Systemic Risk (Clarendon Lectures 3)

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Date
November 2001
Clarendon Lectures

Lecture 3

SYSTEMIC RISK

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28 November 2001
agents $A_1, A_2, \ldots, A_i, \ldots$

credit chain only if
- each agent holds gross financial positions
  (simultaneously debtor and creditor)

liquidity shock propagates through chain only if
- agents are liquidity constrained
  (gross positions cannot be netted out)

propagation causes damage only if
- default creates inefficiency at each link
exchange and credit: ordering, delivery and payment do not all happen simultaneously
(time-to-build + time-to-pay)

\( A_i \) simultaneously supplies on debit to \( A_{i-1} \)
and purchases on credit from \( A_{i+1} \)

\[ A_{i-1} \xrightarrow{\text{goods, payments}} A_i \xrightarrow{\text{goods, payments}} A_{i+1} \]

\( A_i \)'s balance sheet:

\begin{center}
\begin{tabular}{lc}
assets & liabilities \\
accounts receivable & accounts payable \\
(from \( A_{i-1} \)) & (to \( A_{i+1} \))
\end{tabular}
\end{center}
MODEL

3 dates: \( t = 0, 1, 2 \)

single good, storable: \( z_t \rightarrow Rz_t \)

identical agents, measure 1

consume only at date 2; risk neutral

endowments (per capita):

\begin{align*}
\text{date 0:} & \quad e \\
\text{date 1:} & \quad \text{either } -c \quad \text{with probability } \lambda \\
& \quad \text{or } d \quad \text{with probability } 1-\lambda \\
\text{date 2:} & \quad \text{none}
\end{align*}

date 1 endowment risk is uninsurable

assume \( c > 0, \ d > 0 \) and \( e > 0 \)

also \( (1-\lambda)d - \lambda c > 0 \)
production by one agent ("purchaser" P)
requires inputs from measure ε of other agents ("suppliers" S)
inputs are required in equal proportion (Leontief)
constant returns; per composite unit supplied:

\[ \text{date 0} \quad \text{date 1} \quad \text{date 2} \]

purchaser P

\[ \text{output } a \text{ (by P)} \]
\[ \text{output } b \text{ (by P)} \]

\[ S \text{ complete at date 1} \]

suppliers S

\[ \text{input } w \text{ (by S)} \]

\[ S \text{ postpone} \]

\[ S \text{ complete at date 2} \]

assume \( a > b > R^2 w \)
ex ante competition

at date 0, each agent

chooses level of (composite) purchase, $x^p$
and is free to purchase from anyone else

chooses level of supply, $x^s$
and is free to supply to anyone else

except that no-one can supply to their own supplier

($\Rightarrow$ no counter-trade)

ex post lock-in

at dates 1 or 2,

only supplier S can costlessly complete his own job

purchaser P can complete without S if P inputs 1, per composite unit

(backstop technology for P)

assume $b > 1$
incomplete contracts

suppose S can demand payment from P at time of completion, irrespective of any formal contract

and S has all ex post bargaining power

however, P has an outside option: backstop technology

⇒ ex post, P pays S a price of 1 for completion

ex ante market clears through price, q say, that P pays S

implicit contract (per unit):

<table>
<thead>
<tr>
<th>date 0:</th>
<th>P places order with S</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P makes downpayment q to S</td>
</tr>
<tr>
<td></td>
<td>S inputs w</td>
</tr>
</tbody>
</table>

| date 1 or 2:  | S completes, and is paid 1 by P |

price q will be sensitive to anticipated date of completion
symmetric equilibrium $x^p = x^s = x$

date 0 flow-of-funds:

$$qx + wx = e + qx$$

downpayment made on purchases
input for supplies
endowment

downpayment received for supplies

guess: no storage ($z_0 = 0$) at date 0
(to be verified later)
date 1 flow-of-funds:

constrained agents, \( C \) (those with negative endowment)

\[
\hat{x} = \bar{x} - c
\]

payment for purchases completed at date 1
average endowment from supplies completed at date 1
receipts

deep pockets, \( D \) (those with positive endowment)

\[
x + z_1 = \bar{x} + d
\]

payment for all purchases
storage
average endowment from supplies completed at date 1
receipts

where \( \bar{x} = \) average supplies completed at date 1

\[
= \text{average purchases completed at date 1}
\]

\[
= \lambda \hat{x} + (1-\lambda)x
\]
credit chains:

\[ \hat{x} = \lambda \hat{x} + (1-\lambda)x - c \]

\[ = x - \frac{c}{1-\lambda} \]

\[ \uparrow \]

shortage of liquidity
is propagated through
credit chain

\[ \lambda \sim \text{length of chain} \]

example \( \lambda = 2/3 \)

```
C  C
C  C
D  D
C
C
D
C
C
D
```

from any C, average length of chain to first D \( \approx 3 \)
if positions of contrained agents in credit chain weren’t random, then aggregate response would change

examples (keeping $\lambda = 2/3$):

from any C, average length of chain to first D = $2\frac{1}{4}$

from any C, average length of chain to first D = $1\frac{1}{2}$
date 2 utilities:

constrained agents C:

\[ \hat{U} = a\hat{x} + (b - 1)(x - \hat{x}) + (x - \bar{x}) \]

output from purchases completed at date 1
output minus payment for purchases completed at date 2
receipts from supplies completed at date 2

deep pockets D:

\[ U = ax + Rz_{1} + (x - \bar{x}) \]

output from purchases
output from storage
receipts from supplies completed at date 2
C's balance sheet at start of date 2

<table>
<thead>
<tr>
<th>assets</th>
<th>liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>accounts receivable $x - \bar{x}$</td>
<td>accounts payable $x - \hat{x}$</td>
</tr>
<tr>
<td>output from purchases completed at date 1 $\alpha\hat{x}$</td>
<td>net worth (consumption) $\hat{U}$</td>
</tr>
<tr>
<td>output from purchases to be completed at date 2 $b(x - \hat{x})$</td>
<td></td>
</tr>
</tbody>
</table>
ex ante expected utility:

\[ EU = \lambda \hat{U} + (1 - \lambda)U \]

\[ = a\overline{x} + b(x - \overline{x}) + R(1-\lambda)z_1 \]

output from completion at date 1
output from completion at date 2
output from storage

\[ = \left( ax + R\left[ (1-\lambda)d - \lambda c \right] \right) - \left( a - b \right) \frac{\lambda c}{1 - \lambda} \]

first best output (no postponement)
welfare loss from postponement

\[ \uparrow \]

increases with \( \lambda \)
securitization at date 1

S can securitize a fraction $\theta$ of his accounts receivable, by spending $F(\theta)$ per unit of uncompleted (postponed) supply $F(\cdot)$ increasing and strictly convex

deep pockets D won’t securitize their accounts receivable

constrained agents C will choose $\theta$ to maximize

$$\hat{U} = a \hat{x} + (b - 1)(x - \hat{x}) + (1 - \theta)(x - \bar{x})$$

Non-securitized receipts from supplies completed at date 2

subject to

$$\hat{x} = \bar{x} + \left[\frac{\theta}{R} - F(\theta)\right](x - \bar{x}) - c$$

↑

sales of securitized accounts receivable

↑

accounts receivable
assume an interior choice of $\theta$, for which $\hat{x} < x$

first-order condition for $\theta$:

$$F'(\theta) = \frac{1}{R} - \frac{1}{a - b + 1}$$

F.O.C.

date 1 value of date 1 value of
date 2 goods to D date 2 goods to C

securitization shortens the credit chain:

$$\hat{x} = x - \frac{c}{1 - \lambda + \lambda \left( \frac{\theta}{R} - F(\theta) \right)}$$
individual choices in the date 0 market

prices:
\[ q_1 = \text{downpayment anticipating completion at date 1} \]
\[ q_2 = \text{downpayment anticipating completion at date 2} \]

supplies:
\[ x_1^s = \text{units supplied anticipating completion at date 1} \]
\[ x_2^s = \text{units supplied anticipating completion at date 2} \]

purchases:
\[ x^p = \text{units ordered} \]
\[ \tau = \text{expected proportion of purchases } x^p \text{ completed at date 2} \]
\[ = \lambda \left( 1 - \frac{\hat{x}^p}{x^p} \right) \]  
(see below for \( \hat{x}^p \))
date 0 flow of funds:

\[
(1 - \tau) q_1 + \tau q_2 x^p + w(x^s_1 + x^s_2) + z_0 = e + q_1 x^s_1 + q_2 x^s_2
\]

- downpayment made on purchases
- input for supplies
- storage
- endowment received for supplies completed at date 1
- downpayment received for supplies completed at date 2
date 1 flow of funds:

with negative endowment shock:

\[
\tilde{x}_p = x_1^s + \left( \frac{\theta}{R} - F(\theta) \right) x_2^s + Rz_0 - c \\
\uparrow \\
\theta \text{ given} \\
\text{by F.O.C.}
\]

with positive endowment shock:

\[
x_p + z_1 + \frac{1}{R} \frac{\lambda}{1 - \lambda} \theta x_2^s = x_1^s + Rz_0 + d
\]

purchases of securitized accounts receivable
date 2 utilities:

with negative endowment shock at date 1

\[ \hat{U} = a\hat{x}^p + (b - 1)(x^p - \hat{x}^p) + (1 - \theta)x^s_2 \]

with positive endowment shock at date 1

\[ U = ax^p + x^s_2 + Rz_1 + \frac{\lambda}{1-\lambda} \theta x^s_2 \]

maximise expected utility \( EU = \lambda \hat{U} + (1-\lambda)U \)

subject to date 0 flow of funds constraint
indifference conditions:

\[
\frac{EU}{e} = \frac{\lambda(a - b + 1) + (1-\lambda)R}{w - (1-\lambda)q_1 - \lambda q_2}
\]

\[
= \frac{\lambda(a - b + 1) \left( \frac{\theta}{R} - F(\theta) \right) + 1 - \lambda \theta}{w - q_2 - \lambda(q_2 - q_1) \left( \frac{\theta}{R} - F(\theta) \right)}
\]

\[
= \frac{\lambda(b - 1) + (1-\lambda)(a - R)}{(1-\lambda)q_1 + \lambda q_2}
\]

\[
> \frac{\lambda(a - b + 1) + (1-\lambda)R}{\frac{1}{R} - \lambda(q_2 - q_1)}
\]
government

- has endowment at date 0
- cannot produce
- consumes $U^g$ at date 2

social welfare $'= \lambda \hat{U} + (1 - \lambda)U + U^g$

PROPOSITION:

If, at date 1, the government purchases securitized accounts receivable to raise their price above $1/R$, then social welfare will increase.