Sequential Group Lending with Moral Hazard

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Abstract

In Grameen Bank's group lending arrangement, all agents within a group do not borrow at the same time. Agents within a group, queue for credit and their credit is conditional on successful repayments of the previous loans. In a group lending model, where all group members borrow in the same time period with joint liability contracts, if monitoring is costly and the effort is not observable to other agents within the group, the agents are able to obtain higher rents with the threat that they would collude not to monitor each other. These higher rents limit this group lending arrangement’s ability to finance low productivity projects. An increase in monitoring efficiency has virtually no effect on the group lending arrangement’s ability to finance low productivity projects. The paper suggests that within the group, if the agent's projects are financed sequentially, the advantage is that the threat of collusion does not keep rents high along with the disadvantage that expected output is lower. Therefore, we find that between the two group lending arrangements, sequential group lending allows the lender to finance a greater proportion of the socially viable projects if the monitoring technology satisfies a certain efficiency condition.

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1 Introduction

Grameen Bank is the best known example of a Microfinance Institutions. These institutions focus on providing credit to the poor through innovative unconventional mechanisms. In doing so, they offer the poor an invaluable opportunity to lift themselves out of the poverty trap. Much has been written about Grameen Bank since it pioneered the socially driven agenda of poverty alleviation through the provision of credit more than two decades ago. The frequency, with which attempts have been made to replicate its lending mechanism, can only be a sign of its success in capturing people's imagination with its social agenda.

The deprived section in any society lack the assets that are needed as collateral to obtain conventional credit. Thus, any loan to such an agent without collateral is intensive in information. A significant amount of literature has evolved from the Grameen Bank’s experiment. This has tried to examine the reason behind Grameen Bank's success in terms of repayment rates in spite of lending in such an information-intensive lending environment. The literature identifies group lending with joint liability contracts, intensive monitoring of borrowers and promises of repeat loans as reasons behind this success. (Ghatak and Guinnane (1999))

A salient feature of Grameen Bank's unconventional lending arrangement, which might be a significant factor behind its success, has either escaped notice or has not been given its due importance in the literature. Varian (1990) is the only exception to this1. Before discussing this salient feature, lets review Grameen Bank’s lending mechanism with a quote from Farnsworth (1988).

"Although loans are made to individual entrepreneurs, each individual is in a group of four or five others who are in line for similar credits. Together they act as co-guarantors. If one individual is unable to make timely payments, credit for the entire group is jeopardized which results in heavy peer-group pressure on the delinquent. At first only two members of the group are allowed to apply for a loan. Depending on their repayments, the next two borrowers can apply and then the fifth."

- Farnsworth (1988) and quoted again in Varian (1990)

It is remarkable feature of Grameen Bank’s lending arrangement that borrowers are not provided credit in the same time period. The group lending arrangement is sequential in nature and can terminate with a willing2 or unwilling3 default. Even though joint-liability has virtually become synonymous with group lending in this literature, very little space has been devoted to the analysis of the sequential nature of Grameen Bank’s lending arrangement. This paper shows that under some reasonable conditions Sequential group lending is an optimal arrangement to alleviate the

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1 Varian (1990) analyses the advantages of Sequential lending in revealing hidden information. The paper shows that the sequential lending given agents incentives of reveal secret of efficiency to their peers when group members are randomly interview and offered separating contracts.
2 case where the borrower defaults strategically in spite of a favourable project outcome
3 the borrower default due to unfavourable outcome of the project
problem of ex ante moral hazard\(^4\) by providing the group member appropriate incentives to undertake peer-monitoring\(^5\).

The principal and all agents in our model are risk-neutral. The agents have no wealth and are homogenous in all respects except for the (publicly observable) productivity of projects that they are endowed with. Their hidden-action influences the expected outcome of the project. Borrower’s non-diligent action is also associated with private benefits, which can only be curtailed through costly monitoring, thus giving monitoring an integral role in the model. Monitoring also is an unobservable task. This makes it a double moral hazard problem. Agents do not observe each other’s actions but make conjecture about it through known terms of the contract and choose their actions accordingly.

The objective of the paper is to evaluate the various lending arrangements in terms of their ability to finance extremely low productivity projects. We focus on the least productive project feasible (or financed) under each lending arrangement with the belief that increasing the range of socially viable projects financed under a lending scheme has welfare improving consequences for the economy. The efficiency of monitoring technology is defined in terms of its efficacy in curtailing private benefits through monitoring. We use a simple method to rank various monitoring technologies so that we can examine the effect of varying monitoring technology’s efficiency on the range of socially viable projects financed under different lending arrangements.

To simplify the analysis we model a two-member group. The crux of the paper involves a comparison of Sequential Group Lending with Static Group Lending. We show that the rents retained by agents in the two arrangements would have been identical if there was no threat of collusion. With the threat of collusion, Sequential group lending outperforms Static group lending in term of the range of socially viable projects financed if the monitoring technology satisfies certain efficiency condition. This is because contracts offered by the lender in Static group lending are constrained by the threat of group members coordinating on their collective decision to monitor. In Sequential group lending, such coordination is not possible and all benefits of inexpensive information acquisition are realised.

In a single task multi-agent model, it is easier for the lender to induce collusion that is beneficial to him. In a multi-agent multi-task model, ensuring that all possible coordination tasks are beneficial to him is more difficult. In order to avoid the type of collusion the lender does not like, the lender has to restrict the contracts he can offer, in turn, limiting his ability to induce collusion that is beneficial to him. Laffont and Rey (2002) show that in group lending, it is the information sharing among agents and not the collusion per se that is desirable from the lender’s perspective. We show that inexpensive information acquisition in monitoring does not necessarily benefit either the lender or the economy in static group lending but all its benefits are realised in Sequential group lending.

\(^4\) that is, unwilling default

\(^5\) Other papers dealing with ex ante moral hazard in group lending are Stiglitz (1990), Conning (2000), Conning (1996) and Laffont and Rey (2002)
1.1 Joint Liability

In Static Group Lending, all the group members borrow in the same time period and undertake their respective projects simultaneously. In Sequential Group Lending, the projects are financed one after another. A project is financed only if the previous project succeeds. Joint liability is a crucial feature of both group lending arrangements.

In any group lending arrangement, joint-liability gives agents a stake in each other's projects. With zero-wealth agents, limited liability constrains the lender's ability to punish the agent as the lender is confined to non-negative payoffs. Joint liability allows the lender to punish an agent's success when accompanied by a peer's failure, thus, inducing them to behave cooperatively. The agents can side-contract on actions observable to them and ex post transfers. We assume that agent’s actions are not observable to each other. Therefore, the only way in which agents can influence each other’s actions is by curtailing each other’s private benefits through monitoring. With this assumption, instead of assuming the free flow of information between the agents, we actually derive a more realistic relationship between agents. An example to illustrate this would be a father trying to prevent his teenage son from getting into trouble. He can do it either by trying to observe his teenage son’s actions or by monitoring him. We know anecdotally that often parents take the latter root of monitoring. Our justification for this assumption is that observing a complicated action, which involves pinning down exact intentions, might not be feasible, however easy or inexpensive information acquisition is. Monitoring might be the next best available alternative. This is certainly only true for ex ante moral hazard. With ex post moral hazard the outcome is easily observed and pinning down the intentions is not complicated.

1.2 Individual Liability Loans

The two group lending arrangements are also compared with conventional Individual Liability Loans. In Individual Liability Loans, the lender either monitors the borrower directly or delegates the task of monitoring to another agent. The pre-requisite for direct monitoring is that the lender possesses a viable monitoring technology to monitor the borrower. In the absence of the lender possessing a viable monitoring technology, the lender has no option but to delegate the task of monitoring to an agent who possesses a viable monitoring technology.

When the lender delegates the task of monitoring, he has to allocate limited liability rent to the agent who takes on the task of monitoring in order to give the agent incentives to monitor. The lender can induce greater monitoring intensity by allocating larger rents to the monitor. As monitoring intensity increases, the borrower’s private benefits decreases which in turn lowers his opportunity cost of diligence. Thus, with increased monitoring, the lender needs to allocate a lower limited liability rent to the borrower to induce him to be diligent. The total agency cost in this case is the sum of the borrower’s and monitor’s rent, which decreases till the marginal cost of monitoring equals the marginal benefit of monitoring, namely lower borrower’s rent at the margin. Therefore, the lender would obviously like to induce monitoring intensity that minimizes the total agency cost.

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6 with increasing monitoring intensity
Low productivity projects have lower expected surplus over and above the opportunity cost of the funds invested in the projects. As productivity decreases, this surplus decreases, leaving smaller total rent that can be allocated between the borrower and the monitor. Thus, with decreasing productivity, increasing the monitor’s rent becomes imperative for the lender. Consequently, the minimum monitoring intensity required for the project to be feasible increases with decreasing productivity. The least productive project that can be financed is the one with just enough expected surplus to cover the minimum agency cost.

We define monitoring technology’s efficiency in terms of the rate at which it is able to decrease borrowers private benefits. A more efficient monitoring technology curtails the borrowers private benefits at a faster rate with increasing monitoring intensity. This suggests that the more efficient the monitoring technology, the lower the total agency cost and smaller the lower bound on project-productivity.

There is no threat of collusion in the delegated monitoring model because the agent’s benefits from being negligent or slack are private and therefore either non-pecuniary or non non-transferable or both.

1.3 Static Group Lending

In Static group lending, the agents undertake both tasks in the same time period. They borrow, implement the project and monitor the peer. The advantage is that due to the risk of punishment inherent in a joint liability contract, the agent’s compensation for monitoring is being able to keep his rent when his peer is successful. Therefore, he works hard to generate the surplus and monitors with sufficient intensity to ensure that he can keep his rent. The agents do not have to be separately compensated for borrowing and monitoring. Just compensating them for the more expensive task is enough.

Since monitoring decisions are made simultaneously, if the payoffs from investing in the decision to monitor and being diligent subsequently is not high enough, the agents would like to coordinate (or collude) on the decision to monitor by not monitoring at all. By doing so, although they lower the chances of a favourable outcome, by not monitoring they save the cost of monitoring and avail of the non-monitored private benefits. Since they can get a significant payment from colluding on the decision to monitor, the payoffs from monitoring and subsequent diligence should be high enough to compensate.

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7 We could still get collusion if there existed a strong bilateral commitment device between the two agents. The borrower could promise a large chunk of his payoff to the monitor in return for the promise of no monitoring. Thus, the borrower would be slack and if favourable outcome occurs in spite of his negligence, he would share his payoff and enjoy private benefits. Of course, to make the borrower share his payoff ex post, a strong bilateral commitment device is needed.

8 surplus over opportunity cost of funds invested when the project is successful

9 The reason both agents can coordinate is because both agents can simultaneous obtain their respective private benefits by not monitoring. In the delegated monitoring model or sequential group lending, the absence of this simultaneous decision ensures that coordinating not longer a lucrative option.

10 This form of collusion is not a cooperative attempt to conspire. It emerges in a non-cooperative set up due to coordination on a set of payoffs.
This threat of collusion ensures that there is a lower limit on the agent’s rent. If efficiency of monitoring technology increases, even though the agent’s rents, which are compensation for private benefits, fall more rapidly with monitoring, they cannot fall beyond a limit. Thus, greater the efficiency of monitoring technology, the more the lower limit binds. The threat of collusion keeps the agent’s rents high in static group lending and consequently the lower bound on project-productivity does not decrease with increased efficiency in monitoring technology11.

1.4 Sequential Group Lending

Sequential lending is a hybrid version of the delegated monitoring model and the static group lending model. It retains the joint liability element of the static group lending but eliminates the simultaneous and symmetrical nature of the decision-making process by making the decision making process sequential. In a two agent group, lets say Agent 1 takes on the role of a monitor and Agent 2 the role of a borrower. If the project ends with a failure, both agents get a symmetrical payoff of zero due to binding limited liability constraints. If the project is a success, the roles of the agents are reversed. The payoff now depends on the outcome of the second project. Agent 2’s project is now financed by the lender and Agent 1 now takes on the role of a monitor. If the project is a failure, both agents get a symmetrical payoff of zero. If the project is successful, both agents get a symmetrical non-negative payoff. The sequential nature of the decision making process ensures that there is no possibility of collusion in this scheme.

The paper shows that the rents required to incentivize the agents in this scheme are identical to the rents required in the static group lending if there was no threat of collusion within the group. The lender just has to ensure that the more expensive task is compensated. Therefore, the rents go down with monitoring until the monitoring cost overtakes the opportunity cost of diligence.

The obvious advantage is that there is no scope for collusion. Conversely, the disadvantage is that for identical projects, the expected outcome in the sequential group lending scheme is lower than in the static group lending scheme. The reason is that in the static group lending scheme, both projects are undertaken without failure. In sequential group lending, if the first project fails, the second project is not undertaken. This leads to loss of expected output. This implies that the same project has a lower surplus over the opportunity cost of its investment funds in sequential group lending in comparison with the static group lending. Lower surplus means lower rents left for agents which could potentially mean some low productivity projects that can be implemented in static group lending may not be implemented in sequential group lending.

Which institutional arrangement performs better depends on the efficiency of the monitoring technology. As we saw earlier, there is a lower limit on rents that can be offered to the agents in the static group lending arrangement due to the threat of

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11 Continuing on with our analogy above, if the father has two teenage sons and wants to keep them both out of trouble, he can do so by inducing each to monitor the other by introducing a joint liability in punishment, that is, both get punished if one gets into trouble. Joint liability is not enough. Even if the sons are able to monitor each other costlessly, the father still has to find some way of ensuring that the sons do not collude on the decision to monitor.
collusion. This lower limit does not exist in sequential group lending. Thus, we show that for all monitoring technologies that are more efficient than a certain technology, sequential group lending can finance much lower productivity projects than static group lending.

The Grameen Bank's arrangement is in line with this result. It uses the advantages of both static group lending and sequential group lending. The first two agents that take the loan are borrowers who monitor each other and are at the same time being monitored by the agents waiting in line for credit. If their projects are successful, the next two in line get the credit and the first two monitor these borrowers because of joint liability. The last person to take the loan is normally someone who has a credit history with the Grameen Bank and this might be for the purposes of eliminating auditing cost, an issue we do not consider in this paper. Thus the lower threat of collusion is due to the sequential nature of the decision making process. The expected output though would be in between the expected output of the two group lending schemes mentioned above.

Section 2 starts with the description of the basic model. We then obtain results for the standard direct and delegated monitoring models. Section 3 describes the static group lending scheme. Section 4 describes the sequential group lending model. Section 5 compares the two group lending models and derives conditions under which the sequential group lending model outperforms the static group lending model. Conclusions follow.

2 Model

Each entrepreneur is an agent with access to only one project which requires an initial lump-sum investment of $I$ units of capital and produces an uncertain outcome. If the project is successful, the value of project’s outcome is the variable $x^s$. If the project fails the value of the outcome is zero. Agents can exert either a high or a low level of effort for the project. An agent's effort level is unobservable to anyone except himself. The projects vary only in terms of $x^s$, the value of favourable outcome. The project available to each agent can be publicly observable.

By being diligent and exerting a high effort level the agent can increase the probability of favourable outcome $x^s$. With diligence, $x^s$ is realised with probability $\pi$ and 0 with probability $1-\pi$. When the agent slacks and exerts a low level of effort, $x^l$ is realised with probability $\bar{\pi}$ and 0 with probability $1-\bar{\pi}$. A project with a higher value of $x^s$ is deemed to be a more productive project than one with a lower value of $x^s$. For sections ahead, we use $x^s$ to refer to productivity in general since the expected productivity of a project increases with $x^s$ and effort level exerted by the agent.

All agents have zero wealth and are homogenous ex ante in all respects expect the project available to them. Thus if the particular agent wants to undertake a project, he can only do so by borrowing $I$ from a lender if the lender is ready to lend to him, knowing the productivity of his project. A project is considered feasible if a lender is ready to finance a project.
By exerting a low level of effort the agent can obtain private benefits of value $B(c)$ from the project. The private benefits are non-pecuniary and non-transferable among agents. The private benefits are also the agent’s opportunity cost of being diligent. For instance the agent could be non-diligent by being deceptive and secretly diverting the cash from the project for other non-productive purposes or use other techniques to deliver private consumption benefits for himself. An agent’s action effects the probability of the outcomes but not the value of outcomes.

We assume that the cost of high or low effort is the same and we normalise it to zero. The only difference between high and low effort from an agent’s perspective, is the private benefits $B(c)$ the low effort brings with it. There are, of course, no private benefits associated with high effort.

A problem of moral hazard arises because borrower can effectively choose $\pi$ with his actions and since his choice of action is unobservable, no action or effort contingent financial contract can be enforced or verified. Therefore, the only avenue available to the uninformed lender is to incentivize the borrower to be diligent is through borrower's payoffs.

Borrower’s private benefits $B(c)$ for choosing low effort level can be curtailed by monitoring him with intensity $c$. The rate at which these benefits are curtailed diminishes as monitoring intensity $c$ increases. The cost for anyone monitoring with intensity $c$ is $c$ itself. The slope and curvature of $B(c)$ depends on the available monitoring technology. In section 5 we vary the monitoring technology but till then, lets assume that the monitoring technology is given.

We also assume that once information is gathered by act of monitoring, it becomes public knowledge. As we see later, when an agent monitors a fellow agent, he can possibly collude with the agent and not reveal the information learnt during monitoring. To abstract from this we assume that once the monitor learns something, he cannot decide to withhold the information from the principal.

This assumption can be justified on the basis that the agent would not like to run the risk of being publicly humiliated if his knowledge of collusion is revealed. So the agent can collude by on the decision to monitor but they cannot do so by withholding information gained during the process of monitoring.

To summarise the assumption made about the monitoring technology.

Assumptions:
- $B_c(c) < 0$
- $B_{cc}(c) > 0$
- Cost of monitoring with intensity $c$ is $c$ itself
- Knowledge gained during monitoring is in public domain

The uninformed lender or the principal, if he so chooses, can safely earn a $\gamma$ rate of return on funds $I$.

\footnote{whether to monitor or not to monitor at all}
We also assume that 
\[ E[x|\pi] - \gamma I \geq 0 \geq E[x|\pi] - \gamma I + B(0) \]

The assumption implies that if the project is self-financed, the entrepreneur would have to be diligent and exert a high effort level. Conversely, if the borrower slacks or chooses a low effort level, the ex ante expected value of project outcome and the private benefits is less than the opportunity cost of funds \( I \) for the lender. Thus, the lender would not lend \( I \) to the borrower unless he can ensure that the borrower would choose to be diligent in implementing his project.

### 2.1 The complete information optimal contract

As a benchmark case, let's first assume that the lender and an enforcement authority can both observe the effort level of the borrower. The lender can thus stipulate an effort-contingent financial contract that takes the form \((s_s, s_f)\). Thus, the problem (P1) is as follows:

\[
\begin{align*}
\max_{s_i} & \quad E[x|\pi] - E[s_i|\pi] \\
\text{subject to} & \quad E[R_i|\pi] \geq \gamma I \quad (1) \\
& \quad E[s_i|\pi] \geq 0 \quad (2) \\
& \quad s_i \geq 0 ; i = s, f \quad (3) \\
& \quad x_i = R_i + s_i 
\end{align*}
\]

where \( i \) indicates whether the project undertaken has been a successful (s) or a failure (f), \( R_i \) is the lender’s return in state \( i \), \( s_i \) the borrower’s payoff in state \( i \) and \( x_i \) the value of project outcome in state \( i \). (1) is the lender’s participation constraint or the break-even condition, (2) is the borrower’s participation constraint, (3) is the borrower’s limited liability constraint.

If we allocate all the bargaining power to the lender then the borrower's participation constraint (2) will bind and lender's participation constraint would be slack. Borrowers binding participation constraint along with limited liability constraint implies that he gets zero payoff in each state. The lender offers borrower a contract \((0,0)\) and the project is implemented if the borrower accepts the contract.

The lenders break-even condition implies that
\[
E[R|\pi] = E[x|\pi] - E[s_i|\pi] \geq \gamma I \\
E[x|\pi] = \gamma I + E[s_i|\pi] \\
\pi x' \geq \gamma I \\
x' \geq \frac{\gamma I}{\pi}
\]

Thus (4) gives us the set of all social viable projects that can be potentially undertaken with perfect information. In addition, the cost of implementing all the socially viable projects in the first best world is zero.
2.2 The Optimal Second-Best Contract with No Monitoring

We return to the information problem described in the beginning of this section. With incomplete information about borrower's actions, we add the borrowers incentive compatibility constraint to the problem P1.

\[
\begin{align*}
\text{P2} & \quad \max_{s_i} E[x_i|\vec{x}] - E[s_i|\vec{x}] \\
& \quad E[R_i|\vec{x}] \geq \gamma I \\
& \quad E[x_i|\vec{x}] \geq 0 \\
& \quad s_i \geq 0, \quad i = s, f \\
& \quad E[s_i|\vec{x}] \geq E[s_i|\vec{x}] + B(0) \\
& \quad x_i = R_i + s_i
\end{align*}
\]

Constraint (5) is the incentive compatibility constraint, which states that the borrower would choose to be diligent only if his expected payoff from being diligent is greater or equal to his expected payoff and private benefits from being slack. Note that we have not introduced any monitoring and so the private benefits \(B(0)\) are at its maximum value.

Borrowers incentive compatibility constraint (5) can be rewritten as \(s_s - s_f \geq \frac{B(0)}{\Delta \pi}\). Borrower's limited liability constraints are \(s_s \geq 0, \quad s_f \geq 0\).

To give the borrower the maximum amount of incentive to choose high effort level at the same time minimize the agents rent, the lender would choose to design the contract \((s_s, s_f)\) in such way that borrower’s incentive compatibility constraint (5) binds and borrower’s limited liability constraint (3) binds for state \(f\) leaving the borrower positive rent.

With \(s_f = 0\), and borrowers incentive compatibility constraint binding, \(s_s = \frac{B(0)}{\Delta \pi}\) and the borrower is left with economic rent \(E[s_s|\vec{x}] = \frac{\pi B(0)}{\Delta \pi}\).

Substituting the borrower’s rent and rewriting the lender’s participation constraint

\[
\begin{align*}
E[R_i|\vec{x}] &= E[x_i|\vec{x}] - E[s_i|\vec{x}] \geq \gamma I \\
E[x_i|\vec{x}] &\geq \gamma I + E[s_i|\vec{x}] \\
x^* &\geq \frac{\gamma I}{\pi} + \frac{B(0)}{\Delta \pi}
\end{align*}
\]

(6) shows us that there would always be some socially viable projects that would not be financed due to the information problem. Limited liability constraint ensures that borrower’s pay off needs to be non-negative in all states further implying that the borrower is left with some positive economic rent as incentive. Paying this rent implies the set of feasible projects gets restricted and some socially viable projects are no longer feasible.

Let's define \(x^*_{\text{MM}} = \frac{\gamma I}{\pi} + \frac{B(0)}{\Delta \pi}\)
2.3 Directly Monitored Loans

The model has been specified in a such a way that monitoring would make it easier to give the borrower incentive to choose a high effort level by lowering his opportunity cost of diligence and thus lowering borrower’s limited liability rent. This benefit has to be traded-off against the cost of monitoring, which absorbs some of the social surplus and curtails the set of feasible projects.

Let’s assume in this section lender has a viable monitoring technology, which is the same as the other agents. This example is especially pertinent to the moneylenders in the rural economy who, either due to frequency of interaction with the borrower or due to common social network links, are in as good a position to monitor the borrower as any other agent in the social network. To incorporate this in P2 above we would have to modify the lender participation constraint (1) and incorporate the cost of monitoring the agent in (1). Thus, the new problem would be as follows.

P3

$$\max_{i} E[x_i | \pi] - E[s_i | \pi]$$

$$E[R_i | \pi] - c \geq \gamma I$$  \hspace{1cm} (1')

$$E[s_i | \pi] \geq 0$$  \hspace{1cm} (2)

$$s_i \geq 0 \hspace{0.5cm} i = s, f$$  \hspace{1cm} (3)

$$E[s_i | \pi] \geq E[s_i | \pi] + B(c)$$  \hspace{1cm} (5)

$$x_i = R_i + s_i$$

As above, (3) binds for state f and (5) has to bind leaving the borrower a lower limited liability rent of $E[s_i | \pi] = \frac{\pi - B(c)}{\Delta \pi} \leq \frac{\pi - B(0)}{\Delta \pi}$ since $B(c) < 0$ for all $c$. Substituting this rent in the lender’s new participation constraint (1’) we get

$$E[R_i | \pi] = E[x_i | \pi] - E[s_i | \pi] - c \geq \gamma I$$

$$E[x_i | \pi] \geq \gamma I + c + E[s_i | \pi]$$

$$x^s \geq \frac{\gamma I + c + B(c)}{\Delta \pi}$$  \hspace{1cm} (7)

As before (7) gives us the lower bound on the productivity of the project that would be financed for a given amount of monitoring intensity $c$. It also tells gives us the minimum amount of monitoring required to make a given project implementable.

Monitoring would decrease the lower bound given by (7) if $B_s(0) \leq -\frac{\gamma I + c}{\Delta \pi}$. This lower bound would keep decreasing until we reach $c_{DM}$ where $c_{DM}$ is defined by $B_s(c_{DM}) = -\frac{\Delta \pi}{\pi}$. Let’s also define $x^s_{DM} = \frac{\gamma I + c_{DM}}{\pi} + \frac{B(c_{DM})}{\Delta \pi}$

So agents with projects with productivity $x^s \in (x^s_{DM}, x^s_{NM})$ can potentially get their projects financed if the lender monitors directly. Note that $c_{DM}$ depends on efficiency of monitoring technology in decreasing borrowers private benefits. The more efficient the monitoring technology, the faster it decreases the private benefits implying efficient monitoring technology is associated with lower $c_{DM}$ and lower $x_{DM}$. 

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2.4 Loans with Delegated Monitoring

If the lender does not have the requisite monitoring technology, he has no alternative but to delegate the monitoring to another agent who has a viable monitoring technology. This is equivalent to assuming that for lender’s monitoring technology $B_L^C(0) < -\Delta \pi / \pi$ and for monitoring technology for all agents $B_L^A(0) < -\Delta \pi / \pi$.

We also assume that agents monitoring intensity is unobservable to the uninformed lender. Given that all the agents have zero wealth, the lender would allocate the agent some limited-liability rent in order to give him the requisite incentive to monitor the borrower. Agent’s monitoring intensity being unobservable is in line with the assumption that the borrower’s actions are unobservable. The new problem is as follows

$$\begin{align*}
\text{P4} & \quad \max_{s_i, w_j} E[x_i|\pi] - E[s_i|\pi] - E[w_j|\pi] \\
& \quad E[R_i|\pi] \geq \gamma I \tag{1} \\
& \quad E[s_i|\pi] \geq 0 \tag{2} \\
& \quad s_j \geq 0 \quad i = s, f \tag{3} \\
& \quad E[s_i|\pi] \geq E[s_i|\pi] + B(c) \tag{5} \\
& \quad E[w_j|\pi] - c \geq 0 \tag{8} \\
& \quad E[w_j|\pi] - c \geq E[w_j|\pi] \tag{9} \\
& \quad w_j \geq 0 \quad i = s, f \tag{10} \\
& \quad x_i = R_i + s_i + w_j
\end{align*}$$

where (8) is the monitor’s participation constraint, (9) his incentive compatibility constraint and (10) his limited liability constraint. For the same reason as stated above for the borrower, limited-liability constraint for state $f$ would bind for the monitor such that $w_f = 0$. We can rewrite (9) as $w_s = \frac{\gamma}{\Delta \pi}$ implying that (8) would slack giving the monitor an expected payoff of $E[w_j|\pi] = \frac{\pi}{\Delta \pi}$ and limited liability rent of $E[w_j|\pi] - c = \frac{\pi}{\Delta \pi}$. The borrower’s rent as above is $E[s_i|\pi] = \frac{\pi B(c)}{\Delta \pi}$. Substituting monitor’s and borrower’s respective payoff in lender’s break-even condition (1) we get

$$\begin{align*}
E[R_i|\pi] & = E[x_i|\pi] - E[s_i|\pi] - E[w_j|\pi] \geq \gamma I \\
E[x_i|\pi] & \geq \gamma I + c + E[s_i|\pi] + E[w_j|\pi] \\
\pi x^* & \geq \frac{\gamma I}{\Delta \pi} + \frac{\pi B(c)}{\Delta \pi} \\
x^* & \geq \frac{\gamma I}{\Delta \pi} + \frac{B(c) + c}{\Delta \pi} \tag{11}
\end{align*}$$

Again (11) gives us the lower bound on the productivity of projects that would be feasible given $B(c)$. We can call (11) productivity-monitoring intensity locus. It tells us the cost of each project in terms of its minimum monitoring cost given $B(c)$. By comparing (7) and (11) we see that the productivity-monitoring intensity locus for direct monitoring (7) is lower than one for delegated monitoring (11). This implies that delegated monitoring is more expensive compared to direct monitoring in terms
of monitoring cost due to the cost of delegation or the limited liability rent paid out to the monitor.

Delegating monitoring would be effective if $B_i(0) \leq -1$. The *productivity-monitoring intensity locus* would be downward sloping until $B_i(c) = -1$. Let's define $c_M$ such that $B_i(c_M) = -1$ and $x_M$ such that $x_M^i = \frac{\gamma I}{\pi} + \frac{b(c_M) + c}{\Delta \pi}$. Note that $c_M < c_{DM}$ and $x_M^i \leq x_M^i$.

Thus we can see from Figure 1 below that the Direct Monitoring dominates Delegated monitoring and for a given project. Direct monitoring would require lower amount of minimum monitoring intensity to make the project feasible. Proposition 1 follows.

**Proposition 1**

(i) A project with $x^s \in (x_M^s, x_{NM}^s)$ is feasible only with Direct Monitoring by the lender or Delegated Monitoring by an agent. A project with $x^s \in (x_M^s, x_{DM}^s)$ is only feasible with Direct monitoring by the lender, where $x_{DM}^s < x_M^s < x_{NM}^s$.

(ii) The minimum of monitoring intensity required to make a project feasible is lower when the lender monitors directly.

![Figure 1. Minimum Productivity - Monitoring Intensity Locus](image)

**Figure 1. Minimum Productivity - Monitoring Intensity Locus**

for Direct ($x_M^s$) and Delegated Monitoring ($x_{DM}^s$)
2.4.1 Possibility of collusion under Delegated Monitoring

We have abstracted from issues like auditing or ex post moral hazard\(^{13}\) to focus on ex ante moral hazard issues instead. With ex ante moral hazard, monitoring entails sinking the cost of monitoring before the outcome is realised. Private benefits obtained by the borrower when he slacks are non-pecuniary and non-transferable. Under these conditions, delegated monitoring is collusion-proof even if the agents can fully side-contract on monitoring intensity and ex post transfers\(^{14}\).

Let’s assume that agents can fully-side contract costlessly. That would imply that agents choose the monitoring intensity and effort level together to maximise their expected wage payments. The lender offers the agents contracts \((w^*_s, 0)\) and \((s^*_s, 0)\) to induce a monitoring intensity of \(c^*\). The agent would not collude if

\[
\pi\left[w^*_s + s^*_s\right] - c^* \geq \pi\left[w^*_s + s^*_s\right]
\]

The left hand side is the total expected payment agents get if they do not collude. The right hand side is the total expected surplus from collusion when the monitor monitors with zero intensity.

\[
\pi\left[c^* + \frac{B(c^*)}{\lambda \pi}\right] - c^* > \pi\left[c^* + \frac{B(c^*)}{\lambda \pi}\right]
\]

Given the contracts offered by the lender, not colluding is preferred by the agents primarily because the monitor has to sink in the effort monitor before the outcome is realised. If he monitors with intensity 0, he just lowers the expected surplus by more than the amount he saves in terms of cost of monitoring. So he would prefer to monitor with \(c^*\) and not collude.

Conversely, if the private benefits were pecuniary and transferable (and thus not so private) then the agents would prefer to collude. Collusion would take place if

\[
\pi\left[w^*_s + s^*_s\right] - c^* \leq \pi\left[w^*_s + s^*_s\right] + B(0)
\]

As above, the right-hand side is the total surplus from colluding.

\[
\pi\left[c^* + \frac{B(c^*)}{\lambda \pi}\right] - c^* < \pi\left[c^* + \frac{B(c^*)}{\lambda \pi}\right] + B(0)
\]

\[
\left[c^* + \frac{B(c^*)}{\lambda \pi}\right] < c^* + \frac{B(0)}{\lambda \pi}
\]

This is true since \(B(c^*) < B(0)\) for all \(c^* > 0\).

Thus, the monitor does not monitor and agents prefer to collude when the benefits accruing to the borrower are pecuniary and transferable. This leads us to Proposition 2

\(^{13}\) strategic default by the borrower
\(^{14}\) that is if they do not have a bilateral strong commitment device
Proposition 2

With non-pecuniary and non-transferable private benefits accruing to the borrower\(^1\), there is no possibility of collusion between the borrower and the monitor when the uninformed lender delegates the task of monitoring to an agent.

3 Joint Liability Loans under Static Group Lending\(^2\)

For an uninformed lender, static group lending is an alternative to Delegated monitoring. A group consists of two agents ready to accept the joint liability contract offered by the lender. For this section, we assume that if they accept the lender’s contract, both of them invest in their project in the same time period. We assume that both agents in a group have access to the same project \((x^s, 0)\) where \(x^s \in (0, \infty)\). The term static refers to the fact that both borrowers invest in the same time period\(^3\).

The lender cannot directly observe agents actions and can only observe the outcome. Thus, from the lender’s perspective, there are four distinguishable states.

- \(ss\) Both projects undertaken are successful
- \(sf\) Agent 1’s project is successful but Agent 2’s project fails
- \(fs\) Agent 1’s project fails but Agent 2’s project is successful
- \(ff\) Both projects undertaken fail

A contract offered by the lender specifies agent’s payoff in all 4 states \((ss, sf, fs, ff)\). Since the lender is unable to contract on effort, he has to use non-negative payoffs (due to limited liability constraints) as rewards to entice the agent to exert a high effort level.

Making agents jointly liable for each other’s outcome can provide the lender with the opportunity to punish the agent, that is if his project succeeds and his peer’s project fails. The lender does not have the ability to punish in any of the models of Section 2. The possibility of punishment gives the agent a stake in his peer’s project. This stake in peer’s project gives the agent an incentive to monitor his peer and reduce the gap between peer’s expected payoff due to high and low effort.

If the agents could observe each others efforts, they would have side-contracted on effort level and transfers and maximised the payoffs together. We restrict agents ability to observe each others effort levels. The effort decision is observable only to themselves and no one else can observe them. We also assume that agents can costlessly observe whether they are being monitored or not but cannot observe the actual monitoring intensity of the peer. Thus, they infer the effort decision and monitoring intensity from the terms of the contract.

\(^1\) and absence of a bilateral commitment device
\(^2\) This section is largely based on the model from Conning (2000). Conning’s overall emphasis is different. He focuses on the trade-off between collateral and monitoring intensity. Our emphasis on the problems collusion creates in financing low productivity projects in group lending. His model has been adapted for the objectives of this paper. The proof in the Appendix is entirely from Conning (1996)
\(^3\) as opposed to the term sequential where agents borrower in a sequence
As a result, the only way in which agents can influence each others effort level decisions is by monitoring the private benefits and thus reducing the opportunity cost of diligence.\textsuperscript{18}

Agents inability to side-contract on effort limits collusion but it still leaves the possibility of collusion through coordinating on payoffs. The agents can still \textit{effectively collude} by coordinating on either the effort or the monitoring decision if the contract offered gives them the opportunity to so. The lender needs to ensure that any contract offered does not give the agent any opportunity to coordinate in a way that is detrimental to the lender and at the same time encourage the beneficial coordination through the payoffs.

Coordination on effort can have positive consequence though coordination on monitoring can have negative consequences for the lender in the model described in this section. The possibility of collusion eventually restricts the amount of monitoring that can be undertaken by the agents and therefore restrict the set of feasible projects, in turn, limiting the benefits of group lending.

3.1 Model

The game is played in two stages. The borrowers play a non-cooperative game in monitoring intensities at the first stage and effort choices at the second stage. Any given pair of monitoring intensities chosen at the first stage \((c_1,c_2)\) determines the payoff structure of a subgame \(\xi(c_1,c_2)\) in the effort decisions at the second stage.

Our \textit{desired outcome} is that each agent chooses an equilibrium monitoring intensity \(c\), which implements a high level of effort choice at the second stage of the game. We look for the conditions that ensure that the desired outcome is the pure strategy subgame perfect Nash equilibrium (SPNE) of the game.

Let \(x_i\) describe the outcome \(i\) of project for Borrower \(n\)

Let \(s_{ij}\) and \(s_{ij}'\) denote the payoff to Borrower 1 and Borrower 2 following outcome \((x'_i,x'_2)\). When the payoff appears without any superscript, it denotes the payoff for Agent 1, e.g. \(s_{ij}\).

We also assume that borrower’s project returns are statistically independent and likelihood of output pair \((x'_1,x'_2)\) occurring is simply \(\Pi(p_1',p_2')=p_1'p_2'\).

Let the monetary payoff of Agent 1 be \(E(s_{ij}|p_1,p_2)\). For the final payoff, we add private benefits and subtract monitoring costs from the monetary payoffs.

\textsuperscript{18} So in essence we replace the assumption of observability of effort prevalent in the literature with the assumption that agents influence each others decisions through costly monitoring.
Agent 1’s final payoff when he chooses a Low effort level ($\pi$) while monitoring at the intensity $c_1$ and his peer chooses a High effort level ($\bar{\pi}$) while monitoring with intensity $c_2$ is given by $LH(c_1, c_2) = E\left(s_1^1|\pi, \bar{\pi}\right) - c_1 + B(c_2)$

The following functions give the payoffs of subgame $\xi(c_1, c_2)$:

\[
\begin{align*}
HH(c_1, c_2) &= E\left(s_1^1|\pi, \pi\right) - c_1 \\
HL(c_1, c_2) &= E\left(s_1^1|\pi, \bar{\pi}\right) - c_1 \\
LH(c_1, c_2) &= E\left(s_1^1|\bar{\pi}, \pi\right) - c_1 + B(c_2) \\
LL(c_1, c_2) &= E\left(s_1^1|\bar{\pi}, \bar{\pi}\right) - c_1 + B(c_2)
\end{align*}
\]

It is obvious that $ss$ and $ff$ are the two most informative states as far as the lender is concerned. $ss$ tells the lender that the two agents are most likely to have monitored with sufficient intensity to induce high effort level in their respective peer and $ff$ vice versa.

Therefore, the lender should reward $ss$ and punish $ff$ as much as possible. Consequently, for the state $ff$, the limited liability constraint would bind and $s_{ff} = 0$. From symmetry required of the payoff structure $s_{ff} = s_{fs}$. The joint liability reward structure stipulates that $s_{ss} \geq s_{ff}$ and $s_{fs} \geq 0$ ($= s_{ff}$) with at least one strict inequality.

In a competitive lending market, uninformed lender’s participation constraint would bind. Thus, given lender’s binding participation constraint (expected output exhausts expected payoffs and lender’s opportunity cost of funds), $s_{ss}$ can only be increased if $s_{ff}$ and $s_{fs}$ are reduced. Since this sharpens the incentive for agents to reach the desired outcome and given that agents are risk neutral, it makes sense for the lender to give a positive reward only when state $ss$ is achieved and leave no monetary reward for any other states.

To implement a high effort level as Nash equilibrium of subgame $\xi(c_1, c_2)$ the following two conditions need to be met.

The first condition is a no-collusion condition, which ensures that the final payoffs when both agents do not monitor and both agents exert low effort level, is not greater than the final payoffs of the desired outcome.

\[\text{GL Condition 1: } HH(c,c) \geq LL(0,0)\]

\[
\begin{align*}
E\left(s_1^1|\pi, \pi\right) - c &\geq E\left(s_1^1|\pi, \pi\right) + B(0) \\
\pi^2 s_{ss} - c &\geq \pi^2 s_{ss} - c + B(c) \\
s_{ss} &\geq \frac{B(0) + c}{\pi^2 - \pi^2}
\end{align*}
\]

So the Expected monetary payoff for Borrower 1 (and symmetrically borrower 2) under this condition should be
\[ E\left(s_{ij} \mid \pi \pi \right) \geq \pi^2 \frac{B(0) + c}{\pi^2 - \pi^2} \]

Substituting this in the lender’s binding break-even condition we get

\[
E(R_{ij} \mid \pi \pi) = E(x \mid \pi) - E(s_{ij} \mid \pi \pi) = \gamma I
\]

\[
E(x \mid \pi) = \gamma I + E(s_{ij} \mid \pi \pi) = \frac{\gamma I + B(0) + c}{\frac{1}{\pi^2} - \frac{1}{\pi^2}}
\]

(GL1)

Which means that all the projects satisfying (GL1) conditions are bounded below by

\[
x^s_{GL1} = \frac{\gamma I}{\pi} + \frac{B(0) + c}{\pi^2 - \pi^2}
\]

(GL1')

GL1’ is the lower boundary of the set of all project-monitoring combination that satisfy GL condition 1 and rises quite sharply with \(c\).

The second condition that needs to be met is the following incentive compatibility constraint for borrower 1 (and symmetrically for borrower 2).

**GL Condition 2:**

\[ HH(c, c) \geq LH(c, c) \]

\[
E(s_{ij} \mid \pi \pi) - c \geq E(s_{ij} \mid \pi \pi) - c + B(c)
\]

\[
\pi^2 s_{ss} - c \geq \pi \pi s_{ss} - c + B(c)
\]

\[
s_{ss} \geq \frac{B(c)}{\pi \Delta \pi}
\]

Thus, expected monetary payoff for Borrower 1 (and symmetrically for borrower 2) is

\[
E(s_{ij} \mid \pi \pi) \geq \frac{B(c)}{\Delta \pi}
\]

Substituting this in the lender’s break-even condition we get

\[
E(R_{ij} \mid \pi \pi) = E(x \mid \pi) - E(s_{ij} \mid \pi \pi) \geq \gamma I
\]

\[
E(x \mid \pi) = \gamma I + E(s_{ij} \mid \pi \pi) = \frac{\gamma I + B(c)}{\Delta \pi}
\]

(GL2)

Which means all the projects satisfying (GL2) condition are bounded below by

\[
x^{s}_{GL2} = \frac{\gamma I}{\pi} + \frac{B(c)}{\Delta \pi}
\]

(GL2')

GL2’ is the lower boundary of the set of all project-monitoring combination that satisfy GL condition 2. Note that \(\frac{dx_{GL2}}{dc} = \frac{B(c)}{\Delta \pi}\).

Since a given projects expected output is exhausted by lender’s opportunity cost of funds and borrower’s expected monetary payoffs, as project’s productivity and
expected output decreases, the minimum monitoring intensity required to make lower productivity projects feasible increases. This increased monitoring lowers the borrowers opportunity cost of diligence thus reducing the compensation required for choosing high effort level. Consequently, as productivity decreases, the minimum amount of monitoring required to ensure that the project is feasible, increases (GL2).

Concomitantly, the collective opportunity cost of decision to monitor (as opposed to not monitor at all) increases (GL1) with $c$. We define $c_{GL}$ as the monitoring intensity beyond which the latter opportunity cost overtakes the former, namely the collective opportunity cost of the decision to monitor overtakes the opportunity cost of diligence $(x^s_{GL} \geq x^s_{GL2})$. For $c > c_{GL}$, colluding on the decision to monitor becomes a natural choice for the agents. This restricts the amount monitoring that can be undertaken in static group lending to $(0, c_{GL})$ where $c_{GL}$ is defined by

$$\frac{\gamma l}{\pi} + \frac{\pi L(0) + c_{GL}}{\pi^2 - \beta^2} = \frac{\gamma l}{\pi} + \frac{B(c_{GL})}{\Delta \pi}.$$

The lowest project that can be undertaken under static group lending is given by

$$x^s_{GL} = \frac{\gamma l}{\pi} + \frac{\pi L(0) + c_{GL}}{\pi^2 - \beta^2} = \frac{\gamma l}{\pi} + \frac{B(c_{GL})}{\Delta \pi} \quad \text{where} \quad B(c_{GL}) = \frac{\pi L + \pi}{\pi^2 + \beta^2}[B(0) + c_{GL}].$$

The appendix shows that if GL1 and GL2 are met, the SPNE of the game is the desired outcome. The proposition that follows is an application of Proposition 1 in Conning (2000) adapted to the model in this section.

**Proposition 3**

The set all project monitoring combinations that can be financed by static group lending is given by the following two condition

**GL1:**

$$x^s \geq \frac{\gamma l}{\pi} + \left(\frac{\pi}{\pi^2 - \beta^2}\right)\left[\frac{B(0) + c}{\Delta \pi}\right]$$

**GL2:**

$$x^s \geq \frac{\gamma l}{\pi} + \left[\frac{B(c)}{\Delta \pi}\right]$$

Projects with favourable outcome $x^s \in (x^s_{GL}, x^s_{NM})$ are feasible with group lending leading to monitoring levels $c \in (0, c_{GL})$. For $c \in (0, c_{GL})$ GL1 is slack and GL2 binds.

Projects $x^s \in (x^s_{NM}, \infty)$ are feasible through direct lending without any monitoring.

Projects $x^s \in (x^s_{NM}, x^s_{DM})$ are feasible with direct or delegated monitoring as well as static group lending. Projects $x^s \in (x^s_{DM}, x^s_{GL})$ would not be feasible without static group lending. Projects $x^s \in (0, x^s_{GL})$ are not feasible under group lending partly due to lender’s fear of collusion among the agents. For $c \in (c_{GL}, \infty)$ GL1 binds and GL2 is slack.

---

19 This proposition is a restatement of Proposition 1 in Conning (2000). Conning’s emphasis though is on the trade-off between collateral offered by the borrower and monitoring intensity the lender would like to induce. We assume that agents have no collateral and instead focus on the lowest productivity project financed under this group lending arrangement.
Therefore, for $x^s \in (x_{NM}^s, x_{GL}^s)$, the minimum monitoring intensity is given by condition $GL2$ binding. For project below $x_{GL}^s$, the monitoring intensity required implies that collective opportunity cost of the decision to monitor (or not monitor at all) would be higher than the benefits and thus the lenders prefers not to offer a contract to the agents in this range due to fear of collusion among the borrowers. We also note that $x_{GL}^s < x_{DM}^s < x_M^s$.

3.2 Discussion

In delegated monitoring model, the two tasks of monitoring and borrowing are compensated separately in terms of $w_i$ and $s_i$. The level of monitoring induced by the lender is such that the expected value of the outcome is exhausted by the expected payoffs for both the tasks and the lender’s opportunity cost of funds. The lowest productivity project undertaken in this model is $x_M^s$ where the marginal benefit from monitoring (in terms of reducing the opportunity cost of borrower’s diligence) is matched by the marginal increase in the cost of monitoring (which remains constant at 1). Thus $B(c) \left|_{c_m^s} = -1 \right.$.

In the group-lending model given above, the borrower carries out both tasks and is not compensated separately for the two tasks. He is compensated only for the more...
expensive task\(^20\). As a result, for a project, minimum monitoring intensity required for it to be feasible, is lower in static group lending as compared to delegated monitoring. As discussed above, the risk of collusion among the borrowers restricts the maximum amount of monitoring that can be undertaken under static group lending.

We note that the expected value of the outcome is the same for delegated monitoring and static group lending if high effort is implemented.

\[
\pi(2\chi) = \pi^2(2\chi^2) + 2\pi(1-\pi)\chi^4
\]

where the left hand side is the expected output from delegated monitoring and right hand side is the expected output from static group lending if \(2I\) is lent out by the lender. Therefore, the difference between the two models is that group lending requires less monitoring to induce high effort level and therefore leaves lower expected rent with the agents.

4 Sequential Group Lending

Sequential Group-Lending is a two period model. Agents take only one task per period and thus alternate between borrowing and monitoring. The uninformed lender finances projects sequentially, one project at a time with \(I\) units of capital. Therefore, if Agent 1 borrows first, Agent 2 just monitors him in period one while Agent 1 is working on his project. The lender promises Agent 2 that his project would get financed if Agent 1’s project is successful. Since Agent 2 is wealth constrained, he has no alternative but to wait for the lender to finance his project.

Once Agent 1’s project outcome is realised, if the project fails then both agents receive \(s_f\) and the game terminates. Conversely, if the project succeeds, Agent 2’s project is financed by the lender in period two. Agent 1 takes on the role of monitoring Agent 2 in period two. If Agents 2’s project succeeds, both agents get a symmetrical payoff of \(s_{ss}\). If Agent 2’s project fails, then both agents get a symmetrical payoff of \(s_{sf}\).

From the uninformed lender’s perspective, he can differentiate effectively between the following three states and thus offers the agents a state-contingent contract with three state-contingent payoffs (\(s_{ss}, s_{sf}, s_{sf}\)).

- \(s_f\) in state \(f\) (if the first project undertaken fails and the game terminates)
- \(s_{sf}\) in state \(sf\) (if the first project undertaken succeeds and the second project undertaken fails)
- \(s_{ss}\) in state \(ss\) (if both the projects undertaken succeed)

As before, the agents have zero wealth and their respective limited liability constraint applies in all states. Thus \(s_{ij} \geq 0\) for all \(i,j\). Joint liability requires \(\max [s_{ss}, s_{sf}] \geq s_f\) and \(s_{ss} \geq s_{sf}\) with at least one with a strict inequality.

\(^{20}\) in terms of opportunity cost terms
Sequential Group Lending with Moral Hazard

Let’s suppose that Agent 1 decides to borrow first and Agent 2 takes on the role of a monitor. We use the term $M_i$ to refer to Agent $i$ taking on a role of monitoring while $B_j$ refers to Agent $j$ taking on the role of a borrower.

The timing of the game is as follows:

$t = 0$  The lender offers Agent 1 and Agent 2 a contract ($s_{ss}$, $s_{sf}$, $s_f$). If the agents accept, the game continues. Otherwise, it terminates.

$t = 1$  $M_2$ decides on $c_2$
Agent 2, in his role as a monitor, decides on the monitoring intensity

$t = 2$  $B_1$ decides on $\pi_1$
Agent 1, in his role as a borrower, decides whether he wants to exert high or low level of effort

$t = 3$  $B_1$’s project outcome is realized
- If $B_1$’s project fails both agents get a symmetrical payoff of $s_f$ and the game terminates
- If $B_1$’s project succeeds then the game continues

$t = 4$  $M_1$ decides on $c_1$
Agent 1, in his role as a monitor, decides on his monitoring intensity

$t = 5$  $B_2$ decides on $\pi_2$
Agent 1, in his role as a borrower, decides whether he wants to exert high or low level of effort

$t = 6$  $B_2$’s project outcome is realized
- If $B_2$’s project fails both agents get a symmetrical payoff of $s_{sf}$ and the game terminates
- If $B_2$’s project succeeds both agents get a symmetrical payoff of $s_{ss}$ and the game terminates

Our desired outcome is the one where both agents exert high effort for their respective projects and choose to monitor with intensity, which is sufficient to make the other agent exert high effort level. We find the subgame perfect Nash equilibrium (SPNE) of the game by the process of backward induction. In the process, we accumulate conditions under which the desired outcome is the SPNE is the game.

Let’s analyse the subgame $\xi(c_2, \pi_1, c_1)$ which starts at $t = 5$ following the decisions on $c_2$ at $t = 1$, $\pi_1$ at $t = 2$ and $c_1$ at $t = 4$.

At $t = 5$, $B_2$ decides on $\pi_2$. Agent 2’s expected payoff at this stage is:

$$S_5^2[\pi_1, \pi_2, c_1, c_2] = \pi_1[\pi_2s_{ss} + (1-\pi_2)s_{sf}] - c_2 + \left[\frac{\pi - \pi_1}{\pi - \pi_2}\right]B(c_1)$$
So \( B_2 \) chooses
\[
\pi_2 = \bar{\pi} \quad \text{if} \quad \Delta S \geq \frac{B(c_1)}{\pi_1 \Delta \pi} \quad \text{or}
\]
\[
\pi_2 = \pi \quad \text{otherwise}
\]

Subgame \( \xi (c_2, \pi_I) \):

At \( t = 4 \), \( M_1 \) decides on \( c_1 \). Agent 1’s expected payoff at this stage is:
\[
S_{\xi}[\pi_1, \pi_2, c_1, c_2] = \pi_1 \left[ \pi_2 s_{ss} + (1 - \pi_2) s_{sf} \right] - c_1 + \left[ \frac{\pi - \pi_1}{\pi - \pi} \right] B(c_2)
\]

\( M_1 \) chooses \( c_1 \) in such way that
\[
S_{\xi}[\pi_1, \pi_2, c_1, c_2] \geq S_{\xi}[\pi_1, \pi, 0, c_2]
\]
\[
\Delta S \geq \frac{c_1}{\pi_1 \Delta \pi}
\]
where \( \Delta S = s_{ss} - s_{sf} \)

Therefore, we can summarise \( B_2 \) and \( M_1 \)’s decision in one condition.

**SL condition 1:**
\[
\Delta S \geq \frac{1}{\pi_1 \Delta \pi} \max \left[ B(c_1), c_1 \right]
\]

Agents 2 chooses high effort in subgame \( \xi (c_2, \pi_I, c_1) \) if the **SL condition 1** is satisfied. Since **SL condition 1** depends on \( \Delta S \) and not on individual values of \( s_{ss} \) and \( s_{sf} \), rewarding the agents when the second project undertaken fails is unnecessary. Taking the limited liability constraint should bind in this state and \( s_{sf} \) should be set to zero which implies that \( \Delta S = s_{ss} \).

So let’s restate SL condition 1 incorporating \( \Delta S = s_{ss} \)

**SL condition 1’**:
\[
s_{ss} \geq \frac{1}{\pi_1 \Delta \pi} \max \left[ B(c_1), c_1 \right]
\]

From the uninformed lender’s perspective, the state \( ss \) (both successive projects are successful) is the most informative state. Therefore **SL condition 1’** states that given agent’s risk-neutrality, the lender can make \( ss \) state as lucrative as possible and at the same make the limited liability constraint for the agent bind if this state is not reached.

Subgame \( \xi (c_2) \)

Let’s analyse Agent 1’s decision to exert effort for his project at \( t = 2 \).

At \( t = 2 \), \( B_1 \) decides on \( \pi_I \). Agent 1’s expected payoff at this stage is:
\[
S_{\xi}[\pi_1, \pi_2, c_1, c_2] = \pi_1 \left[ \pi_2 s_{ss} + (1 - \pi_2) s_{sf} \right] - c_1 + \left[ (1 - \pi_1) s_{sf} \right] + \left[ \frac{\pi - \pi_1}{\pi - \pi} \right] B(c_2)
\]

So \( B_1 \) will choose high effort if
\[
\pi_2 s_{ss} - s_{sf} \geq \frac{B(c_1)}{\Delta \pi}
\]
For the same reasons as above, \( s_f \) can be set to zero without hurting \( B_1 \)’s incentive to choose high effort. Therefore, \( B_1 \)’s choice would be as follows

\[
\begin{align*}
\pi_1 &= \pi & \text{if } s_{ss} \geq \frac{B(c_2)}{\pi_2 \Delta \pi} \\
\pi_i &= \pi & \text{otherwise}
\end{align*}
\]

If SL Condition 1’ holds, \( B_1 \) would choose to exert high effort if \( s_{ss} \geq \frac{B(c_2)}{\pi \Delta \pi} \)

Now let’s analyse Agent 2’s decision at \( t = 1 \).

At \( t = 1 \), \( M_2 \) decides on \( c_2 \). Agent 2’s expected payoff at this stage is:

\[
S_1^2[\pi, \pi_2, c_1, c_2] = \pi_1 \left[ \pi_2 s_{ss} + (1 - \pi_2) s_{sf} \right] - c_2 + (1 - \pi_1) s_{sf} + \left[ \frac{\pi - \pi_2}{\pi - \pi_1} \right] B(c_1)
\]

So \( M_2 \) chooses \( c_2 \) such that:

\[
s_{ss} \geq \frac{c_2}{\pi_2 \Delta \pi}
\]

We can summarise \( B_1 \) and \( M_2 \)’s decision in one condition.

Let SL condition 2 be: \( s_{ss} \geq \frac{1}{\pi \Delta \pi} \max[B(c_2), c_2] \)

If both SL conditions hold together, we can summarise them in one condition.

SL condition: \( s_{ss} \geq \frac{1}{\pi \Delta \pi} \max[B(c), c] \)

If SL condition holds, it ensures that the game would have a SPNE in which both the agents would monitor with sufficient intensity \( c \) to ensure that both the agents exert high effort level towards their respective project.

So \( E[s_f | \pi, \pi] \geq \frac{\pi}{\Delta \pi} \max[B(c), c] \) using the SL condition

The expected output per borrower in sequential group lending is given by

\[
E[x | \pi, \pi] = \frac{1}{2} \left[ \pi x + \pi (1 - x) \right] = \frac{1}{2} \left[ \pi (1 + \pi x) \right]
\]

If we substitute the above expression in the lenders participation constraint:
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\[ E[R_{ij} | \pi, \bar{\pi}] = E[x | \pi, \bar{\pi}] - E[s_{ij} | \pi, \bar{\pi}] = \gamma I \]

\[ E[x | \pi, \bar{\pi}] \geq \gamma I + \frac{\pi}{\Delta \pi} \max[B(c), c] \]

\[ x^* \geq \frac{\gamma I}{\pi(\pi + 1)} + \max \left[ \frac{2 \cdot B(c)}{\Delta \pi(\pi + 1)}, \frac{2 \cdot c}{\Delta \pi(\pi + 1)} \right] \]

Therefore, the productivity-monitoring intensity locus for Sequential group lending is

\[ x^* = \frac{\gamma I}{\pi(\pi + 1)} + \frac{2 \cdot B(c)}{\Delta \pi(\pi + 1)} \text{ for } c \in (0, c_{SL}) \text{ where } B_c(c_{SL}) = c_{SL} \]

Let's analyse the equation above. Given a project with outcomes \((x^*, 0)\) minimum monitoring required is given by

\[ x^* = \frac{\gamma I}{\pi(\pi + 1)} + \max \left[ \frac{2 \cdot B(c)}{\Delta \pi(\pi + 1)}, \frac{2 \cdot c}{\Delta \pi(\pi + 1)} \right] \]

If \( c \leq c_{SL} \Rightarrow B(c) \geq c \), then

\[ x^* = \frac{\gamma I}{\pi(\pi + 1)} + \frac{2 \cdot B(c)}{\Delta \pi(\pi + 1)} \]

If \( c > c_{SL} \Rightarrow B(c) < c \), then

\[ x^* = \frac{\gamma I}{\pi(\pi + 1)} + \frac{2 \cdot c}{\Delta \pi(\pi + 1)} \text{ where } B(c_{SL}) = c_{SL} \]

The resultant productivity-monitoring locus for sequential group lending is decreasing (since \( B_c(c) < 0 \) ) function of \( c \) till \( c = c_{SL} \) and starts increasing after \( c_{SL} \). So the least productive project that can be financed by the uninformed lender is given by

\[ x^*_{SL} = \frac{\gamma I}{\pi(\pi + 1)} + \frac{2 \cdot B(c_{SL})}{\Delta \pi(\pi + 1)} = \frac{\gamma I}{\pi(\pi + 1)} + \frac{2 \cdot c_{SL}}{\Delta \pi(\pi + 1)} \]

So sequential group lending can finance project \((x^*, 0)\) where

\[ x^* \in \left( \begin{array}{c}
\frac{\gamma I}{\pi(\pi + 1)} + \frac{2 \cdot B(c_{SL})}{\Delta \pi(\pi + 1)}, \frac{\gamma I}{\pi(\pi + 1)} + \frac{2 \cdot B(0)}{\Delta \pi(\pi + 1)}
\end{array} \right) \]
Figure 1. Productivity-Monitoring Intensity Locus for Sequential Lending
5 Comparing the Two Lending Arrangements

5.1 Economic Rents

Let's look at the economic rent the agents get in the various models.

Economic Rent for Agent $i$ under Sequential group lending:

$$E[g_i|\pi, \pi] - c = \pi^2 s_{si} - c$$

$$= \frac{\pi^2}{\pi \Delta \pi} B(c) - c \quad \text{for } c \in (0, c_{SL})$$

$$= \frac{\pi}{\Delta \pi} [B(c) - c] + \frac{\pi}{\Delta \pi} c \geq 0$$

Agent $i$'s economic rent is decreasing in $c$ and which means more intensive the monitoring, the higher the cost of monitoring and lower the value of private benefits. So as the lender induces greater monitoring intensity, Agent $i$ rents decrease.

It is evident that without the threat of collusion, rents that agent can retain under sequential group lending is identical to the rent he could retain under static group lending as compensation payment for private benefits forgone is almost identical under both schemes, that is $s_{si} = \frac{B(c)}{\Delta \pi}$ in state $ss$ and zero in any other state.

Let's compare this to the rent the borrower and monitor were able to keep in the delegated monitoring model due to their respective limited liability constraint.

Borrower's rent:

$$E[\delta_j|\pi] = \frac{\pi}{\Delta \pi} B(c)$$

Monitor's rent:

$$E[\omega_i|\pi] = \frac{\pi}{\Delta \pi} c - c = \frac{\pi}{\Delta \pi} c$$

In the delegated monitoring model, the borrower and monitor had to be offered separate incentives for engendering the favourable outcome of the project. In group lending models, without the threat of collusion, since both the tasks are undertaken by the same agent, agent needs to be compensated only for the more expensive task.

With the threat of collusion, rents retained by agents increase with monitoring once the no-collusion constraint ($GL1'$) starts binding. In sequential group lending, the rents unconditionally decrease as monitoring intensity increases till the cost of monitoring becomes greater than private benefits. ($SL1'$ binds)

5.2 Expected Value of the Outcome

The expected value of the outcome (per period per unit of capital) once high effort has been implemented is lower in Sequential group lender as compared to static group lending or delegated monitoring model.

$$\pi^2 [2.x] \leq \pi^2 [2.x] + 2 \pi (1 - \pi) x = \pi [2.x]$$
From the right hand side, the first expression is the expected output from lending \( I \) for two periods in a sequential lending scheme, the second expression the expected output if \( 2I \) is lent for one period in a static group lending scheme and the third expression is the if \( 2I \) is lent to two agents in delegated monitoring model.

This as we see ahead does not give us a clear intuition for the lower bound on project productivity for sequential group lending. What is clear though is that as monitoring technology becomes more efficient, sequential group lending’s lower bound decreases at a faster rate than static group lending’s. Therefore, at some point sequential group lending is able to finance projects with lower productivity. We need to derive conditions under which this is true.

5.3 Possibility of Collusion

There is no chance of collusion in the sequential group-lending model if the assumption that private benefits \( B(c) \) are non-pecuniary and non-transferable is maintained. The sequential nature of the decision making process at each stage ensures that the same result would be attained whether the game is played cooperatively or non-cooperatively. Therefore, collusion does not limit the scope of sequential group-lending model in any way.

The lack of possibility of collusion in sequential group lending is similar to the result we obtained in delegated monitoring model. The delegated monitoring model can be thought of as a subgame in the sequential group lending model.

Collusion plagues the Static group lending scheme because of the simultaneous nature of the decision making process. Even though the cost and benefits for these wealthless agents are non-pecuniary and non-transferable, there is a possibility of collusion through coordination. The uninformed lender can do nothing more than offer high enough rents so that the collective opportunity cost of the decision to monitor remains lower than the opportunity cost of diligence. Since low productivity project can only be undertaken with intensive monitoring, some socially viable projects are not financed.

In conclusion, the sequential group lending scheme has the advantage of being collusion proof. The limit to the implementability of sequential group lending model is that after \( c = c_{SL} \) the compensation required for monitoring activities becomes greater than compensation required to forgo monitored private benefits and thus there is no point in inducing monitoring intensity beyond \( c_{SL} \). On the other hand limits to static group lending is due to threat of collusion beyond \( c = c_{GL} \).

Sequential group lending is disadvantaged by the fact that expected output is lower for it compared to the expected output in the static group lending scheme. In sequential group lending scheme, the opportunity to undertake the second project in the second time period is missed if the first project does not succeed. In static group lending, both projects are undertaken and outcome of both projects realised. Given that the project returns are statistically independent, this is akin to lost expected output.
To summarise even though the sequential group lending projects are not limited in scope by collusion, they can be more expensive to implement due to lower expected output.

It is left for us to show the conditions under which a much greater proportion of socially viable projects can be implemented under sequential group lending than in static group lending scheme. That is, we need to show conditions under which the lowest-productivity project feasible under sequential group lending is lower in value than under static group lending scheme. ($x_{SL}^s < x_{GL}^s$)

### 5.4 Group lending with varying efficiency of Monitoring Technology

All results obtained till now were for a given monitoring technology $B(c)$. To investigate the effect of monitoring technology’s efficiency on our result we need to separate the private benefits into its two components namely the initial private benefit $B(0)$ and the actual effect of monitoring intensity $c$ in decreasing private benefits.

Therefore $B(c) = B(0) + \beta_c b(c)$

where $b(c) < 0$; $b_c(c) < 0$; $b_{cc}(c) \geq 0$;

$b(0) = 0$ and $b(\infty) = -B(0)$

We can use $\beta_c$ to define the efficiency of the monitoring technology. Higher values of $\beta_c$ are associated with greater efficiency in monitoring technology and faster rate of decreases in private benefits due to monitoring. Similarly, smaller values of $\beta_c$ are associated with less efficient monitoring technology.

For any given $\beta_c$ ($x_{GL}^s, c_G$) is determined by

\[
x_{GL}^s = \frac{\gamma I}{\pi} + \frac{B(c_G)}{\Delta \pi} \quad \text{GL2'}
\]

\[
x_{GL}^s = \frac{\gamma I}{\pi} + \alpha \frac{B(0) + c_{GL}}{\Delta \pi} \quad \text{GL1'}
\]

or $x_{GL}^s = \min \left\{ \frac{\gamma I}{\pi} + \max \left[ \frac{B(c_G)}{\Delta \pi}, \frac{\alpha(B(0) + c_G)}{\Delta \pi} \right] \right\}$

where $\alpha = \left[ \frac{\pi}{\pi + \frac{c_G}{\Delta \pi}} \right]$ and similarly for any given $\beta_c$ ($x_{SL}^s, c_S$) is determined by

\[
x_{SL}^s = \frac{\gamma I}{\pi(\pi + 1)} + \frac{2. B(c_S)}{\Delta \pi(\pi + 1)} \quad \text{SL2'}
\]

\[
x_{SL}^s = \frac{\gamma I}{\pi(\pi + 1)} + \frac{2. c_S}{\Delta \pi(\pi + 1)} \quad \text{SL1'}
\]

or $x_{SL}^s = \min \left\{ \frac{\gamma I}{\pi(\pi + 1)} + \max \left[ \frac{2. B(c_S)}{\Delta \pi(\pi + 1)}, \frac{2. c_S}{\Delta \pi(\pi + 1)} \right] \right\}$
Proposition 4

(i) For a large enough value of $\beta$, $x_{GL}^* > x_{SL}^*$. 
Thus for extremely efficient monitoring technology, sequential group lending can finance projects with lower productivity compared to static group lending.

(ii) As $\beta$ becomes smaller, $x_{GL}^*$ increases at a slower rate than $x_{SL}^*$. 

(iii) For a small enough value of $\beta$, $x_{GL}^* < x_{SL}^*$. 
Thus for very inefficient monitoring technology, static group lending can finance projects with lower productivity compared to sequential group lending.

For all $\beta$, SL2’ and GL2’ cross at $x^* = \frac{\gamma_l}{\pi(1-\pi)}$. Lets define $\tilde{x}$ as $\tilde{x} = \frac{\gamma_l}{\pi(1-\pi)}$.

Lets call $x^* = \tilde{x}$ (= $\frac{\gamma_l}{\pi(1-\pi)}$) the cross-over line. By comparing SL2’ and GL2’ we know that below this line, for any given $c$ and $\beta$, $x_{GL2} \geq x_{SL2}$.

Let us define $(\hat{c}, \hat{x})$ on SL1’ as the point where GL1’s crosses SL1’.

where $\hat{c} = \frac{\Delta \pi \gamma_l + \alpha.(\pi + 1)B(0)}{2 - \alpha(\pi + 1)}$

$\hat{x} = \frac{\gamma_l}{\pi(\pi + 1)} + \frac{2}{\pi(\pi + 1)} \left[ \frac{\Delta \pi \gamma_l + \alpha.(\pi + 1)B(0)}{2 - \alpha(\pi + 1)} \right]$

Lets also define $(\tilde{x}, \tilde{c})$ on SL1’ as the point where SL1’ crosses the cross-over line.

where $\tilde{x} = \frac{\gamma_l}{\pi(1-\pi)}$

$\tilde{c} = \frac{\gamma_l \Delta \pi}{1-\pi}$

So, we have defined two points $(\tilde{x}, \tilde{c})$ and $(\hat{x}, \hat{c})$ on SL1’. $(\hat{x}, \hat{c})$ depends on $\gamma_l, \pi, \pi$ and $B(0)$. $(\tilde{x}, \tilde{c})$ also depends on $\gamma_l, \pi, \pi$ but not on $B(0)$. Thus the value of $B(0)$ determines whether GL1’ crosses SL1’ below or above the cross-over line.

Proof

To show that for a large enough value of $\beta$, $x_{GL} \geq x_{SL}$ we consider two cases.

Case I: $(\hat{x}, \hat{c})$ is right of $(\tilde{x}, \tilde{c})$ or when $\tilde{x} \leq \hat{x}$
This happens when GL1’ crosses SL1’ above the cross over line.

Case II: $(\hat{x}, \hat{c})$ is left of $(\tilde{x}, \tilde{c})$ or when $\hat{x} < \tilde{x}$
This happens when GL1’ crosses SL1’ below the cross over line.
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Case I: $\tilde{x} \leq \hat{x}$

Case II: $\hat{x} < \tilde{x}$
Case I: We can find a $\beta$ such that $SL_2'$ and $GL_2'$ meet at $(\tilde{x}, \tilde{c})$. Thus for such a $\beta$, $x_{SL}^e = \tilde{x}$. We know from the construction of the case, that $SL_1'$ is below $GL_1'$ at $c = \tilde{c}$. Given that $GL_2'$ is downward sloping, it meets $GL_1'$ at $x_{GL}^e > \tilde{x}$ implying $x_{GL}^e > x_{SL}^e$.

Case II: We can find a $\beta$ such that $GL_2'$ meet at $GL_1'$ at $(\hat{x}, \hat{c})$. Therefore, $x_{SL}^e = \hat{x}$ The $SL_2'$ for the same $\beta$ will cross the cross-over line at some point $A$ (refer to figure below) where $c_A < \hat{c}$. This same will $SL_2'$ will cross $SL_1'$ at at point where again $c < \hat{c}$. Since by construction of the case, for $c < \hat{c}$, $SL_1'$ is lies below $GL_1'$, we can conclude that $x_{SL}^e < \hat{x}$ implying $x_{SL}^e < x_{GL}^e$.

Given $GL_1'$ and $GL_2'$, $c_{GL}$ gets determined by $\alpha [B(0) + c_{GL}] = B(c_{GL})$

Substituting $B(c) = B(0) + \beta_i b(c)$ in above, we get

$$\alpha [B(0) + c_{GL}] = B(0) + \beta_i b(c_{GL}) \quad (eq. 5.1)$$

To examine the change in $c_{GL}$ due to change in $\beta$, we differentiate the above given expression and obtain

$$\frac{dc_{GL}}{d\beta} = \frac{b(c_{GL})}{\alpha - \beta b'(c_{GL})} \leq 0 \quad (eq. 5.2)$$

This tells us the rate at which $c_{GL}$ decreases when $\beta$ decreases (monitoring technology becomes more efficient). Similarly given $SL_1'$ and $SL_2'$, $c_{SL}$ gets determined by

$$c_{SL} = B(c_{SL}) = B(0) + \beta_i b(c_{SL}) \quad (eq. 5.3)$$

and by differentiating the above we obtain

$$\frac{dc_{SL}}{d\beta} = \frac{b(c_{SL})}{1 - \beta b'(c_{SL})} \leq 0 \quad (eq. 5.4)$$

Given the value of $\beta$, eq. 5.1 and 5.3 solve for $c_{GL}$ and $c_{SL}$ respectively and gives us $c_{GL} < c_{SL}$ and $b(c_{GL}) > b(c_{SL})$

Thus for a large enough $b$ namely $\beta > (1 - \alpha) / b'(c_{SL}) - b'(c_{GL})$

$$\frac{dc_{GL}}{d\beta} > \frac{dc_{SL}}{d\beta}$$
which tells us that as $\beta$ decreases (monitoring technology becomes less efficient) both $c_{GL}$ and $c_{SL}$ increase, but $c_{SL}$ increases at a faster rate as compared to $c_{GL}$. The restriction on $\beta$ becomes significant only for very small values of $\beta$ when the value of $c_{GL}$ is very close to $c_{SL}$ and thus $b'(c_{SL})$ is close to $b'(c_{GL})$.

Using GL1' 
$$x_{GL}' = \frac{\gamma I}{\pi} + \frac{\alpha}{\Delta \pi} (B(0) + c_{GL})$$

Using SL1' 
$$x_{SL}' = \frac{\gamma I}{\pi(\pi + 1)} + \frac{2}{\Delta \pi(\pi + 1)} c_{GL}$$

From the above equations thus

$$\frac{dx_{GL}'}{d\beta} > \frac{dx_{SL}'}{d\beta}$$
and $$\left|\frac{dx_{GL}'}{d\beta}\right| < \left|\frac{dx_{SL}'}{d\beta}\right|$$

This tells us that as $\beta$ decreases (monitoring technology becomes less efficient) both $x_{GL}$ and $x_{SL}$ increase, but $x_{SL}$ increases at a faster rate as compared to $x_{GL}$. Given that at a large $\beta$, $x_{GL}$ > $x_{SL}$ and as $\beta$ decreases, $x_{SL}$ decreases at a faster rate than $x_{GL}$, there must exist a $\beta^{SL}$ at which when $x_{SL} = x_{SL}'$. Thus for $\beta \geq \beta^{SL}$ a greater proportion of socially viable projects are feasible under sequential group lending. For $\beta \leq \beta^{SL}$ a greater proportion of projects are feasible under static group lending.

Conclusion

We found that if there was no threat of collusion, the rents under Static and Sequential group lending would have been identical. The threat of collusion in Static group lending implies that agents are able to obtain excessively high rents. These rents do not vary much with increased efficiency of monitoring technology. Thus, the counter-intuitive result followed that increased efficiency of monitoring technology does not significantly drive down the rents obtained by agents. This, if compared to standard result that states information sharing on effort is beneficial to the lender suggest that at least cheaper information acquisition is not always beneficial to the lender.

An important question that needs to be addressed is whether in group lending observing effort is feasible in spite of agent’s close proximity to each other. In the reality, effort is complex and multi-dimensional. Observing effort might entail not just merely observing actions but also trying to pin down intentions. Conversely, monitoring could be passive where only deviation beyond an acceptable range need be acknowledged. The difference between the two is that observing effort is a bottom
up approach and monitoring a *top down* approach. As our example suggested in the introduction, a father might find pinning down his son’s exact effort more difficult than monitoring him. In this example at least, the framework for evaluating monitoring signals would be far less complex than a framework for evaluating numerous effort signals. Similarly, a neighbour might find pinning down his neighbour’s intention more difficult than just monitoring him. The problem is far less complicated with ex post moral hazard in group lending. In ex post moral hazard, the outcome is clearly visible to the fellow group member and thus pinning down intentions is far easier.

With our assumption of non-observability of effort and costly monitoring, it is evident that sequential group lending leaves lower rents with agents in comparison with static group lending. In spite of lower expected output in Sequential group lending, it is able to finance less productive project if the monitoring technology is sufficiently efficient. Thus with sequential group lending all benefits of inexpensive *information acquisition* regarding monitoring are realized.

**Appendix**

There are four possible subgame. \( \xi(c,c) \), \( \xi(c,0) \), \( \xi(0,c) \), \( \xi(0,0) \)

For subgame \( \xi(c,c) \) if the contract meets the condition GL2 it follows that that

\[
HH(c,c) \geq LH(c,c)
\]

leading us to conclude that HH would be an Nash equilibrium. For LL to be a Nash equilibrium of this subgame, , we need to show that

\[
LL(c,c) \geq HL(c,c)
\]

\[
E(s_i|\pi\pi) - c + B(c) \geq E(s_i|\pi\pi) - c + B(c)
\]

\[
\pi s_{ss} \geq \pi s_{ss} + B(c)
\]

\[
s_{ss} \leq \frac{B(c)}{\pi\Delta\pi}
\]

Thus showing that LL would also be an Nash equilibrium. The agents coordinate on HH \( (c,c) \) because HH \( (c,c) > LL(c,c) \).

For rest of three subgames, we can show that LL can be Nash equilibrium. Let take the subgame \( \xi(c,0) \) where Agent 1 monitors with intensity \( c \) and Agent 2 does not monitor.

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21 The proof in this appendix is entirely based on Conning (1996)
Let's analyse Agent 1’s effort level decision in this subgame. Given that Agent 2 has chosen high effort level, Agent 1 would also choose high effort level if

\[ HH(c,0) \geq LH(c,0) \]

\[ E(s_j|\pi,\pi) - c \geq E(s_j|\pi,\pi) - c + B(0) \]

\[ \pi^2s_{ss} \geq \pi\pi s_{ss}^2 + B(0) \]

\[ s_{ss} \geq \frac{B(0)}{\pi \Delta \pi} \]

but we know that \( s_{ss} = \frac{B(c)}{\pi \Delta \pi} \) for \( c \in (\xi, c_{GL}) \) which contradicts the above condition leading to the conclusion that Agent 1 would certainly deviate to low effort level.

Now let's analyse Agent 2’s effort level decision. Given Agent 1’s low effort level, Agent 2 will choose high effort if

\[ HL(c,0) \geq LL(c,0) \]

\[ E(s_j|\pi,\pi) - 0 \geq E(s_j|\pi,\pi) - 0 + B(c) \]

\[ \pi\pi s_{ss} \geq \pi^2 s_{ss} + B(c) \]

\[ s_{ss} \geq \frac{B(c)}{\pi \Delta \pi} \]

but we know that \( s_{ss} = \frac{B(c)}{\pi \Delta \pi} \leq \frac{B(c)}{\pi \Delta \pi} \) which contradicts the above condition and leads us to the conclusion that Agent 2 will not deviate from low effort level. This implies that LL is the Nash equilibrium of subgame \( \xi(c,0) \).

Similarly, we can show that for subgame \( \xi(0,c) \) and \( \xi(0,0) \) the Nash equilibrium would be LL.

For subgame \( \xi(c,0) \) and \( \xi(0,0) \), HH cannot be a Nash Equilibrium because

\[ LH(c,0) \geq HH(0,c) \]

and

\[ LH(0,0) \geq HH(0,0) \]

Moving up the game tree since \( HH(c,c) > \max \{ LL(0, c), LL(c, 0) \} \) we can state that \( c \) is the best response to the \( c \) at the first stage.

That leaves us with \( HH(0,0) \) and \( LL(0,0) \). But from the no-collusion constraint we know that \( HH(cc) \geq LL(0,0) \). Thus if the two conditions hold, \( HH(cc) \) is the SPNE of the game.
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