Does Market Concentration Preclude Risk Taking in Banking?

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Date
February 2004
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February 2004

Abstract

We analyse risk-taking behaviour of banks in the context of a model based on spatial competition. Banks mobilise deposits by offering deposit rates. We show that when the market concentration is low, banks invest in the gambling asset. On the other hand, for sufficiently high levels of market concentration, all banks choose the prudent asset to invest in, and some depositors may even be left out of the market. Our results suggest a discontinuous relation between market concentration and social welfare.

JEL Classification numbers: G21, L11, L13.
Keywords: Financial intermediation, Risk-taking, Market concentration.

*The current version of the paper has benefited from helpful comments of Matias Fontenla, David Pérez-Castrillo, Inés Macho-Setdier and Rosella Nicolini, and seminar participants at the Universitat Autonoma de Barcelona, and University of Edinburgh. The authors gratefully acknowledge the financial supports from projects BEC 2000-0172 and 2000 SGR-00054. The usual disclaimer applies.
1 Introduction

When banks are able to raise deposits to invest in assets with uncertain returns, excessive deposit might induce banks to take more risk. Involvement in high risk activities is viewed as one of the principal causes of several instances of banking crises that the world economy has witnessed during the last two decades.\textsuperscript{1}

The main goal of this paper is to analyse the role of market concentration in determining the risk-taking behaviour of banks. The banking sector described here consists of a finite number of banks. Banks are identical with respect to their equity capital. They raise deposits by offering deposit rates. Banks choose between a prudent asset and a gambling asset to invest their total fund (equity plus deposit). The gambling asset on average yields lower return than the prudent asset, but if the gambling is successful it gives higher return. There is a continuum of depositors. The depositors have one unit of monetary fund apiece, which they may place in a bank to earn the deposit rate offered. The depositors are not insured in case the gamble fails.

We analyse a model of bank competition in the context of a circular city model à la Salop\textsuperscript{[7]}. Both the banks and the depositors are located uniformly on a unit circle. The depositors incur a per unit transport cost to travel to a bank. In this model banks compete in deposit rates and each bank enjoys certain market power in the deposit market. We introduce an element of product differentiation across banks. Market power stems from transportation cost. The transport cost should not literally be interpreted as the cost (or, time) a depositor spends travelling to a bank. Banks could be differentiated because of differences in ATM facilities, availability in various regions, internet banking services, etc.\textsuperscript{2} In general, most of our findings do not depend particularly on the source of market power.

\textsuperscript{1}Recall the great S&L debacle in the United States. In 1985 S&L failures in Maryland caused loss to state deposit insurance fund and Maryland taxpayers of $185 million. The country-wide crisis had a cost of about $600 billion in bailouts.

\textsuperscript{2}Sometimes banks gift frying pans or wine glasses to their clients.
What is more important is that the amount of deposit in each bank is a continuous function of the deposit rates offered by him and his rivals. Our objective is to analyse the effects of market concentration, and not to model its sources. Ours is possibly one of the simplest formulations in order to capture the essence of a monopolistically competitive market structure.

Two types of symmetric equilibria might arise. A prudent equilibrium, where all banks invest in the prudent asset, and a gambling equilibrium, where all banks invest in the gambling asset. We use the unit transport cost relative to the number of banks as a measure of market concentration. We characterise the equilibrium of the banking sector. We show that when concentration is low, banks compete aggressively in order to capture greater market share by offering higher deposit rates. In this case, all the depositors participate, and a covered market is said to arise. Here, for very low concentration, all banks invest in the Gambling asset (i.e., a Covered Gambling Equilibrium exists). As concentration increases, all banks investing in the prudent asset can also be supported in equilibrium (i.e., a Covered Prudent Equilibrium co-exists along with a Covered Gambling Equilibrium).

For high levels of market concentration, banks never invest in the gambling asset. This is because the banks are interested in preserving the higher rent, which stems from increased market power, by behaving prudently. Hence, for moderately high concentration, competition with banks investing in the prudent asset is the only equilibrium. For even higher levels of concentration, a uncovered market emerges where deposit rate is so low that some depositors find it unprofitable to place their funds in banks and prefer to stay out of the deposit market.

To summarise, we show that higher market concentration is efficiency enhancing. As market concentration increases, the resulting lower competition induces banks to invest only in the prudent asset.

The equilibrium outcomes of the market also have significant implications for social welfare. It is clear that, for a fixed level of equity capital and a given level of concentration, social welfare under a prudent equilibrium is higher than that under a gambling equilibrium.
(when both co-exist for these levels). In the characterisation equilibrium we show that beyond a certain level of market concentration a gambling equilibrium ceases to exist. Hence, at this particular level of concentration social welfare has a discontinuous leap. Although, quite naturally welfare decreases as market concentration increases.

Prudential regulations of banks are viewed as instruments to prohibit banks from investing in risky projects. To this end, several mechanisms are used by the central banking authorities. Two popular instruments are minimum capital requirements and deposit rate ceilings which are often used to curb fierce competition among banks. Hellmann, Murdock and Stiglitz [4] show, using a dynamic model of prudential regulation, that freely determined deposit rates are inconsistent with Pareto efficiency, and that an optimal regulation policy combines minimum capital requirements and deposit rate ceiling. Repullo [6] uses a dynamic model of banking based on spatial competition à la Salop [7], to show that for very low levels of market concentration all banks invest only in the gambling asset if there is no regulation. Chiaporri, Pérez-Castrillo and Verdier [1] analyse the regulation of deposit rates in a circular city model of bank competition in both the deposit and loan markets.

Apart from the above two measures, central banks often insure the depositors in case of bank failure. Diamond and Dybvig [3] find that deposit insurance system prevents sunspot runs. With deposit insurance, Depositors become (ex-ante) indifferent between placing their funds in a prudent bank and a gambling bank. On the other hand, deposit insurance reduces bank’s incentives to behave prudently by increasing the moral hazard at the bank level when they are protected by a limited liability clause. Hence, the effect of deposit insurance on social welfare remains ambiguous. Demirgüç-Kunt and Detragiache [2] find empirical evidence that explicit deposit insurance has provoked bank failure.

Another goal of the paper is to analyse a scenario where the depositors are not insured. It is clear that the banks will invest in the prudent asset only if they earn higher profits

\^3\textsuperscript{3}Capturing the phenomenon of bank run, which is associated with withdrawal of funds by the depositors, is beyond the scope of our model which is a static one.
from doing so. This condition crucially depends on the network size (i.e., the number of depositors) of each bank. We show that if there are too many depositors with a particular bank he is more likely to gamble. In a regime of full deposit insurance, the depositors are paid back the promised deposit rates; whereas under no deposit insurance when a bank gambles his depositors are paid off less (in expected terms) compared to a prudent bank. Hence, depositors preferences for banks depend not only on the deposit rates posted by all banks, but also on the network size of each bank which gives rise to a coordination problem among the depositors. Comparing our findings with that of a deposit insurance system we show that when the depositors are insured, a gambling equilibrium is more likely to occur.

The current setup is closest to that of Repullo [6] and Keeley [5]. The latter shows, using a static model, that increased competition may lead to higher risk-taking in banking. We also use a static model assuming away any sort of deposit insurance. Standard portfolio choice theory asserts that positive risk premium (the difference between the expected return from the risky asset and the return from the safe asset) is necessary to induce (risk averse) individuals to invest positive amount in a risky asset. In the current model with negative risk premium, a gambling equilibrium might arise since a successful gamble yields more than a prudent asset. Hence, in order to ensure a prudent equilibrium, one needs an additional restriction that, given deposit rates and total volume of deposit, the profit from investing in the prudent asset must exceeds that from the gambling asset (the No Gambling Condition). In our model, banks invest all their equity capital. This can be thought as if there is fixed capital standard posed by a regulatory authority. But, as Hellmann, Murdock and Stiglitz [4] show, capital requirements are not enough to achieve Pareto efficiency unless combined with deposit rate ceiling. In our model, unlike their work, total volume of deposit with a bank determines whether bank is going to choose a gambling asset or a prudent asset. Hence, the uninsured depositors play a major role in determining banks’ risk-taking behaviour.

The main body of the paper shows that increased market concentration ensures that all banks invest in the prudent asset. Much of the debates on bank mergers pose the view
that mergers are able to enhance efficiency in case of speculative lending by providing the banks with more market power. One important thing to note is that our results would differ significantly if we allowed the number of banks to vary. One appropriate way to do this is accommodating entry of new banks in a dynamic context. But, in our understanding, market concentration is essentially a short run concept since, with free entry, the market power each bank enjoys is merely temporary. Suárez and Perotti [8] show that, in the presence of last bank standing effects, appropriate mergers and regulatory policies can enhance efficiency by giving banks incentives to behave prudently.

The paper is organised as follows. In Section 2, we lay out the basic model. In the next section, we analyse prudent and gambling equilibria. In Section 4, we discuss the relation between market concentration and social welfare. We conclude in Section 5.

2 The Model

Consider a banking sector with $n$ risk neutral banks distributed uniformly on a unit circle. A bank $i$ has equity capital $k$ which is assumed to be fixed. Banks compete in deposit rates in order to mobilise deposit. Let $r = (r_1, ..., r_i, ..., r_n)$ be the deposit rates offered by the banks. Bank $i$’s demand for deposit is given by $D(r_i, r_{-i})$, $r_{-i}$ being the rates offered by the other banks. Banks invest in assets their deposits together with the equity capital.

There is a continuum of depositors, also uniformly distributed on the unit circle, with a unit of fund apiece. A depositor can deposit her fund in a bank and earns deposit rate next period. We assume away any sort of deposit insurance. A depositor incurs a per unit transport cost $t$ to travel to a bank.

Banks can choose between a prudent asset and a gambling asset to invest their total fund. The prudent asset yields a constant return $\alpha$, and the gambling asset gives a return $\gamma$ with

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4In Section ? we comment on deposit insurance.
probability \( \theta \) and zero with probability \( 1 - \theta \). The prudent asset has higher expected return \((\alpha > \theta \gamma)\), but if the gamble pays off it yields higher private return \((\gamma > \alpha)\). The bank invests \( k + D(r_1, r_{-1}) \) in an asset.\(^5\)

If bank \( i \) chooses to invest in prudent and the gambling assets then his expected profits, respectively are:

\[
\begin{align*}
\pi^P(r_1, r_{-i}, k) &= \alpha k + (\alpha - r_1)D(r_1, r_{-i}), \\
\pi^G(r_1, r_{-i}, k) &= \theta (\gamma k + (\gamma - r_1)D(r_1, r_{-i})).
\end{align*}
\]

The timing of the game is as follows: Banks simultaneously offer deposit rates. Depositors then choose the bank in which to deposit their funds. The deposit mobilisation is followed by the portfolio choice by the banks. Finally, project outputs are realised and the depositors are paid.

3 Equilibrium

In this section, we characterise the equilibrium of the economy where banks compete in the deposit market by offering deposit rates and choose a prudent asset or a gambling asset to invest in, and the depositors choose banks to place their funds. We will focus on two types of symmetric equilibria. A prudent equilibrium, where all banks choose to invest in the prudent asset, and a gambling equilibrium, where all banks invest in the gambling asset. The natural solution concept used here is Subgame Perfect Equilibrium. Since, the depositors have to incur a per unit transport cost \( t \) to travel to a bank, the transport cost relative to the number of banks in the economy \( \left( \frac{t}{n} \right) \) can be used as a proper measure of market concentration.\(^6\) This is because, if the transport cost increases relative to the number of banks, given the total

\(^5\)We may assume that a bank can invest a fraction of total capital in each asset. It is easy to show that optimality would imply that banks choose only one asset to invest in.

\(^6\)See Salop [7].
number of depositors, each bank can lower the deposit rate to earn higher rent.

When market concentration is relatively low, there is no depositor who finds it profitable not to place her fund in any of the banks. In this case, all the depositors are served and a covered market is said to arise. On the other hand, when the transport cost relative to the number of banks is very high, there might be some depositors who, in equilibrium, would not find it profitable to travel to a bank to place their fund (since, deposit rate would not be high enough in order to compensate for the increased travelling cost). In that case, we say that an uncovered market arises. Prior to characterising the equilibrium, we first analyse the necessary conditions for existence of prudent and gambling equilibria in both types of market structures.

Banks will choose to invest in prudent asset if the expected profits from prudent asset exceeds the expected profits from gambling asset ($\pi^P \geq \pi^G$), i.e., if the deposit of a bank satisfies the following No Gambling Condition.

\[ D_i \leq \frac{mk}{(1 - \theta)r_i - m}, \]  

(NGC)

where $m = \alpha - \theta\gamma$. We assume that $(1 - \theta) - m > 0$ in order that the term in the right hand side of the above inequality is positive. If the above inequality is reversed, i.e., a Gambling Condition (call that (GC)) holds, then an individual bank will invest in the gambling asset.

Now recall the timing of events. The above conditions (NGC and GC) determine banks’ portfolio choice which follows the decision taken by the depositors. If there is a small number of depositors place their funds in a particular bank, then this bank is more likely to invest in the prudent asset (since, (NGC) is more likely to be satisfied). Hence, the conditions (NGC) and (GC) are endogenous, rather than being exogenous constraints. In a regime where the depositors are not insured in case of bank failure, this fact bears important consequences on the market equilibria. If there was deposit insurance, the depositors would have been indifferent between banks investing in the prudent asset and in the gambling asset.
3.1 A Covered Market

A covered market is a situation where all the depositors place their funds in any of the \( n \) banks rather than keeping their funds idle.

First, consider a Covered Prudent Equilibrium (CPE).\(^7\) We first compute the demand for deposit issued by bank \( i \) when he offers \( r_i \) and all the other banks offer \( r \). If the depositors anticipate that all banks are going to choose the prudent asset, then the demand for deposit of bank \( i \) is given by:

\[
D_{CP}(r_i, r) = \frac{r_i - r}{t} + \frac{1}{n}.
\]

Here, one should take two restrictions into account. First, all the banks must comply with the No Gambling Condition in order that the market structure arises at equilibrium is indeed a prudent equilibrium. Second, there is no depositor who has an incentive to keep her fund idle, i.e., for any depositor at a distance \( x \) from any bank one must have \( i, r_i - 1 \geq tx \). This restriction implies the following Participation Condition:

\[
D_{CP}(r_i) \leq \frac{2(r_i - 1)}{t} \quad \text{(PC)}
\]

Hence, bank \( i \)'s shareholders will solve the following problem:

\[
\max_{r_i} \left\{ \alpha k + (\alpha - r_i) \left( \frac{r_i - r}{t} + \frac{1}{n} \right) \right\}
\]

subject to \((NGC)\) and \((PC)\).

Let \( r_i = r = r_{CP} \) be the candidate optima for the above maximisation problem, which are

\(^7\)The detailed calculations and proofs of the results in Section 3 are in the appendix.
summarised below.

\[
\begin{align*}
    r^{CP} = & \begin{cases} 
        \bar{r} & \text{if } \frac{t}{n} \leq \alpha - \bar{r} \\
        \alpha - \frac{t}{n} & \text{if } \alpha - \bar{r} \leq \frac{t}{n} \leq \frac{2(\alpha - 1)}{3} \\
        1 + \frac{t}{2n} & \text{if } \frac{2(\alpha - 1)}{3} \leq \frac{t}{n} \leq 2(\bar{r} - 1),
    \end{cases}
\end{align*}
\]

where \( \bar{r} = \frac{m(1+nk)}{1-\theta} \) is the deposit rate which makes the (NGC) bind with equal deposit for all banks. In order to interpret the above, first consider the corner solution \( \bar{r} \). This deposit rate must satisfy (PC), which implies \( \frac{t}{n} \leq 2(\bar{r} - 1) \). Also at \( \bar{r} \), the profit function must have a non-negative slope which implies \( \frac{t}{n} \leq \alpha - \bar{r} \). Then consider the interior solution \( \alpha - \frac{t}{n} \). This must satisfy both (NGC) and (PC), which implies \( \alpha - \bar{r} \leq \frac{t}{n} \leq \frac{2(\alpha - 1)}{3} \). Finally, consider the other corner solution \( 1 + \frac{t}{2n} \) which must satisfy (NGC) and at this point the profit function must have a negative slope. These two together implies that \( \frac{2(\alpha - 1)}{3} \leq \frac{t}{n} \leq 2(\bar{r} - 1) \). And also, a symmetric CPE exists only if

\[
\frac{t}{n} \leq 2(\bar{r} - 1) \equiv \phi^P,
\]

Next, we analyse the Covered Gambling Equilibrium (CGE). Note that if a bank \( i \) promises a deposit rate \( r_i \), a depositor in this bank gets (in expected terms) \( \theta r_i \). If the depositors anticipate that all banks are going to choose the gambling asset, the deposit of bank \( i \) is given by:

\[
D^{CG}(r_i, r) = \frac{\theta (r_i - r)}{t} + \frac{1}{n}.
\]

\footnote{Here we consider a banking sector where the depositors are not insured. Had there been deposit insurance system, the depositors would have received the promised deposit rate \( r_i \), and hence the demand function faced by the banks would not change. In this model, since deposit mobilisation is followed by banks’ portfolio choice, the depositors in a way influence the (NGC). Because the depositors choose banks simultaneously, a coordination problem among them arises in this stage.}
Bank $i$’s shareholders will solve the following problem:

$$\max_{r_i} \left\{ \theta \gamma k + \theta (\gamma - r_i) \left( \frac{\theta (r_i - r)}{t} + \frac{1}{n} \right) \right\}$$

subject to \((GC)\) and \((PC)\).

Let $r_i = r = r^{CG}$ be the candidate optima for the above maximisation problem. These are summarised below.

$$r^{CG} = \begin{cases} 
\gamma - \frac{1}{n} & \text{if } \frac{1}{n} \leq \min \left\{ \frac{2(\theta \gamma - 1)}{3}, \theta (\gamma - \bar{r}) \right\} \\
\bar{r} & \text{if } \theta (\gamma - \bar{r}) \leq \frac{1}{n} \leq 2(\theta \bar{r} - 1) \\
\frac{1}{n} (1 + \frac{1}{2n}) & \text{if } \frac{1}{n} \geq 2(\theta \bar{r} - 1)
\end{cases}$$

First consider the interior solution $\gamma - \frac{1}{n}$. This must satisfy both the reversed \((NGC)\) and \((PC)\), which implies $\frac{1}{n} \leq \theta (\gamma - \bar{r})$ and $\frac{1}{n} \leq \frac{2(\theta \gamma - 1)}{3}$. Then consider the corner solution $\bar{r}$. This deposit rate must satisfy \((PC)\), which implies $\frac{1}{n} \leq 2(\theta \bar{r} - 1)$. Also at $\bar{r}$, the profit function must have a negative slope which implies $\frac{1}{n} \geq \theta (\gamma - \bar{r})$. Finally, consider the other corner solution $\frac{1}{n} (1 + \frac{1}{2n})$ which must satisfy the reversed \((NGC)\) and at this point the profit function must have a negative slope. These two together imply that $\frac{1}{n} \geq 2(\theta \bar{r} - 1)$. In Section 3.3 we will show that the corner solutions can be ruled out.

### 3.2 An Uncovered Market

In an uncovered market between any two consecutive banks on the circle there is a non-empty subset of depositors who do not place their funds in either of the banks. Consider a bank $i$ offering deposit rate $r_i$. A depositor at distance $x$ from $i$ will prefer to stay home if $r_i - 1 < tx$. Hence, bank $i$ will get a maximum deposit of $x \leq \frac{r_i - 1}{t}$ from either side and he

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9For a gambling equilibrium, we need that for no bank the \((NGC)\) is satisfied.
will have following amount of deposit:

\[ D(r_i) = \frac{2(r_i - 1)}{t}. \]

In this section, we focus on two possible kinds of equilibria: an Uncovered Prudent Equilibrium (UPE) and an Uncovered Gambling Equilibrium (UGE).

First, we look for the conditions under which does a UPE exist. In such equilibrium, banks simply maximise profits subject to the \textit{No Gambling Condition}, and the \textit{No Participation Constraint}.\textsuperscript{10} Since banks offer deposit rate which is independent of the other banks’ decisions, it is sufficient to check that the depositor at \( x = \frac{1}{2n} \) does not deposit in either of the banks. Hence, the \textit{No Participation Constraint} boils down to

\[ r_i \leq 1 + \frac{t}{2n}. \]

Therefore, bank \( i \)'s shareholders will solve the following problem:

\[
\max_{r_i} \left\{ \alpha k + (\alpha - r_i) \left( \frac{2(r_i - 1)}{t} \right) \right\}
\]
\[
\text{subject to } (NGC) \text{ and } (NPC).
\]

Let \( r_i = r = r^{UP} \) be the candidate optima for the above programme, which are summarised below.

\[
r^{UP} = \begin{cases} 
1 + \frac{t}{2n} & \text{if } \frac{t}{n} \leq \min\{\alpha - 1, 2(\hat{r} - 1)\}, \\
\hat{r} & \text{if } \frac{t}{n} \geq 2(\hat{r} - 1) \text{ and } \alpha - 1 \geq 2(\hat{r} - 1), \\
\frac{\alpha + 1}{2} & \text{if } \frac{t}{n} \geq \alpha - 1 \text{ and } \alpha - 1 \leq 2(\hat{r} - 1),
\end{cases}
\]

\textsuperscript{10}In case of an uncovered market, the Participation Constraint must be reversed.
where $\tilde{r}$ is defined by:

$$\frac{2(\tilde{r} - 1)}{t} = \frac{mk}{(1 - \tilde{r})\tilde{r} - m}.$$ 

Note that when $\frac{t}{n} \geq \min\{\alpha - 1, 2(\tilde{r} - 1)\}$, the deposit rate offered by a bank is $r^{MP} = \min\{\frac{\alpha + 1}{2}, \tilde{r}\}$. This form of deposit rate depends on the slope of the profit function at $\tilde{r}$. If $\tilde{r} \geq \frac{\alpha + 1}{2}$, then the slope is negative, and hence the deposit rate that maximises bank’s profit is simply $\frac{\alpha + 1}{2}$.

Also a UPE exists only if:

$$\frac{t}{n} \geq \min\{\alpha - 1, 2(\tilde{r} - 1)\} \equiv \psi^P.$$

In the rest of this section, we analyse the Uncovered Gambling Equilibrium (UGE). Bank $i$, operates on the part of the demand curve $\frac{2(\theta r_i - 1)}{t}$ above $\frac{mk}{(1 - \theta)\tilde{r} - m}$ (i.e., No Gambling Condition is reversed). Also, in this case, the No Participation Constraint is slightly different from the case of a prudent bank.

$$r_i \leq \frac{1}{\theta} \left(1 + \frac{t}{2n}\right).$$ (NPC')

Hence, bank $i$’s shareholders will solve the following problem:

$$\max_{r_i} \left\{ \theta \gamma k + \theta (\gamma - r_i) \left(\frac{2(\theta r_i - 1)}{t}\right) \right\}$$

subject to (GC) and (NPC').

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11Notice that when $r^{UP} = 1 + \frac{t}{2n}$, this is same as a CPE with deposit rate $1 + \frac{t}{2n}$. Hence, the part of UPE, when $\frac{t}{n} \leq \min\{\alpha - 1, 2(\tilde{r} - 1)\}$, will be referred to as a CPE.

12While interpreting the above necessary conditions, notice that $\tilde{r}$ is a function of $\frac{t}{n}$. After several steps of tedious algebra one can show that $\frac{t}{n} \geq 2(\tilde{r} - 1)$ if and only if $\frac{t}{n} \geq 2(\tilde{r} - 1)$.
The candidates for \( r_1 = r = r^{UG} \) are

\[
r^{UG} = \begin{cases} 
\frac{1}{\sigma} (1 + \frac{t}{2n}) & \text{if } \frac{1}{n} \leq \theta \gamma - 1 \text{ and } \frac{1}{n} \geq 2(\theta \hat{r} - 1), \\
\hat{r} & \text{if } \frac{1}{n} \geq 2(\theta \hat{r} - 1) \text{ and } \theta \gamma - 1 \leq 2(\theta \hat{r} - 1), \\
\frac{\theta \gamma + 1}{2\sigma} & \text{if } \frac{1}{n} \geq \theta \gamma - 1 \text{ and } \theta \gamma - 1 \geq 2(\theta \hat{r} - 1),
\end{cases}
\]

where \( \hat{r} \) is defined by:

\[
\frac{2(\theta \hat{r} - 1)}{t} = \frac{mk}{(1 - \theta)\hat{r} - m}.
\]

Also when \( \frac{1}{n} \geq \max\{\theta \gamma - 1, 2(\theta \hat{r} - 1)\}, \)

\[
r^{UG} = \max\left\{ \frac{\theta \gamma + 1}{2\theta}, \hat{r} \right\}.
\]

In the following proposition, we show that none of the candidates for \( r^{UG} \) can constitute an equilibrium.\(^{13}\)

**Proposition 1** An Uncovered Gambling Equilibrium never exists.

**Proof** We are going to provide one deviation from each of the above candidates, \( \hat{r} \) and \( \frac{\theta \gamma + 1}{2\sigma} \). All these deviations have the common feature of generating same deposits as the candidates they deviate from.

First consider the deposit rate \( \hat{r} \). A bank would be strictly better off by choosing a prudent asset and offering a deposit rate \( \theta \hat{r} \). Hence, the UGE with deposit rate \( \hat{r} \) cannot survive as an equilibrium. Next, consider the deposit rate \( \frac{\theta \gamma + 1}{2\theta} \). In the similar fashion, this is also dominated (in terms of profits) by a deposit rate \( \frac{\theta \gamma + 1}{2\theta} \) and a bank choosing a prudent asset.

The only thing remains to be checked is that any non-participant who can alter bank’s investment decision will stay out. This may happen only when \( r^{UP} = \hat{r} \) (when (NGC) is

\(^{13}\)In the same vein as the UPE, the UGE with deposit rate \( \frac{1}{\sigma} (1 + \frac{t}{2n}) \) will be called a CGE.
binding). Let $x$ be the maximum distance that a depositor travels from. We call $x$ the marginal consumer, for whom $tx = \tilde{r} - 1$. She will not participate if $\theta\tilde{r} - 1 \leq tx$. Hence, it is sufficient to show that $\theta\tilde{r} - \tilde{r} \leq 0$. This is always true, since $m > 0$ and $\theta < 1$. □

Till now we have provided only the necessary conditions for the existence of different types of equilibria and showed that an UGE never exists. In the following section, we also provide the sufficient conditions for existence.

### 3.3 Characterisation of Equilibrium

In the following proposition, we characterise the equilibrium. Recall that the term $\frac{1}{n}$ is used as a measure of market concentration.

**Proposition 2** For a given level equity capital of each bank, $k$,

(a) there exists a threshold $\tilde{\phi}$ such that if $\frac{1}{n} \leq \tilde{\phi}$ (low market concentration), only the Competitive Gambling Equilibrium exists, with the banks offering deposit rate $\gamma - \frac{1}{\theta n}$;

(b) if $\frac{1}{n} \in [\tilde{\phi}, \phi^G]$ (intermediate levels of market concentration), both a Competitive Gambling Equilibrium and a Competitive Prudent Equilibrium exist, with banks offering $\gamma - \frac{1}{\theta n}$, and $\tilde{r}$ or $\alpha - \frac{1}{n}$, respectively;

(c) if $\frac{1}{n} \in [\phi^G, \psi^P]$ (moderately high levels of concentration), only Competitive Prudent Equilibrium exists, with banks offering $\alpha - \frac{1}{n}$ or $1 + \frac{1}{2n}$;

(d) if $\frac{1}{n} \geq \psi^P$ (very high concentration), only Monopoly Prudent Equilibrium exists, with banks offering $\alpha + \frac{1}{2}$ or $\tilde{r}$.

**Proof** First we show that in case of CGE, only the interior solution survives. Consider the solution $\frac{1}{\theta} (1 + \frac{1}{2n})$. Notice that this deposit rate is optimal only if $\frac{1}{n} \geq 2(\theta\tilde{r} - 1)$. It is easy to check that $\pi^{CP} (1 + \frac{1}{2n}) > \pi^{CG} (\frac{1}{\theta} (1 + \frac{1}{2n}))$. Hence, a bank will have incentive to reduce
the deposit rate, attracting the same deposit, and becoming a local (prudent) monopolist. Next, when the deposit rate $\tilde{r}$ is a candidate optimum, a bank can gain strictly higher profit by reducing the deposit rate to $r' = \frac{\tilde{r}}{2}$, attracting the same demand, and switching to prudent asset. Hence, only the interior optimum survives. In this region, no bank can gain by offering a different deposit rate and choosing a prudent asset. a CGE exists if and only if:

$$\frac{t}{n} \leq \min \left\{ \frac{2(\theta \gamma - 1)}{3}, \theta(\gamma - \tilde{r}) \right\} \equiv \phi^G.$$

Next, there exists a threshold value $\tilde{\phi} \leq \alpha - \tilde{r}$ such that if $\frac{t}{n} \leq \tilde{\phi}$, a bank will find it profitable to switch to a gambling asset by offering a deposit rate which is his best response to $\tilde{r}$. Hence, for $\frac{t}{n} \leq \tilde{\phi}$, there is no CPE.

Recall that $\psi^P \equiv \min \{\alpha - 1, 2(\tilde{r} - 1)\} = \min \{\alpha - 1, \phi^P\}$. Therefore, CPE and MPE might co-exist if $\alpha - 1 < \phi^P$. However, this does not occur, since a bank has incentive to choose to become a local monopolist by offering a deposit rate $\frac{\alpha + 1}{2}$. In this region, no bank can gain by offering a different deposit rate and choosing a gambling asset. Therefore, a symmetric CPE exists if and only if:

$$\tilde{\phi} \leq \frac{t}{n} \leq \psi^P.$$

Notice that if $\tilde{\phi} < \phi^G$, then CPE and CGE do not exist together.

Finally, when $\frac{t}{n} \geq \psi^P$, only MPE exists since, by Proposition 1, there is no MGE. □

The intuition behind the above proposition is fairly straightforward. When the market concentration is very low, competition erodes banks’ profit, thus leaving little incentive for them to invest in prudent asset. Also, with fierce competition, banks offer high deposit rate which compensate for the travelling cost of the depositors (although they receive only $\theta$

\[^{14}\text{The threshold is given by } \tilde{\phi} \equiv \left( \sqrt{(1 - \theta)\tilde{r}} - \sqrt{\theta(\gamma - \tilde{r})} \right)^2.\]
fraction of it). On the other hand, for very high degree of concentration, banks gain monopoly rent, and hence they have incentive to choose prudent asset in order to preserve that. For, even a higher values of $\frac{t}{n}$, the market becomes monopolistic, i.e., banks offer even lower deposit rate which is not conducive to attract the depositors located at a longer distance. The above proposition is summarised in the following figure.

[Insert Figure 1 about here]

Also, for intermediate levels of concentration, banks might invest in the prudent asset by offering a lower deposit rate or in the gambling asset offering a higher rate which compensates for the expected loss for the depositors due to a possible failure in gambling.

4 Social Welfare

In this section, we discuss the connection between market concentration and welfare. In the current set up social welfare is simply the total consumer’s surplus, since the deposit rate is transfer from the banks to the depositors.

For covered markets welfare is independent of deposit rates

\[
W^{CPE}(t) = \alpha kn + \alpha DT - 2nt \int_0^{\frac{1}{n}} xdx;
\]

\[
= \alpha (nk + 1) - \frac{t}{4n};
\]

\[
W^{CGE}(t) = \theta \gamma (nk + 1) - \frac{t}{4n}.
\]

Hence, in a covered market social welfare is always higher under the prudent equilibrium,\(^{15}\) since

\[
W^{CPE} - W^{CGE} = m(nk + 1) > 0.
\]

\(^{15}\)One interesting result would be if $W^{CPE}(n\tilde{\phi}) < W^{CGE}(0)$. However, this does not always hold true since depend on the value of the parameters.
For the UPE take the case where the equilibrium deposit rate is $r^{UP} = \frac{\alpha + 1}{2}$. Let $x^I$ be the last consumer who deposits with a bank. Hence, $x^I$ must satisfy the following:

\[
\frac{\alpha + 1}{2} - tx^I = 1
\]

\[
x^I = \frac{\alpha - 1}{2t}
\]

And total deposit is given by $D^T = n\left(\frac{\alpha - 1}{t}\right)$, and the social welfare is given by:

\[
W^{UP}(t) = \alpha kn + \alpha D^T - 2nt \int_0^{x^I} x dx
\]

\[
= \alpha kn + \alpha n\frac{\alpha - 1}{t} - 2nt \int_0^{\frac{\alpha - 1}{2t}} x dx
\]

\[
= n[\alpha k + \frac{\alpha - 1}{t}(3\alpha + 1)]
\]

So if the lower bound on concentration beyond which we have a UPE is $\alpha - 1$, one can show that

\[
W^{UP}((\alpha - 1)n) = W^{CPE}((\alpha - 1)n)
\]

Once can do the same with the other equilibrium candidate $2(\beta - 1)$.

The above can be summarised as follows. First, social welfare decreases with the level of market concentration. Second, for a given level of market concentration where both the CPE and CGE exist (for $\frac{t}{n} \in [\bar{\phi}, \phi^G]$), social welfare is higher in case all banks behaving prudently.

Next, at the level of concentration beyond which no bank invests in the gambling asset (at $\frac{t}{n} = \phi^G$), welfare takes a discontinuous leap.

5 Conclusions

In this paper, we use a model of baking sector based on spatial competition, and analyse the role of market concentration in influencing the risk-taking behaviour of banks. Using a
static model we show that, for a very low level of market concentration, banks invest in the gambling asset. On the other hand, when the market concentration increases, banks invest only in the prudent asset. We assert that, more market concentration works as a device to refrain banks from being involved in high risk activities. We also show a discontinuous relationship between concentration and social welfare.

References


Figure 1: Characterisation of Equilibrium