Edinburgh School of Economics
Discussion Paper Series
Number 127

Consumption, Status and Redistribution

Ed Hopkins (University of Edinburgh)
Tatiana Kornienko (University of Stirling)

Date
November 2004

Published by
School of Economics
University of Edinburgh
30 -31 Buccleuch Place
Edinburgh EH8 9JT
+44 (0)131 650 8361
http://www.ed.ac.uk/schools-departments/economics

THE UNIVERSITY *of* EDINBURGH

Consumption, Status and Redistribution*

Ed Hopkins†
Department of Economics
University of Edinburgh
Edinburgh EH8 9JY, UK

Tatiana Kornienko‡
Department of Economics
University of Stirling
Stirling FK9 4LA, UK

November, 2004

Abstract

This paper considers the effect of inequality when there are concerns for status. We analyse the effects of linear redistributive taxes in an economy where agents' utility depends both on consumption and on their rank in the distribution of consumption of a positional good. This increase in equality increases the degree of social competition. The equilibrium level of expenditure on the positional good rises for most agents with the possible exception of some with above average income. Equilibrium utility falls for those with average and above income, while the utility of the poor may (or may not) rise.

Journal of Economic Literature classification numbers C72, D11, D31, D62
Keywords: Status, relative standing, income inequality, consumption externalities.

---

*Comments are welcome. Errors remain our own. Ed Hopkins acknowledges support from the Economic and Social Research Council, award reference RES-000-27-0065.

†E.Hopkins@ed.ac.uk, http://homepages.ed.ac.uk/ehk

‡Tatiana.Kornienko@stir.ac.uk, http://staff.stir.ac.uk/tatiana.kornienko

1 Introduction

There is increasing acceptance amongst economists that people may care about their relative position as well as the absolute level of their consumption. For example, a survey of empirical research on happiness goes as far as to conclude that in determining happiness "It is not the absolute level of income that matters most but rather oneŠs position relative to other individuals" (Frey and Stutzer, 2002, p411). If this is the case, it seems a natural conclusion that greater inequality would worsen the degree of social competition (Frank (1999)) and, therefore, give a new justification for policies to reduce inequality. In earlier work (Hopkins and Kornienko (2004a)), we were able to elaborate the model of relative concerns of Frank (1985) so that we could characterise how equilibrium behaviour and equilibrium utility changed in response to changes in the distribution of income. We found that, surprisingly, greater equality could lead to an increase in conspicuous consumption and a reduction in welfare for those with middle and low incomes. Thus, even in the presence of relative concerns, greater equality is not necessarily welfare-enhancing. A limitation of this analysis, however, was that it was based on comparisons at a fixed level of income. That is, in effect we investigated the behaviour of agents whose income did not change as the income of those around them did.

In this paper, we analyse the effects of a general redistribution of income in an economy where agents' utility depends both on consumption and on status, defined as rank in the distribution of consumption of one positional good. In particular, we investigate linear income tax schemes, where the money raised is redistributed equally across the population. This reduces inequality while maintaining the relative position of each individual. We find that the equilibrium level of expenditure on the positional good rises for most agents with the possible exception of some with above average income. Equilibrium utility falls for most of the population, with the possible exception of some of those with below average income. That is, even the gainers in terms of income may lose in terms of welfare.

In our current work and our earlier paper, Hopkins and Kornienko (2004a), we use the same model derived from that of Frank (1985), where individuals must decide how to divide their income between consumption of a normal good and a positional good. For example, one might care about the characteristics of one's car, but also about how it compares to those of own's neighbours. The choice of the positional good is therefore strategic, in that consumption choices of my neighbours affect my payoffs, as does my choice affects theirs. The symmetric Nash equilibrium of the resulting game will be Pareto inefficient in that all will spend more on the positional good than in the absence of status concerns, but resulting in no net change in relative position. That is, everyone increases conspicuous consumption in order to improve status, but any gain in status is cancelled out by the similarly increased expenditure of others. A salient question in this context is how the distribution of income effects the degree of social competition and, hence, the amount of excess consumption.

In such a model, however, changes in the income distribution can affect agents through three channels: one's own income, one's relative position or rank in the distribution, and the shape of the distribution all matter. Of necessity, any analysis must hold at least of one of these constant. In our previous work, we considered changes in the distribution of income that left some people's incomes unchanged. For example, imagine a change in taxes on earned income that does not affect those who do not work. We then analysed how someone with unchanged income would respond to the change in the distribution of income. We found that a reduction of inequality of this type would lead to a fall in utility for those with low income.

It has been argued to us, however, that this "equality hurts the poor" result may be misleading. Often, when one thinks of a reduction in inequality, one thinks, for example, of an increase in income of the poor, rather than a change that keeps the incomes of some poor people constant. In response to this argument, we here consider changes in the distribution of income that affect every individual. Those with below average income have their income raised, and those with above average income have their net income decreased. The linear schemes we consider, however, do have the advantage is that they keep constant the relative position of all individuals. That is, in contrast to our earlier work, we keep relative position constant, not income.

We find as we did before that an increase in equality increases the degree of social competition. The closer individuals are together, the easier it is to overtake others in status, thus giving a greater incentive to indulge in conspicuous consumption. This means that the overall effect for the poor is ambiguous: they have greater income, but more of it may be spent on wasteful consumption. We can find examples where the poor are worse off in a more equal society, even with higher income, and other examples where they benefit. However, the effect on those richer than average is definitely negative. This is in contrast to the effect of a reduction in inequality in Hopkins and Kornienko (2004a), where the effect on the rich was ambiguous, and on the poor was definitely negative.

We hope these contrasting results may help to explain why it has been difficult to establish empirically whether greater equality does in fact lead to greater happiness. Clark (2003), using British panel data, finds a positive relationship between inequality and self-reported happiness while Senik (2004) finds that inequality has no statistical influence on life satisfaction in post-reform Russia. In contrast, Alesina *et al.* (2004) find a negative relationship between inequality and happiness for both Europe and the US. Our results suggest that, even in the presence of relative concerns, whether greater equality does increase utility or happiness may depend quite sensitively on the measure of equality considered.

2 The Model

Following Frank (1985), we consider a simple model where individuals care about their social status as determined by their (conspicuous) consumption of a visible (positional) good, as well as absolute level of (conspicuous) consumption of this visible (positional) good x and absolute level of (non-conspicuous) consumption of another (non-positional) good y, the consumption of which is not directly observable by other agents. We assume an economy consisting of a continuum of agents, identical except in terms of income. Each agent is endowed with a level of income z which is private information and is an independent draw from a common distribution. This is described by a distribution function $G(z)$ which is twice continuously differentiable with a strictly positive density on some interval $[\underline{z}, \bar{z}]$ with $\underline{z} \geq 0$.

Agents' choices of conspicuous consumption are aggregated in a distribution of conspicuous consumption $F(\cdot)$, with $F(x)$ being the mass of individuals with consumption less than or equal to x. Following Frank (1985) and Robson (1992), an agent's status will be determined by her position in the distribution of conspicuous consumption, with higher consumption meaning higher status. Following Hopkins and Kornienko (2004a), we define status as follows:

$$S(x, F(\cdot)) = \delta F(x) + (1 - \delta) F^{-}(x) + S_0 \tag{1}$$

where x is individual's consumption, $\delta \in [0, 1)$, $F(x)$ is the mass of individuals with consumption less or equal to x, and $F^{-}(x) = \lim_{x' \to x^{-}} F(x')$ is the mass of individuals with consumption strictly less than x. The current formulation is a way of dealing with ties. For example, if all agents chose the same level of consumption in one sense they would all be "equal first", but it is unlikely that they would gain the same level of satisfaction as someone who was uniquely first. To reflect this, the current assumption would award them status equal to δ which is strictly less than one.[1] In contrast, if the distribution of consumption $F(x)$ is continuous, there are no ties, the above measure of status is identical to rank in consumption, or $S(x, F(\cdot)) = F(x)$. The parameter $S_0 \geq 0$ is a constant representing a guaranteed minimum level of status, reflecting the intensity of social pressures. We discuss its role below.

Individuals have identical Cobb-Douglas-style preferences over absolute consumptions and status as follows:

$$U = x^{\alpha} y^{\beta} S(x, F(\cdot))^{\gamma}$$

They make a simultaneous allocation of their incomes z between conspicuous consumption x and non-conspicuous consumption y, so that each agent faces the following problem,

$$\max_{x,y} x^{\alpha} y^{\beta} S(x, F(\cdot))^{\gamma} \text{ subject to } x + y \leq z, \ x \geq 0, \ y \geq 0. \tag{2}$$

[1] This also gives agents an incentive to break any ties, so, as we will see, there will be no ties in equilibrium. See Hopkins and Kornienko (2004a) for a full rationale of this specification.

The unit price of the two goods are taken to be equal, and are normalised to one.

Notice that if $\gamma = 0$, there is no concern with status, and the optimal choices are simply:

$$x^*(z_i) = \frac{\alpha}{\alpha+\beta} z_i, \qquad y^*(z_i) = \frac{\beta}{\alpha+\beta} z_i \tag{3}$$

But if individuals are in competition for status, that implies that their choice of consumption of different types of goods is strategic. Note that the distribution of conspicuous consumption $F(\cdot)$ is endogenously determined, so is the social status $S(x, F(\cdot))$. Thus, a rational individual makes a consumption choice in anticipation of consumption choices of all other individuals, i.e. is engaged into a game of status.

It is possible to solve the resulting game but the solution will, however, depend on the distribution of income in society. This approach is developed in more generality and detail in Hopkins and Kornienko (2004a), so here we will only briefly sketch the main features of the game. Here, a strategy for an agent is a choice of a mapping from income z to conspicuous consumption x. This game has a formal resemblance to a first-price auction, as increasing one's conspicuous consumption leads to a trade-off between the increase in status and the decrease in non-conspicuous consumption component of utility, just as a bidder in an auction trades off an increase in probability of winning for lower realized profits in the event of winning.

In this context, a *symmetric equilibrium* will be a Nash equilibrium in which all agents use the same strategy, that is, the same mapping $x(z)$ from income to conspicuous consumption. Suppose all agents adopt the same increasing, differentiable strategy $x_i = x(z_i)$ and consider whether any individual agent has an incentive to deviate. Suppose that instead of following the strategy followed by the others, an agent with wealth z_i chooses $x_i = x(\hat{z})$, that is, she consumes as though she had wealth $\hat{z}$. Note first that $F(x_i) = G(x^{-1}(x_i)) = G(\hat{z})$, resulting in $S_i = S_0 + G(\hat{z})$, and second that her utility would be equal to

$$U_i = x(\hat{z})^{\alpha}(z - x(\hat{z}))^{\beta}(S_0 + G(\hat{z}))^{\gamma}$$

We differentiate with respect to $\hat{z}$. Then, given that in a symmetric equilibrium, the agent uses the equilibrium strategy and so $\hat{z} = z_i$, this gives the first order condition,

$$\frac{\alpha x'(z_i)}{x(z_i)} - \frac{\beta x'(z_i)}{z_i - x(z_i)} + \frac{\gamma g(z_i)}{S_0 + G(z_i)} = 0. \tag{4}$$

This first order condition therefore defines a differential equation,

$$x'(z_i) = \left(\frac{\gamma}{\alpha+\beta}\frac{x(z_i)(z_i - x(z_i))}{x(z_i) - x^*(z_i)}\right)\left(\frac{g(z_i)}{S_0 + G(z_i)}\right). \tag{5}$$

An important point to recognise is that this differential equation and the equilibrium strategy, which is its solution, both depend on the distribution of income G.

We also need to specify the boundary conditions for the differential equation. As Hopkins and Kornienko (2004a) show, the boundary condition depends on the "intensity" of concern with status, and in particular, the level of S_0. When social norms are such that being lowest in society results in complete social exclusion, $S_0 = 0$, then low ranked individuals are desperate to avoid that fate and spend heavily on visible consumption.[2] Specifically, if $S_0 = 0$, then $x(\underline{z}) \leq \underline{z}$ with $\lim_{z \to \underline{z}^+} x(z) = \underline{z}$, or those close to the bottom must spend close to all their income on visible consumption. However, if $S_0 > 0$, then low ranked individuals are no longer desperate, and the choice of the individual with the lowest income $\underline{z}$ is as though she has no concern for status at all. She knows she will come last, so why try? That is, her choice is given by the equations (3). We have,

$$x(\underline{z}) = \begin{cases} \underline{z} & \text{if } S_0 = 0 \\ \frac{\alpha}{\alpha+\beta}\underline{z} = x^*(\underline{z}) & \text{if } S_0 > 0 \end{cases} \tag{6}$$

$$y(\underline{z}) = \begin{cases} 0 & \text{if } S_0 = 0 \\ \frac{\beta}{\alpha+\beta}\underline{z} = y^*(\underline{z}) & \text{if } S_0 > 0 \end{cases} \tag{7}$$

As we show later, these boundary conditions play an important role in our analysis. It is also important to stress that, as Frank (1985) was the first to point out, Nash equilibrium consumption of the positional good is higher than in the absence of status concerns. Formally, Hopkins and Kornienko (2004a, Proposition 1) show the following.

Proposition 1 *The differential equation (5) with boundary conditions (6) has a unique solution which is an essentially unique symmetric Nash equilibrium of the game of status. Equilibrium conspicuous consumption $x(z_i)$ is greater than in the absence of status concerns, $x(z_i) > x^*(z_i)$ on $(\underline{z}, \bar{z}]$.*

The equilibrium is only "essentially" unique as when $S_0 = 0$, as there is other possible equilibrium behaviour for the agent with the lowest income. Specifically, given the necessary condition for equilibrium $\lim_{z \to \underline{z}^+} x(z) = \underline{z}$, the agent with income $\underline{z}$ will always have rank zero and always have zero utility and so is indifferent between any choice of x on the range $[0, \underline{z}]$. The specific boundary conditions (6) and (7) are not crucial to our analysis but are used for convenience.

3 Introducing Redistribution

As we saw in the previous section, individuals' consumption choices depend on the distribution of income. Thus, if the distribution changes, consumption choices will

[2] Note that $S_0 = 0$ represents social rather than material exclusion. The lowest ranked individual could still have an income $\underline{z}$ that is significantly greater than zero.

change as well. Thus, the question arises: what will happen to consumption choices if the income distribution becomes more equal - say, as a result of redistributive taxation? In this section, we introduce a novel treatment of linear income taxes. This treatment, together with an assumption of Cobb-Douglas-style preferences, will allow us to do a comparative static analysis of consumption choices of an individual at the same *rank* for *any* income distribution. This is in contrast to Hopkins and Kornienko (2004a), who performed comparative static analysis for more general type of preferences but for specific pairs of income distributions that satisfy strong refinements of stochastic dominance. Also, the comparative static analysis was done for individuals at fixed *income level* rather than at the same *rank*.

We will consider a pure redistributive taxation scheme of the following form: individuals with above-average income pay a tax of $\tau(z-\mu)$, while those with below-average income receive a subsidy of $\tau(\mu - z)$, where $\tau \in [0, 1)$ is the tax rate and μ is the average income in the society. The redistribution scheme is equivalent to the one in which everyone is taxed at a flat rate τ, and the tax revenue is redistributed back equally. Let $\tilde{z}_i$ represent the post-tax income of someone with initial income z_i, then

$$\tilde{z}_i = (1-\tau)z_i + \tau\mu \Rightarrow z_i = \frac{\tilde{z}_i}{1-\tau} - \frac{\tau\mu}{1-\tau} \tag{8}$$

As the result, the initial before-tax distribution $G(z)$ is a mean-reserving spread of the after-tax distribution $\tilde{G}(\tilde{z})$ (see Figure 1). In other words, the above pure redistributive taxation scheme is inequality-reducing.

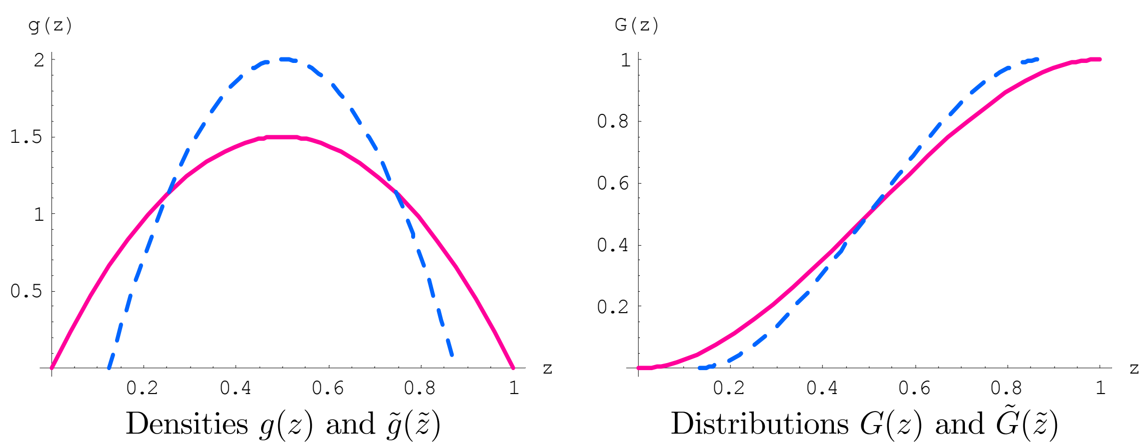


Figure 1: Before-tax income distribution G (solid curves) is a mean-preserving spread of after-tax income distribution $\tilde{G}$ (dashed curves), and the two distributions cross at the mean income μ.

Being a lump-sum tax, it might be thought that the policy only affects the incentives to consume different types of goods insofar as it changes the current income of the individual. However, it also changes the return to conspicuous consumption through the change in returns to status as it is determined by the income distribution. To see that, observe that the after-tax distribution of income $\tilde{G}(\tilde{z})$ is *the same type*[3] as the before-tax distribution $G(z)$, so that for a fixed individual i with income z_i[4] we have

$$\begin{aligned}
\tilde{G}(\tilde{z}_i) = G(z_i) &= G\left(\frac{\tilde{z}_i}{1-\tau} - \frac{\tau\mu}{1-\tau}\right) \\
\tilde{g}(\tilde{z}_i) = \frac{1}{1-\tau}g(z_i) &= \frac{1}{1-\tau}g\left(\frac{\tilde{z}_i}{1-\tau} - \frac{\tau\mu}{1-\tau}\right)
\end{aligned}$$

That is, while the linear redistributive tax scheme is rank-preserving (that is, individual i has the same rank in the after-tax income hierarchy as in the before-tax income hierarchy), the density of individuals is, however, increased at every income level, which, in turn, will increase the returns to conspicuous consumption.

In what follows, we explore how redistributive taxes change the allocation between conspicuous and non-conspicuous consumption in the presence of status concerns. To do so, let us first denote the after-tax optimal choice functions as $\tilde{x}(\cdot)$ and $\tilde{y}(\cdot)$, respectively. We fix an individual i with before-tax income z_i and post-tax income $\tilde{z}_i$, and compare her optimal choices of before and after tax. Let us employ the following notation:

$$\begin{aligned}
\hat{x}(z_i) &= \tilde{x}((1-\tau)z_i + \tau\mu) = \tilde{x}(\tilde{z}_i) \\
\hat{y}(z_i) &= \tilde{y}((1-\tau)z_i + \tau\mu) = \tilde{y}(\tilde{z}_i)
\end{aligned}$$

where $\tilde{x}(\tilde{z}_i)$ and $\tilde{y}(\tilde{z}_i)$ are optimal choices of an individual i after the tax is imposed. Importantly, what we will analyze will be $\hat{x}(z_i)$ and $\hat{y}(z_i)$ which denote after-tax consumption decisions for individual i with before-tax income z_i.

We also are interested in the comparison of the before and after-tax equilibrium welfare for each individual. For an individual i with with before-tax income z_i, let $U(z_i) = x(z_i)^\alpha y(z_i)^\beta (S_0 + G(z_i))^\gamma$ be the utility gained in the symmetric equilibrium before the tax is imposed, and let $\tilde{U}(\tilde{z}_i) = x(\tilde{z}_i)^\alpha y(\tilde{z}_i)^\beta (S_0 + G(\tilde{z}_i))^\gamma$ be the equilibrium utility after the tax is imposed. Using similar notation, we write:

$$\tilde{U}(\tilde{z}_i) = \tilde{U}((1-\tau)z_i + \tau\mu) = \hat{U}(z_i)$$

4 Comparative Static Analysis

We now turn to the comparative static analysis.[5] In this section, we consider an individual i with pre-tax income z_i. After the linear distributive tax scheme is implemented,

[3] Distribution functions F and G with density functions f and g, respectively, are of the same *type* if there exist constants $a > 0$ and b, such that $G(x) = F(ax+b), g(x) = af(ax+b)$ for all x (see, for example, Feller (1968).

[4] We can think of an index i as individual's rank in the before-tax income hierarchy, i.e. $i = G(z_i)$.

[5] The results in this section are a generalization of those in Hopkins and Kornienko (2004b).

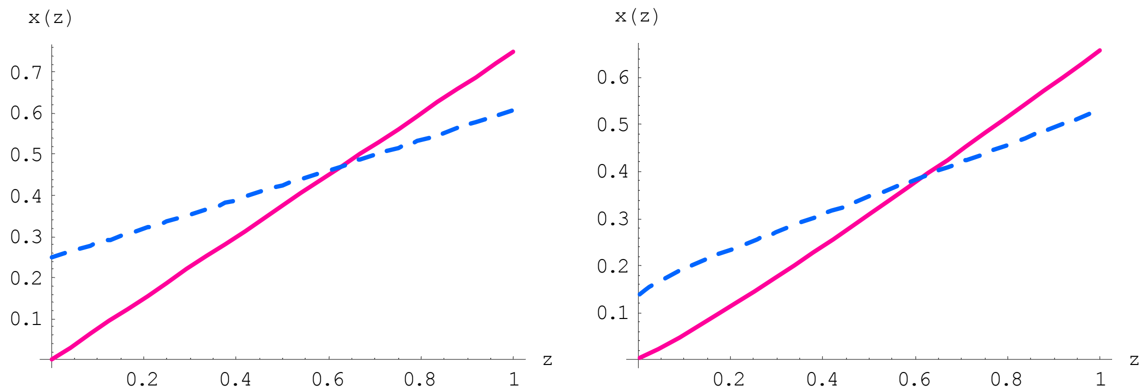
her post-tax income is $\tilde{z}_i = (1 - \tau)z_i + \tau\mu$. As it was noted in the previous section, the linear tax scheme is rank-preserving, so that an individual's rank in pre-tax income hierarchy coincides with post-tax income hierarchy. In turn, this implies that, in the equilibrium, her rank in the distribution of conspicuous consumption will also be unchanged by the redistributive taxation.

Yet, her equilibrium choice of consumption will change because of two reasons. First this happens for a conventional reason - because her income changed. Recall that as the portion of the utility that depends on the absolute levels of consumption imply that both positional and non-positional goods are normal. Thus, in the absence of status concerns we would expect that, after the tax is imposed, those with below-average income will consume more of both positional and non-positional goods as their incomes have increased, while those with above average income will consume less of both goods as their incomes have decreased, and we will also expect that the individual with mean income will not change her consumption choices as her income stayed the same.

However, there is a second effect namely that, in the equilibrium, the return to status changes as the income distribution changes. As the previous section shown, the post-tax income distribution is different from the pre-tax income distribution. Moreover, this change is such that the redistributive tax increases the density of individuals at each income level. This, in turn, increases social pressures, and thus provides an upward pressure to consume in a conspicuous way at the expense of non-positional consumption. As we will show below, the relative strength of the two effects is important. We first start with spending on positional goods.

Proposition 2 *Suppose $x(z_i)$ and $\hat{x}(z_i)$ are the equilibrium choices of conspicuous consumption before and after tax, respectively, for an individual i with before-tax income z_i. Then, for any $S_0 \geq 0$, $\hat{x}(z_i)$ crosses $x(z_i)$ at most once at some $z^x_\times$. Moreover, $\hat{x}(z_i) > x(z_i)$ for all $z_i \in (\underline{z}, \mu]$, with a possible crossing on $(\mu, \bar{z}]$.*

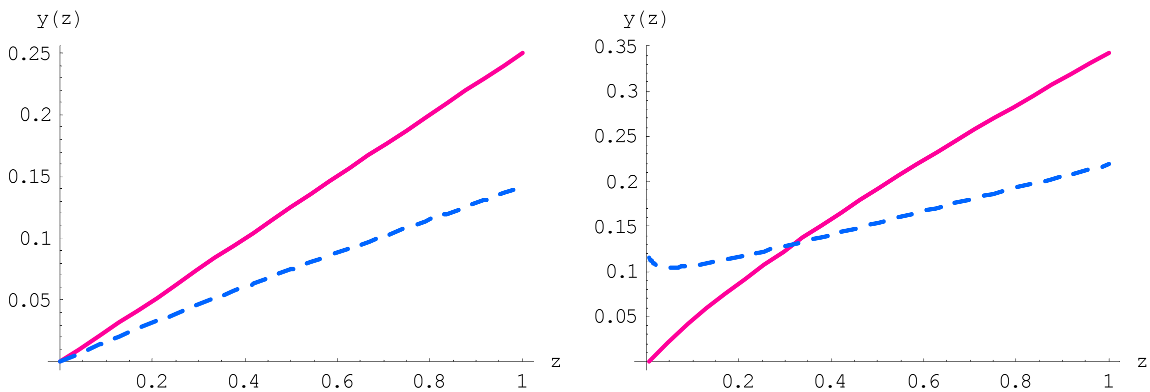
The above statement offers a clear-cut result: the equilibrium conspicuous consumption of those with below-average income (with a possible exception for the poorest individual) is always higher after tax relatively to that before tax. This is hardly surprising given that these people have higher income after the redistributive tax scheme is imposed. What is surprising that the individual with average income, whose income is unchanged by the redistributive tax scheme, nevertheless has higher conspicuous consumption after tax is imposed. This is because after the tax is imposed, most of the individuals feel increased social pressure by those with lower incomes, and, thus, in order to "keep up", they have to spend more on positional goods. For the same reason, people with incomes slightly higher than the average income, will generally also spend more on positional goods after the tax is imposed, even though they incomes are lower now (see Figure 2). In this and in following figures, average income is at 0.5.



"Harsh" competition $S_0 = 0$ "Mild" competition $S_0 > 0$

Figure 2: Equilibrium choices of conspicuous consumption before tax (solid curves) and after tax (dashed curves) as a function of pre-tax income.

The increased incentives to consume positional goods for those at the lower end of the distribution arise because of greater return to status arising as the consequence of redistribution. This, in turn, may reduce incentives to consume non-positional goods for some individuals (see Figure 3), as the next statement shows.[6]



"Harsh" competition $S_0 = 0$ "Mild" competition $S_0 > 0$

Figure 3: Equilibrium choices of non-positional consumption before tax (solid curves) and after tax (dashed curves) as a function of pre-tax income.

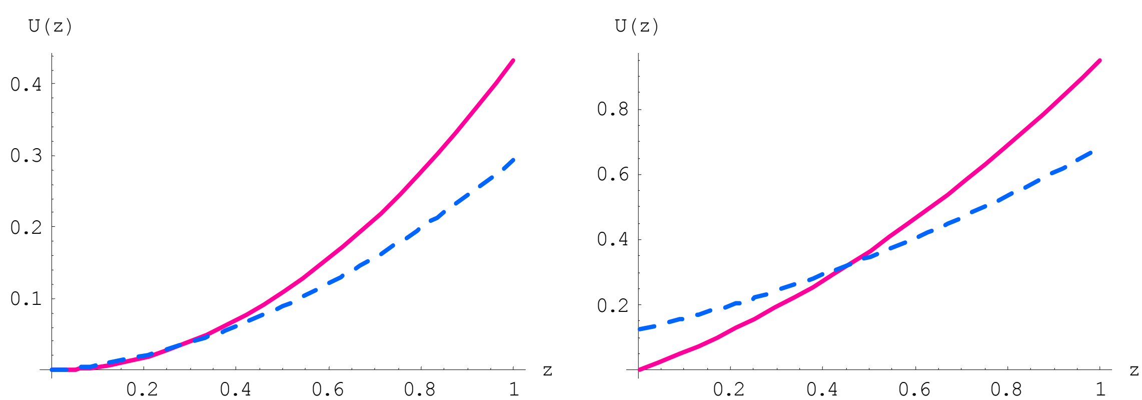
Proposition 3 *Suppose $y(z_i)$ and $\hat{y}(z_i)$ are the equilibrium choices of non-positional consumption before and after tax, respectively, for an individual i with before-tax income z_i. From the boundary condition (7), we have $\hat{y}(\underline{z}) \geq y(\underline{z})$. Furthermore,*

[6]It is important to mention that, while the equilibrium conspicuous consumption schedules are increasing with income, the equilibrium non-conspicuous consumption schedules may be non-monotone. In particular, when social competition is not very "harsh" (i.e. $S_0 > 0$), non-conspicuous consumption schedules of some relatively poor individuals may decrease with income.

(*i*) *for* $S_0 > 0$, $\hat{y}(z_i)$ *crosses* $y(z_i)$ *exactly once and from above at some point* $z_\times^y$, *where* $\underline{z} < z_\times^y < \mu$;
(*ii*) *for* $S_0 = 0$, *either* $\hat{y}(z_i)$ *crosses* $y(z_i)$ *exactly once and from above at some point* $z_\times^y$, *where* $\underline{z} < z_\times^y < \mu$ *or* $\hat{y}(z_i) < y(z_i)$ *for all* z_i *on* $(\underline{z}, \bar{z}]$.

The above statement says that, regardless of the importance of status, the "middle" and "upper" classes (those with near-average and above average incomes) spend less of their income on non-positional goods after the linear tax schedule is imposed. While for those with above-average incomes this is hardly a surprise, as their after-tax income is lower than their before-tax income, the consumption behavior of those with average income is startling as their income stays unchanged. But this is because linear taxes result in a more equal after-tax income distribution, with a bigger mass of people bunching around the mean income, which in turn result in a more fierce social competition. The same goes for those few individuals with incomes immediately below the mean income who also spend less on non-positional goods even those they have higher after-tax income than before taxes were imposed. We will now turn to the comparative statics analysis of equilibrium utility before and after taxes.

Proposition 4 *Suppose* $U(z_i)$ *and* $\hat{U}(z_i)$ *are the equilibrium utilities before and after tax, respectively, for an individual* i *with before-tax income* z_i. *First, if* $S_0 = 0$, *then* $U(\underline{z}) = \hat{U}(\underline{z}) = 0$, *but if* $S_0 > 0$, *then* $U(\underline{z}) < \hat{U}(\underline{z})$. *Second, for any* $S_0 \geq 0$, *then* $U(\mu) > \hat{U}(\mu)$. *Furthermore,* $\hat{U}(z_i)$ *may cross* $U(z_i)$ *at most once - from above at some* $\underline{z} \leq z_\times^y \leq z_\times^U < \mu$.



"Harsh" competition $S_0 = 0$ "Mild" competition $S_0 > 0$

Figure 4: Equilibrium utility before tax (solid curves) and after tax (dashed curves) as a function of pre-tax income.

The above proposition has three important points. First, it is clear that the individual with mean income is worse off with redistribution. Second, it clarifies what happens to the beneficiaries of the greater equality, i.e. those at the bottom of the income distribution. They indeed may be better off with redistribution. Once the linear tax is imposed, their after-tax income (actually, their after-subsidy income) is higher. However, how much of this increased income is spent on positional goods is determined by the intensity of social competition (characterized by S_0). If this intensity is very high (i.e. $S_0 = 0$), the very poor invest nothing out of their increased after-tax income, and spend it all on conspicuous consumption. Yet, if the social competition is not so cut-throat ($S_0 > 0$), the very poor spend a fixed proportion of their income on non-positional goods, so that their after-tax consumption of non-positional goods is higher simply because the after-tax income is higher. Eventually, however, the increased income is not enough to deal with the increased social pressure, so that, starting at some income level, increase their conspicuous consumption at the expense of their non-positional one. As the social pressure builds up further as we move along the income line closer to the mean income, the increase in conspicuous consumption results in a decrease in equilibrium utility (see Figure 4).

Third, it offers insights to what happens to the "upper" classes. As they see their incomes decreased by the redistribution, they unambiguously spend less on non-positional goods. Yet, as a consequence of redistribution, the social pressures ease up at the top of the distribution, some people may spend less on conspicuous consumption. Nonetheless, overall the rich are worse off after tax.

5 Conclusion

It is often assumed that social preferences, where individual agents care about what others have, imply a distaste for inequality. Here, when individuals care about their social position as indicated by their rank in visible or conspicuous consumption, we find that greater equality can make people worse off. In fact, even those who receive net subsidies in the redistribution schemes that we consider can end up with lower utility. The reason being that greater equality increases the incentives to differentiate oneself, raising the equilibrium level of conspicuous consumption.

That is, our results are in broad concordance with those in our earlier work (Hopkins and Kornienko (2004a)) which used a different type of change in equality to generate its comparative statics results. It is true that the distribution of gains and losses is different here. In particular, in the current analysis, the very poorest will gain from redistribution except in the extreme situation where the lowest ranked individuals are completely socially excluded. Under the type of changes considered in Hopkins and Kornienko (2004a), the poor were always worse off. However, in both cases, we have the unexpected result that the middle classes are hurt by greater equality.

Appendix: Proofs

Lemma 1 *Consider three pairs of positive functions: $\psi_A(t)$ and $\psi_B(t)$; $\phi_A(t)$ and $\phi_B(t)$; $\xi_A(t)$ and $\xi_B(t)$, all continuous in t on some interval $(\underline{t}, \bar{t})$. Suppose that these function stand in the following relationship:*

$$\frac{\psi_A(t)}{\psi_B(t)} \frac{\phi_A(t)}{\phi_B(t)} \frac{\xi_A(t)}{\xi_B(t)} = 1 \tag{9}$$

and suppose that $\xi_A(t) < \xi_B(t)$ for all $t \in (\underline{t}, \bar{t})$. Then,
(i) a pair of functions $\psi_A(t)$ and $\psi_B(t)$ cross at most once;
(ii) a pair of functions $\phi_A(t)$ and $\phi_B(t)$ cross at most once;
(iii) if these crossings do occur, then $\psi_A(t)$ crosses $\psi_B(t)$ from below at some point $t_1 \in (\underline{t}, \bar{t})$ and $\phi_A(t)$ crosses $\phi_B(t)$ from above at some point $t_2 \in (\underline{t}, \bar{t})$ with $\underline{t} < t_1 < t_2 < \bar{t}$.

Proof of Lemma 1: Since $\xi_A(t) < \xi_B(t)$ on $(\underline{t}, \bar{t})$, then

$$\frac{\psi_A(t)}{\psi_B(t)} \frac{\phi_A(t)}{\phi_B(t)} > 1$$

on this interval. Suppose that $\psi_A(t)$ crosses $\psi_B(t)$ on this interval at some point t_1. Then it must be that $\phi_A(t_1) > \phi_B(t_1)$. Furthermore, suppose that $\phi_A(t)$ crosses $\phi_B(t)$ on this interval at some point t_2. Then it must be that $\psi_A(t_2) > \psi_B(t_2)$. Thus, each pair of functions $\psi_A(t)$ and $\psi_B(t)$; $\phi_A(t)$ and $\phi_B(t)$; can cross only once on this interval. Suppose that these crossings occur, and, without loss of generality, suppose that $\psi_A(t)$ crosses $\psi_B(t)$ from below at t_1. Then it must be that $\phi_A(t)$ crosses $\phi_B(t)$ from above at t_2. Moreover, it is clear that $t_1 < t_2$. ∎

Proof of Proposition 2: Let us start by looking more closely into the first-order condition for the after-tax economy:

$$\begin{aligned}
\frac{\partial \tilde{x}(\tilde{z}_i)}{\partial \tilde{z}_i} &= \left(\frac{\gamma}{\alpha+\beta} \frac{\tilde{x}(\tilde{z}_i)(\tilde{z}_i - \tilde{x}(\tilde{z}_i))}{\tilde{x}(\tilde{z}_i) - x^*(\tilde{z}_i)}\right) \left(\frac{\tilde{g}(\tilde{z}_i)}{S_0 + \tilde{G}(\tilde{z}_i)}\right) = \\
&= \left(\frac{\gamma}{\alpha+\beta} \frac{\tilde{x}(\tilde{z}_i)(\tilde{z}_i - \tilde{x}(\tilde{z}_i))}{\tilde{x}(\tilde{z}_i) - x^*(\tilde{z}_i)}\right) \left(\frac{1}{1-\tau} \frac{g(z_i)}{S_0 + G(z_i)}\right)
\end{aligned}$$

Notice also that

$$\frac{\partial \hat{x}(z_i)}{\partial z_i} = \frac{\partial \tilde{x}(\tilde{z}_i)}{\partial \tilde{z}_i} \frac{\partial \tilde{z}_i}{\partial z_i} = (1-\tau) \frac{\partial \tilde{x}(\tilde{z}_i)}{\partial \tilde{z}_i}$$

so that the marginal change in equilibrium *after-tax* conspicuous consumption arising from a change in a *before-tax* income is:

$$\hat{x}' = \frac{\partial \hat{x}(z_i)}{\partial z_i} = \left(\frac{\gamma}{\alpha+\beta} \frac{\tilde{x}(\tilde{z}_i)(\tilde{z}_i - \tilde{x}(\tilde{z}_i))}{\tilde{x}(\tilde{z}_i) - x^*(\tilde{z}_i)}\right) \left(\frac{g(z_i)}{S_0 + G(z_i)}\right)$$

Thus, the ratio of the marginal changes in equilibrium *after-tax* and *before-tax* conspicuous consumption arising from a change in a *pre-tax* income is:

$$\frac{\hat{x}'}{x'} = \frac{\tilde{x}(\tilde{z}_i)}{x(z_i)} \frac{\tilde{z}_i - \tilde{x}(\tilde{z}_i)}{z_i - x(z_i)} \frac{x(z_i) - x^*(z_i)}{\tilde{x}(\tilde{z}_i) - x^*(\tilde{z}_i)} \tag{10}$$

Now, if $\hat{x}(z_i)$ and $x(z_i)$ do cross at all, it must be that at the points of crossing $(z^x_\times, x_\times)$ we have:

$$\frac{\hat{x}'}{x'} = \frac{\tilde{z}^x_\times - x_\times}{z^x_\times - x_\times} \frac{x_\times - x^*(z^x_\times)}{x_\times - x^*(\tilde{z}^x_\times)}$$

which is greater than 1 if $z^x_\times < \mu$ and less than 1 if $z^x_\times > \mu$. Thus, $\hat{x}(z_i)$ crosses $x(z_i)$ at most twice - from below for $z_i \in (\underline{z}, \mu)$ and from above for $z_i \in (\mu, \bar{z})$. According to the boundary condition (6), for all $S_0 \geq 0$, $\tilde{x}(\underline{z}) > x(\underline{z})$. Thus, for all $S_0 \geq 0$ only one crossing is possible and to the right of μ. ∎

Proof of Proposition 3: We first show that, for any $S_0 \geq 0$, $\hat{y}(z_i) < y(z_i)$ for all $z_i \in [\mu, \bar{z}]$, with possible crossings on $(\underline{z}, \mu)$. Notice that

$$\frac{\hat{x}'}{x'} = \frac{(1-\tau) - \hat{y}'}{1 - y'} = \frac{\hat{y}(z_i)}{y(z_i)} \frac{\tilde{z}_i - \hat{y}(z_i)}{z_i - y(z_i)} \frac{y^*(z_i) - y(z_i)}{y^*(\tilde{z}_i) - \hat{y}(z_i)}$$

Thus, if $y(z_i)$ and $\hat{y}(z_i)$ cross at some point $z^y_\times$, then at the point of crossing $z^y_\times$ we have:

$$\frac{(1-\tau) - \hat{y}'}{1 - y'} = \frac{\tilde{z}^y_\times - y_\times}{z^y_\times - y_\times} \frac{y^*(z^y_\times) - y_\times}{y^*(\tilde{z}^y_\times) - y_\times}$$

Let us first suppose that $\hat{y}(z_i)$ crosses $y(z_i)$ from below, so that $\hat{y}' > y'$ at this point of crossing $z^y_\times$. In this case,

$$\frac{\tilde{z}^y_\times - y_\times}{y^*(\tilde{z}^y_\times) - y_\times} \frac{y^*(z^y_\times) - y_\times}{z^y_\times - y_\times} < 1 \Leftrightarrow \frac{y^*(z^y_\times) - y_\times}{z^y_\times - y_\times} < \frac{y^*(\tilde{z}^y_\times) - y_\times}{\tilde{z}^y_\times - y_\times}$$

which is true for $\tilde{z}^y_\times > z^y_\times$. Thus, $\hat{y}(z_i)$ can cross $y(z_i)$ from below only to the left of μ. This, in turn, implies that $\hat{y}(z_i)$ and $y(z_i)$ cannot cross to the right of μ because, by Proposition 2, $\hat{y}(\mu) = \mu - \hat{x}(\mu) < y(\mu) = \mu - x(\mu)$.

Now, let us turn to the interval $(\underline{z}, \mu)$. Notice that in the equilibrium, $S(z_i) = \tilde{S}(\tilde{z}_i)$, we have that

$$U(z_i) - \hat{U}(z_i) = \left(x(z_i)^\alpha y(z_i)^\beta - \hat{x}(z_i)^\alpha \hat{y}(z_i)^\beta\right)(S_0 + G(z_i))^\gamma$$

which can be written as

$$\frac{\hat{U}(z_i)}{U(z_i)} \frac{x(z_i)^\alpha}{\hat{x}(z_i)^\alpha} \frac{y(z_i)^\beta}{\hat{y}(z_i)^\beta} = 1 \tag{11}$$

By Proposition 2, $\hat{x}(z_i) > x(z_i)$ for all $z_i \in (\underline{z}, \mu)$. By Lemma 1, there can be at most one crossing of $y(z_i)$ and $\hat{y}(z_i)$ to the left of μ.

As it was shown above, $\hat{y}(\mu) < y(\mu)$, thus if $\hat{y}(z_i)$ crosses $y(z_i)$ to the left of μ, then it must be from above. By the boundary condition (7), we have that for $S_0 > 0$, $\hat{y}(\underline{z}) = y^*(\underline{\tilde{z}}) > y(\underline{z}) = y^*(\underline{z})$. Thus, for $S_0 > 0$, $\hat{y}(z_i)$ crosses once $y(z_i)$ from above to the left of μ. But for $S_0 = 0$, we have that $\hat{y}(\underline{z}) = y(\underline{z}) = 0$, so in this case $y(z_i)$ and $\hat{y}(z_i)$ may or may not cross to the left of μ. ∎

Proof of Proposition 4: First, by the boundary conditions (6), when S_0, $U(\underline{z}) = \hat{U}(\underline{z}) = 0$. But, when $S_0 > 0$, we have $U(\underline{z}) = U(x^*(\underline{z}), y^*(\underline{z}), S_0) < U(x^*((1-\tau)\underline{z} + \tau\mu), y^*((1-\tau)\underline{z}+\tau\mu), S_0) = \hat{U}(\underline{z})$. Second, pre-tax and post-tax income are the same at z, and by Proposition 2, we have $x(\mu) < \hat{x}(\mu)$. Since, by Proposition 1, $x(\mu) > x^*(\mu)$, and utility is decreasing in x for $x > x^*$, and so $U(\mu) > \hat{U}(\mu)$.

Let us consider the interval $(\underline{z}, \mu]$. From the Proposition 2 and the application of Lemma 1 to the equation (11), we have that there can be at most one crossing of $U(z_i)$ and $\hat{U}(z_i)$ to the left of μ. By Proposition 3, if $\hat{y}(z_i)$ crosses $y(z_i)$ to the left of μ, then it must be from above, so that if $\hat{U}(z_i)$ crosses $U(z_i)$ to the left of μ, then it must cross from above as well.

Now let us turn to the interval $(\mu, \bar{z}]$, where $z < \tilde{z}$. By Propositions 2 and 3, we have $y(z) > \hat{y}(z)$ on the whole interval but $x(z)$ and $\hat{x}(z)$ may cross once. If at any z in $(\mu, \bar{z}]$ we have both $\hat{x}(z) < x(z)$ and $\hat{y}(z) < y(z)$, then $\hat{U}(z) < U(z)$, simply because U is strictly increasing. If we have instead $\hat{x}(z) > x(z)$ and $\hat{y}(z) < y(z)$, we then can find a pair (x_0, y_0) such that $x_0 + y_0 = \tilde{z}$ but $x^*(\tilde{z}) < x_0 < x(z)$ and $y_0 < y(z)$. But then, $U(\hat{x}(z), \hat{y}(z), S_0) < U(x_0, y_0, S_0) < U(x(z), y(z), S_0)$, and the result follows. ∎

References

Alesina, Alberto; di Tella, Rafael and MacCulloch, Robert (2004) , "Happiness and Inequality: Are Europeans and Americans Different?" *Journal of Public Economics*, August 2004, 88: 2009-2042.

Clark, Andrew E. (2003), "Inequality-Aversion and Income Mobility: A Direct Test." Working paper, DELTA, Paris.

Feller, William (1968), "Introduction to Probability Theory and Its Applications", Vol.II, Wiley.

Frank, Robert H. (1985), "The demand for unobservable and other positional goods", *American Economic Review* 75(1): 101-116.

Frank, Robert H. (1999), *Luxury Fever: Why Money Fails to Satisfy in an Era of Excess*, Free Press, New York.

Frey, Bruno and Alois Stutzer (2002), "What Can Economists Learn from Happiness Research?" *Journal of Economic Literature*, June 2002, 40, pp. 402Ű435.

Hopkins, Ed and Tatiana Kornienko (2004a), "Running to Keep in the Same Place: Consumer Choice as a Game of Status", *American Economic Review*, September 2004, 94 (4), pp. 1085-1107.

Hopkins, Ed and Tatiana Kornienko (2004b), "Status, Inequality and Growth", working paper.

Robson, Arthur (1992), "Status, the distribution of wealth, private and social attitudes to risk", *Econometrica* 60 (4): 837-857.

Senik, Claudia (2004), "When Information Dominates Comparison. Learning from Russian Subjective Panel Data," *Journal of Public Economics*, August 2004, 88: 2099-2123.