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Methods of Social Comparison in Games of Status*

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Abstract

This paper considers the effects of changes in the income distribution in an economy where agents' utility depends both on consumption and on their rank in the distribution of consumption of a positional good. We introduce a new methodology to compare the behavior of agents that occupy the same rank in the two different income distributions but typically have different levels of incomes, and analyze equilibrium choices and welfare of every member of the society for continuous distributions with arbitrary, even disjoint, ranges. If an income transformation raises incomes at the lower end of the income distribution, the poor will typically be better off. But because such an income transformation also increases the degree of social competition, the middle class will typically be worse off - even if they have higher incomes as well. An increase in incomes can make all better off, but only if it is accompanied by an increase in income dispersion. Our new techniques highlight the importance of density of social space as we demonstrate that one can have an increase both in income and relative position but still be worse off.

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Keywords: Status, relative standing, conspicuous consumption, consumption externalities, income inequality, social competitiveness, first price auctions, dispersive orderings.

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1 Introduction

There is increasing acceptance amongst economists that people may care about their relative position as well as the absolute level of their consumption. For example, a survey of empirical research on happiness goes as far as to conclude that in determining happiness “It is not the absolute level of income that matters most but rather one’s position relative to other individuals” (Frey and Stutzer, 2002, p411). The importance of relative comparisons finds further support in more recent research (Brown et al. (2004); Luttmer (2005)). If this is the case, it seems a natural conclusion that greater inequality would worsen the degree of social competition (Frank (1999)) when the middle classes are confronted with the greater affluence of the rich. This would seem to give a new justification for policies to reduce inequality. However, it turns out that this argument is problematic. In earlier work (Hopkins and Kornienko (2004)), we were able to extend the analysis of the model of relative concerns of Frank (1985) so that we could characterize how equilibrium behavior and equilibrium utility changed in response to changes in the distribution of income. We found that, surprisingly, greater equality could lead to an increase in conspicuous consumption and a reduction in welfare for those with low and middle incomes. Thus, even in the presence of relative concerns, greater equality is not necessarily welfare-enhancing. Furthermore, we found that greater social wealth decreases satisfaction at any given level of income.

It has been argued to us, however, that our “equality hurts the poor” and “economic growth hurts everybody” results may be misleading. Often, when one thinks of a reduction in inequality, one thinks, for example, of an increase in income of all the poor, rather than a change that keeps the incomes of some poor people constant. Or when one thinks of economic growth, one thinks of an increase in income of all. Yet our earlier methodology did not allow us to address such situations. Instead, it only allowed us, in effect, to investigate the behavior of agents whose income did not change as the income of those around them did. In fact, such limitations are inevitable if one follows the established tradition and analyzes consumer behavior as a function of her income (as we did). Besides, as the model of relative concerns initially considered by Frank (1985) turned out to be formally similar to first price auctions, we borrowed our earlier techniques from auction theory. These borrowed techniques were fruitful insofar they allowed us to solve the game and to make comparative statics predictions. However, they only allowed us to compare equilibrium outcomes under two different income distributions for a given level of income, and thus, only for the distributions with the same support. We now believe that we can do better.

In this paper, we develop a new set of techniques created specifically for problems of social comparison. We re-examine games of status where agents care about their relative position in terms of their rank in expenditure on a visible positional good, and analyze consumer choice as a function of *rank* rather than *income*. This new methodology allows us to compare the equilibrium behavior and well-being of *every* individual for a pair of continuous distributions with *arbitrary*, even disjoint, ranges. We still find that greater equality provides greater incentives to spend on the visible positional good in

order to differentiate oneself. If an income transformation raises incomes at the lower end of the income distribution, the poor will typically be better off. But as an increase in equality increases the degree of social competition, the middle class will typically be worse off - even if they have higher incomes as well. We find that to make all better off it is sufficient to raise incomes provided at the same time there is an increase in income dispersion.

In our current work and our earlier paper, Hopkins and Kornienko (2004), we use the same model derived from that of Frank (1985), where individuals must decide how to divide their income between consumption of a normal good and a positional good. For example, one might care about the characteristics of one's car, but also about how it compares to those of one's neighbours. The choice of the positional good is therefore strategic, in that consumption choices of my neighbours affect my payoffs, as my choice affects theirs. The symmetric Nash equilibrium of the resulting game will be Pareto inefficient in that all will spend more on the positional good than is privately optimal, but will result in no net change in relative position. That is, everyone increases conspicuous consumption in order to improve status, but any gain in status is cancelled out by the similarly increased expenditure of others. A salient question in this context is how the distribution of income affects the degree of social competition and, hence, the amount of excess consumption.

In such a model, however, changes in the income distribution can affect agents through three channels. One's own income, one's relative position or rank in the distribution, and the shape of the distribution all matter. Of necessity, any analysis must hold at least of one of these constant. In our earlier work, we considered changes in the distribution of income that left some people's incomes unchanged. (For example, imagine a change in taxes on earned income that does not affect incomes of those who do not work.) We then analysed the effect of the change in the distribution of income on equilibrium behavior and utility at each income level. We found that a reduction of inequality of this type would lead to a fall in utility at low income levels.

Here, instead, we perform the comparative statics analysis for a fixed rank, and find, as we did before, that an increase in equality increases the degree of social competition. The closer individuals are together, the easier it is to overtake others in status, thus giving a greater incentive to indulge in conspicuous consumption. This means that the overall effect of redistribution on the poor is ambiguous: they have greater income, but more of it may be spent on wasteful consumption. Typically, with their income increased, the poor are better off in a more equal society, yet the lower middle class are worse off - even though they may also have higher income. This is because the increase in affluence of those at the bottom results in their higher expenditure on visible positional goods, forcing everyone "further up" the social ladder to increase their spending as well in order to "keep up" - and for most, this increased expenditure on positional goods comes at the expense of their expenditure on non-positional goods, leading to a decline in post-redistribution welfare. The effect on those with average or greater income is definitely negative. This is in contrast to the effect of a reduction in inequality in

Hopkins and Kornienko (2004), where the effect on the rich was ambiguous, and on the poor was definitely negative.

We hope these contrasting results may help to explain why it has been difficult to establish empirically whether greater equality does in fact lead to greater happiness. Clark (2003), using British panel data, finds a positive relationship between inequality and self-reported happiness while Senik (2004) finds that inequality has no statistical influence on life satisfaction in post-reform Russia. In contrast, Alesina *et al.* (2004) find a negative relationship between inequality and happiness for both Europe and the US. Our results suggest that, even in the presence of relative concerns, whether greater equality does increase utility or happiness may depend quite sensitively on the measure of equality considered and on the method of comparison.

In fact, the new techniques also allow us to look at the following contentious issue. Prior to 1970s, the economics profession held a consensus regarding the benefits of economic growth, as it seemed obvious that an increase in real income of every individual (and thus an increase in the level of consumption) leads to an increase in happiness of every individual. However, Easterlin (1974) pointed out that data on happiness across time and countries did not support this simple hypothesis.¹ Concerns with relative position are a likely culprit, in that happiness increases significantly in cross-section even if average happiness does not rise strongly in response to increases in average income. In a way, Easterlin's observation posed a challenge to the profession as to whether one can find a condition for an increase of happiness of every individual when relative concerns are present. Contrary to the standard neoclassical result, our earlier result suggests that one will be unhappy with an unchanged income when other people's incomes grow. But our earlier methodology did not allow us to explore whether an increase in incomes of everyone increases everyone's happiness. Here, we suggest that it is possible for every agent to be made better off when everyone experiences an increase in income, provided at the same time, incomes become no less dispersed and so there is no increase in social competition. Another way to see our sufficient condition for happiness, imagine an income transformation whereby the income of the lowest member in the society increased (or stayed the same) while everyone else faces such an increase in income that results in a decrease in "social competitiveness" (by which we mean a decrease in the density of the income distribution at every rank).

Notice that our techniques allows us to avoid the problems of interpersonal comparisons as we can make ordinal comparisons of utilities for the same individual before and after an income transformation. If the income transformation is rank-preserving, each individual will have the same rank before and after the income transformation, and thus we can apply our rank-indexing technique directly. We find that even if the income transformation is not rank-preserving, one can still carry out ordinal comparisons for the same individual by correcting for ranks by means of a rank transformation function. Non-rank-preserving transformations highlight the importance of the density

¹Easterlin's findings have been found significant support in subsequent research. See Frey and Stutzer (2002) for a survey.

of the social space, as we provide an example where individuals may have both higher income and higher rank, but because of the increased density of their social space they may be worse off.

Finally, both the model presented here and the model in our earlier work (Hopkins and Kornienko (2004)) can also be interpreted as models of labor supply, as they can apply to situations where individuals decide on how to allocate their endowment of productivity (rather than of income) between labor and leisure, and where status or prestige is assigned to the most productive workers. In equilibrium individuals choose a level of labor which is higher than that which would be privately optimal - that is, in terms of labor supply, they overwork. As increased education changes the productivity endowments (making the “social space” to be less dispersed), the increased competition leads to further worker dissatisfaction.²

The paper is organized as follows. In the next section we present the model and show that strategies and equilibrium outcomes can be written in terms of rank in the distribution of income. In Section 3 we compare the two methodologies in games of status - one indexing individuals by their level of income, and the other indexing by rank. In Sections 4 and 5 we utilize the concept of dispersive ordering and conduct comparative static analysis. In Section 6 we show that our techniques can be useful for comparing individual’s choices and welfare for non-rank-preserving income transformations. In the conclusion, we discuss the applicability and limitations of both methodologies.

2 The Model

Following Frank (1985) and Hopkins and Kornienko (2004) (hereafter “HK”), we consider a simple model where individuals care about their social status as determined by their (conspicuous) consumption of a visible (positional) good, as well as absolute level of (conspicuous) consumption of this visible (positional) good x and absolute level of (non-conspicuous) consumption of another (non-positional) good y , the consumption of which is not directly observable by other agents. We assume an economy consisting of a continuum of agents, identical except in terms of income. Each agent is endowed with a level of income z which is private information and is an independent draw from a common distribution. This is described by a distribution function $G(z)$ which is twice continuously differentiable with a strictly positive density on some interval $[\underline{z}, \bar{z}]$ with $\underline{z} \geq 0$.

Agents’ choices of conspicuous consumption are aggregated in a distribution of conspicuous consumption $F(\cdot)$, with $F(x)$ being the mass of individuals with consumption less than or equal to x . Following Frank (1985) and Robson (1992), an agent’s status will be determined by her position in the distribution of conspicuous consumption, with

²See Schor (1991) and (1998) for a vivid description of both overspending and overworking behavior of modern Americans.

higher consumption meaning higher status. Following HK, we define status as follows:

$$S(x, F(\cdot)) = \delta F(x) + (1 - \delta)F^-(x) + S_0 \quad (1)$$

where x is individual's consumption, $\delta \in [0, 1)$, $F(x)$ is the mass of individuals with consumption less or equal to x , and $F^-(x) = \lim_{x' \rightarrow x^-} F(x')$ is the mass of individuals with consumption strictly less than x . The current formulation is a way of dealing with ties. For example, if all agents chose the same level of consumption in one sense they would all be "equal first", but it is unlikely that they would gain the same level of satisfaction as someone who was uniquely first. To reflect this, the current assumption would award them status equal to δ which is strictly less than one.³ In contrast, if the distribution of consumption $F(x)$ is continuous, there are no ties, the above measure of status is identical to rank in consumption, or $S(x, F(\cdot)) = F(x)$. The parameter $S_0 \geq 0$ is a constant representing a guaranteed minimum level of status, reflecting the intensity of social pressures. We discuss its role below.

We follow HK, and assume that individuals have identical preferences over absolute consumptions and status as follows:

$$U(x, y, S(x, F(\cdot))) = V(x, y)S(x, F(\cdot)) \quad (2)$$

In effect, $V(\cdot)$ is a conventional utility function over the two goods, x and y , and we assume that it is non-negative, strictly increasing in both its arguments, strictly quasiconcave and twice differentiable. We further assume that $V_{ii} \leq 0$ for $i = 1, 2$ and that $V_{ij} \geq 0$ for $i \neq j$. As agents simultaneously decide how to allocate their endowment z between consumption x and saving y , each agent faces the following problem,

$$\max_{x, y} V(x, y) (\delta F(x) + (1 - \delta)F^-(x) + S_0) \quad \text{subject to } px + y \leq z, \quad x \geq 0, \quad y \geq 0 \quad (3)$$

where p is the price of the positional good. The price of the non-positional good is normalized to one.

As individuals are in competition for status, that implies that their choice of consumption of different types of goods is strategic. Note that the distribution of conspicuous consumption $F(\cdot)$ is endogenously determined, so is social status $S(x, F(\cdot))$. Thus, a rational individual makes a consumption choice in anticipation of consumption choices of all other individuals, i.e. is engaged in a game of status. It is possible to solve the resulting game but the solution will, however, depend on the distribution of income in society. This game has a formal resemblance to a first-price auction, as increasing one's conspicuous consumption leads to a trade-off between the increase in status and the decrease in non-conspicuous consumption component of utility, just as a bidder in an auction trades off an increase in probability of winning for lower realized profits in the event of winning.

HK concentrated on a symmetric Nash equilibrium in which all agents use the same strategy, defined as a mapping $x(z)$ from income to expenditure. However, given an

³This also gives agents an incentive to break any ties, so, as we will see, there will be no ties in equilibrium. See Hopkins and Kornienko (2004) for a full rationale of this specification.

income distribution $G(z)$, an agent of income \tilde{z} has rank $\tilde{r} = G(\tilde{z})$ and it is equally valid to think of his type as being \tilde{r} as much as it is \tilde{z} .⁴ We can also write his income as a function of his rank or $\tilde{z} = G^{-1}(\tilde{r})$ (i.e. \tilde{z} is at the \tilde{r} -quantile). Note an important distinction. While in everyday speech, “rank” and “status” are almost synonymous, here we use the two terms to signify two different things. Rank we take to be an agent’s true, exogenous rank in the underlying distribution of income, or $r = G(z)$. Status S , however, is endogenously determined by visible expenditure on positional goods in comparison with the expenditures of others according to the function (1). In any case, we now consider a symmetric Nash equilibrium defined as a mapping $x(r)$ from rank to expenditure.

That is, a *symmetric equilibrium* will be a Nash equilibrium in which all agents use the same strategy, that is, the same mapping $x(r)$ from rank in income to conspicuous consumption. Suppose all agents adopt the same increasing, differentiable strategy $x(r)$ and consider whether any individual agent has an incentive to deviate. Suppose that instead of following the strategy followed by the others, an agent with rank r_i chooses $x_i = x(\tilde{r})$, that is, she consumes as though she had rank \tilde{r} . Note first that $F(x_i) = x^{-1}(x_i) = \tilde{r}$, resulting in $S_i = S_0 + \tilde{r}$, and second that her utility would be equal to

$$U = V(x(\tilde{r}), G^{-1}(r_i) - px(\tilde{r}))(S_0 + \tilde{r}).$$

We differentiate this with respect to \tilde{r} . Then, given that in a symmetric equilibrium, the agent uses the equilibrium strategy and so $\tilde{r} = r_i$, this gives the first order condition,

$$V(x_i, G^{-1}(r_i) - px_i) + (S_0 + r_i)x'(r_i) (V_1(x_i, G^{-1}(r_i) - px_i) - pV_2(x_i, G^{-1}(r_i) - px_i)) = 0. \quad (4)$$

This first order condition therefore defines a differential equation,⁵

$$x'(r) = \frac{V(x, G^{-1}(r) - px)}{pV_2(x, G^{-1}(r) - px) - V_1(x, G^{-1}(r) - px)} \frac{1}{S_0 + r} = \frac{\phi(x, G^{-1}(r))}{S_0 + r} \quad (5)$$

An important point to recognize is that this differential equation and the equilibrium strategy, which is its solution, both depend on the distribution of income $r = G(z)$.

Our next step is to specify what Frank (1985) and HK call the “cooperative choice”, which is an optimal consumption choice $(x^c(r), y^c(r))$ when an individual does not or cannot affect her social status. This choice corresponds to the standard tangency

⁴We have assumed that $G(\cdot)$ is strictly increasing on its support so that there is a one-to-one relation between income and rank.

⁵As we will see, the analysis of equilibrium choices under rank-indexing approach is much simpler than under income-indexing approach. To see that, compare equation (5) to the equation 6 in Hopkins and Kornienko (2004):

$$\frac{dx(z)}{dz} = \left(\frac{g(z)}{S_0 + G(z)} \right) \left(\frac{V}{pV_2 - V_1} \right) = \frac{dx(r)}{dr} \frac{dr}{dz} = \frac{dx(r)}{dr} g(z)$$

In other words, under income-indexing, the analysis of equilibrium decisions is complicated by the presence of income density $g(z)$ in the differential equation. As we will see later on (see footnote 10), the reverse is true of the analysis of equilibrium utilities.

condition: $V_1(x^c, y^c)/V_2(x^c, y^c) = p$. Given the budget constraint $px^c(r) + y^c(r) = G^{-1}(r)$, one can rewrite the above tangency condition as:

$$\frac{V_1(x^c(r), G^{-1}(r) - px^c(r))}{V_2(x^c(r), G^{-1}(r) - px^c(r))} = p. \quad (6)$$

Let $x^c(r)$ be the strategy implied by the above condition. The cooperative strategy also enables us to fix the appropriate boundary condition for the differential equation (5). This is not a purely technical question. As we will see, equilibrium behavior is quite different in the two different cases, where S_0 minimum guaranteed status is zero, and when it is positive. Thus, the initial conditions, or the choices of the individual with the lowest rank zero are:

$$\begin{aligned} S_0 = 0 &\Rightarrow x(0) = \frac{G^{-1}(0)}{p} \\ S_0 > 0 &\Rightarrow x(0) = x^c(0) \end{aligned} \quad (7)$$

When S_0 is positive, the lowest ranked individual does not take part in social competition and spends only the cooperative amount on visible consumption. In complete contrast, when $S_0 = 0$, low ranked individuals are desperate and spend all their endowment in a futile attempt to get ahead. While these boundary conditions were of crucial importance in the endowment-based approach of HK, with rank-based indexing these play less prominent role. Hopkins and Kornienko (2004, Proposition 1) also showed the following. In the original, the equilibrium strategy was expressed as a function of income, but as there is smooth mapping $z = G^{-1}(r)$, the results are easily carried over.

Proposition 1 *The differential equation (5) with boundary conditions (7) has a unique solution which is an essentially unique symmetric Nash equilibrium of the game of status. Equilibrium conspicuous consumption $x(r)$ is greater than in the absence of status concerns, that is $x(r) > x^c(r)$ on $(0, 1]$.*

The equilibrium is only “essentially” unique as when $S_0 = 0$, as there is other possible equilibrium behavior for the agent with the lowest income. Specifically, given the necessary condition for equilibrium $\lim_{z \rightarrow \underline{z}^+} x(z) = \underline{z}$, the agent with income \underline{z} will always have rank zero and always have zero utility and so is indifferent between any choice of x on the range $[0, \underline{z}/p]$. The specific boundary condition that $x(0) = G^{-1}(0)/p$, she spends all her income, is not crucial to our analysis but is used for convenience.

3 Indexing by Rank vs. Indexing by Income

In this paper as in our earlier work (Hopkins and Kornienko (2004)) (HK) we look at games of status. In HK we found that changes in income distributions translate into changes in individual consumption choices. The techniques we used to show this

were first developed in the literature on the first price auctions. In general in games of incomplete information, it is customary to identify individuals by their type, a draw from an exogenous distribution of characteristics. In the games of status we consider, agents only differ in terms of income. So, at first, we found it to be natural to write equilibrium consumption choices and equilibrium utility as functions of income, or $x(z)$ and $U(x(z), z - px(z), S(z)) = U(z)$ respectively. We then analyzed how a change in income distributions affects the choices made and utility received at each income level. However, while this approach to games of status has proved to be useful, it has some limitations.

Our principal interest is the effect of changes in income distribution on equilibrium behavior and well-being. Yet, take, for example, a redistributive scheme which raises the income of all those with below average income, while taxing all those with above average income. Since the income of every agent changes, it is difficult to make comparisons at a given level of income. In particular, there are income levels that no longer exist after the change. For example, imagine the lowest income ex ante was \$5000, but ex post was \$6000. We cannot compare how it feels to have \$5000 before and after the changes, as after the changes there is no-one with that income. Thus, to avoid this problem, in HK we looked at changes in the income distribution that did not change its support.

This is important despite the conventional assumption is that the support should not matter.⁶ Yet in the games of status the density of incomes matters a great deal, and this conventional wisdom turns out to be misleading. One of the principal findings of HK was that a higher density of the income distribution increased the degree of social competition: the closer people are together, the easier it is to surpass them with a slight increase in conspicuous consumption. When one keeps the support of the income distribution constant, a stochastically higher income distribution is only possible if there is an increase in density among the relatively well-off. That is, it is not possible to increase incomes without increasing the degree of social competition. However, if one is allowed different supports, then this is no longer necessary. In the current work, we look at a larger set of changes to income distributions, including those that alter the support of the distribution. For example, if everyone in society receives an extra \$1 then the income distribution moves to the right, with its density unchanged. In this case, we have higher incomes without greater social competition, and thus everyone is better off.

The second issue is whether comparison at a constant level of income makes sense. In our original work (HK), it helped to contrast the behavior of agents with concerns for status with behavior under standard preferences. If the incomes of others change, this would normally have no effect on the consumption choices of an agent with unchanged income. But we were able to show that, when there are concerns for relative position, changes in the distribution of income can change outcomes at every level of income. For example, in Figure 1 one of these results is illustrated. An individual with income

⁶Given two distributions on different supports, one can approximate either distribution arbitrarily closely with a distribution which has support on the union of the two original supports.

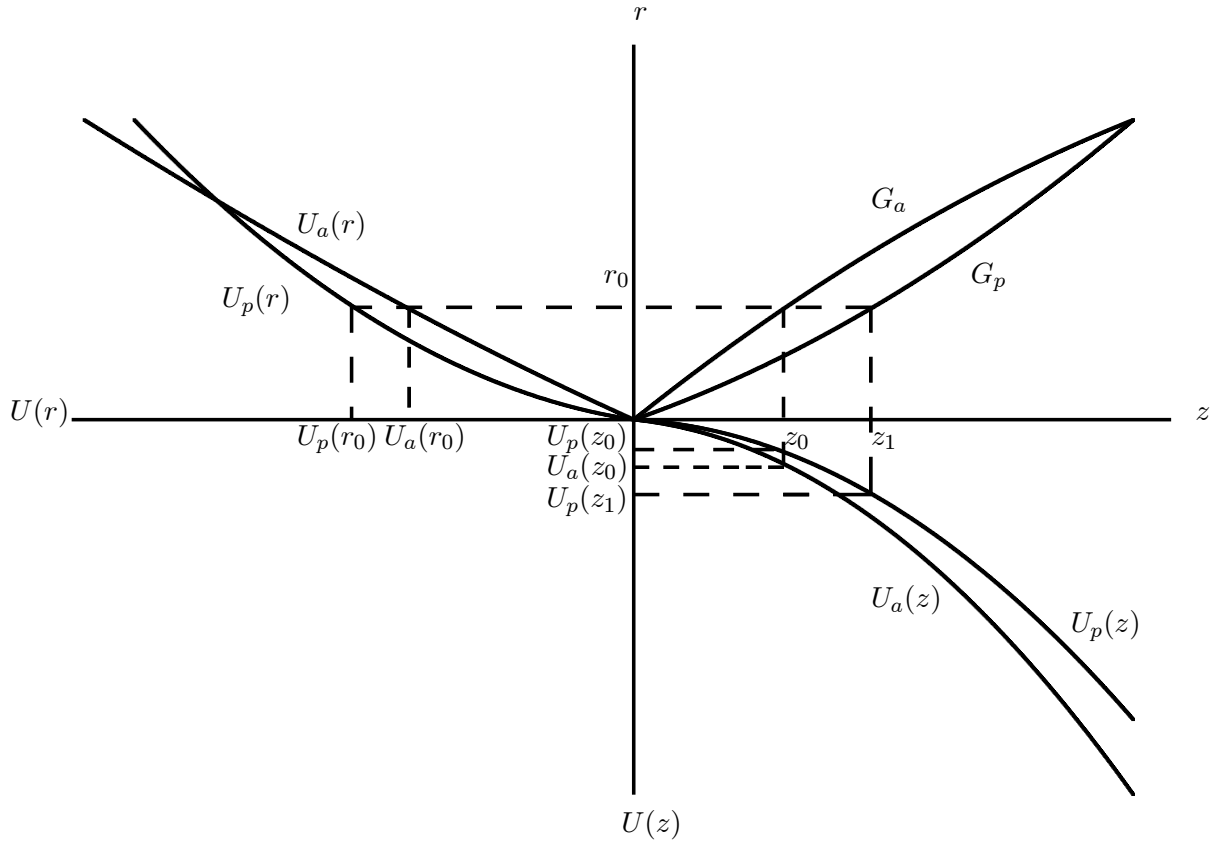


Figure 1: Comparative statics: income indexing vs. rank indexing. The top right panel represents the relationship between income and rank for two distributions G_a and G_p , with G_p being stochastically higher than G_a . The bottom right panel represents the comparative statics of welfare under the income indexing approach of Hopkins and Kornienko (2004). The top left panel represents the comparative statics of welfare under the new rank-indexing approach.

z_0 has higher rank under the initial income distribution $G_a(z)$ than under the stochastically higher income distribution $G_p(z)$ (in the top right panel $G_a(z_0) > G_p(z_0)$), and subsequently has higher utility under the initial income distribution $G_a(z)$ than under the stochastically higher income distribution $G_p(z)$ (in the bottom right panel $U_p(z_0) < U_a(z_0)$). That is, someone whose own income is unchanged, is made worse off by the improved situation of others. Nonetheless, such comparisons may not be the most appropriate for policy questions. For the income distribution to improve in the sense of stochastic dominance, someone must have gained in income. Yet, our previous methodology by making comparisons at constant income in effect looked at those who were left behind by such a change rather than those that benefitted. More generally, the assessment of some policy is often based on what happens to the average or median individual, or what happens to the bottom or top decile. That is, the comparison is often at a particular social position, not a level of income (presumably for similar reasons that income changes over time and because of policy interventions).

Thus, in this paper we index individuals by their rank in the income distribution, rather than by their income. In particular, we use the relation $r = G(z)$ or $z = G^{-1}(r)$ to write strategies and equilibrium utilities as functions of an agent's rank not her income. That is, $x(r)$ and $U(x(r), G^{-1}(r) - px(r), S_0 + r) = U(r)$ respectively. As in HK, equilibrium strategies depend on the distribution of income and, once the distribution changes, so does equilibrium behavior. Rank-based indexing actually allows us to consider a broader class of distributional changes than in HK. In particular, the rank-indexing approach is devoid of this unequal domain problem and is quite capable of comparing two income distributions that have different, even disjoint, supports.⁷

These two different approaches generate results that sometimes can seem contradictory. For example, in HK we have the result that if the income distribution changes so that the new distribution (first order) stochastically dominates the old, equilibrium utility $U(z)$ falls at every level of income. This is illustrated in Figure 1, where $U_a(z) > U_p(z)$ for almost all incomes z (in the bottom right panel). Compare again an individual with income z_0 having rank r_0 in the original distribution G_a . If after income transformation she still has the same income z_0 in the new distribution G_p , she would be worse off in this new distribution, i.e. $U_a(z_0) > U_p(z_0)$ (in the bottom right panel). But suppose we compared utilities not at a constant level of income, but at a constant rank. That is, suppose our individual with initial income z_0 has the same rank r_0 both in the initial distribution G_a and in the new distribution G_p . Figure 1 shows a result that we will go on to prove: this individual can have greater utility in the new distribution, i.e. $U_p(r_0) > U_a(r_0)$ (in the top left panel). However, this is not a contradiction to the result in HK. This is because since she still occupies the same position in the new distribution G_p , she will have the higher income z_1 .⁸ Her utility of her new income z_1 in the new distribution G_p exceeds her utility of her old income z_0 in the old distribution G_a (the bottom right panel). Thus, the same change analysed using the same model gives different results when different forms of comparisons are used.

There is another important feature of games of status. Observe that because income and rank stand in a reciprocal relationship to each other, the density of the individual's "rank space" is reciprocal of the density of her "income space":

$$\frac{dG^{-1}(r)}{dr} = \frac{1}{g(G^{-1}(r))} \quad (8)$$

Thus, the two "sides of the social coin" affect the return to happiness in a reciprocal way. In other words, take people around a given individual with rank r . The more densely packed are these individuals in the "rank space" - and, reciprocally, the more sparsely packed they are in the "income space" - the higher is the marginal return to

⁷We hope that the techniques developed here could also be used to advantage in first price auctions as well.

⁸By the definition of stochastic dominance $G_p(z) \leq G_a(z)$ for all z . This implies that, for a given income one's rank is lower in the stochastically higher distribution, and for a given rank, one's income is higher in the higher distribution.

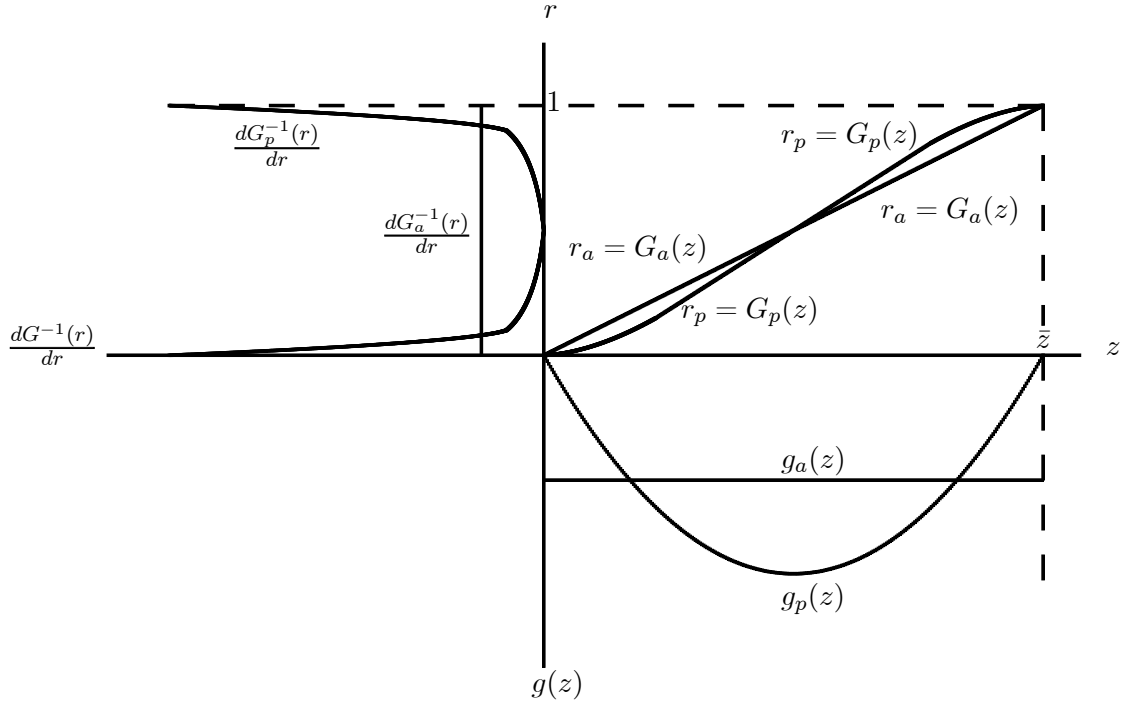


Figure 2: Densities vs. inverse densities. The top right panel represents the relationship between income and rank for two distributions G_a and G_p , with G_p being more “equal” than G_a . The bottom right panel represents the densities of the two distributions, $g_a(z)$ and $g_p(z)$. The top left panel represents the inverse densities of the two distributions, $\frac{dG_a^{-1}(r)}{dr}$ and $\frac{dG_p^{-1}(r)}{dr}$.

happiness from rank, and vice versa (see Figure 2). This observation will be important for our argument to follow.

In order to get clear-cut analytical results, for each method, we employ a corresponding partial ordering of income distributions. Under income-indexing we used the concepts of first and second order stochastic dominance, as well as their strengthenings based on the likelihood ratio order, or the ratio of densities (which is the ratio of competitiveness in the income space). Here we use another way of ordering distributions which is based on ranking distributions based on the ratio of the inverse densities (which is the ratio of competitiveness in the rank space).

In particular, here we use the dispersive ordering discussed in detail by Shaked (1982) (see also references therein) and Shaked and Shanthikumar (1994). Let F and G be two arbitrary continuous distribution functions with arbitrary, even disjoint, supports and let F^{-1} and G^{-1} be the corresponding left-continuous inverses (so that $F^{-1}(t) = \inf\{x : F(x) \geq t\}, t \in [0, 1]$ and $G^{-1}(r) = \inf\{x : G(x) \geq r\}, r \in [0, 1]$), and let f and g be the respective densities. Income in the society with income distribution F is said to be smaller in the dispersive order (or less dispersed) than income in the society with income distribution G (denoted as $F \prec_{disp} G$) whenever

$$F^{-1}(r_2) - F^{-1}(r_1) \leq G^{-1}(r_2) - G^{-1}(r_1) \text{ whenever } 0 < r_1 \leq r_2 < 1$$

That is, the income difference between individuals ranked r_1 and r_2 in both societies is no greater in the society with distribution F than in the society with distribution G . This implies that the income increase that an individual with rank r and income z gets by moving from the distribution F to more disperse distribution G is progressive with both rank r and income z , i.e. $G^{-1}(r) - F^{-1}(r)$ is increasing in rank r for $r \in (0, 1)$ and $G^{-1}(F(z)) - z$ increases in z . Finally, when both distributions have finite means, if F is less dispersed than G then $Var_F(z) \leq Var_G(z)$ whenever $Var_G(z) < \infty$.

In what follows we will use heavily the location-free feature of the dispersive order, which implies that adding an arbitrary constant to everyone's income in one society ($z \rightarrow z + c$ where c is a real number) will preserve the dispersive order. This would imply that $F \prec_{disp} G$ if and only if $F(z - c)$ crosses $G(z)$ at most once and when it does cross then from below. Shaked (1982, Remark 2.3) pointed out at the following important consequence of the location-free features of the dispersive order:

$$F \prec_{disp} G \text{ if and only if } f(F^{-1}(r)) \geq g(G^{-1}(r)) \text{ whenever } r \in (0, 1) \quad (9)$$

That is, for a fixed rank, the more disperse distribution is less dense than the less dispersed one when compared at the corresponding incomes. Note that because the condition (9) is expressed in terms of ranks, there is no problem in comparing distributions with different, even disjoint, supports.

The location-free features of the dispersive order imply that the dispersive ordering does not have a clear relationship with first and second order stochastic dominance, concepts that may be more familiar to economists. To see that, suppose $G(z)$ is uniform on $[0, 1]$ and $F_1(z)$ is uniform on $[0, 0.5]$, while F_2 is uniform on $[0.25, 0.75]$. One can verify that the distributions F_1 and F_2 are equal in the dispersive order, but G dominates both F_1 and F_2 in a sense of dispersive order (i.e. $F_1 \sim_{disp} F_2 \prec_{disp} G$). However, G first (and thus second) order stochastically dominates F_1 , but F_2 second (but not first) order stochastically dominates G (and of course F_2 first (and thus second) order stochastically dominates F_1). As will be seen, for the problems that we are concerned with here the dispersive order will be more useful than stochastic dominance.

There is one final point to make. One might be interested in total or average social welfare as well as the welfare of individuals. Under the income based approach, it was difficult to make comparisons. While average social welfare is equal to $\int U(z)dG(z)$, it is often difficult to calculate whether it rises. For example, in the situation illustrated in the right hand side of Figure 1, it is not clear whether average welfare is higher under income distribution G_a or distribution G_p . Individual utility is lower at each level of income under G_p . However, as G_p is stochastically higher, there are more rich people, who have high utility. Overall, the comparison is ambiguous. However, the equivalent expression for social welfare under the rank approach is $\int U(r)dr$. Therefore, if utility rises at each rank, it is necessarily true that social welfare must have risen.

4 The Effects of a Change in the Distribution of Income on Equilibrium Behavior

Suppose a society experiences a change in the distribution of income, or an *income transformation* - for example, because of economic growth, or because of redistributive taxation, or for any other reason. Let us denote the ex-ante cumulative distribution of income as $G_a(z)$, and the ex-post cumulative distribution of income as $G_p(z)$, each distribution being continuously differentiable with densities $g_a(z)$ and $g_p(z)$ that are strictly positive on their respective supports. In addition, we assume that the two distributions are distinct from each other except at a finite number of points (i.e. there is no interval where the two distributions coincide). Note that, in contrast to Hopkins and Kornienko (2004), here we do not require the two distributions to have the same support, even being able to perform the analysis on disjoint supports.

We will consider how changes in the distribution of income affect conspicuous consumption and welfare. We first start with equilibrium strategies. We show that an increase in income for those with low rank will raise their expenditure on conspicuous consumption.⁹

Proposition 2 *Suppose that $G_p^{-1}(r) \geq G_a^{-1}(r)$ on an interval $[0, \hat{r}]$ where \hat{r} is the point of first crossing of $G_p^{-1}(r)$ and $G_a^{-1}(r)$. Then $x_p(0) \geq x_a(0)$ and $x_p(r) > x_a(r)$ on $(0, \hat{r}]$.*

Note that if the ex post distribution (first order) stochastically dominates the ex ante distribution this would imply

$$G_p^{-1}(r) \geq G_a^{-1}(r), r \in [0, 1] \Leftrightarrow G_p(z) \leq G_a(z), z \in [0, \infty). \quad (10)$$

Note that this implies that after the change in question, income is (weakly) higher at every rank in society. Combined with the previous proposition we can see that in a stochastically higher distribution of income (almost) all individuals will spend more on conspicuous consumption.

Corollary 1 *Suppose that $G_p^{-1}(r) \geq G_a^{-1}(r)$ for all $r \in [0, 1]$. Then $x_p(0) \geq x_a(0)$ and $x_p(r) > x_a(r)$ on $(0, 1]$.*

Consider now an income transformation where the poorest individual has an increase in income, and the ex-post distribution is more dispersed than the ex-ante one, in the

⁹Strictly speaking, the inequalities of Proposition 2 and subsequent results should be qualified as only holding almost everywhere. Specifically, equality between x_p and x_a is possible at isolated points where both $G_a^{-1}(r) = G_p^{-1}(r)$ and $g_a(G_a^{-1}(r)) = g_p(G_p^{-1}(r))$ hold simultaneously. Of course, we could rule this out with further technical assumptions designed specifically to exclude this possibility. However, the only result where such a non-generic situation may be qualitatively important is Proposition 3 below, where we cannot rule out the possibility of a such a non-generic crossing of x_p and x_a at the point of interest.

sense of the dispersion order introduced in the previous section. In fact, it is easy to see that such an income transformation results in an ex-post distribution first-order dominating the ex-ante one. Thus we have the following simple result.

Corollary 2 *Suppose that*

$$G_p^{-1}(0) \geq G_a^{-1}(0) \quad (11)$$

and also

$$g_p(G_p^{-1}(r)) \leq g_a(G_a^{-1}(r)) \text{ for all } r \in (0, 1) \Leftrightarrow G_p \succ_{disp} G_a \quad (12)$$

then, $x_p(0) \geq x_a(0)$ and $x_p(r) > x_a(r)$ on $(0, 1]$.

However, as we will show in the next section, the increased affluence of those at the “bottom”, by pushing those “further up” to spend more on conspicuous goods, will tend to adversely affect the welfare of those in the “middle” - regardless of what happened to their incomes. Thus, even if everyone in the economy is richer, it does not necessary mean that everyone is happier.

Proposition 2 is also instrumental in understanding what happens if the distribution of income becomes more equal, for example, when a redistributive taxation scheme is imposed. Using similar reasoning, we now look at consumption decisions when income becomes more equal, or less dispersed in a sense of the dispersive ordering. Such a transformation implies that the distributions cross at some point \hat{r} (the dispersive order and our assumption that the two distributions are almost everywhere distinct imply that there would be at most one such point). Then everyone at the lower end of the income hierarchy up to the individual with rank \hat{r} will have a higher income and spend more on conspicuous consumption. This is over and beyond the standard effect of increased income on the demand for a normal good, because the individual with rank \hat{r} will spend more on conspicuous consumption even though her income has not changed. All those who are “below” her in the income hierarchy have become richer after the transformation, and thus are able to afford more conspicuous consumption. As the result, in order to “keep up” her social rank (as determined by the relative position in the consumption hierarchy), the individual with rank \hat{r} now has to spend more.

Corollary 3 *Suppose that*

$$G_p^{-1}(0) \geq G_a^{-1}(0) \quad (13)$$

and also

$$g_p(G_p^{-1}(r)) \geq g_a(G_a^{-1}(r)) \text{ for all } r \in (0, 1) \Leftrightarrow G_a \succ_{disp} G_p \quad (14)$$

and also suppose that

$$G_p^{-1}(1) \leq G_a^{-1}(1) \quad (15)$$

Then, $x_p(0) \geq x_a(0)$ and $x_p(r) > x_a(r)$ on $(0, \hat{r}]$ where \hat{r} is the only point of crossing of $G_a^{-1}(r)$ and $G_p^{-1}(r)$.

Here, everyone at the lower end of the income hierarchy, and possibly everyone, spends more on conspicuous consumption - including the individual with rank \hat{r} whose income does not change (the location-free properties of the dispersive order together with the smaller range of G_p ensures that there is exactly one point \hat{r}). This result implies, by continuity of the equilibrium strategy $x(r)$, that those with slightly higher ranks (and thus lower incomes) also will have to spend more on conspicuous consumption even though they have lower ex-post income. In other words, the increased affluence of those at the “bottom” of the social hierarchy forces those in the “middle” to spend more on conspicuous consumption in order to “keep up” their social “place”. This can be further demonstrated by the following example.

Example 1 *Suppose $z_p = (1 - \tau)z_a + \tau\mu_a, \tau \in (0, 1)$, where μ_a is the mean income of the ex-ante income distribution G_a . This is a mean-preserving income transformation (so that $\mu_p = \mu_a$) and it is equivalent to a redistributive scheme whereby everyone is taxed at a flat rate of τ and given a lump transfer of $\tau\mu_a$. Those with income that is initially lower than average (i.e. with $z_a < \mu_a$) get an income subsidy, while those with above average income see their incomes taxed away. An arbitrary ex-ante distribution G_a crosses the corresponding ex-post distribution G_p from above at the mean income μ_a , so that the individual with mean income μ_a sees no change in neither rank nor income. However, by Proposition 2 she spends more on conspicuous consumption, i.e. $x_p(G_p^{-1}(\mu_a)) > x_a(G_a^{-1}(\mu_a))$. By continuity, those with slightly above average incomes will also spend more on conspicuous goods, even though their incomes are lower.*

5 The Effects of a Change in the Distribution of Income on Equilibrium Utility

In this section, we will explore equilibrium welfare in the game of status when individuals are indexed by their rank. We begin with the following important question. Given the existence of relative concerns, what kind of income transformation would guarantee that *everyone* in the economy is better off? Recall that if we take the standard self-centered approach to the consumer choice problem, the answer is trivial - increase everyone’s income, and everyone in the economy would be better off. Yet a number of empirical studies pointed out that economic growth is not unequivocally happiness-enhancing. While most researchers agree that this may be because of the existence of relative concerns, there is still little understanding of why relative concerns may reduce the benefits from economic growth (increased income).

In this section we will offer a sufficient condition for an increase in the happiness level for *everyone* when relative concerns are present. We demonstrate that an increase in each individual’s income by itself cannot constitute a sufficient condition for a uniform increase in happiness. Instead, what one needs is a reduction in “competitive pressures” which follows from a greater dispersion of income, coupled with the Rawlsian-style

requirement that the individual at the bottom of the social hierarchy becomes richer.

In this section, we consider equilibrium utility which is $U(r) = V(x(r), G^{-1}(r) - px(r))(S_0 + r)$. Using the envelope theorem, one can find that the marginal change in the equilibrium utility from change in rank is:¹⁰

$$U'(r) = \frac{dU(r)}{dr} = \frac{\partial U(x, y, S)}{\partial r} = V_2(x, G^{-1}(r) - px)(S_0 + r) \frac{dG^{-1}(r)}{dr} \quad (16)$$

That is, the marginal return to happiness from rank depends on the density of the individual's "social space". Remember that from (8), $dG^{-1}/dr = 1/g(z)$.

Again we compare outcomes under an ex ante distribution of income G_a and an ex post distribution G_p . The respective equilibrium utilities are $U_a(r) = V(x_a(r), G_a^{-1}(r) - px_a(r))(S_0 + r)$ and $U_p(r) = V(x_p(r), G_p^{-1}(r) - px_p(r))(S_0 + r)$. We first look at an arbitrary income transformation that leaves some people with both unchanged incomes and ranks, and among these individuals, we look at the poorest one. The next proposition shows that such an individual will be worse off whenever those below her in the income hierarchy become more affluent.¹¹

Proposition 3 *Suppose that $G_p^{-1}(r) \geq G_a^{-1}(r)$ on an interval $(0, \hat{r})$, where \hat{r} is the point of the first crossing of G_a^{-1} and G_p^{-1} . Then $U_p(\hat{r}) < U_a(\hat{r})$.*

This result says that if incomes rise but some individual at rank \hat{r} does not have her income increased, she is not indifferent but is strictly worse off. This is because the social pressure of others' higher incomes forces her to increase conspicuous consumption. Note that, by continuity, some people with rank just less than \hat{r} will also be worse off, even though they now have higher income.

One might think that in order to avoid this problem and for everyone in the society to be better off, we simply have to increase the income of everyone. Yet, as the following example demonstrates, under relative concerns, higher income at every rank in society does not imply greater happiness for all.

Example 2 *Suppose that $U = xyS$, with $S_0 = 0$ and G_a is uniform on $[0, 1]$, and consider a linear income transformation of income $z_p = 0.25z_a + 1.5$, so that G_p is*

¹⁰As we mentioned before in footnote 5, the analysis of equilibrium utility under rank-indexing approach is more difficult than under income-indexing approach. To see that, compare equation (16) with the analogous equation under income-indexing approach (found in the proof of the Proposition 2 of Hopkins and Kornienko (2004)):

$$\frac{dU(z)}{dz} = \frac{dU(r)}{dr} \frac{dr}{dG^{-1}(r)} = \frac{dU(r)}{dr} g(z) = V_2(S_0 + G(z))$$

Thus, under rank-indexing, the analysis of equilibrium utilities is complicated by the presence of the reciprocal of the income density, $1/g(z)$.

¹¹An implication of the issue we raised in footnote 9 is that it is possible that $U_p(\hat{r}) = U_a(\hat{r})$ in the non-generic situation we identified there.

uniform on $[1.5, 1.75]$. Then $U_p(r) > U_a(r)$ for all $r \in (0, 0.9)$, while $U_p(r) < U_a(r)$ for all $r \in (0.9, 1]$.

Notice that in the above example G_p first order stochastically dominates G_a , and *everyone* is (vastly) richer ex-post. Under the standard self-centered paradigm, that would imply that everyone would be happier ex-post as well. Yet this is not the case and indeed only 90 percent of the population is happier. This is because the transformation that raised incomes also compressed them. More compressed distributions give rise to greater social competition.

That is, in the games of status not only income, but also the degree of social competitiveness that matter. This gives rise to simple sufficient conditions for everyone to be happier. Incomes must be raised but at the same time, social competition must not rise. For this to be true, the distribution of income must not become any more compressed in the sense of the dispersive order.

Proposition 4 *Suppose that*

$$G_p^{-1}(0) \geq G_a^{-1}(0) \tag{17}$$

and also

$$g_p(G_p^{-1}(r)) \leq g_a(G_a^{-1}(r)) \text{ for all } r \in (0, 1) \Leftrightarrow G_p \succ_{disp} G_a \tag{18}$$

then $U_p(0) \geq U_a(0)$ and $U_p(r) > U_a(r)$ for all $r \in (0, 1]$.

In other words, in order for everyone to be happier, we need, first, the poorest person to be no worse in terms of income (and thus of happiness - reminiscent of Rawls' criterion) and, second, a decrease in "competitive pressures" (as represented by the income density of people with similar rank). Note that together they imply that incomes are strictly higher at every rank. (Intuitively, since the lowest person is no worse off, the only way to make incomes more dispersed is to spread them upwards.) However, recall that we saw earlier in Example 2 that a general increase in income may not be sufficient to increase happiness if the "competitiveness criterion" of the equation (18) is not satisfied. That is, the reduction in social pressure implicit in (18) is essential for greater income to be beneficial for all.

We have looked at the case where the income transformation results in an increase in incomes. We now look at what happens when there is redistribution of a fixed level of total income.

Example 3 *Suppose that $U = xyS$, with $S_0 = 0$, and G_a is uniform on $[0, 1]$, and consider a linear income transformation $z_p = 0.5z_a + 0.25$. Then $U_p(r) > U_a(r)$ for all $r \in (0, 0.35)$, while $U_p(r) < U_a(r)$ for all $r \in (0.35, 1]$.*

Notice that the income transformation of the above example is a mean-preserving redistribution scheme of the type considered in the Example 1. Here, the bottom half of

the population is richer ex-post, yet only the poorest 70 percent of them (and thus the poorest 35 percent of the entire population) is better off! The remaining 30 percent of those whom become richer (which is 15 percent of the population) and all of those who became poorer (that is the 50 percent of the entire population) is worse off. That is, the above mean-preserving scheme, by redistributing income from the richest half of the population to to the poorest half, makes happier the poorest 35 percent of population, at the expense of the decreased happiness of 65 percent of the population.

We can generalize the above example as follows. Suppose we change the income distribution so that it less dispersed than before. We do this by increasing the incomes of those with low ranks and decreasing the incomes of those with high ranks, and increasing social competitiveness. Of course, this is consistent with the examples of redistributive policies above. Then, we can show that the middle and upper classes are worse off in this more equal society.

Proposition 5 *Suppose that*

$$G_p^{-1}(0) \geq G_a^{-1}(0) \tag{19}$$

and also

$$g_p(G_p^{-1}(r)) \geq g_a(G_a^{-1}(r)) \text{ for all } r \in (0, 1) \Leftrightarrow G_a \succ_{disp} G_p \tag{20}$$

and also suppose that that

$$G_p^{-1}(1) \leq G_a^{-1}(1) \tag{21}$$

Then, $U_p(0) \geq U_a(0)$ and $U_p(r) < U_a(r)$ for all $r \in [\hat{r}, 1]$ where \hat{r} is the only point of crossing of $G_p^{-1}(r)$ and $G_a^{-1}(r)$.

Notice that G_p is less dispersed than G_a in the sense of the dispersive order. The result implies that a simple redistributive scheme, which takes from those with above average income to supplement the income of those with below average income, will reduce the utility of the middle and upper classes. Of course, it is unsurprising that the rich would not benefit from such a scheme. However, an individual whose income is unchanged (the one with rank \hat{r}) is strictly worse off and by continuity this will also apply to some “lower middle class” individuals who have less than average income. This is even though they have higher income post distribution. This is because the more compact income distribution after tax implies greater social competition, greater expenditure on conspicuous consumption and lower utility.

We lastly point out that as long as the the social competition is not too extreme (i.e. $S_0 > 0$) individuals at the very bottom of the distribution will be strictly better off when the very poorest individual is strictly richer.

Proposition 6 *Suppose that*

$$G_p^{-1}(0) > G_a^{-1}(0) \tag{22}$$

If $S_0 > 0$, then there exist $\tilde{r} \in (0, 1]$ such that $U_p(r) > U_a(r)$ on $[0, \tilde{r})$.

That is, as long as the poorest individual does not face extreme social competition, additional income for the poorest will make them better off. This is because when $S_0 > 0$, the lowest ranked do not take part in social competition (see (7)) and so do not waste the additional income on conspicuous consumption. One should not take the result that the very poor may benefit from extra income as trivial, because, as the next example shows, once the poorest individual faces extreme social competition of $S_0 = 0$, those at the very bottom of the distribution can be worse off, as the benefits of higher income are dissipated by high social competition.

Example 4 *Suppose $U = yx^a S$, with $S_0 = 0$. Then we have $U(0) = 0$ irrespective of income and $U'(r) = x^a r/g(r)$. Suppose we look at an income transformation such as in Proposition 6 that raises the income of the poorest at the same time as increasing the density. The utility of the poorest is unchanged at zero, but if a is small and the change in density large, $U'_p(r) < U'_a(r)$ in the neighbourhood of $r = 0$ and so those at the very bottom of the distribution are worse off in the more equal situation.*

6 Non-Rank-Preserving Income Transformations

We have been trying to emphasize that, when relative concerns are present, changes in the income distribution can affect agents through three channels: one's own income, one's relative position or rank in the distribution, and the shape of the distribution. Our earlier methodology (HK) allows one to analyze individual decisions and utility for fixed incomes, but is limited to comparing distributions with the same support. Our current methodology allows one to make this analysis for fixed ranks, and thus is particularly suitable for analyzing rank-preserving income transformations (such as linear transformations of the form $z_p = a + bz_a$).

In this section, we present some situations which would seem to defeat both methods. For example, suppose that incomes change in a manner that is not rank preserving and which changes the support of the distributions. Nonetheless, one can still get some understanding of what happens employing the rank-indexing approach augmented by the use of a rank transformation function. Moreover, these situations highlight the importance of the third factor of analysis - the social competitiveness (defined as $g(z)$ in the income space or $\frac{dG^{-1}(r)}{dr}$ in the rank space).

Let us start with the following example.

Example 5 *Suppose $U = xyS$, with $S_0 \geq 0$. Suppose G_a is uniform on $[1, 2]$, and consider an inequality-reducing transformation whereby that the “lower half” (or everyone with ex ante incomes of $[1, 1.5)$) receives an income subsidy of 0.5, but the “upper half” (or everyone with ex ante incomes of $[1.5, 2]$) sees no change in incomes. Then all incomes are now distributed according to G_p , which is uniform on $[1.5, 2]$.*

Here ex post everyone faces a uniform distribution on $[1.5, 2]$, and their conspicuous consumption choices are still determined by the equation (5). Given that the ex post distribution first order dominates the ex ante distribution, by Corollary 1, for every fixed rank, everyone spends more on conspicuous consumption. However, here, the rank-indexing approach alone is not sufficient to analyze what happens to equilibrium utility of every individual. This is because this is an example of non-rank-preserving income transformation since in the new, post-transformation, society, individuals with ex-ante ranks of 0 and 0.5 both now have rank of 0, individuals with ex-ante rank of 0.1 and 0.6 both now have rank of 0.2, individuals with ex-ante rank of 0.49 and 0.99 both now have rank of 0.98, and so on.

However what we can do is to carry on the analysis based on rank indexing by constructing a rank transformation $\mathcal{R} : r_a \rightarrow r_p$ implied by this income transformation. Here it is, $r_p = 2r_a$ for $r_a \in [0, 1/2)$ and $r_p = 2r_a - 1$ for $r_a \in [1/2, 1]$. As the result, the ex-ante “lower half” sees an increase in both income and rank, while the “upper half” sees a decrease in rank and no change in income. Clearly, the bottom individual (i.e. the one with ex ante rank of 0) will have an ex post rank of 0 and a higher income, and thus will be no worse off after the income transformation. But those near the top of the bottom half (e.g. the one with ex ante rank just below 0.5) will have an ex post rank of just below 1 and higher income and will be better off. In contrast, everyone in the ex-ante “upper half” sees no change in incomes, but a decrease in ex-ante rank, and an increase in density of individuals around them. In fact, one can show that they will be worse off after the income transformation. Thus, using rank-indexing method, combined with rank transformation, one can do analysis for non-rank-preserving income transformations.

Non-rank-preserving transformations in fact highlight the importance of the third factor affecting one’s choices and well-being, namely, the density of the individual social space, or social competitiveness. As we pointed out before, an increase in equality implies an increase in social competitiveness. While in the above example the effect was not strong enough to offset the effect of increased income, Example 6 below demonstrates that one should not discard the importance of the social density. That is, one can experience an increase in income and increase in rank, but a decrease in utility because of an increase in social density, and thus an increase in competitive pressures.

Example 6 *Suppose $U = xyS$, with $S_0 = 0.1$. Suppose G_a is uniform on $[1, 2]$, and consider the following inequality-reducing transformation of income: everyone with incomes of $[1, 1.1)$ receive an income subsidy of 0.9, everyone with incomes of $[1.1, 1.2)$ receive an income subsidy of 0.8, and so on, with everyone with incomes of $[1.8, 1.9)$ receiving an income subsidy of 0.1, while those with incomes of $[1.9, 2.0]$ face no change in income. Then all incomes are now distributed with G_p , which is uniform on $[1.9, 2]$.*

Again, this is not a rank-preserving income transformation since in the new, post-transformation, society, everyone with ex-ante incomes of 1, 1.1, 1.2, \dots , 1.9 (and thus ex-ante ranks of 0, 0.1, 0.2, \dots , 0.9) now have ex-post rank of 0, everyone with incomes of

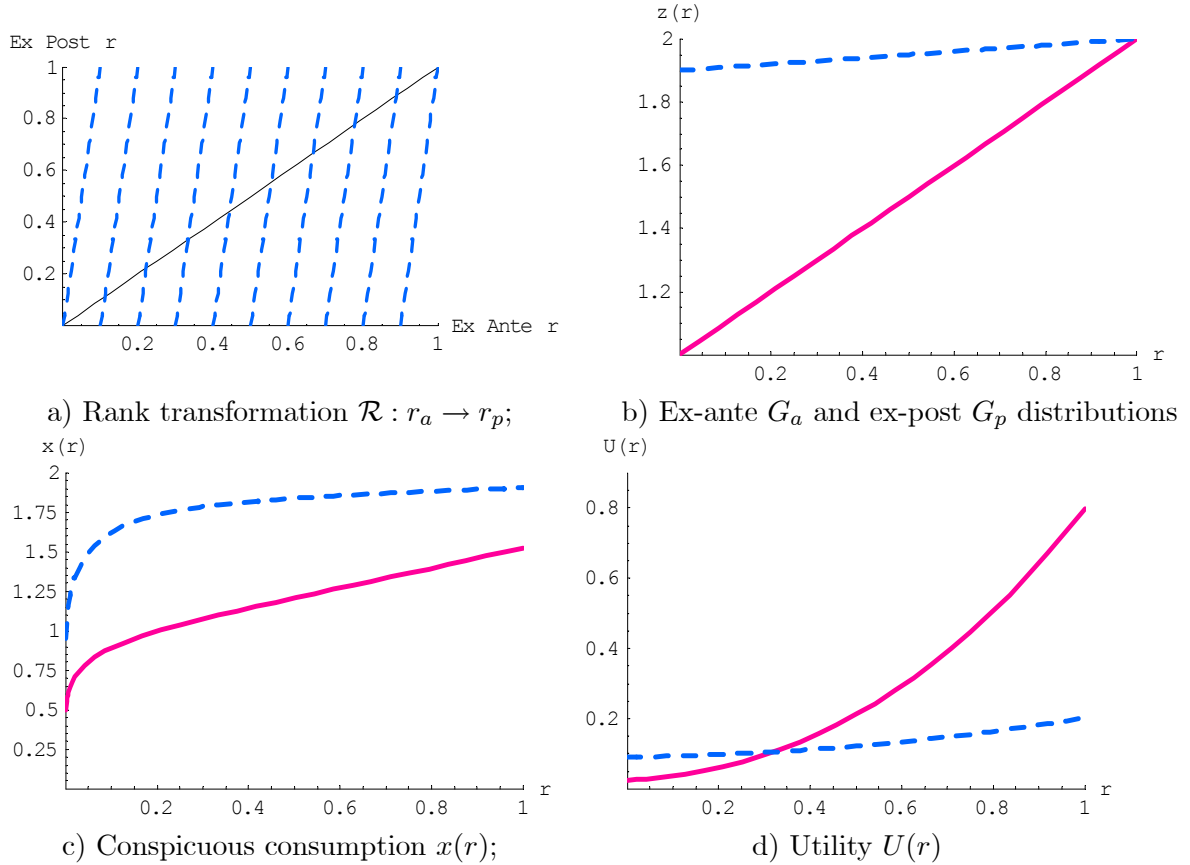


Figure 3: Social competitiveness $g(z)$ is important. When most individuals receive an increase in income, while being “squeezed” into a small income range, the increased affluence of the “bottom” of the society pushes the rest of the society to increase their conspicuous consumption $x(r)$ in order to “keep up”. This may result in some individuals getting lower utility even though both their income and rank increase. Ex-ante choices and utilities are represented by solid curves, while ex-post ones are represented by dashed curves. (Example 6: G_a is uniform on $[1, 2]$, G_p is uniform on $[1.9, 2]$, $U = xyS$, $S_0 = 0.1$).

1.05, 1.15, 1.25, \dots , 1.95 (and thus ranks of 0.05, 0.15, 0.25, \dots , 0.95) now have income rank of 0.5, everyone with incomes of 1.099, 1.199, 1.299, \dots , 1.999 (and thus ranks of 0.099, 0.199, 0.299, \dots , 0.999) now have ex-post rank of 0.99. The corresponding rank transformation $\mathcal{R} : r_a \rightarrow r_p$ implied by this income transformation is as follows: $r_p = 10r_a$ for $r_a \in [0, 0.1)$; $r_p = 10(r_a - 0.1)$ for $r_a \in [0.1, 0.2)$, \dots , and $r_p = 10(r_a - 0.9)$ for $r_a \in [0.9, 1]$ (Figure 3a). Here, ex post everyone faces a uniform distribution on $[1.9, 2]$ (Figure 3b), and thus, given their ex-post income and rank, their conspicuous consumption choices are determined by the equation (5) (Figure 3c).

One can see from Figure 3d that almost two thirds of the society are worse off after the transformation, even though 90 percent faced an increase in income. While the individuals varied in what happened to their rank (those above the 45 degree line in

Figure 3a faced an increase in rank, those below the line faced a decrease, those on the line faced no change), yet uniformly all faced an increase in social competitiveness - which as we argue, is important in games of status. For example, consider an individual with ex-ante income of 1.45 and rank 0.45, whose ex-ante utility is 0.17. After she receives a subsidy of 0.5, her ex-post income increases to 1.95, her rank increases to 0.5, but her utility goes down to 0.12. Needless to say, those with ex ante rank $[0.9, 1]$ who had no change in income, now have lower rank, and face higher competitive pressures, and thus are worse off.

7 Conclusions

In this paper we presented an alternative approach to the games of status introduced in our earlier paper (Hopkins and Kornienko (2004)). There we followed tradition and looked at consumer choice as a function of income. Thus the comparative static analysis of the effects of social affluence and income equality on consumer choices and welfare was done at a fixed level of income. Here, we look at the same situation but instead we index agents by their rank. The results in this context look surprisingly different, even though the fundamental insights remain robust to the change in the indexing paradigm.

The main new result is a sufficient condition for an increase in each individual's equilibrium utility. This involves non-decreased income for the society's poorest individual, plus an increase in the dispersion of incomes that leads to a decrease in social competitiveness. Together, these two conditions ensure that income at each rank level increases (and thus an *increase* of social affluence in terms of first-order stochastic dominance), and in equilibrium, results in higher utility at each rank level. Compare this to our earlier results for the income-indexing approach, where we had the apparently contradictory result that an increase in social affluence in terms of first-order stochastic dominance leads to a *decrease* in equilibrium utility at every level of income.

It is important to understand that these contrasting results are nonetheless consistent. When one examines the effect of an increase in social affluence using the income-indexing approach, one is looking at individuals whose income did not change, and so who did not gain from this social transformation. In fact, these individuals also face greater social competition from those who gained and their utility falls. In contrast, when one compares ex ante and ex post utilities at a constant rank, under this type of transformation an individual of a given rank will have a higher income. If the benefits of higher income are not outweighed by greater social competition, utility will rise. But this does not conflict with the result that individuals whose income is unchanged would see a fall in utility.

Another important insight is that, regardless whether we index individuals by their rank or their income, those whose income is left unchanged by the income transformation, tend to be adversely affected by the increased affluence of those who are below them in the income hierarchy. This is because, in order to "keep up" their social posi-

tion, they have to increase their consumption of conspicuous goods further above what would be privately optimal, leading to a decline in welfare. That is, it is the unchanged income of the poor that was the main culprit behind our earlier result that “equality hurts the poor”. Once we allow the income of the poor to increase (which sometimes may lead to greater equality), the poor will be better off.

Our third main insight is that in the games of status not only income and rank, but also social density, or competitiveness, matter. We demonstrate that one can have higher income and higher rank, but nevertheless have lower well-being because of the increase in social competitiveness. Note that such income transformation is non-rank preserving, yet we still were able to utilize the rank-indexing approach by augmenting it with a rank transformation function. In fact, income indexing approach can be reduced to a rank indexing approach with a suitable rank transformation.

Thus, given the similarities and differences in the two approaches, the question arises - if one is interested in analyzing what happens when relative concerns are present, which approach should one take? We think that our earlier approach based on indexing by income or endowment is well suited for some problems. The first such case is where income transformations differentially affect different types of income - for instance, it affects only unearned income but does not affect earned income, and vice versa, without changing the lowest income. Given that individuals may vary in their income composition, income-indexing can help to understand what happens to individuals who have only one type of income (say, earned). Another situation would be one where individuals make decisions based on their unchangeable *abilities*, or talents, and suppose that immigration changed the composition of abilities in the country. Here, the endowment-based approach would allow one to compare the decisions and well-being of domestic workers whose abilities were left unchanged by the influx of new people.

However, we now think that the rank-indexing approach is likely to be more generally fruitful in problems of social comparison. This is because it happens to be easier, allows for a wider set of comparative static predictions, and simply seems to make better sense. In particular, it avoids the most pessimistic of conclusions of the previous approach: that greater equality hurts the poor. We also hope that this theoretical investigation will prove useful for empirical analysis of the links between the distribution of income and happiness. In particular, it makes clear that the results one obtains will depend on one’s choice of the method of social comparison.

Nonetheless, both our earlier and present papers show that, when relative concerns are present, people will respond to an increase in the affluence of those who are below them by increasing their own conspicuous consumption in order to maintain their social position. Thus, whether we take the rank- or income-indexing approach, any policy that increases incomes of the lower classes without a sufficient increase in the incomes of the middle classes, will tend to decrease the latter’s welfare. Depending on the extent of the policy, it may make the upper classes worse off as well. In other words, whichever method of comparison is used, in these models of relative concerns, the middle classes may be worse off with policies that change the distribution of endowments in a way

that has been traditionally considered to be progressive.

We should point out, however, that this work is best interpreted as an investigation of the logical consistency of the argument that social preferences such as desires for rank or status imply that greater equality is necessarily beneficial. While we find that in fact greater equality in this type of model increases social pressures to consume, this is not in itself an argument for maintaining or extending existing inequality. Rather the policy implications are that interventions to increase equality would work better if accompanied by consumption taxes to alleviate the resulting increased social pressures. A similar point is made by Corneo (2002) who suggests that similar considerations may lie behind the Scandinavian model. That is, the high levels of taxation in these countries may serve to reduce visible consumption that otherwise would be highly competitive given their low pre-tax income inequality. Finally, further research (Hopkins and Kornienko (2005)) indicates that, in a similar model of social competition, different forms of equality can have quite different effects.

Appendix

Lemma 1 *Consider a pair of distributions $G_a(z)$ and $G_p(z)$ which are distinct from each other except at a finite number of points, both continuously differentiable with positive densities on the respective supports. If the corresponding equilibrium strategies are $x_a(r)$ and $x_p(r)$, then at any point \check{r} such that $x_a(\check{r}) = x_p(\check{r})$, the sign of $x'_a(\check{r}) - x'_p(\check{r})$ is equal to the sign of $G_a^{-1}(\check{r}) - G_p^{-1}(\check{r})$.*

Proof of Lemma 1: First note that, given the equation (5), we have that

$$\frac{x'_a(r)}{x'_p(r)} = \frac{\phi(x_a, G_a^{-1}(r))}{\phi(x_p, G_p^{-1}(r))} \quad (23)$$

so that any point where $x_a = x_p$ the relative slope only depends on G_a^{-1} and G_p^{-1} , and thus the slopes are equal whenever G_a^{-1} and G_p^{-1} are equal. Furthermore, given our assumptions that $V_{ii} \leq 0$ and $V_{ij} \geq 0$, we have that

$$\frac{\partial \phi}{\partial G^{-1}(r)} = \frac{V_2}{pV_2 - V_1} - \frac{V(pV_{22} - V_{12})}{(pV_2 - V_1)^2} > 0 \quad (24)$$

Thus, at any point where $x_a(r) = x_p(r)$ we have that $x'_a > x'_p$ (so that x_a is steeper than x_p and thus crosses x_p from below) whenever $G_a^{-1}(r) > G_p^{-1}(r)$ (i.e. whenever ex-ante income exceeds ex-post income), and vice versa. ■

Proof of Proposition 2: By the boundary conditions (7), the condition $G_a^{-1}(0) \leq G_p^{-1}(0)$ implies that $x_p(0) \geq x_a(0)$ (i.e. that the poorest individual, now that she has more income, spends more on conspicuous consumption). Given our assumption that G_a and G_p are distinct it follows that $G_p^{-1}(r) > G_a^{-1}(r)$ almost everywhere on $(0, \hat{r}]$.

Thus, by Lemma 1, $x_p(r)$ can only cross $x_a(r)$ from below except perhaps at the finite number of points where $G_p^{-1}(r) = G_a^{-1}(r)$.

We first rule out that there is an interval where $x_p(r) \leq x_a(r)$. Suppose on the contrary there exist at least one interval $[r_1, r_2] \subseteq [0, \hat{r}]$ such that $x_p(r) \leq x_a(r)$. By the continuity of x_a and x_p , it must be that $x_p(r_1) = x_a(r_1)$. Note that

$$\frac{\partial \phi}{\partial x} = \frac{-(V_1 - pV_2)^2 - V(pV_{21} - pV_{22} - V_{11} + pV_{12})}{(pV_2 - V_1)^2} < 0. \quad (25)$$

By a combination of Lemma 1 and (25), it would follow that $x'_p(r) < x'_a(r)$ almost everywhere on $[r_1, r_2]$, which combined with $x_a(r_1) = x_p(r_1)$ is a contradiction to $x_p(r) \leq x_a(r)$ on the interval. Thus, $x_p(r) > x_a(r)$ almost everywhere on $[0, \hat{r}]$.

We next rule out that $x_p(r) = x_a(r)$ at individual points. By Lemma 1 and the previous argument that excludes intervals where $x_p(r) \leq x_a(r)$, this is only possible at the isolated points where $G_p^{-1}(r) = G_a^{-1}(r)$. But at any such point \tilde{r} on $(0, \hat{r}]$, as $G_p^{-1}(r) > G_a^{-1}(r)$ almost everywhere, we have that $g_p(G_p^{-1}(\tilde{r})) \geq g_a(G_a^{-1}(\tilde{r}))$. Now, note that $G_p^{-1}(\tilde{r}) = G_a^{-1}(\tilde{r}) = \tilde{z}$. Next, we invoke the income indexing approach and consider solutions to the game in terms of income z , that is, solutions to the differential equation

$$\frac{dx(z)}{dz} = \left(\frac{g(z)}{S_0 + G(z)} \right) \left(\frac{V}{pV_2 - V_1} \right) = \frac{dx(r)}{dr} \frac{dr}{dz} = \frac{dx(r)}{dr} g(z) \quad (26)$$

that we write $x_p(z)$ and $x_a(z)$ for the respective distributions. Then if $x_p(\tilde{r}) = x_a(\tilde{r})$, it must be that $x_p(\tilde{z}) = x_a(\tilde{z})$. As $x_p(r) > x_a(r)$ for r in $(\tilde{r} - \epsilon, \tilde{r})$ for some $\epsilon > 0$, we must have $x_p(z) > x_a(z)$ for incomes slightly less than \tilde{z} . Note that by Lemma 1, $x'_p(\tilde{r}) = x'_a(\tilde{r})$, and for the case of $g_p(\tilde{z}) > g_a(\tilde{z})$, it must be that $x'_p(\tilde{z}) > x'_a(\tilde{z})$ so that $x_p(z)$ crosses $x_a(z)$ from below, which is a contradiction. This leaves us with the possibility, as was mentioned in footnote 9, that it is possible that $x_p(r) = x_a(r)$ in a non-generic case of $g_p(G_p^{-1}(\tilde{r})) = g_a(G_a^{-1}(\tilde{r}))$. ■

Proof of Proposition 3: Notice that since \hat{r} is the point of crossing of G_a^{-1} and G_p^{-1} , this implies that $G_p^{-1}(\hat{r}) = G_a^{-1}(\hat{r}) = \hat{z}$: an agent of this rank has the same income \hat{z} ex ante and ex post. Note that as G_p^{-1} crosses G_a^{-1} from above, we have $g_p(G_p^{-1}(\hat{r})) \geq g_a(G_a^{-1}(\hat{r}))$. Again, for the non-generic case of $g_p(G_p^{-1}(\hat{r})) = g_a(G_a^{-1}(\hat{r}))$, we cannot rule out that $x_p(\hat{r}) = x_a(\hat{r})$ and thus that $U_p(\hat{r}) = U_a(\hat{r})$. Yet for the case of $g_p(G_p^{-1}(\hat{r})) > g_a(G_a^{-1}(\hat{r}))$, Proposition 2 implies that $x_p(\hat{r}) > x_a(\hat{r})$, so this agent now spends more on conspicuous consumption. By Proposition 1, in equilibrium $x > x_c$, consumption exceeds the level that maximizes regular utility V . By the strict quasiconcavity of V , given a fixed level of income, $V(x, z - x)$ is strictly decreasing in x for $x > x_c$. This implies that $U_p(\hat{r}) > U_a(\hat{r})$. ■

Proof of Proposition 4: First, as income is (weakly) higher at $r = 0$ by the boundary conditions (7), $x_p(0) \geq x_a(0)$ and $y_p(0) \geq y_a(0)$, so that $U_p(0) \geq U_a(0)$ (i.e. as the poorest individual has no reduction in income she will not be worse off). Second, notice that $U_p(r) > U_a(r)$ if and only if $V_p(r) = V(x_p(r), y_p(r)) < V(x_a(r), y_a(r)) = V_a(r)$.

Finally, the condition (18) implies that

$$g_p^{-1}(r) = \frac{dG_p^{-1}(r)}{dr} \geq \frac{dG_a^{-1}(r)}{dr} = g_a^{-1}(r) \text{ for all } r \in [0, 1]$$

In other words, $G_p^{-1}(r)$ is (weakly) steeper than $G_a^{-1}(r)$ on $[0, 1]$, so that clearly $G_p^{-1}(r) \geq G_a^{-1}(r)$ for $r \in [0, 1]$. Suppose that $U_p(0) > U_a(0)$, and suppose, in contradiction to the claim we are trying to prove, that $U_p(r)$ equals $U_a(r)$ at least once on $(0, 1)$. Denote the first such point as $r_1 \in (0, 1)$ and notice that it must be that $V(x_p(r_1), y_p(r_1)) = V(x_a(r_1), y_a(r_1))$. But since by Corollary 1, $x_p(r) > x_a(r)$ on $(0, 1]$, it must be that $y_p(r) < y_a(r)$ in the neighborhood of r_1 . Furthermore, $dV_2 = V_{21}dx + V_{22}dy$, and, given our original assumptions on V , it thus must be that $V_2(x_p(r), y_p(r)) > V_2(x_a(r), y_a(r))$ in a neighborhood of r_1 . Using the marginal utility condition (16), combined with the density condition (18), it must be that $U'_p(r) > U'_a(r)$ in a neighborhood of r_1 , so that $U_p(r)$ can only be steeper than $U_a(r)$, and thus can only cross from below. Given $U_p(0) > U_a(0)$, we are done.

If instead we have that $U_p(0) = U_a(0)$, then, by the above argument which rules out that U_p can cross U_a from above, the claim can only fail if there is an interval $(0, \tilde{r})$ on which $U_p(r) \leq U_a(r)$. Then, there must exist a point $r_2 \in (0, \tilde{r})$ such that $U'_p(r_2) \leq U'_a(r_2)$ and $V(x_p(r_2), y_p(r_2)) \leq V(x_a(r_2), y_a(r_2))$. But given (16) and the density condition (18), if $U'_p(r_2) \leq U'_a(r_2)$ then $V_2(x_p(r_2), y_p(r_2)) \leq V_2(x_a(r_2), y_a(r_2))$ at r_2 , which can only happen if $y_p(r_2) \geq y_a(r_2)$. But this, combined with the fact that $x_p(r_2) > x_a(r_2)$ (by Proposition 2) implies that $V(x_p(r_2), y_p(r_2)) > V(x_a(r_2), y_a(r_2))$, which is a contradiction. ■

Proof of Proposition 5: Notice again that $U_a(r) > U_p(r)$ if and only if $V_a(r) = V(x_a(r), y_a(r)) > V(x_p(r), y_p(r)) = V_p(r)$. From Proposition 2, we have $x_p(\hat{r}) > x_a(\hat{r})$. We can then consider two cases. First, suppose that $x_p(r) \geq x_a(r)$ on $[\hat{r}, 1]$. Then, as wealth for individuals with rank $(\hat{r}, 1]$ is strictly lower ex-post than ex-ante, we have necessarily $y_p(r) < y_a(r)$ on $[\hat{r}, 1]$. Now, as $x_p(r) \geq x_a(r)$ and $y_p(r) < y_a(r)$, we then for some \tilde{r} can find a pair (\tilde{x}, \tilde{y}) such that $p\tilde{x} + \tilde{y} = px_p + y_p$ (that is, (\tilde{x}, \tilde{y}) are feasible given ex-post wealth) but $x_p^c < \tilde{x} < x_p$ and $\tilde{y} = y_a$. But then, $V(x_p(r), y_p(r)) < V(\tilde{x}, \tilde{y}) < V(x_a(r), y_a(r))$, and the result follows.

Suppose now instead that $x_p(r) < x_a(r)$ for some r in (r_1, r_2) with $r_1 > \hat{r}$. If $y_p(r) \leq y_a(r)$ on that interval, it is clear that $V_p(r) < V_a(r)$ and we are done. Suppose instead that $y_p(r) > y_a(r)$ on some interval (r_3, r_4) with $r_4 \leq r_2$ (as incomes are lower ex post for $r > \hat{r}$, it must be that $r_3 > r_1$). We want to rule out the possibility of $U_p(r) \geq U_a(r)$ somewhere on this interval. Now, it must be the case that $V_p(r_3) < V_a(r_3)$ as $x_p(r_3) < x_a(r_3)$ and $y_p(r_3) = y_a(r_3)$. We have $g_p(r) \geq g_a(r)$ everywhere. Furthermore, $dV_2 = V_{21}dx + V_{22}dy$. Given that x decreases and y increases ex post on (r_3, r_4) and our original assumptions on V , it can be calculated that, given (16), that $U'_p(r) < U'_a(r)$ on this interval. Combined with $U_p(r_3) < U_a(r_3)$, the result follows. ■

Proof of Proposition 6: If $S_0 > 0$ we must have $U_p(0) > U_a(0)$, simply because by assumption $G_p^{-1}(0) > G_a^{-1}(0)$, the lowest ranked individual has strictly higher income

ex post. Since by the boundary condition (7), the lowest ranked individual spends the cooperative amount and behaves like a neoclassical consumer, a strictly higher income must make her strictly better off. Higher utility on some interval $[0, \tilde{r})$ then follows by continuity of $U(r)$. ■

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