-period flow of demand and supply:  $P^d(x^c) = P^s(x^c) = p^c$ . The competitive benchmark for our *dynamic* model can then be straightforwardly defined as the outcome where the trades in every period are given by  $x^e$  ( $x^c$  in the limit) AND where there are no traders waiting (overnight) at any time.

We start our analysis by establishing some lower bounds on the gains from trade in order to make trade possible in our model.

**Lemma 1**  $\lambda G(0) > c_b$  and  $(1 - \lambda)G(0) > c_s$  are necessary conditions for a steady state equilibrium with a positive measure of trade.

**Proof.** See the Appendix. ■

The basic intuition is that  $\lambda G(0) < c_b$  would imply that the marginal buyer makes a loss and  $(1 - \lambda)G(0) < c_s$  would imply that the marginal seller makes a loss. In what follows, we assume that  $\lambda G(0) > c_b$  and  $(1 - \lambda)G(0) > c_s$  as otherwise there would be no trade. Note that when  $c_s + c_b$  is small relative to  $G(0)$  – the maximum gains from trade possible in a match – then these conditions would not be satisfied only if the bargaining powers were too asymmetric.

Let  $\bar{s}(x) = \frac{1}{x} \int_0^x P^s(y) dy$  denote the average seller valuation in the market where  $x$  is the quantity traded, and define the quantity  $z = G^{-1}(\frac{c_s}{1-\lambda})$ .

**Proposition 1** *i) If  $P^s(z) - \bar{s}(z) > (\frac{c_b}{\lambda} - \frac{c_s}{1-\lambda}) \frac{1-\lambda(1-d)}{d}$  then there is a unique steady state (perfect Bayesian) equilibrium with trade. In this equilibrium the measure of sellers in the market is  $z$ , while the stock of buyers is*

$$B^* = \left( \frac{(P^s(z) - \bar{s}(z)) d}{1 - \lambda(1 - d)} + \frac{c_s}{1 - \lambda} \right) \frac{\lambda}{c_b} z > z. \quad (4)$$

*ii) If  $P^s(z) - \bar{s}(z) < (\frac{c_b}{\lambda} - \frac{c_s}{1-\lambda}) \frac{1-\lambda(1-d)}{d}$  then there exists at least one steady state (perfect Bayesian) equilibrium with trade. In all such equilibria the buyers are the*

short side of the market. The quantity traded and the probability of a seller being matched  $(x^*, \pi_s^*)$  is a solution to

$$\begin{aligned} G(x) &= \frac{c_s}{\pi_s(1-\lambda)} \\ P^s(x) - \bar{s}(x) &= \left( \frac{c_b}{\lambda} - \frac{c_s}{(1-\lambda)\pi_s} \right) \frac{d + (1-\lambda)\pi_s(1-d)}{d}, \end{aligned} \quad (5)$$

while the stock of sellers in the market is  $S^* = \frac{x^*}{\pi_s^*}$ .

iii) If  $P^s(z) - \bar{s}(z) = \left( \frac{c_b}{\lambda} - \frac{c_s}{1-\lambda} \right) \frac{1-\lambda(1-d)}{d}$  then there is a unique steady state (perfect Bayesian) equilibrium with trade. In this equilibrium the measure of buyers and sellers in the market is  $z$  and the market is balanced ( $\pi_s = \pi_b = 1$ ).

**Proof.** See the Appendix. ■

**Corollary 1** *In any steady state equilibrium, the expected transaction price between (trading) seller  $s$  and any (trading) buyer is*

$$p^*(s) = \frac{\lambda(ds - (1-d)c_s) + (1-\lambda)(d + (1-d)\pi_s) P^d \left( G^{-1} \left( \frac{c_s}{\pi_s(1-\lambda)} \right) \right)}{d + (1-\lambda)(1-d)\pi_s}$$

and this price is increasing in  $s$ .

The proofs are given in the appendix. Here we look at how the steady state equilibrium is constructed and give some intuition. In the a steady state equilibrium the marginal traders must have participation values of zero. Since all the buyers have the same cost  $c_b$ , the same bargaining power  $\lambda$  and the same probability of trading  $\pi_b$  and they face the same distribution of sellers, it follows that the difference between a buyer's valuation  $b$  and steady state participation value  $V^B(b)$  must be the same for all buyers. Combining this with the fact that the marginal buyer,  $b^*$ , has a participation value of zero we have  $b - V^B(b) = b^*$ . This implies that a seller will ask the same price,  $b^*$ , from every buyer. This intuition does not apply to the sellers because the seller does not trade in the next period with probability  $d$ . Consider the extreme situation, when the good lasts only a single day ( $d = 1$ ). In this case, the buyer only has to offer

the seller her cost,  $s$ . Hence a buyer will make a greater surplus when trading with a seller with a lower cost. When the perishing rate is lower, the buyer has to offer more than  $s$  but will continue to make a greater surplus when trading with a seller with a lower cost. Hence for  $d > 0$  the price will be increasing in  $s$  and will be independent of  $b$  and, in agreement with the literature on durable goods (see, for example, Mortensen and Wright, 2002), when  $d = 0$ , this price dispersion disappears.<sup>8</sup>

Consider the situation where the amount of trade is  $z$  and there are no queues. The marginal seller,  $s^*$ , will get  $G(z)(1 - \lambda) - c_s$  which does indeed give her a participation value of zero. If the marginal buyer has a positive participation value at this level of trade then a buyer queue is necessary to have a steady state equilibrium. Intuitively, buyers with lower valuations (but positive participation values) will enter until this participation value is zero. If, on the other hand, the marginal buyer has a negative participation value at this level of trade then we must have less trade,  $x^* < z$ , in the steady state equilibrium. Intuitively, the marginal buyer will not wish to participate when the level of trade is  $z$  so trade contracts until we have a marginal buyer who has a zero participation value. Since the marginal seller will now have a positive participation value without a queue, a seller queue will be necessary to have a steady state equilibrium. Hence, whether we have a buyer or seller queue will depend on whether  $V^B(P^d(z))$  is positive or negative with no queues. Now, if trade with the marginal seller results in a positive surplus then  $V^B(P^d(z))$  must be positive (since  $p^*(s)$  is increasing in  $s$ ) and we must have a buyer queue. If trade with the marginal seller results in a negative surplus, then whether  $V^B(P^d(z))$  is positive or negative will depend on the difference between the marginal seller cost and average seller cost. If the difference is sufficiently large then  $V^B(P^d(z))$  will be positive (buyer queue). Intuitively, the marginal buyer faces the prospect of meeting sellers with relatively low costs that will more than make up for making a loss when matched with sellers with high costs. If however the supply curve is almost flat then all the sellers are

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<sup>8</sup>More precisely, the price will be the independent outcome of the same lottery in every match.

similar to the marginal seller and  $V^B(P^d(z))$  will be negative (seller queue).

We now take a closer look at the inefficiency of the steady-state equilibrium. As in the simple example, there is an inefficiency that arises from the queue which has the effect of inflating the cost of each queuing trader. However, unlike in the simple example, the volume of trade is generically inefficient, which gives rise to an additional dead-weight loss. In fact, the steady-state volume of trade could be either greater than or less than  $x^e$  but it will always be less than  $x^c$ .

**Corollary 2** *In the steady-state equilibrium the amount of trade is strictly less than  $x^c$ , the quantity that clears the flow of supply and demand.*

**Proof.** From Proposition 1, the amount of trade in equilibrium is always  $G^{-1}\left(\frac{c_s}{\pi_s(1-\lambda)}\right)$ . The corollary follows from  $\frac{c_s}{\pi_s(1-\lambda)} > 0$  and  $x^c = G^{-1}(0)$ . ■

**Corollary 3** *The steady state measure of trade will be greater than  $x^e$  if and only if*

$$\frac{c_s}{(1-\lambda)(c_b + c_s)} < \pi_s. \quad (6)$$

**Proof.** Equation (6) follows from comparing  $G^{-1}\left(\frac{c_s}{\pi_s(1-\lambda)}\right)$  with  $G^{-1}(c_b + c_s)$ . ■

Linking this to our earlier discussion, if  $z < x^e$  then  $V^B(P^d(z))$  must be positive because trade with the marginal seller results in a positive surplus for the buyer (since the marginal seller gets  $G(z)(1-\lambda) - c_s$  which gives her a zero participation value and the total surplus  $G(z)$  is greater than  $c_s + c_b$ ). The steady state equilibrium will have a buyer queue. If  $z > x^e$  then trade with the marginal seller results in a negative surplus for the buyer and whether we have a buyer or seller queue will depend on the difference between the marginal and the average seller costs.

In the durable goods case it is well known that, if the parameters satisfy the Hosios condition,<sup>9</sup> then we have a balanced market (with no queues). We reproduce this result here.

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<sup>9</sup>This condition was first derived in Hosios (1990).

**Corollary 4** *If  $d = 0$ , in steady state the Hosios condition holds:*

$$\frac{\pi_b}{\pi_s} = \frac{(1 - \lambda)c_b}{\lambda c_s}.$$

**Proof.** This follows directly from (13) – in the Appendix – (multiplying through by  $d$  and then setting  $d = 0$ ). ■

In words, we have a balanced market ( $\pi_b = \pi_s = 1$ ) if and only if the bargaining powers are inversely proportional to the costs of staying in the market. The corollary also implies that in a balanced market with durable goods, we also have  $z = x^e$ . That is, not only is there no loss due to waiting (by definition), but given the fixed costs, there is also the efficient amount of trade. To see this, note that by Proposition 1, in a balanced market it must be the case that  $G(z) = \frac{c_s}{1-\lambda}$ . Using the Hosios condition (evaluated at  $\pi_s = \pi_b = 1$ ), we also have  $G(x^e) = c_b + c_s = \frac{\lambda c_s}{(1-\lambda)} + c_s = \frac{c_s}{1-\lambda}$ .

However, the efficient outcome is not possible when  $d > 0$ .

**Corollary 5** *If  $d > 0$ , then it is not possible to have both a balanced market ( $\pi_s = \pi_b = 1$ ) and the efficient measure of trade ( $x^e$ ).*

**Proof.** In a balanced market the measure of trade must be  $z = G^{-1}(\frac{c_s}{1-\lambda})$  and efficiency requires that the measure of trade is  $x^e = G^{-1}(c_s + c_b)$ . Hence  $\frac{c_s}{1-\lambda} = c_s + c_b$ . Re-arranging gives  $\frac{c_b}{\lambda} - \frac{c_s}{1-\lambda} = 0$ . This implies that the conditions for a buyer queue in Proposition 1 are satisfied (since  $P^s(z) - \bar{s}(z) > 0$ ) and we cannot have a balanced market. ■

The intuition for this follows from our earlier discussion. If we have a steady state where the volume of trade is  $x^e$  and there is no seller queue, then the marginal buyer and marginal seller generate a surplus of  $c_s + c_b$ . We know that the marginal seller will make zero on average when matched with the marginal (or any other) buyer, so the marginal buyer must also make zero on average, when matched with the marginal seller. However, if this is the case then  $V^B(P^d(z))$  must be positive as the marginal buyer will make positive surpluses when matched with sellers with lower costs and

we must have a buyer queue. Note that price dispersion is crucial here, as without price dispersion the last part of the argument would not apply. In that case, if the marginal buyer makes zero on average when matched with the marginal seller, then he will make zero on average when matched with any seller (as the expected price would be the same) and no queue would be necessary.

We now look at the steady-state equilibrium in the limit as the participation costs go to zero. First, observe that, in the limit, the marginal buyer will earn zero when matched with the marginal seller. As the price dispersion does not disappear (c.f. Corollary 1), it must be the case that we have a buyer queue.

**Lemma 2** *When  $d > 0$ , in the limit as the participation costs tend to zero (along any path) the market has a unique steady state equilibrium with trade. In it the buyers are queuing.*

**Proof.** Note that  $\lim_{(c_b, c_s) \rightarrow (0,0)} G^{-1}(\frac{c_s}{1-\lambda}) = G^{-1}(0) = x^c$ . Now, since  $P^s(z) - \bar{s}(z) > 0$  and since – when  $d > 0$  –  $\lim_{(c_b, c_s) \rightarrow (0,0)} (\frac{c_b}{\lambda} - \frac{c_s}{1-\lambda}) \frac{1-\lambda(1-d)}{d} = 0$ , the necessary and sufficient conditions identified in Proposition 1 for a unique equilibrium leading to a sellers’ market are satisfied. ■

It is worth emphasizing that for  $d = 0$  the above result does not apply. By Corollary 4, in the durable good case it is the Hosios condition that determines which side of the market has a queue. This demonstrates that the Hosios condition is not robust to the order in which we take limits. In cases where perishability is important and participation costs are insignificant, it is more appropriate to use the limit of the equilibrium with positive perishability.

Using Lemma 2, the limiting equilibrium is straightforward to characterize.

**Proposition 2** *In the limiting equilibrium as the costs of participation tend to zero, the amount of trade taking place coincides with the competitive one ( $x^c$ ). Unless  $d = 0$ , the limiting queue of the buyers is infinite and the aggregate waiting cost (per period) is given by  $L = \frac{(p^c - \bar{s}(x^c))d\lambda x^c}{1-\lambda(1-d)} > 0$ . Finally, the limiting price distribution is  $\lim_{(c_b, c_s) \rightarrow (0,0)} p^*(s) = \frac{\lambda ds + (1-\lambda)p^c}{1-\lambda(1-d)}$ , which collapses to  $p^c$  if (and only if)  $d = 0$ .*

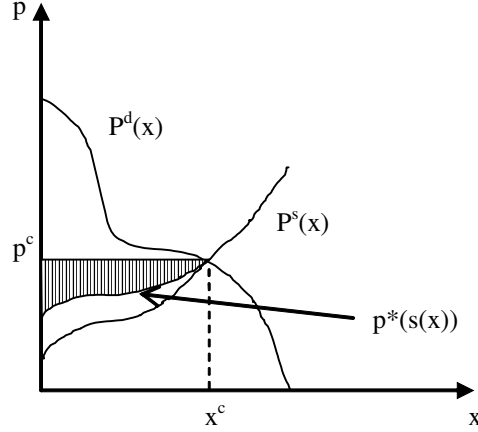


Figure 2: The dead-weight loss in the limit as the costs disappear

**Proof.** When  $d > 0$ , from Lemma 2 we must have a buyer queue. It is straightforward from the definition of  $z$  that in a sellers' market the limiting amount of trade is  $G^{-1}(0) = x^c$ . When  $d = 0$ , then the same result follows from the discussion of Corollary 4. The welfare loss caused by the permanent excess demand is the aggregate participation cost of all buyers in each period,<sup>10</sup> which – from (4) – is given by

$$B^*c_b = \left( \frac{(s^*(z) - \bar{s}(z))d}{1 - \lambda(1 - d)} + \frac{1 - d}{1 - \lambda}c_s \right) \lambda z. \quad (7)$$

$$\lim_{(c_b, c_s) \rightarrow (0, 0)} B^*c_b = \frac{(p^c - \bar{s}(x^c))d\lambda x^c}{1 - \lambda(1 - d)}.$$

■

In Figure 2 one can visualize the functioning of the limit equilibrium. Note that the per period loss,  $L$ , is the (shaded) area between the expected price curve,  $\lim_{(c_s, c_b) \rightarrow (0, 0)} p^*(s(x))$ , and the competitive price.

The intuition for this is simple. The marginal buyer makes an expected surplus of  $b^* - \bar{p}^*$ , where  $\bar{p}^*$  is the average price in the market. This must be balanced with

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<sup>10</sup>Alternatively, we could define the welfare loss as the participation cost of traders who do not actually trade in that period. As the limiting amount of trade is finite, the limiting value of the aggregate cost would be unaltered.

an expected loss (which is the same for all buyers) of  $\sum_{n=1}^{\infty} c_b + (1 - \pi_b)^n c_b = \frac{c_b}{\pi_b}$ . In the limit, we have

$$p^c - \lim_{(c_s, c_b) \rightarrow (0,0)} \bar{p}^* = \lim_{(c_s, c_b) \rightarrow (0,0)} \frac{c_b}{\pi_b}.$$

Now, note that the aggregate loss “newly generated” in every period is  $x^c \lim_{(c_s, c_b) \rightarrow (0,0)} \frac{c_b}{\pi_b}$ . Since we are in steady state, this aggregate loss must equal the aggregate delay cost suffered in every period,  $L$ . Putting these together, we have that

$$L = \left( p^c - \lim_{(c_s, c_b) \rightarrow (0,0)} \bar{p}^* \right) x^c,$$

which corresponds to the shaded area in Figure 2.<sup>11</sup> This reinforces the point (made after Corollary 5) that price dispersion is crucial to the inefficiency of the market. The figure illustrates that any measures that reduce the price dispersion (reducing  $\lambda$  or  $d$ ) will also reduce the loss.

Finally, it is perhaps interesting to spend a few words on how the system reaches a steady state. In the simple example the valuations were set so that the number of sellers in the market is always 2, which makes it relatively easy to construct the equilibrium by looking at the marginal buyer’s participation decision and letting the queue grow for the required number of periods. In the general (continuous) case the dynamics will be more complicated as the marginal buyer and marginal seller will be changing over time.

## 5 Implications for the strategic foundations of competitive equilibrium

The above result may sound like a serious blow to the non-cooperative foundations of competitive equilibrium. After all, why should perishability prevent the market from

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<sup>11</sup>An alternative way of thinking of the marginal buyer’s problem at the limiting equilibrium is that she is given a lottery ticket at cost  $c_b$  and with probability  $\pi_b$  she earns a prize of  $p^c - \lim_{(c_s, c_b) \rightarrow (0,0)} \bar{p}^*$ .



reaching efficiency as per period frictions disappear? In this section, we examine both the intuition for and the robustness of our inefficiency result, further clarifying our understanding of competitive equilibrium.

One way to understand the inefficiency caused by perishability is to realize that – given the normalization of the sellers’ outside option to zero – technically, perishability is equivalent to the discounting of the sellers’ future utility. In other words, our results would not change a bit if we reinterpreted  $1 - d$  as the sellers’ discount factor (holding the buyers’ discount factor at one) and assumed that the good was durable.<sup>12</sup> Since discounting is a friction, it is not surprising that it should lead to inefficiency.

One has to be careful with how far to take this argument though. For example, we should not say that in light of the above we should only be interested in the limit as the period length – and therefore both discounting and additive costs of delay – tends to zero. The reasons for this are twofold.

First, there are markets where looking at the limit as perishability disappears would make no sense. Think of markets for (individualized) services.<sup>13</sup> These neatly fit our model, if we allow the sellers to return to the market after having served a customer. Note that the assumption of repeating sellers makes no difference to the analysis as long as the market interaction is anonymous. Now, if a dentist, say, cannot treat a patient between 9:30 and 10:00, she cannot postpone delivery of the treatment to 10:00-10:30, since in equilibrium she expects to treat another patient by then. In other words, if she does not get to an agreement with the patient signed up for 9:30, she loses that trade. To model this case, we would take  $d = 1$  and in the limit the

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<sup>12</sup>Note that the fact that changes in the characteristics of goods traded in a market only matter insofar they affect the traders’ preferences is a general truism. So, it is only natural that the time-varying nature of perishable goods can be captured via the traders’ time preferences. On the other hand, this technical equivalence does not imply that talking about different classes of goods cannot be a better way of discussing things, especially since the goods are observable and the preferences are not.

<sup>13</sup>See Ponsatí and Sákovics (2005) for a related model with vertically differentiated service providers.

aggregate loss would be  $(p^c - \bar{s}(x^c)) \lambda x^c$ . The bottom line is that in a decentralized model of services, there will be an important dead-weight loss even in the limit as the waiting costs tend to zero. Consequently, it should not be thought of – even approximately – as a competitive market.

The second reason against hastily taking limits is that it is not necessary. Consider a model where the death rate is changing over time – but the buyers cannot tell the age of the good.<sup>14</sup> Although such a process is not stationary, we now show that the steady-state equilibrium of the stationary model where the death rate is constant is also an equilibrium of this model, whenever the market has the buyers queueing. Intuitively, holding the traders’ strategies fixed on either side of the market, the traders on the other side have no new incentives to deviate.

**Proposition 3** *For waiting costs close enough to zero, the model where the perishing rate is variable but non-decreasing over time, and the buyers cannot tell the age of the good has a steady-state (Perfect Bayesian) equilibrium, which leads to the same outcome as the unique steady-state (subgame-perfect) equilibrium (with trade) in the model where the death rate is constant at  $d_1 > 0$ .*

**Proof.** Let  $V(\cdot)$  be the steady-state equilibrium value function of a trader in the case where the death rate is always  $d_1$ . From Lemma 2 we have a buyer queue for low enough costs of waiting. Hence, in the steady-state equilibrium the sellers always trade in period one and thus the buyers are always matched with a new entrant:

$$\begin{aligned} V(s) &= (1 - \lambda)E_b[\max\{b - s - V(b), (1 - d_1)V(s)\}] + \lambda(1 - d_1)V(s) - c_s \\ &= (1 - \lambda)E_b[b - s - V(b)] + \lambda(1 - d_1)V(s) - c_s, \\ V(b) &= \pi_b \lambda E_s[b - s - (1 - d_1)V(s)] + (1 - \pi_b \lambda)V(b) - c_b. \end{aligned}$$

Now, consider whether the same outcome is supported by an equilibrium of the non-stationary model. Let  $V_i(s)$  be the value function for the owner of a good in

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<sup>14</sup>Note that in our main model knowing the age of the good provides no additional information (by the stationarity of the perishing process).

the  $i^{th}$  period of its life. By hypothesis, the sellers are always matched in the first period. As the buyers cannot tell the age of the good, they will always offer the sellers  $(1 - d_1)V_2(s)$ . Hence the value functions are

$$\begin{aligned}
V_1(s) &= (1 - \lambda)E_b[\max\{b - s - V(b), (1 - d_1)V_2(s)\}] + \lambda(1 - d_1)V_2(s) - c_s \\
V_2(s) &= (1 - \lambda)E_b[\max\{b - s - V(b), (1 - d_2)V_3(s)\}] + \lambda \max\{(1 - d_1)V_2(s), (1 - d_2)V_3(s)\} - c_s \\
&\quad \vdots \\
V_n(s) &= (1 - \lambda)E_b[\max\{b - s - V(b), (1 - d_n)V_{n+1}(s)\}] + \lambda \max\{(1 - d_1)V_2(s), (1 - d_n)V_{n+1}(s)\} - c_s \\
&\quad \vdots \\
V(b) &= \pi_b \lambda E_s[\max\{b - s - (1 - d_1)V_2(s), V(b)\}] + (1 - \pi_b \lambda)V(b) - c_b.
\end{aligned}$$

Now, since  $1 - d_n \geq 1 - d_{n+1}$ , we have that  $(1 - d_1)V_2(s) \geq (1 - d_n)V_{n+1}(s)$ , for all  $n$ . As a result, all the max operations are resolved in favor of the (stationary) left-hand argument – corresponding to trade occurring in every match, thereby validating the hypothesis that all the sellers trade in their first period in the market –, implying that  $V_1(s) = V_n(s) = V(s)$  for all  $n$ .

■

Intuitively, since in the hypothetical equilibrium the buyers are queuing, all the sellers are matched and thus, as long as they always trade, the buyers are always faced with sellers of “new” goods, and thus have no new incentive to deviate. The only thing that can go wrong is that a seller who in period one prefers to trade rather than to wait, in a later period prefers to wait (because the death rate has dropped), but this is ruled out by the assumption of a non-decreasing death rate. As a result, a seller indeed never stays in the market for more than one period, and thus her entry decision will also only depend on her first-period perishing rate.

By Proposition 3, if we take the limit as  $d_1 \rightarrow 0$  in the non-stationary model, then the welfare loss converges to zero even though  $d_n > 0$ ,  $n = 2, 3, \dots$ . That is, we

do not need all the frictions to disappear in order to regain efficiency. The (almost) *certain* option to costlessly switch bargaining partners *at least once* is necessary and sufficient to obtain the competitive outcome.

One way to think of this in terms of our dentist example, is that each dentist needs to take a break once in every two periods, but she is indifferent between taking this break now or in the next period. If she fails to sell in the first period, she uses this as her break but then definitely wants to provide the service in the next period. This market is equivalent to  $d_i = 0$  and  $d_{i+1} = 1$ , for  $i$  odd. In the efficient equilibrium half the dentists would work in odd periods and half in even periods.

It is eye-opening to note that Proposition 3 does not hold with  $d_1 = 0$ . That is, the equilibrium of the standard durable-good model is in general not an equilibrium of the non-stationary model. To see this, note that in the durable good case the long side of the market is decided by the Hosios condition. Consequently, no matter how low the costs are, we may have the sellers queuing. In that case, in the non-stationary model there *is* a new incentive to deviate in period 2, as it is no longer a credible belief that all the sellers are “new.”

## 6 Appendix

We begin by setting up some steady state equations. Denote the buyers’ and sellers’ value functions in a stationary equilibrium by  $V(b)$  and  $V(s)$ , respectively. We then have<sup>15</sup>

$$\begin{aligned}
 V(s) = & \pi_s(1 - \lambda)E_b[\max\{b - s - V(b), (1 - d)V(s)\}] + \\
 & (1 - \pi_s(1 - \lambda))(1 - d)V(s) - c_s
 \end{aligned}
 \tag{8}$$

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<sup>15</sup> $E_b$  ( $E_s$ ) denotes the expectation operator over the buyer (seller) types who are in the market in any given period in equilibrium.

and

$$V(b) = \pi_b \lambda E_s[\max\{b - s - (1 - d)V(s), V(b)\}] + (1 - \pi_b \lambda)V(b) - c_b \quad (9)$$

These value functions are constructed as follows. With probability  $\pi_s(1 - \lambda)$ , a seller gets matched and is allowed to name the price. In this case she will either make an offer, which leaves the buyer indifferent about accepting ( $b - s - V(b)$ ), or makes an unacceptable offer and waits for next period. If she is either unmatched or matched but has to take the buyer's offer or leave it, then her payoff is the same as if she had to wait until next period  $(1 - d)V(s)$ . A symmetric argument explains the buyers' value function. Rearranging (9), we obtain

$$b - V(b) = \frac{c_b}{\pi_b \lambda} + E_s[\min\{s + (1 - d)V(s), b - V(b)\}]. \quad (10)$$

Since (10) holds for all (participating) buyers, the minimum operator has to select the same argument all the time (otherwise, the derivative with respect to  $b$  would differ on the two sides of the equation). Since choosing the second argument leads to an inconsistency, we have that  $b - V(b)$  must be constant, which implies that we can drop the expectations operator from (8). The above argument also implies that  $b - s - V(b) \geq (1 - d)V(s)$  for all participating suppliers. Consequently, the maximum operator in (8) always selects its first argument. Rearranging the "operatorless" (8), we have

$$V(s) = \frac{\pi_s(1 - \lambda)}{d + \pi_s(1 - \lambda)(1 - d)}(b - s - V(b)) - \frac{c_s}{d + \pi_s(1 - \lambda)(1 - d)}. \quad (11)$$

Note that  $V(s)$  is decreasing in  $s$  and  $V(b)$  is increasing in  $b$ . Denoting the marginal traders by  $b^*$  and  $s^*$ , by definition we have that  $V(b^*) = V(s^*) = 0$ . Therefore,  $b - V(b) \equiv b^* - V(b^*) \equiv b^*$ . Plugging this back into (11) evaluated at  $s = s^*$  yields

$$b^* - s^* = \frac{c_s}{\pi_s(1 - \lambda)} \quad (12)$$

## 6.1 Proof of Lemma 1

For the marginal buyer

$$V(b^*) = b^* - \frac{c_b}{\pi_b \lambda} - E_s[s + (1-d)V(s)] = 0$$

which implies

$$b^* - E_s[s] \geq \frac{c_b}{\pi_b \lambda} \geq \frac{c_b}{\lambda}$$

Since we require a positive measure of trade  $P^d(0) > b^*$ . Hence

$$G(0) = P^d(0) - P^s(0) > b^* - P^s(0) \geq b^* - E_s[s] \geq \frac{c_b}{\lambda}$$

and  $\lambda G(0) > c_b$ . Similarly, using (12)

$$G(0) = P^d(0) - P^s(0) > b^* - s^* = \frac{c_s}{\pi_s(1-\lambda)} \geq \frac{c_s}{1-\lambda}$$

which gives  $(1-\lambda)G(0) > c_s$ .

## 6.2 Proof of Proposition 1

Since in a steady state the measure of entering buyers and sellers must be equal, we have that

$$P^d(x^*) - P^s(x^*) = \frac{c_s}{\pi_s(1-\lambda)},$$

yielding either the definition of  $z$  or the first equation in (5). Taking expected values of (11) and simplifying (using  $V(s^*) = 0$ ) we obtain

$$E_s[V(s)] = \frac{\pi_s(1-\lambda)(s^* - \bar{s})}{d + \pi_s(1-\lambda)(1-d)}.$$

Substituting into (9), with the max operation resolved, and simplifying yields

$$s^* - \bar{s} = \left( \frac{c_b}{\lambda \pi_b} - \frac{c_s}{(1-\lambda)\pi_s} \right) \frac{d + (1-\lambda)\pi_s(1-d)}{d}. \quad (13)$$

Note that the right-hand side of (13) is increasing in  $\pi_s$  and decreasing in  $\pi_b$ . The right-hand side evaluated at a symmetric market ( $\pi_s = \pi_b = 1$ ) is the marginal

situation. If this is less than the left-hand side then  $\pi_b$  will have to fall to achieve a steady-state with a buyer queue. To obtain  $B^*$  we solve (13) for  $\pi_b$ , (under the assumption that  $\pi_s = 1$ ) and multiply it by the stock of sellers, that is,  $z$  which must be positive for a non-degenerate steady-state ( $G(0) > \frac{c_s}{1-\lambda}$ ). If the right-hand side of (13) is greater than the left-hand side when  $\pi_s = \pi_b = 1$ , then  $\pi_s$  will have to fall – what by the first equation in (5) implies a decrease from  $G^{-1}\left(\frac{c_s}{1-\lambda}\right)$  in the traded quantity as well – to achieve a steady-state with a seller queue. Substituting the first equation into (13), with  $\pi_b$  set to 1, we have:

$$P^s(x) - \bar{s}(x) = \left(\frac{c_b}{\lambda} - G(x)\right) \frac{d + c_s(1-d)/G(x)}{d}. \quad (14)$$

The right-hand side is increasing in  $x$  and it is equal to 0 at  $G^{-1}\left(\frac{c_b}{\lambda}\right) > 0$ . Now,  $P^s(x) - \bar{s}(x) > 0$  for  $x > 0$ , so – by continuity – a solution  $x^* \in \left(G^{-1}\left(\frac{c_b}{\lambda}\right), G^{-1}\left(\frac{c_s}{1-\lambda}\right)\right)$  to (14) must exist.  $\pi^*$  is then given by  $\frac{c_s}{(1-\lambda)G(x^*)}$ , which is less than one as  $\frac{c_b}{\lambda} > \frac{c_s}{1-\lambda}$ , by the assumption that in the balanced case the right-hand side of (13) exceeds the left-hand side. Q.E.D.

### 6.2.1 Proof of Corollary 1

The transaction price between  $b$  and  $s$  is

$$\begin{aligned} p(b, s) &= \lambda(s + (1-d)V(s)) + (1-\lambda)(b - V(b)) \\ &= \lambda \frac{s(d + (1-d)\pi_s(1-\lambda)) + (1-d)(\pi_s(1-\lambda)(b^* - s) - c_s)}{d + (1-d)\pi_s(1-\lambda)} + (1-\lambda)b^* \\ &= \lambda \frac{ds - (1-d)c_s}{d + (1-d)\pi_s(1-\lambda)} + (1-\lambda) \frac{d + (1-d)\pi_s}{d + (1-d)\pi_s(1-\lambda)} b^*. \end{aligned}$$

which is increasing in  $s$ . Observing that  $b^* = P^d\left(G^{-1}\left(\frac{c_s}{\pi_s(1-\lambda)}\right)\right)$ , whether it is a buyers' or a sellers' market, completes the proof. Q.E.D.

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